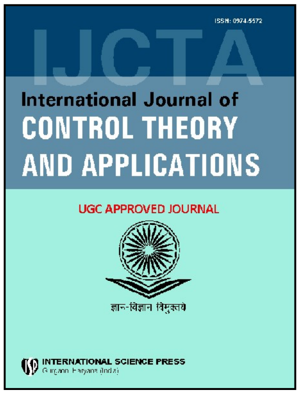

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Fractional PI Controller for Non-Integer Order Systems with Temporal Specifications

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Abstract: This paper deals with control of fractional order systems using a non-integer order Proportional Integral controller (PIë). The controller parameters are computed in order to ensure desired closed loop temporal specifications such as rise time and overshoot. The Characteristic Ratio Assignment (CRA) method is used to find the desired closed loop characteristic equation. Some new results concerning the desired overshoot are also presented based on sufficient stability conditions. Simulation results are given in order to illustrate the efficiency of the proposed controller.

Keywords: PI^{λ} controller, Characteristic Ratio Assignment (CRA), temporal specifications, stability conditions.

1. INTRODUCTION

Several applications of non-integer order models have been studied in different fields such as electrical transmission lines and signal processing. Fractional differentiation is employed in various branches like viscoelasticity and sound waves propagation. Moreover, it is used in biomedicine citing human respiratory system and genetic. Also, fractional calculus is exploited in bioengineering, the multimedia streaming and the finance [1]. These applications can be described by theoretical models quoting Cole-Cole, Cole-Davidson and recently the Havriliak–Negami model which depends on the real values of the parameters α and γ which characterize the distinctive features of the response [2].

Many researches have been done to calculate fractional order systems [3-5]. Approximate methods have been developed by the CRONE team to study these types of systems [6, 7]. Thanks to graphical interfaces, such that FOMCON toolbox, temporal and frequency specifications can be easily elaborated [8].

During the study of fractional order systems, stability of this kind of systems took the attention of researchers. Hurwitz or Routh Hurwitz criterion is employed in [9] to determine the necessary and sufficient condition of stability.

Different types of fractional controller have been exploited in literature [10, 11]. Proportional Integral and Proportional Integral Derivative controllers (PI^{λ} or $PI^{\lambda}D^{\mu}$) were applied to control a first order time delay plant based on the generalized version of the Hermite-Biehler theorem [12, 13]. In [14] a tuning and auto-tuning of fractional order PIëDì controllers is proposed in industrial applications to fulfil different specifications regarding robustness to plant uncertainties, load disturbances and high frequency noise.

Pole placement is one of the methods making it possible to impose temporal specifications (rise time, settling time and overshoot) for both integer and fractional order systems [15]. However, this method is limited to the second order transfer function.

Besides, the Characteristic Ratio Assignments (CRA) method which is a pole placement method, allows us to impose desired performances for higher order transfer function. In fact, by fixing the value of the generalized time constant ô we can obtain the desired rise time and settling time. With the ratio values $\acute{a}_i$, the overshoot can be adjusted [16]. The specified overshoot depends essentially on the value of commensurate order v if the system transfer function contains zeros [17].

The CRA method is used in [18] to design a low order controller for integer order systems. In [19-20] this method is employed in the synthesis of a robust controller. The CRA method is also used to design the RST controller for fractional order systems [16].

This work deals with the design of a fractional PI controller for three types of commensurate non-integer order systems, (first order, first order with zero and second order systems) by imposing temporal specifications. In order to obtain these performances, the CRA method is used.

This paper is divided into 6 sections. Section 2 is reserved to define fractional calculus and to find the interval in which the closed loop step response contain an overshoot. The CRA method is presented in section 3 and sufficient stability conditions for fractional order systems are proposed. Section 4 is reserved to design the controller. Section 5 provides simulation results. Conclusions and perspectives are given in the last section.

2. FRACTIONAL CALCULUS

Many definitions are used to calculate non integer order equations. The most popular one is the Riemann Liouville that is described in the continuous form as follows [3]:

$$ {}_a\mathrm{D}_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{(\alpha-n+1)}} \, , \; (n-1 \le \alpha < n) \tag{1} $$

where a and t denote the limits of the operation, α denotes the fractional order and $\Gamma(\cdot)$ is Euler's gamma function.

The discrete form of this definition is defined as:

$$ {}_a\mathrm{D}_t^{\alpha} f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-\alpha}{n}\right]} (-1)^j \binom{\alpha}{j} f(t-jh) \tag{2} $$

where h is the sampling period and $\begin{pmatrix}\alpha \\ j\end{pmatrix} = \frac{\alpha(\alpha-1)\ldots(\alpha-k+1)}{jl}$

2.1. Laplace Transform and System Representation

The general Laplace transform of the differential integral is given by [16]:

$$L(D_{0t}^{v}f(t)) = \int_0^t e^{-st} D_{0t}^{v} f(t)dt$$
$$= s^{v}F(s) - \left[\sum_{k=0}^{n-1} s^{k}(-1)^{j} D_{0t}^{v-k-1} f(t)\right]_{t=0} \quad (3)$$

where n is an integer such that $n - 1 < v < n$ and $F(s) = L(f(t))$

If all the derivatives are zero the previous expression becomes:

$$\mathrm{L}(\mathrm{D}_{0t}^{v} f(t)) = s^{v}F(s) \quad (4)$$

The generalized transfer function of the commensurate fractional order linear system can be expressed by:

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_{n-1}s^{(n-1)v} + \ldots + b_1 s^{v} + b_0}{s^{nv} + s^{(n-1)v} + \ldots + a_1 s^{v} + a_0} \quad (5)$$

where *N(s)* and *D(s)* have no common zeros and v is the commensurate fractional order.

There are different approximate methods used in FOMCON Matlab toolbox to calculate this kind of systems. The most used in literature is Oustaloup method [6].

