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Analysis of an One Equilibrium Novel Hyperchaotic System and its Circuit Validation

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Abstract: The obective of the paper is to develop a new hyperchaotic system with a single equilibrium point. The system is more hyperchaotic, larger bandwidth and complex than many other reported hyperchaotic systems. The proposed hyperchaotic system has non-uniform contraction and expansion of volume in phase space. Different theoretical and numerical techniques are used to analyse the proposed hyperchaotic system. Hyperchaotic attractor, Poincare map, frequency spectrum, and Lyapunov spectrum are the tools used to analyse the system. The system depicts hyperchaotic, chaotic, periodic and quasi-periodic behaviour for certain set of parameters over a large range of parameters. Circuit design of the proposed system is also presented to show the applicability of the proposed hyperchaotic system in real life.

Keywords: Hyperchaotic system; Lyapunov spectrum; circuit design; one equilibrium.

1. INTRODUCTION

Hyperchaotic systems have more complex and dense structure than chaotic system. Because of many inherent advantages, hyperchaotic systems have been extensively used in many fields like information science, electronics, mathematics, physics, communication, etc. as given in [1]. Any hyperchaotic system must be of minimum 4-D, and have two positive Lyapunov exponents [2]. Construction of new hyperchaotic system and its study are useful to explore the nature of hyperchaos. It is very interesting to construct a new hyperchaotic system with more complex dynamics and hyperchaotic nature.

Rossler reported the first hyperchaotic system in 1979 [3]. In the last three decades, many hyperchaotic systems have been reported [4-10]. Many methods are available for construction of new hyperchaotic system like (a) use of anti-control on a chaotic system [11], (b) addition one state variable with the existing chaotic system [12], (c) designing a new system so as to satisfy the conditions for hyperchaos [13]. Equilibrium point plays an important role in characterization of a hyperchaotic system. Thus, developing a new hyperchaotic system with different characteristics and behaviour is the novel problem which is considered here.

In this paper a novel 4-D, fourteen terms hyperchaotic system with only one real equilibrium point is presented. The system is constructed to be hyperchaotic. The proposed system is more hyperchaotic and complex than many other hyperchaotic systems. Parameter range for hyperchaos of the proposed system is very large. Detailed theoretical and numerical analyses of the system are presented using hyperchaotic attractor, Poincare map, frequency spectrum, Lyapunov. Circuit design of the system is also presented to highlight the real life applications. Novelty of the proposed hyperchaotic system is claimed by the following points:

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1. The proposed hyperchaotic system has only one real equilibrium point, but many available systems like [4, 6, 7, 10, 11, 15, 16] have more than one.
2. The proposed system is comparatively more hyperchaotic than many earlier reported hyperchaotic systems as in [3-11, 13, 15, 16].
3. The proposed system has non-uniform contraction and expansion of volume in phase space, but available systems [6-11, 13, 15, 16] have uniform contraction and expansion of volume.
4. The proposed system has larger parameter range for hyperchaos compared to the systems in [3-11, 13, 15, 16].
5. The proposed system has relatively larger bandwidth compared to the systems in [3-6, 9-11].

Section 2 presents the dynamics of the proposed system. Theoretical analysis of the proposed hyperchaotic system are highlighted in Section 3. Section 4 describes the numerical analysis and discussion of the novel hyperchaotic system. Conclusions are given in Section 5.

2. DYNAMICS OF THE NOVEL HYPERCHAOTIC SYSTEM

The proposed hyperchaotic system is discussed as:

$$\begin{aligned} \dot{x}_1 &= -dx_1 + (d-1)x_2 - x_3 + x_4 \\ \dot{x}_2 &= x_1(r-10x_3) + ax_2 + x_4 \\ \dot{x}_3 &= x_1(10x_2 + x_3 - c) - bx_3 \\ \dot{x}_4 &= px_1 + px_2 \end{aligned} \tag{1}$$

