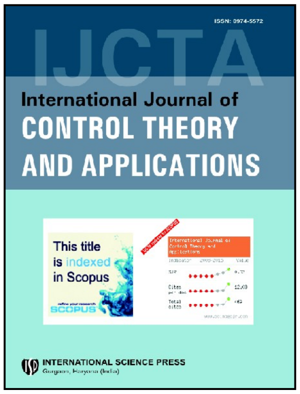

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Assortments of Links in a Kinematic Chains

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Abstract: The wide applications of kinematic chains has led more scope towards synthesis of kinematic chains. In this paper the initiation to be taken before actual enumeration of kinematic chain which helps to judge the degree of freedom of kinematic chain and minimum number of binary links a kinematic chain has been established. Then, the philosophy of the algorithm used in the enumeration process is developed along with the assortment procedure which aids in the enumeration of kinematic chains, using the Maxcode, a canonical number. The link assortment tables for number of links six to ten is constructed.

Keywords/Index words: Enumeration, Kinematic Chain, Assortments of links, links, degree of freedom, Maxcode.

I. INTRODUCTION

For the new design of products, a necessary requirement for the designer is to have many diversified designs so as to make ease in selection of new concepts or aspects of designs. Varieties in available mechanisms as options, makes the selection of mechanisms more easy. The kinematics of a mechanism is defined by the number of links, number, type of joints, the connectivity of the links and joints. Also for a mechanism it is important to know which link is to be considered as frame or input. An understanding of this structure is conducive to a systematic development of methods of enumeration, identification, classification & isomorphism of kinematic chains and mechanisms and at the same time, can be of great help in an optimum selection of one from amongst a large family of kinematic chains. Nevertheless, it is desirable to enumerate the kinematic chains systematically. Max code has been widely accepted by graph theorists as a complete graph invariant and has been used in the current work for differentiating co-spectral, non-isomorphic chains.

In the present work the initial input tables of link assortment has been prepared for computerized method of enumeration of kinematic chains, using identification codes, is proposed for preparing a catalogue of all distinct chains of given number of links and given degree of freedom.

Enumeration or synthesis of distinct kinematic chains is being the major interest of many machine designers since time immemorial. The researchers worked on many aspects of kinematic chains like synthesis, isomorphism, feasibility and many other aspects. The review of kinematic chains was presented by Mruthyunjaya[1] according to which the characteristic polynomial was proposed by Uicker and Raicu[2], but the computations are rather tedious. Mruthyunjaya and Raghavan, [3] proposed the modified matrix notation to identify the distinct kinematic chains. Hamming Number Technique was given by the Rao and Raju [4], Rao[5] for the synthesis of planar kinematic chains. Similarly Hwang and Hwang [6], A.C Rao and Pratap B. Deshmukh[7] Mruthyunjaya [8-10], also synthesized the kinematic chains by different methods. Many researchers have worked on different approaches for enumeration and identification of kinematic chains uniquely and have the selection of non isomorphic chains but methods were lacking in the decodability aspect.

A Remarkable statement was given by Read and Corneil [11] that a good solution to the coding problem provides a good solution to the isomorphism problem , though, the converse is not necessarily true. Hence, it stated that the work is required on the coding method to enumerate the kinematic chains and also to identify non isomorphic kinematic chains. The conceptual introduction of maxcode and mincode was given by Ambekar and Agrawal [12,13]. According to [14], the review of different approaches for the enumeration and identification of kinematic chains has been discussed.

The author's present work is on the input table preparation of coding software for given number of links and degrees of freedom. The software is based on canonical numbering algorithm using maxcode. One important feature of canonical numbers is that they are decodable, and also gives digital identification to kinematic chain with which the testing of isomorphic chains will be easy. The algorithm was generated for the structural enumeration and identification of kinematic chains.[15]. The algorithm has been proved in the earlier publications. The present paper discusses "the assortment procedure which aids in the enumeration of kinematic chains, using the Maxcode, a canonical number. The link assortment tables for number of links 6 to ten is constructed.

Kinematic chains with a given number of links and degree of freedom as related by the Kutzbach-Chebyshev-Gruebler equation [16]; Degree of freedom is the most fundamental structural characteristic of a kinematic chain and, in general, depends only on the kinematic pairing among the links of the chain. Gruebler criterion [17], which to be satisfied by a constrained (single degree of freedom) planar chain. Later, Gruebler came up with the general relation for degree of freedom, F, of a planar chain as

$$F = 3(n-1)-2L \text{ where} \tag{1}$$

L = number of simple turning pairs or simple joints

F = Degrees of freedom(dof)

n = Total number of links

This is now referred to as the Chebyshev–Gruebler equation

From the Grubler's equation the effect of even/ odd number of links on degree of freedom (dof) can be analysed as shown :

Thus, from eqn. 4.1, for 'L' to be an integer F(dof) and n (Total number of links) should be integers.