The Oustaloup's recursive filter in a frequency range [ω_b, ω_h] for $0 < v < 1$ is defined by [6] as:

$$H_f(s) = s^{v} = K\prod_{k=-N}^{N} \frac{s + \omega'_k}{s + \omega_k} \quad (6)$$

where poles, zeros and gain of the filter can be determined as:

$$\omega_k = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+\frac{1}{2}(1+v)}{2N+1}}, \ \omega'_k = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+\frac{1}{2}(1-v)}{2N+1}}, \ K = \omega^{v}_h \quad (7)$$

Where ω_b and ω_h are respectively the low and the high frequency.

For $v > 1$, the approximation can be divided into a product of the integer portion with the fractional one by the following:

2.2. Sufficient Condition for Existence of an Overshoot in the Step Response of Fractional Order Systems

The transfer function *G(s)* defined by (5) is Bounded Input Bounded output (BIBO) stable if and only if the real parts of the denominator zeros are negative [17]. The sufficient condition for the existence of an overshoot in the step response of system (5) proved by [17] is that:

The strictly proper and BIBO stable transfer function *G(s)* with the steady state gain $G(0) \neq 0$ has always an overshoot in its step response if:

$$\lim_{s\to 0} \frac{G(s) - G(0)}{s} = 0 \quad (9)$$

In the same reference [17], it was announced that the step response of each stable fractional order system with commensurate order v where $1 < v < 2$ has an overshoot.

2.3. Fractional-order PI Controller and Overshooting Step Responses

In this section, we will extend the previous conditions for the closed loop transfer function by using a fractional order PI controller whose transfer function is given by:

$$C(s) = k_p + \frac{k_i}{s^{\lambda}} \tag{10}$$

The closed loop transfer function of a system described by an all pole commensurate fractional order transfer function is defined by:

$$F(s) = 1 - \frac{s^{\lambda}(s^{nv} + a_{n-1}s^{(n-1)v} + \ldots + a_1 s^{v} + a_0)}{s^{\lambda}(s^{nv} + a_{n-1}s^{(n-1)v} + \ldots + a_1 s^{v} + a_0) + (b_{n-1}s^{(n-1)v} + \ldots + b_1 s^{v} + b_0)(k_p s^{\lambda} + k_i)} \tag{11}$$

It is clear that $\lim_{s \to 0} \frac{F(s) - F(0)}{s} = 0$ if $\lambda > 1$ and $b_0 \, k_i \neq 0$

The step response of the closed loop system shown in Figure 1 has always an overshoot if $\lambda = v > 1$ and *G(s)* is a transfer function having non zero steady state gain and *C(s)* is a fractional order controller [17].

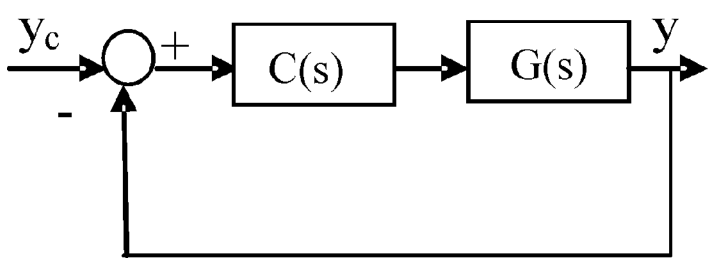

Figure 1: Feedback control system structure

3. CHARACTERISTIC RATIO ASSIGNMENT

In this section, the characteristic ratio and the generalized time constant are defined for an all-pole commensurate fractional order system. Then, the closed loop transfer function is considered as follows:

$$F(s) = \frac{\delta_0}{s^{nv} + \delta_{(n-1)}s^{(n-1)v} + \ldots + \delta_1 s^{v} + \delta_0} \tag{12}$$

where v is the commensurate order.

Definition 1. The characteristic ratios α_i are defined by [16]:

$$\alpha_i = \frac{\delta_i^2}{\delta_{i-1}\delta_{i+1}}, \; i = 1, \ldots, n-1 \tag{13}$$

Where $\delta_n = 1$.

Corollary 1. Parameters δ_i of the system (12) are expressed in function of the characteristic ratios α_i and the generalized time constant τ by [16]:

$$\begin{cases} \delta_1 = \tau^{v}\delta_0 \\ \delta_i = \dfrac{\delta_0 \tau^{iv}}{\alpha_{i-1}\alpha_{i-2}^2\alpha_{i-3}^3\ldots\alpha_1^{i-1}} \end{cases}, \quad i = 2,\ldots,\ n \tag{14}$$

It is also possible to make the previous relation into a recursive form:

$$\begin{cases} \delta_1 = \tau^{v}\delta_0 \\ \delta_i = \dfrac{\delta_{i-1}\tau^{v}}{\prod_{k=1}^{i-1}\alpha_k} \end{cases}, \quad i = 2,\ldots,\ n \tag{15}$$

Knowing that the step response does not have overshoot if the magnitude of the system frequency response is monotonically decreasing. In order to ensure a small amount of overshoot, the characteristic ratios of system (12) must be defined by [16]:

$$\alpha_i = \begin{cases} -2\beta\cos(\pi v) & \text{if } i = 2k+1 \\ \dfrac{-2}{\beta\cos(\pi v)} & \text{if } i = 2k,\ k \in N \end{cases} \tag{16}$$

were β is function of v.

This function is found by [17] for system having a commensurate order $0.5 < v < 1$ and a target transfer function expressed by (12). For systems having zeros in their transfer functions and their commensurate order $1 < v < 1.5$, we were proposed new expressions for such system. The following table allows us finding the values of β by imposing as objective an overshoot less than (*5%, 10%*). β is selected so to limit the peak of overshoot.