where x_1, x_2, x_3, x_4 are the states variable and $a, b, c, d\ r$ are the positive parameters of the system (1) and $p < 0$. The system (1) is constructed to satisfy the properties of hyperchaotic system. The system (1) is more hyperchaotic and has wide range of parameters of hyperchaos. For maximum hyperchaos [17], Lyapunov exponents of the system (1) are $L_i = 1.17, 0.02376, -0.00678, -50.22$ with $d = 55$, $r = 46.6$, $a = 12, b = 6, c = 11, p = -4$. Maximum Lyapunov dimension (Kaplan Yorke dimension) is $L_D = 3.127$ with Lyapunov exponents $L_i = 2.303, 0.0489, 0.0012, -18.4892$ for $d = 16$, $r = 46.6$, $a = 13, b = 6, c = 11, p = -4$. The next section presents the theoretical properties of the system (1).

3. THEORETICAL ANALYSES OF THE PROPOSED SYSTEM

In this section we present some basic properties of the system (1) like equilibrium point, dissipativity.

3.1 Equilibrium Point

The equilibrium point of the proposed system for $d = 55$, $r = 46.6$, $a = 12, b = 6, c = 11, p = -4$ is calculated by equating each variable of the system (1) to zero. We can find that the origin is the only real equilibrium point of the proposed system.

Eigenvalues of the equilibrium point are $\lambda_1 = -81.925$, $\lambda_2 = 38.686$, $\lambda_3 = 0.1807$, $\lambda_4 = -5.9412$ for $d = 55$, $r = 46.6$, $a = 12, b = 6, c = 11, p = -4$. It is apparent that the system has unstable equilibria with saddle node behaviour.

3.3 Dissipativity

To validate the hyperchaotic nature of system (1), dissipative condition is satisfied by calculating the divergence of the vector field $V(x)$ on R^4 as:

$$\nabla V = \frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} + + \frac{\partial \dot{x}_4}{\partial x_4} = x_1 - (-a + b + d) \quad (2)$$

When $x_1(t) = 0$ and $(-a + b + d) > 0$, then $\nabla V < 0$. Thus the system (1) is dissipative and volume of trajectories converges to zero exponentially with $V(t) = V(0)e^{-(-a+b+d)t}$ for any initial volume $V(0)$. For $x_1(\mathrm{t}) \neq 0$, $(-a + b + d) > 0$, and $(\mathrm{t}) < (-a + b + d)$ the system has non-uniform shrinking and expansion of volume in phase space with $V(0)e^{-(-a+b+d)t+x_1(\mathrm{t})}$and thus increases the complex nature of the system. Such nature of hyperchaotic system are rare in literature.

4. NUMERICAL ANALYSES OF THE PROPOSED SYSTEM

The section represent the numerical analyses and discussion of the proposed new hyperchaotic system.

4.1 Hyperchaotic Attractor

Hyperchaotic attractors of the proposed system are given in Fig. 1 (a-d). Following conclusions can be derived from the attractor plots:

1. The system is not symmetric.
2. The system has double scrolls, butterfly shape attractor.
3. Attractors of the system are complex and dense.

4.2 Poincare Maps and Time-Series Plot

Poincare map and time series plot are also important tools to validate hyperchaotic/chaotic nature of a system. Fig. 2 (a, b) represent the Poincare map of the proposed system (1) in two different planes. Time series are plotted in Fig. 2 (c). From both plots it appears that the system has complex and dense structure.

4.3 Frequency Spectrum

Normalized frequency spectrum of $x_2\,(t)$ signal of the proposed system (1) is shown in Fig. 3. It is seen from Fig. 3 that the system has [0-12] Hz bandwidth considering 0.1 spectral normalized value as cut-off. It may be noted that frequency spectrum of the novel system is more than many other hyperchaotic systems like in [3-6, 9-11].