- If dof ' F' is odd(say, 1,3,5 …), (n-1) should also be odd or n must be even.

For eg. In a 4-bar kinematic chain n = 4 ; L = 4 Hence from eqn. 4.1 we have F= 1.

- If dof ' F' is even(say, 2,4,6 …), (n-1) should also be even or n must be odd.

For eg. In a 5-bar kinematic chain(in which all are binary links) n = 5 ; L = 5 Hence from eqn. 4.1 we have F= 2.

To, summarize with , for dof. 'F' to be even, n must be odd and for 'F' to be odd, n must be even.

II.Minimum number of Binary Links to have a feasible kinematic chain

Once it is defined with the number of links and degree of freedom of a kinematic chain then link assortment has to be decided. For that purpose to start with the minimum number of binary links, n_2 in a kinematic chain has to be found. According to Ambekar [18],

Let, n_2 = number of binary links

n_3 = number of ternary links

n_4 = number of quaternary links

.

n_k = number of k-nary links

$$n = n_2 + n_3 + n_4 + n_5 + \ldots\ldots\ldots n_k \quad (2)$$

Since discussions are limited to simple jointed chains, each joint/pair consists of two elements. Thus, if 'e' is total number of elements in the mechanism, then

$$e = 2L$$

By definition binary, ternary, quaternary,…. links consist respectively of 2,3,4, …. etc. elements. Hence, total number of elements are also given by

$$e = 2\,n_2 + 3\,n_3 + 4\,n_4 + 5\,n_5 + \ldots\ldots\ldots k\,n_k \quad (3)$$

Hence, from eqns. (4.2) and (4.3)

$$2L = 2\,n_2 + 3\,n_3 + 4\,n_4 + 5\,n_5 + \ldots\ldots\ldots k\,n_k \quad (4)$$

Substituting n and 2L from eqns (2) and (4) in eqn (1), we have

$$F = 3\,[(n_2 + n_3 + n_4 + n_5 + \ldots + n_k)-1] - [\,2\,n_2 + 3n_3 + 4n_4 + 5n_5 + k\,n_k] \quad (5)$$

After simplification and rearrangement of eqn (5),

$$n_2 = (F+3) - [\,n_4 + 2\,n_5 + 3\,n_6 \ldots \ldots + (k-3)\,n_k] \quad (6)$$

Thus, number of binary links required in a kinematic chain , depend on number of links having elements >3.

Hence, minimum number of binary links that are to be present in a kinematic chain, can be deduced from eqn (6) as:

$$n_2 \geq 4, \text{ for dof } = 1;$$
$$n_2 \geq 5, \text{ for dof } = 2;$$
$$n_2 \geq 6, \text{ for dof } = 3; \text{ etc.}$$

From the results, it is proved that degree of freedom is an important property of a kinematic chain, based on which we can have minimum number of binary links in a kinematic chain. "***This proves that minimum number binary links for dof =1 is 4, while for dof = 2 is 5 and so on".***

III. ASSORTMENTS OF LINKS

The minimum number of binary links are defined then the focus is on to have a feasible kinematic chain is established, next step will be to establish the number of ternary, quaternary,… etc. number of links for a given degree of freedom and number of links in a kinematic chain.

To establish a range of Identification numbers between *MAXU* and *MAXL* such that all feasible chains are sure to be covered in this range for a given number of links '*n*' and given degree of freedom '*F*' as per eqn (1)

The link of highest degrees in the chains of given *n* and *F* :

$$K = n/2 \text{ (when } F \text{ is odd)} \quad (4.7)$$

$$K = (n+1)/2 \text{ (when } F \text{ is even)} \quad (4.8)$$

From equations. (2) and (4) the possible combinations of n_2 , n_3 , n_4 , n_5 is established for a given number of links 'n' and given degree of freedom 'F.'

For the sample calculation of assortments of links consider for example 8- link chain with single degree of freedom .

Number of joints from eqn (1) we have, L = (3(8)-4)/2 =10

Maximum number of elements on one link from eqn (4.8), K = 8/2 = 4

Hence, the chains can have binary, ternary and quaternary links only. From equations (2) and (4) ,

$$n_2 + n_3 + n_4 = 8 \text{ and } 2n_2 + 3\, n_3 + 4\, n_4 = 20$$

Substituting in Grubler's eqn (1) ,

$$3(n_2 + n_3 + n_4) - (2\, n_2 + 3\, n_3 + 4\, n_4) - 4 = 0 \text{ or } (n_2 - n_4) = 4$$

The combination of links, by the trial and error method are found are been tabulated in Table 1.