Table 1
Values of β in function of v

v	1.05	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45
$\beta_{D(\%)<5\,(n=1)}$	3	4	4	5	8	10	16	40	80
$\beta_{D(\%)<10\,(n=1)}$	1.4	1.7	2.2	2.7	3.5	6.5	12	24	55
$\beta_{D(\%)<10\,(n=2)}$	40	50	80	110	200	350	750	2000	8000

From this table, we can deduce relation between β and v for $n = 1$ and $n = 2$:

$$\beta_{(n=1,D<5\%)} = 0.01593.\mathrm{v}^{22.82} + 3.71 \tag{17}$$

$$\beta_{(n=1,D<10\%)} = 0.007391.\mathrm{v}^{23.89} + 1.931 \tag{18}$$

$$\beta_{(n=2,D<10\%)} = 0.003373.\mathrm{v}^{39.46} + 127.9 \tag{19}$$

Corollary 2. For two systems with identical characteristic ratio and different generalized time constants τ_i, the following relations are proposed [16]:

$$\frac{t_{s1}}{t_{s2}} = \frac{t_{r1}}{t_{r2}} = \frac{\tau_1}{\tau_2} \quad (20)$$

where t_{si} and t_{ri} are respectively the settling time and the rise time.

Algorithm

To determine the desired step response using the characteristic ratio method, the following algorithm is proposed. This algorithm is divided in two steps:

Step 1

i) We fix τ_1 and β.

ii) We calculate the characteristic ratio for $k = 1,\ldots, n\text{-}1$ using (16).

iii) We calculate the target polynomial coefficients δ_i, for i=1,…, n ($\delta_0 = 1$) by using (15).

iv) We found the desired transfer function and its rising time t_{r1}.

Step 2

i) We set the desired rising time t_{r2} and we calculate the generalized time constant τ_2 by using (20).

ii) By using the new generalized time constant τ_2 we find the new coefficients δ_i by using (15).

iii) Finally, we determine the new desired transfer function.

This algorithm will be applied in the following example.

Example

Consider two systems described by (12), where $\delta_0 = 1$, $v = 0.8$ and $n = 3$. The characteristic ratios α_i = [0.48 8.24] for both systems are obtained by applying (16). For an initial design, the generalized time constant is chosen as $\tau_1 = 10s$. Then, the coefficients δ_i are determined. Thus, the rise time $t_{r1} = 27.6s$ is deduced. Let the objective be the obtaining of a rise time $t_{r2} = 13.8s$, so by using relation (20), the second generalized time constant is $\tau_2 = 5s$. The relation (15) is used to compute the δ_i of the second transfer function. Then the presented algorithm allows the computation of the coefficients 1, ..., δ_n with $\delta_0 = 1$. Then, we will compute a transfer function that its δ_n coefficient is set to one. So, it is enough to devise the coefficients of the transfer function by δ_n.

Parameters δ_i for the second transfer function are given in Table 2. Figure 2 shows the unit step response for the two systems.

Table 2
Coefficients of the characteristic polynomials for two systems

i	δi $\tau_1 = 10s$	δi $\tau_2 = 5s$
0	0.007	0.041
1	0.048	0.148
2	0.631	1.104

Figure 2 shows the step responses for the transfer function having the characteristic ratios given by table 2. The desired rise time is then found.

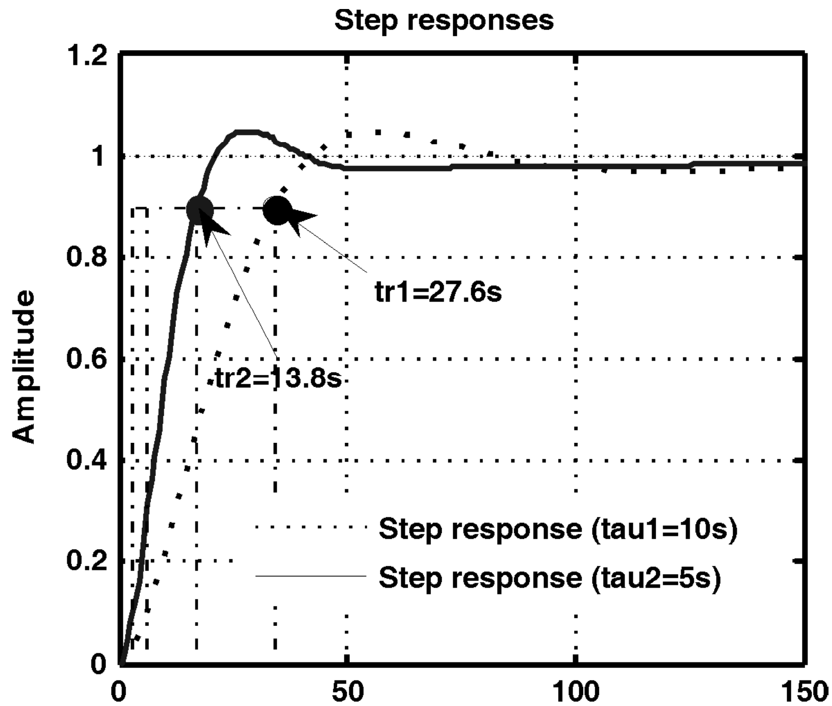

Figure 2: Unit step response for τ_1 =*10s* and τ_2 = *5s*

SUFFICIENT STABILITY CONDITIONS FOR FRACTIONAL ORDER SYSTEMS

System (12) is stable if and only if their characteristic ratios answer the following conditions proposed by Lipatov and Sokolov:

$$\begin{aligned} &i)\ \sqrt{\alpha_i \alpha_{i+1}} > 1.4656 \qquad && i = 1,2,\ldots,\ n-2 \\ &\text{ii)}\ \alpha_i \geq 1.1237\left(\frac{1}{\alpha_{i-1}} + \frac{1}{\alpha_{i+1}}\right) \qquad && i = 1,2,\ldots,\ n-2 \end{aligned} \tag{21}$$

Proof: The sufficient condition for stability is that conditions in (21) are fulfilled for

$0.5 < v < 1$; $1 < v < 1.5$ and $\beta > 1$

i) Case 1: *i=2k+1*

$$1.1237\left(\frac{1}{\alpha_{i-1}} + \frac{1}{\alpha_{i+1}}\right) = -1.1237(\beta\cos(\pi v)) \leq -2\beta\cos(\pi v) = \alpha_i \tag{23}$$

(For $0.5 < v < 1$ and $1 < v < 1.5$, $0 < -\cos(\pi v) < 1$)

ii) Case 2: i=2k

$$1.1237\left(\frac{1}{\alpha_{i-1}} + \frac{1}{\alpha_{i+1}}\right) = \frac{-1.1237}{\beta\cos(\pi v)} \leq \frac{-2}{\beta\cos(\pi v)} = \alpha_i \tag{24}$$

(For $0.5 < v < 1$ and $1 < v < 1.5, \dfrac{-1}{\cos(\pi v)} > 1$)

Thus the system (12) with the characteristic ratios in (16) is stable.