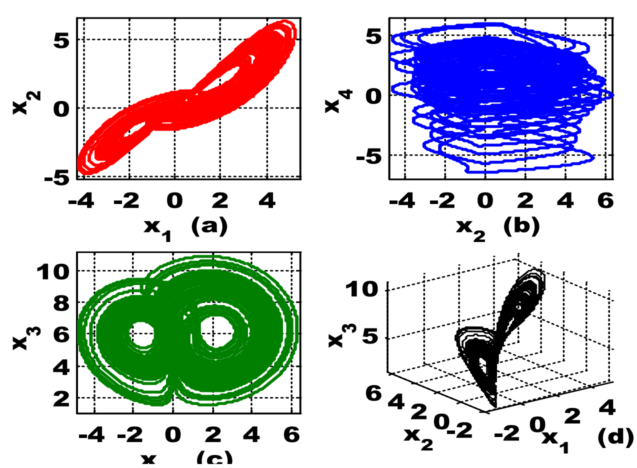


Figure 1: Attractors of the system (1) with parameter $d = 55$, $r = 46.6$, $a = 12, b = 6, c = 11, p = -4$ (a) on $x_1 - x_2$ plane, (b) on $x_2 - x_4$ plane, (c) on $x_2 - x_3$ plane, (d) on $x_1 - x_2 - x_3$ space.

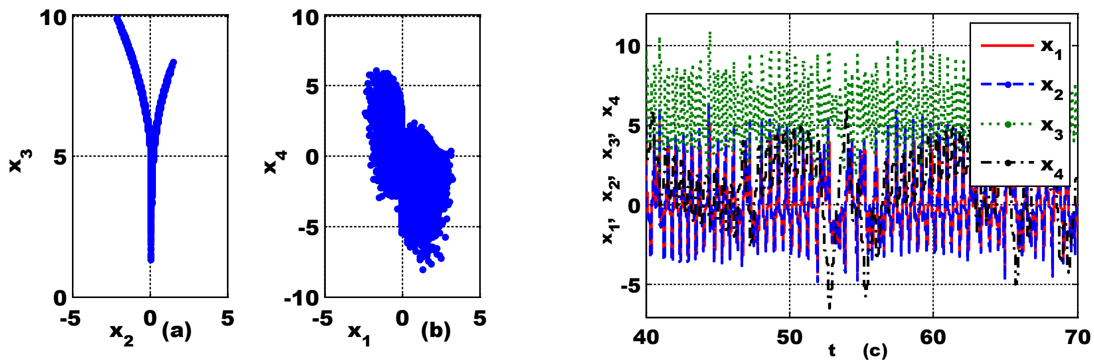


Figure 2: (a) Poincare map in $x_2 - x_3$ plane for $x_1 = 0$ (b) Poincare map in $x_1 - x_4$ plane for $x_2 = 0$, and (c) waveform of the system (1), with $d = 55,\ r = 46.6,\ a = 12, b = 6, c = 11, and\ p = -4$.

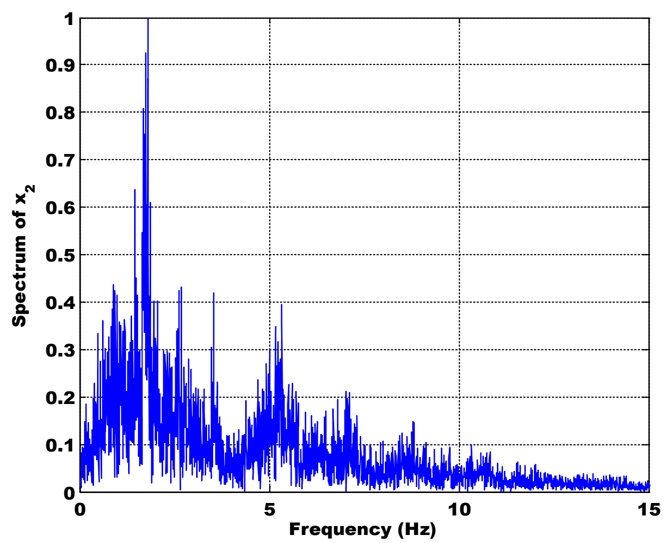


Figure 3: Normalized frequency spectrum of the system (1) for $d = 55,\ r = 46.6,\ a = 12, b = 6, c = 11, p = -4$.