Table 1
Combination of links for 8-linked kinematic chain

n_4 *(assumed)*	$n_2 = n_4 + 4$	$n_3 = 8-(n_2 + n_4)$	*Remark*
4	8	-	Not acceptable as -($n_2 + n_4$) > n
3	7	-	Not acceptable as -($n_2 + n_4$) > n
2	6	-	acceptable
1	5	2	acceptable
0	4	4	acceptable

Thus, the three valid combination of links are :

n_2 = 6 ; n_3 = 0 ; n_4=2

n_2 = 5 ; n_3 = 2 ; n_4=1

n_2 = 4 ; n_3 = 4 ; n_4=0

Thus repeating the procedure for number of links 6,7,8,9,10, and the results are tabulated in Table 2 and 3.

Table 2
Assortments of Six, Seven, Eight and Nine Linked Kinematic Chains

(N) Number of Links	*(DF) Degree of Freedom*	*(N2) Number of Binary links*	*(N3) Number of Ternary links*	*(N4) Number of Quaternary links*	*(N5) Number of Pentagonal links*	*(N6) Number of Hexagonal links*
6	1	4	2	0	0	0
7	2	6	0	1	0	0
7	2	5	2	0	0	0
8	1	6	0	2	0	0
8	1	5	2	1	0	0
8	1	4	4	0	0	0

(N) Number of Links	(DF) Degree of Freedom	(N2) Number of Binary links	(N3) Number of Ternary links	(N4) Number of Quaternary links	(N5) Number of Pentagonal links	(N6) Number of Hexagonal links
8	3	7	0	1	0	0
8	3	6	2	0	0	0
9	2	7	0	2	0	0
9	2	7	1	0	1	0
9	2	6	2	1	0	0
9	2	5	4	0	0	0
9	4	8	0	1	0	0
9	4	7	2	0	0	0

Table 3
Assortments of Ten linked kinematic chain

Number of Links(N)	Degree of Freedom (DF)	Number of Binary links (N2)	Number of Ternary links (N3)	Number of Quaternary links (N4)	Number of Pentagonal links (N5)	Number of hexagonal links (N6)
10	1	4	6	0	0	0
10	1	6	2	2	0	0
10	1	6	3	0	1	0
10	1	5	4	1	0	0
10	1	7	1	1	1	0
10	1	7	0	3	0	0
10	1	8	0	0	2	0
10	3	8	1	0	1	0
10	3	7	2	1	0	0
10	3	6	4	0	0	0
10	3	8	0	2	0	0
10	3	9	0	0	0	1

Once the assortments of links is made, the input table is ready for the program by using which, the feasible kinematic chain is generated. and identification code is assigned to each kinematic chain. To do so, the philosophy of the algorithm (program) is discussed.

IV. PHILOSOPHY OF MAXCODE ALGORITHM FOR GENERATING AND IDENTIFYING KINEMATIC CHAINS [14]

To Listing all assortments of binary, ternary, etc. links in the chains for given N and F. Assuming any of the link (amongst the available) of highest degree, as link 1 and write down first row of UTAM to extract maximum possible number.

The range of Maxcode is established i.e.upperlimit (MAXU) and lower limit(MAXL) is established.

MAXU is established by connecting link 1 to all links of highest degrees from the remaining so as to produce feasible chains.

MAXL is established by connecting link 1 to all binary links, so as to produce feasible chains, Step 3: The range is to be then scanned for every digital number.

The scanned results will give feasible kinematic chain.

For the scanning purpose the steps followed are as below

Firs MAXL value is selected and its decoding is done to generate binary code.

From the binary code adjacency matrix is generated which represents the kinematic chain .

Then to have feasible chains generated without 3-links loop or open chain

Once the adjacency matrix is selected then maxcode labeling scheme is used to identify the Kinematic Chain.

It is done in following way :

Firstly for a single Kinematic Chain if number of schemes of labeling are existing they are printed.

Then establishing the decimal code for each scheme of labeling and pick up only those having maximum value of decimal code for the same Kinematic Chain and that scheme of labeling is known as canonical scheme of labeling

A canonical matrix is derived from a canonical scheme of labeling of links of the given Kinematic chain which is unique for a given chain.

The isomorphism check is done by the comparison of maxcodes of the two chains.

The Kinematic Chain is defined completely with the unique maxcode (maximum) which is checked for the earlier codes to avoid repetitiveness and the it is stored.

V. IMPLIMENTATION

The implementation of the algorithm has been discussed in the enumeration of kinematic chains of 9 links and two degree of freedom [19]

VI. CONCLUSION

The major aspects covered in this paper are basics of kinematic chain like minimum number of links in a chain to have a feasible chain, the link assortments table construction for six to ten links which is required as input to the maxcode algorithm. In all the initial parameters of decisions regarding kinematic chain is explained and proved in this paper. So that by the combination or assortment of various links like binary, ternary etc. the kinematic chains can be generated easily.. The computer program therefore user interactive. Max code has been widely accepted by graph theorists as a complete graph invariant and has been used in the current work for differentiating co-spectral, non-isomorphic chains.

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