The characteristic ratio method will be used in the following examples to find controller parameters.

4. SYNTHESIS OF THE CONTROLLER

In this section, the design of a fractional PI controller with time specification is developed for different forms of the fractional order transfer function.

Model 1:

Let consider the following open loop transfer function:

$$G(s) = \frac{b_0}{a_1 s^v + a_0} \tag{25}$$

By using the PI^λ controller ($\lambda = v$) which is given by (10), the closed loop transfer function is obtained:

$$F(s) = \frac{b_0 k_p s^v + b_0 k_i}{a_1 s^{2v} + (a_0 + b_0 k_p) s^v + b_0 k_i} \tag{26}$$

Then, the characteristic equation is expressed by:

$$E_c = a_1 s^{2v} + (a_0 + b_0 k_p) s^v + b_0 k_i \tag{27}$$

This characteristic equation is identified to the characteristic polynomial *ä(s)* determined using the proposed algorithm:

$$\delta(s) = \delta_2 s^{2v} + \delta_1 s^v + \delta_0 \tag{28}$$

Using relations (13) and (14), we can fix the coefficient δ_0 as:

$$\delta_0 = \frac{\delta_2 \alpha_1}{\tau_1^{2v}} \tag{29}$$

Knowing that $\delta_2 = a_1 = 1$, so relation (29) becomes:

$$\delta_0 = \frac{a_1 \alpha_1}{\tau_1^{2v}} \tag{30}$$

The controller parameters can be determined using relations (27) and (28):

$$\begin{cases} k_p = \dfrac{\delta_1 - a_0}{b_0} \\ k_i = \dfrac{\delta_0}{b_0} \end{cases} \tag{31}$$

Model 2: In this case, the open loop transfer function of the plant is now chosen as a first order function with presence of one zero.

$$G(s)=\frac{b_1 s^v + b_0}{a_1 s^v + a_0} \tag{32}$$

The closed loop transfer function with the fractional order PI controller is given by:

$$\text{F(s)}=\frac{\text{b}_1 k_p s^{2v} + \left(b_1 k_i + b_0 k_p\right) s^v + b_0 k_i}{\left(a_1 + b_1 k_p\right) s^{2v} + \left(a_0 + b_0 k_p + b_1 k_i\right) s^v + b_0 k_i} \tag{33}$$

The characteristic equation is expressed by:

$$\text{E}_\text{c} = \left(a_1 + b_1 k_p\right) s^{2v} + \left(a_0 + b_0 k_p + b_1 k_i\right) s^v + b_0 k_i \tag{34}$$

This equation is identified to the characteristic polynomial δ*(s):*

$$\delta(s) = \delta_2 s^{2v} + \delta_1 s^v + \delta_0 \tag{35}$$

Hence, the controller parameters can be determined using relations (34) and (35):

$$\begin{cases} k_p = \dfrac{\delta_1 - a_0 - b_1 k_i}{b_0} \\ k_i = \dfrac{\delta_0}{b_0} \end{cases} \tag{36}$$

δ_0 is chosen so that $\delta_2 \cong \left(a_1 + b_1 k_p\right) = 1$.

Model 3: The third model is chosen with one zero and two poles. The transfer function is given as follows:

$$\text{G(s)}= \frac{\text{b}_1 s^v + b_0}{s^{2v} + a_1 s^v + a_0} \tag{37}$$

The closed loop transfer function of the system with the fractional order PI controller is:

$$\text{F(s)}=\frac{\text{b}_1 k_p s^{2v} + \left(b_1 k_i + b_0 k_p\right) s^v + b_0 k_i}{s^{3v} + \left(a_1 + b_1 k_p\right) s^{2v} + \left(a_0 + b_0 k_p + b_1 k_i\right) s^v + b_0 k_i} \tag{38}$$

The controller parameters can be determined by using the following identification:

$$\begin{cases} a_1 + b_1 k_p = \delta_2 \\ a_0 + b_1 k_i + b_0 k_p = \delta_1 \\ b_0 k_i = \delta_0 \end{cases} \tag{39}$$

It has been mentioned in [17] that the lower powers coefficients of *s* are the most related to the step response. So, δ_0 and δ_1 have more influence on the step response than δ_2 That implies:

$$\begin{cases} a_1 + b_1 k_p = \delta_2 \\ k_i = \dfrac{\delta_0}{b_0} \\ k_p = \dfrac{\delta_1 - a_0 - b_1 k_i}{b_0} \end{cases} \tag{40}$$

By considering the characteristic ratios α_i and the system parameters δ_i, the following relations are obtained:

$$\alpha_1 = \frac{\delta_1^2}{\delta_0 \delta_2}\ ,\ \alpha_2 = \frac{\delta_2^2}{\delta_1 \delta_3}\ ,\delta_1 = \tau^v \delta_0\ \ ,\ \delta_2 = \frac{\delta_0 \tau^{2v}}{\alpha_1} \tag{41}$$

Using these equations, the system parameter δ_0 can be determined:

$$\delta_0 = \frac{\alpha_1^2 \alpha_2}{\tau^{3v}} \tag{42}$$

5. SIMULATION RESULTS

In order to shed light the effectiveness of the proposed controller, two intervals of v are considered.