4.4 Lyapunov Spectrum Analysis

The Lyapunov spectrum of the proposed system is obtained from time series [14] The simulation is carried out for 300 sec with time steps of for each set of parameters. Five parameters of the system (1) are kept constant and only one is varied within a range to find the influence of the varied varied parameter on the occurrence of hyperchaos.

4.4.1 Fix Parameters r = 46.6, a = 12, b = 6, c = 11, p = –4 and Vary d

Fig. 4 (a, b) shows the LS of the proposed system with variation of parameter. It is observed from Fig. 4 that the system produces different behaviours. Table 1 (a) summarizes the dynamical performance of the proposed system with varying parameter.

Table 1 (a)

Dynamical performance of the system (1) with variation of parameter *d*

Range of parameter	*Behaviour*	*Range of parameter*	*Behaviour*
$d \leq 14.5$	Chaotic	$21.5 \leq d \leq 22.5$	Hyperchaotic
$14.6 \leq d \leq 18$	Hyperchaotic	$22.6 \leq d \leq 22.9$	Chaotic
$18.1 \leq d \leq 20.6$	Chaotic	$23.0 \leq d \leq 64.3$	Hyperchaotic
$20.7 \leq d \leq 21.2$	Hyperchaotic	$64.4 \leq d \leq 70$	Periodic; except few values of
$d = 21.3, 21.4$	Chaotic	$d = 67.2, 67.6, 67.7, 68.3, 68.7$	Quasi-periodic

4.4.2 Fix Parameters d = 33, a = 12, b = 6, c = 11, p = –4 and Vary r

Fig. 5 (a, b) shows the LS of the novel system with variation of parameter. Fig. 5 depicts that the system produces hyperchaotic orbits for the wide parameter range, chaotic orbits and periodic orbits for some values of. Table 1 (b) summarizes of dynamical behaviour of the system (1) with varying parameter.

Table 2 (b)

Different dynamics of the system (1) with variation of parameter *r*

Range of parameter	*Behaviour*	*Range of parameter*	*Behaviour*
$r \leq 0.8$	Periodic	$3.5 \leq r \leq 7.7$	Periodic
$0.9 \leq r \leq 1.4$	Chaotic	$7.8 \leq r \leq 200$	Hyperchaotic
$1.5 \leq r \leq 3.4$	Hyperchaotic		

4.4.3 Fix Parameters r = 46.6, D =33, b = 6, c = 11, p = – 4 4 and Vary Parameter a

Fig. 6 (a, b) shows the LS of the system (1) with variation of parameter. It is observed from Fig. 6 that the system produces different dynamic behaviours. Table 1 (c) shows the summary of dynamical behaviour of the system (1) with variation of parameter.

Table 2 (c)

Dynamical performance of the proposed system (1) with variation of parameter *a*

Range of parameter	*Behaviour*	*Range of parameter*	*Behaviour*
$0 \leq a \leq 23.4$	Hyperchaotic	$a = 23.5, 23.7$	Periodic
$23.8 \leq a \leq 24.8$	Hyperchaotic	$24.9 \leq a \leq 30$	Chaotic

4.4.4 Fix Parameters r = 45.6, d = 33, a = 4, c = 11, p = –4 and Vary Parameter b

LS of (1) with a variation of parameter is plotted in Fig. 7 (a, b). It is observed from Fig. 7 that the system produces hyperchaotic, periodic, and chaotic orbits. Table 1 (d) shows the summary of dynamical behaviour of the system (1) with variation of parameter.

Table 2 (d)

Dynamical performance of the system (1) with variation of parameter *b*

Range of parameter	*Behaviour*	*Range of parameter*	*Behaviour*
$b \leq 0.3$	Periodic	$14.9 \leq b \leq 16.7$	Chaotic
$0.3 \leq b \leq 0.6$	Chaotic	$16.7 \leq b \leq 17$	Periodic
$0.6 \leq b \leq 0.8$	Periodic	$17.1 \leq b \leq 18.5$	Chaotic
$0.9 \leq b \leq 13.9$	Hyperchaotic	$18.6 \leq b \leq 24$	Many periodic; but may produce chaos
$14 \leq b \leq 14.8$	Periodic		

4.4.5 Fix Parameters r = 46.6, d = 33, a = 12, b = 6, p = –4 and Vary Parameter c

Fig. 8 (a, b) shows the LS of the system (1) with variation of parameter. It is apparent from Fig. 8 that the system many dynamic behaviour. Table 1 (e) shows the summary of dynamical behaviour of the system (1) with variation of parameter.

Table 2 (e)

Dynamical performance of the system (1) with variation of parameter *c*

Range of parameter	*Behaviour*	*Range of parameter*	*Behaviour*
$0 < c \leq 0.2$	Chaotic	$18.2 \leq c \leq 19.2$	Hyperchaotic
$0.3 \leq c \leq 17.5$	Hyperchaotic	$19.3 \leq c \leq 19.7$	Chaotic
$c = 0.4, 0.6$	Chaotic	$19.8 \leq c \leq 20.7$	Hyperchaotic
$17.6 \leq c \leq 18.1$	Chaotic	$20.8 \leq c \leq 30$	Chaotic

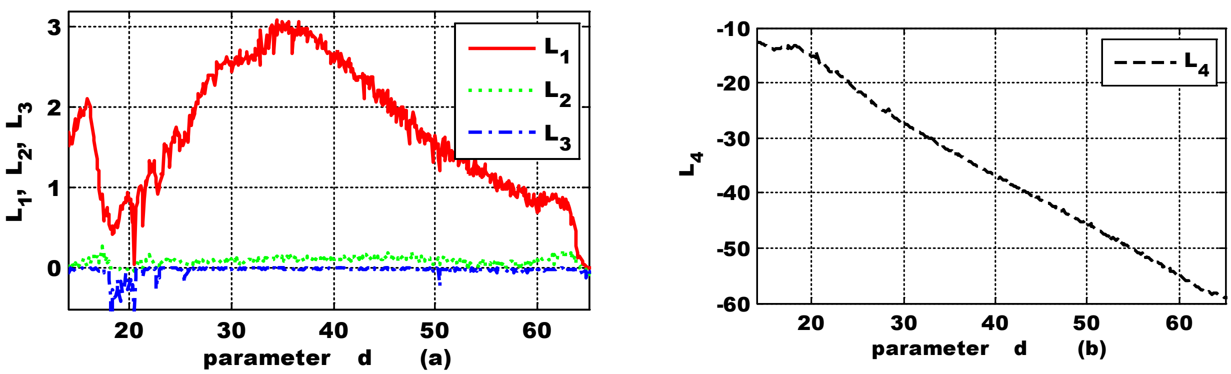


Figure 4: Lyapunov spectrum of the system (1) with $a = 12, r = 46.6, b = 6, c = 11$ for $d \in [14, 65]$ (a) Lyapunov exponents L_1, L_2, L_3 (b) Lyapunov exponents L_4.