5.1. Case *0.5 < v < 1*

As it is mentioned in section 2.2, the closed loop responses do not have overshoot in their transfer functions with these values of the commensurate order. So β will be fixed by simulations in order to illuminate undershoots. Two examples will be chosen to find the appropriate controller insuring the desired temporal specifications.

Example 1: Let consider the following transfer function:

$$G_1(s) = \frac{0.4}{s^{0.7} + 0.2} \tag{43}$$

The unit step open loop system is shown in figure 3. The aim is to design a fractional PI that will be able to guarantee the following closed loop specifications: a rise time equals to 15s with no overshoot.

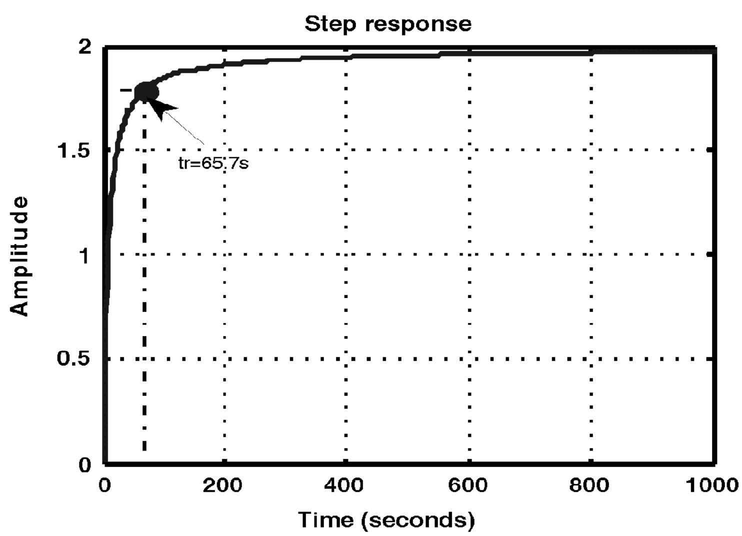

Figure 3: Open loop step response (system described by (43))

From figure 3, we note that the system rise time is *65.7s*. By choosing $\tau_1 = 5s$ and $\beta = 0.6$, the characteristic ratio is $\alpha_1 = 0.7053$. Then the system parameters are $[\delta_2\ \delta_1\ \delta_0] = [1\ 0.2286\ 0.0741]$.

The obtained fractional PI controller can be written as:

$$C_1(s) = 0.0716 + \frac{0.1853}{s^{0.7}} \tag{44}$$

The rise time in initial closed loop system is *21.3s*. To increase the plant speed we will use corollary 2. The new system parameters are $[\delta_2\ \delta_1\ \delta_0] = [1\ 0.2922\ 0.1211]$. The controller parameters are $k_p = 0.2306$ and $k_i = 0.303$. Applying these parameters to the plant, we found the final rise time as $t_{r2} = 12.2s$ which is less than *15s*. To have the rise time equal to *15s*, we set the generalized time constant $\tau_2 = 4.032s$. So that the final system parameters are $[\delta_2\ \delta_1\ \delta_0] = [1\ 0.226582\ 0.1002]$ and then the final controller is founded:

$$C_2(s) = 0.1645 + \frac{0.2504}{s^{0.7}} \tag{45}$$

The unit closed loop step response by applying the controllers $C_1(s)$ and $C_2(s)$ are plotted in Figure 4a. Figure 4b shows the control signals obtained with these controllers. It is noted from Figure 4a, that the controller $C_2(s)$ meets the desired time specifications.

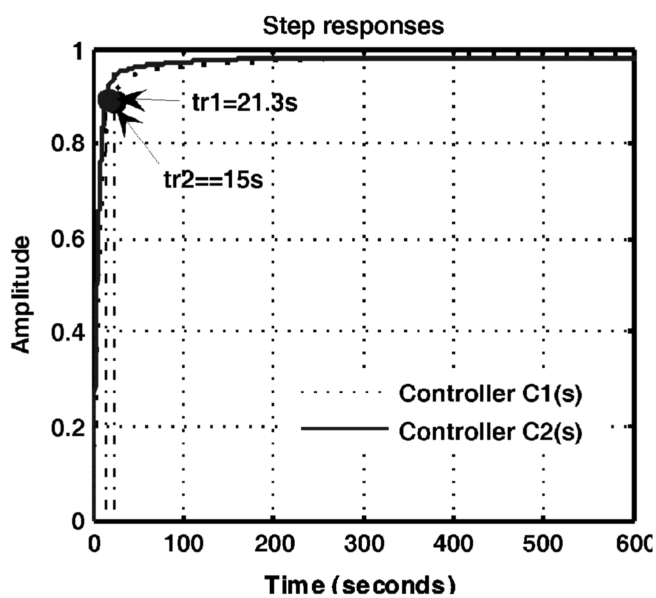

Figure 4: (a) Closed loop responses with controller $C_1(s)$ and $C_2(s)$

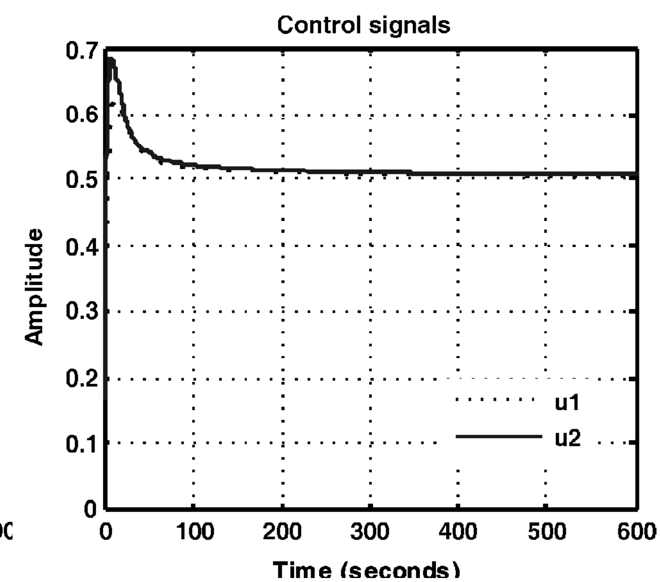

Figure 4: (b) Control signals

Example 2: Let the system transfer function be:

$$G_2(s) = \frac{0.5s^{0.7} + 0.3}{0.8s^{0.7} + 0.2} \tag{46}$$

From the unit open loop step response (Figure 5), we can deduce the rise time $t_r = 48.9$. The objective now is the design of fractional PI to obtain a rise time equals to *25s* with no overshoot. To have no overshoot, β is chosen as $\beta = 0.99$.