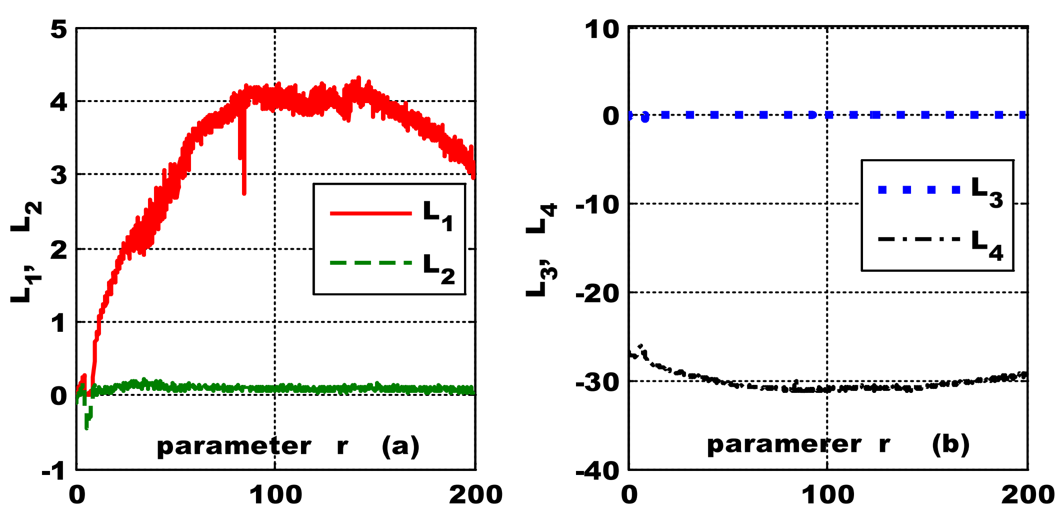


Figure 5: Lyapunov spectrum of the system (1) with $a = 12, d = 33, b = 10, c = 11, p = -4$ for $r \in [-1, 200]$ (a) Lyapunov exponents L_1, L_2 (b) Lyapunov exponents L_4.

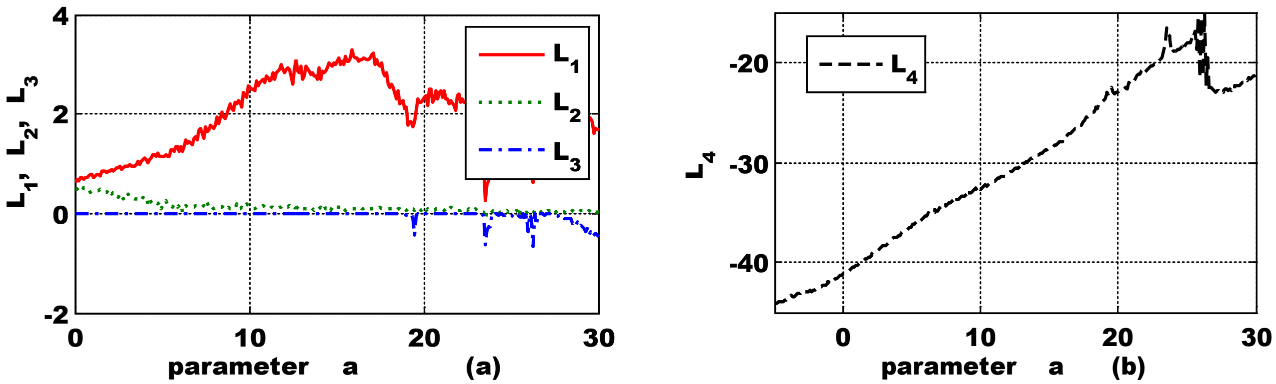


Figure 6: Lyapunov spectrum of the system (1) with $d = 33, r = 46.6, b = 6, c = 11, p = -4$ for $a \in [0, 30]$; (a) Lyapunov exponents L_1, L_2, L_3 and (b) Lyapunov exponent L_4.

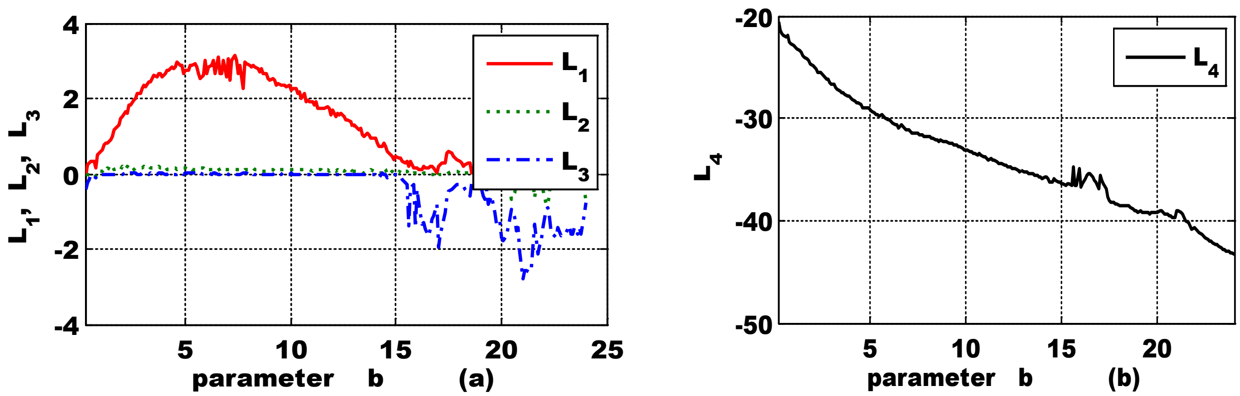


Figure 7: Lyapunov spectrum of the system (1) with $d = 33, r = 46.6, c = 11, a = 12, p = -4$ for $b \in [0, 25]$; (a) Lyapunov exponents L_1, L_2, L_3 and (b) Lyapunov exponent L_4.