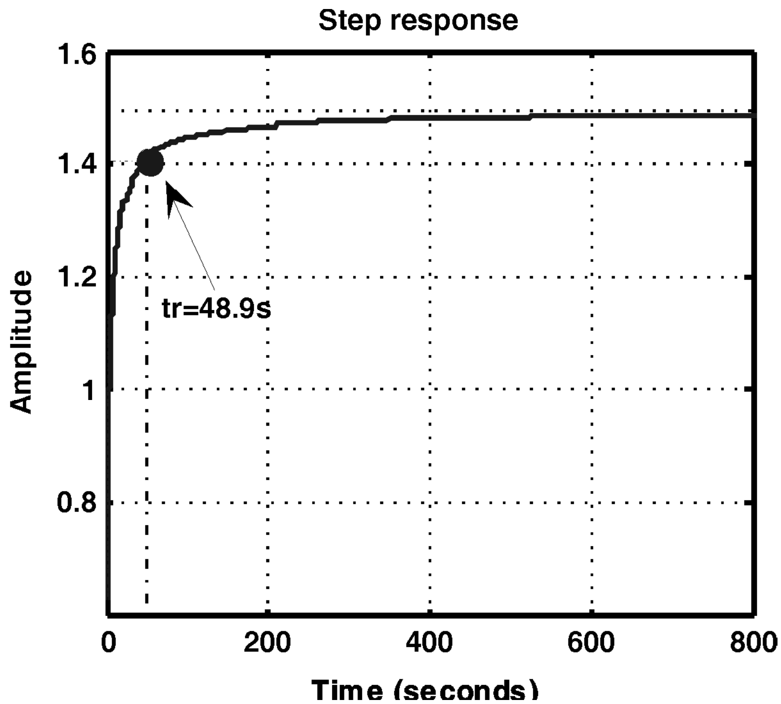

Figure 5: Open loop step response (system described by (46))

For an initial choice $\tau_1 = 10s$, $\beta = 0.99$ and $\delta_0 = 0.06$, the characteristic ratio is $\alpha_1 = 1.164$ and then the system parameters are $[\delta_2\ \delta_1\ \delta_0] = [1\ 0.232\ 0.0463]$. Using these values we found a first fractional order PI controller:

Applying this controller to the system (46), the rise time is $t_{r1} = 34.6s$. In order to obtain the desired rise time, relation (20) is used to compute the new generalized time constant $\tau_2 = 7.512s$. Then, the new system parameters are $[\delta_2\ \delta_1\ \delta_0] = [1\ 0.2839\ 0.0692]$. So, the desired fractional order PI controller is then found:

The closed loop system shown in Figure 6a presents a rise time $t_{r2} = 25s$ with no overshoot.

The control signals obtained by the considered controllers $C_1(s)$ et $C_2(s)$ are depicted in Figure 6b.

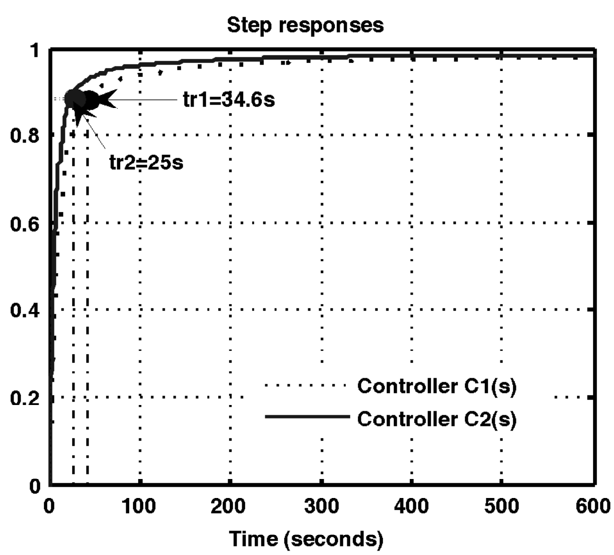

Figure 6: (a) Closed loop responses with controller $C_1(s)$ and $C_2(s)$

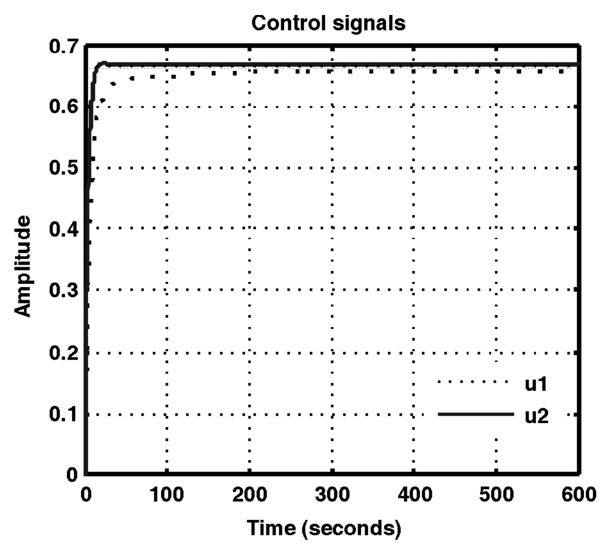

Figure 6: (b) Control signals

5.2. Case 1 < v < 1.5

In this case, we will use the expressions of β defined in section 2 in order to find the desired overshoot.

Example 1: The open loop transfer function is defined by:

$$G_3(s) = \frac{0.02}{s^{1.16} + 0.01} \tag{49}$$

The rise time of the open loop system is $t_{r1} = 85.7s$. The aim is to design a fractional order PI controller that will be able to guarantee the following closed loop specifications: a rise time equals to 24s and an overshoot less than 5%. The open loop step response of this system is shown in Figure 7.

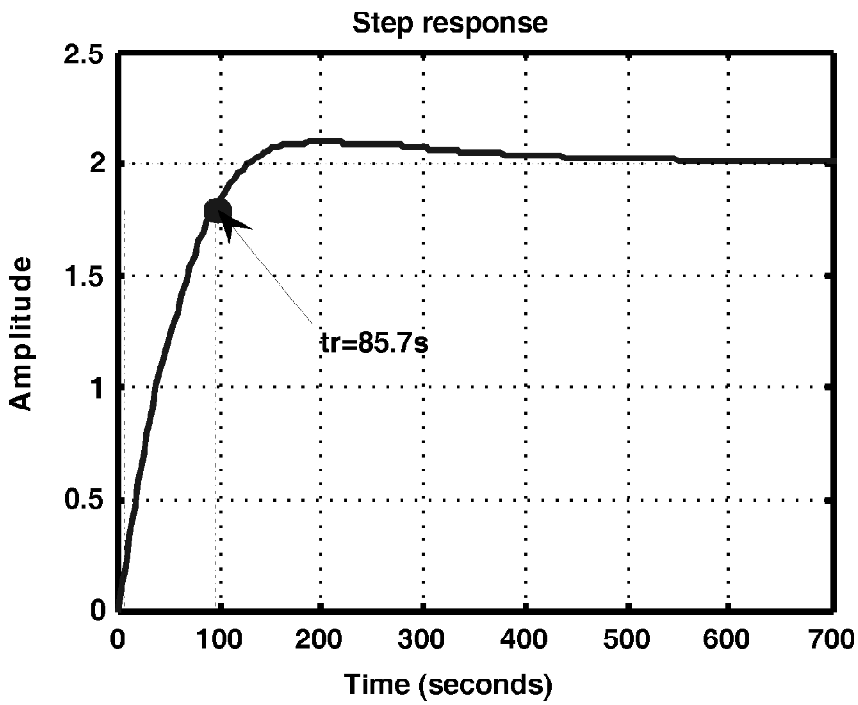

Figure 7: Open loop step response (system described by 49)

Based on CRA method, the characteristic ratios is determined by $\alpha_1 = 7.33$. As initial design the generalized time constant is chosen as τ1 = 100*s* and β is calculated with relation (17). The fractional PI controller can be deduced:

The rise time is then $t_{r1} = 48s$. To obtain the desired rise time, relation (20) is used. The generalized time constant becomes τ1 = 67.5*s* and then the rise time $t_{r2} = 23.8s$ is found and the overshoot is $D(\%) = 4.22$. The final controller is given by:

Figure 8a and Figure 8b show respectively the closed loop responses with controllers $C_1(s)$ and $C_2(s)$ and the control signals.

Example 2: Let consider the following system transfer function:

$$G_4(s) = \frac{0.1s^{1.44} + 0.02}{s^{2.88} + 1.8s^{1.44} + 0.01} \tag{52}$$

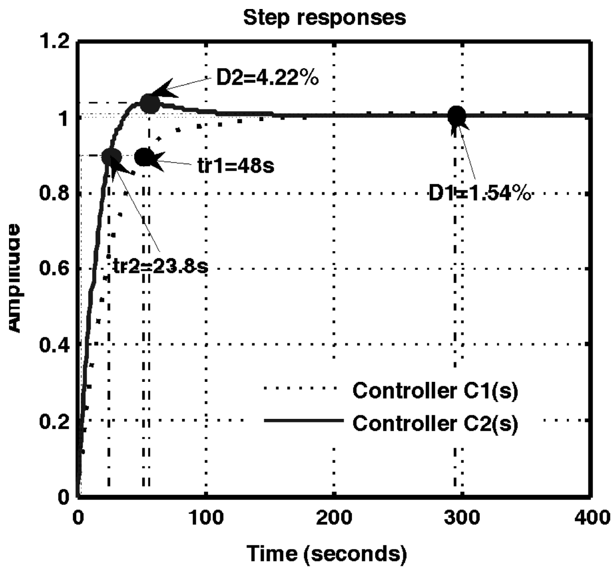

Figure 8: (a) Closed loop responses with controller $C_1(s)$ and $C_2(s)$

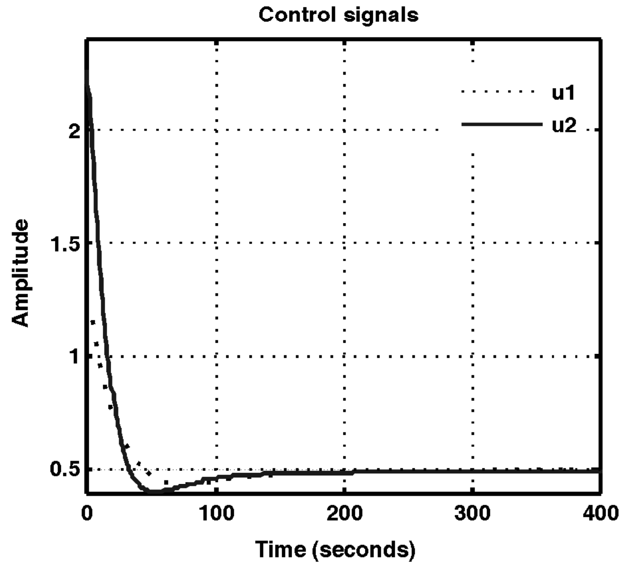

Figure 8: (b) Control signals

The unit step response of the open loop system is shown in Figure 9. The rise time is $t_r = 46.9s$. We would like to increase the speed of the step response and to have a rise time $t_{r2} = 25s$ and an overshoot less than *10%*.