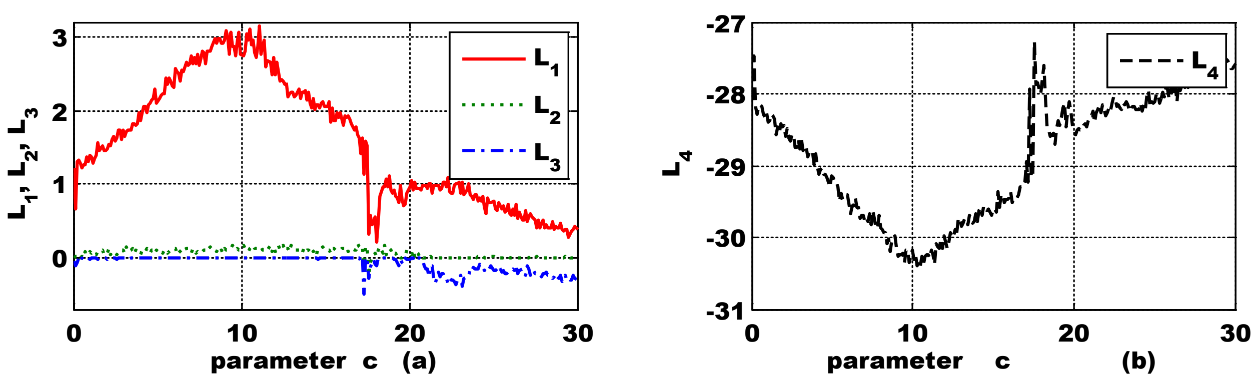


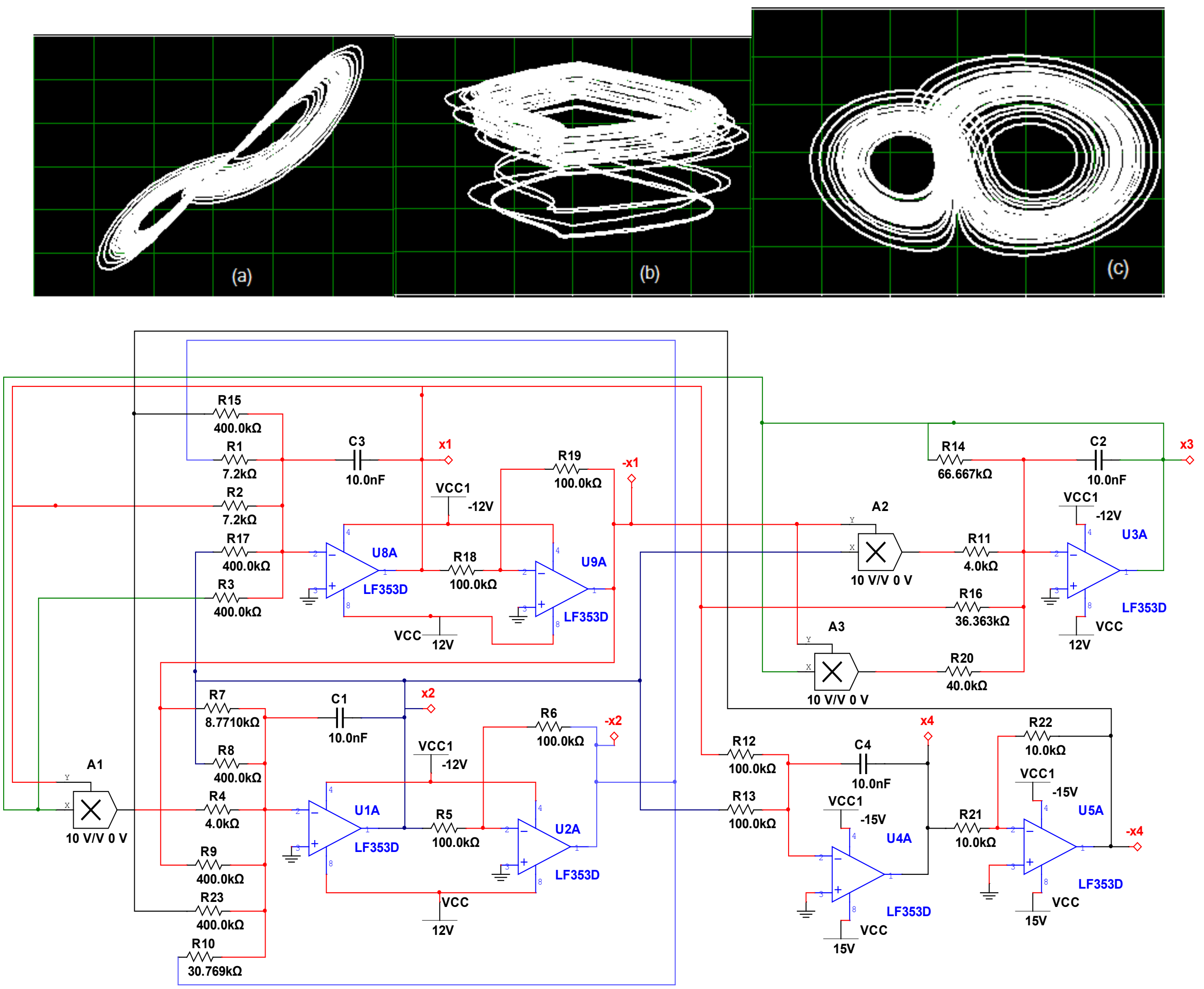
Figure 8: Lyapunov Spectrum of the system (1) with $d = 33, r = 46.6, b = 6, a = 12$ for $c \in [0, 30]$; (a) Lyapunov exponents L_1, L_2, L_3 (b) Lyapunov exponent L_4.

5. CIRCUIT VALIDATION OF THE PROPOSED SYSTEM

Circuit design for implementation of the system (1) is shown in Fig. 9 (a-d). Three multipliers, many resistors, capacitors, seven amplifiers are used to design the circuit. Resistors, capacitors values are shown in the circuit itself. Attractor plot obtained from circuit simulation is shown in Fig. 9 (a-c) and matches with the MATALB simulation results.

6. CONCLUSIONS

A novel hyperchaotic system is reported in this paper. The system has only one real equilibrium point. Results of different analyses confirm that the new hyperchaotic system is more complex and more hyperchaotic than many other reported hyperchaotic systems. Further, it has more bandwidth. The proposed new system may be more suitable for secure communication. Hardware of the ircuit for implementation is kept reserved as future work.



(d)

Figure 9 Attractor plot form circuit simulation of system (1) (a) across $x_1 - x_2$ plane, (b) across $x_2 - x_4$ plane, (c) across $x_2 - x_3$ plane (d) Circuit design.

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