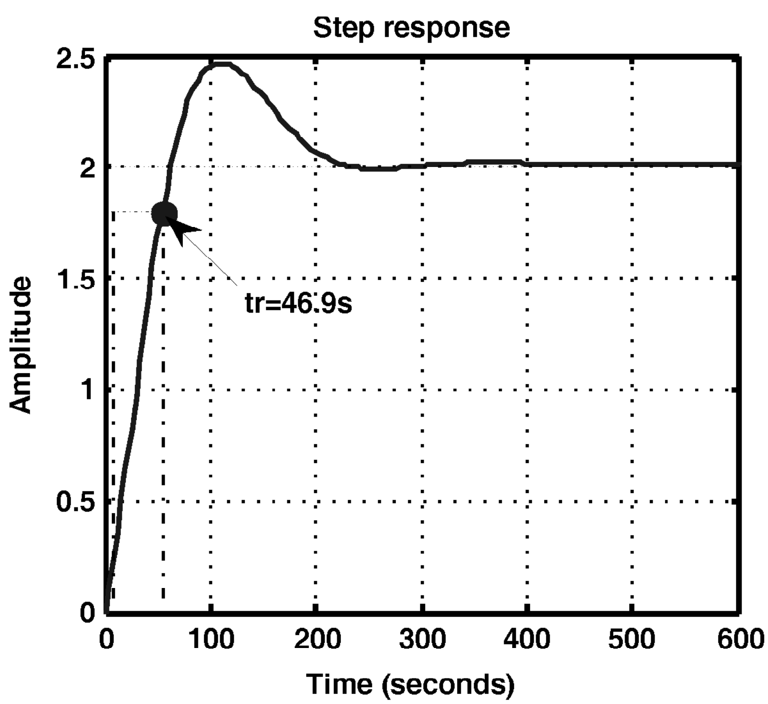

Figure 9: Open loop step response (system described by (52))

For initial design $\tau_1 = 70s$, the controller parameters are calculated $(k_p, k_i) = (1.0001, 0.0028)$. So, the rise time $t_{r1} = 36.8s$ is determined and the overshoot is D_1 (%) = 7.35. Considering the relation (20), the new generalized time constant is defined by $\tau_2 = 69.45s$. Thus, we find the controller parameters $(k_{p2}, k_{i2}) = (1.7489, 0.0051)$.

Figure 9a shows the final step response with the new controller parameters and it indicates that the rise time is $t_{r2} = 24.9s$ and the overshoot is $D_2(\%) = 9.67$ which is less than *10%*. Control signals are shown in Figure 10b.

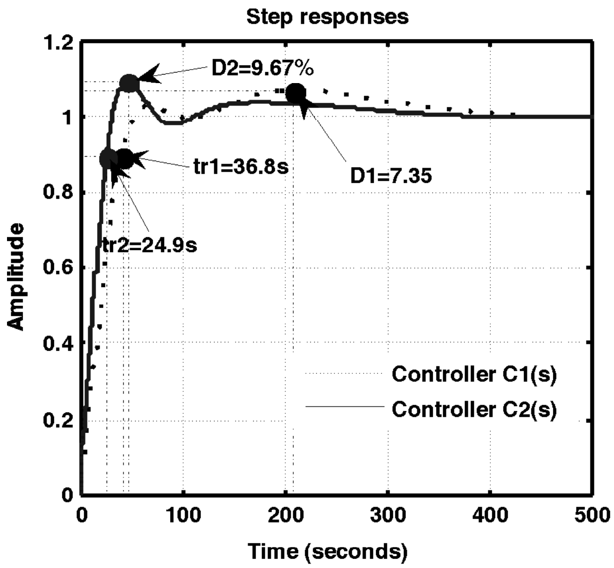

Figure 10: (a) Closed loop responses with controller $C_1(s)$ and $C_2(s)$

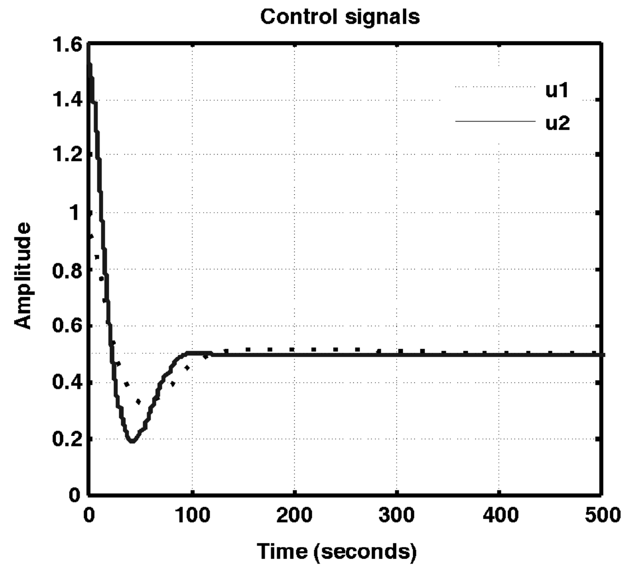

Figure 10: (b) Control signals

From the previous examples, we can deduce that the overshoot is assured for the systems having zeros in their transfer function only if $1 < v < 1.5$.

6. CONCLUSION

In this work, the CRA method is used to design a fractional order PI controller for first and second fractional order systems. The CRA method allows us to obtain the characteristic equation that ensures the desired temporal specifications. A demonstration is done to find the commensurate order interval to have overshoot in closed loop step response respecting the sufficient conditions of stability. Simulation results show the effectiveness of the proposed method. Our next topic is to extend the developed approach in the case of uncertain parameters systems.

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