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SOME SYMMETRIC PROPERTIES OF KENMOTSU MANIFOLD ADMITTING A QUARTER SYMMETRIC METRIC CONNECTION

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Abstract

The objective of the present paper is to study weakly ϕ-symmetric, weakly ϕ -Ricci symmetric and weakly ϕ-concircular Ricci symmetric Kenmotsu manifold admitting quarter symmetric metric connection and we obtained interesting results.

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Keywords: *Weakly ϕ-symmetric, weakly ϕ-Ricci symmetric, weakly ϕ-concircular Ricci symmetric, Kenmotsu manifold, η-Einstein manifold, quarter symmetric metric connection.*

1. INTRODUCTION

In 1972, Kenmotsu [18] introduced and studied a class of contact Riemannian manifolds satisfying some special conditions. Later, we call that manifold as Kenmotsu manifold. Several geometers such as Pitis [25], Binh et al. [4], De and Pathak [10], Ozgur [22], Ozgur and De [23] have studied several properties of Kenmotsu manifold and obtained interesting results. The notion of metric connection with torsion tensor on a Riemannian manifold was introduced by Hayden [14]. Later, Golab [13] defined a quarter symmetric connection on an n-dimensional Riemannian manifold M^n as a linear connection $\overline{\nabla}$ such that its torsion T satisfies

$$T(X,Y) = \overline{\nabla}_X Y - \overline{\nabla}_Y X - [X,Y] \text{ or } T(X,Y) = \eta(Y)\phi(X) - \eta(X)\phi(Y), \quad (1.1)$$

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where, η is a 1-form and ϕ is a (1, 1) tensor field. Further a quarter symmetric connection $\overline{\nabla}$ is said to be a metric connection relative to Riemannian metric g of M if

$$(\overline{\nabla}_X g)(X, Y) = 0 \tag{1.2}$$

for any vector fields $X, Y, Z \in \chi(M)$, where $\chi(M)$ is a Lie algebra of vector fields of M.

Therefore a linear connection $\overline{\nabla}$ satisfying conditions (1.1) and (1.2) is a quarter symmetric metric connection. Many geometers studied various properties of manifolds with Quarter- Symmetric metric connection. Such as Rastogi [[29],[30]], Mishra and Pandey [19], Yano and Imai [41], De and Biswas [5], Bagewadi, Prakasha and Venkatesha [1] etc. In particular if $\phi(X) = X$ and $\phi(Y) = Y$, then the connection $\overline{\nabla}$ becomes a semi-symmetric metric connection [40]. Thus the notion of quarter symmetric connection generalizes the notion of the semi-symmetric connection. Semi-symmetric metric connections have been studied by several authors like Barman [3], De [7], Ozgur and Sular [24], Ozen et al [[21], [23]], Prvanovic [27], Prvanovic and Pusic [28], Smaranda and Andonie [34], Singh and Pandey [33], Bagewadi, Prakasha and Venkatesha [2] and many others.

The weakly symmetric and weakly Ricci-symmetric manifolds were defined by Tamassy and Binh [37] and studied by several authors [[8], [11],[12], [17], [22], [31], [32]] etc. HUI [15] studied weakly ϕ-symmetric and weakly ϕ-Ricci symmetric Kenmotsu manifold. The weakly concircular Ricci symmetric manifolds was introduced by De and Ghosh [9] and these types of notions were studied with Kenmotsu structure in [16].

Definition 1.1 [37]: A non-flat Riemannian manifold M^n $(n > 2)$ is called a weakly symmetric if its curvature tensor R of type $(0, 4)$ satisfies

$$(\nabla_W R)(X, Y)Z = A(W)R(X, Y)Z + B(X)R(W, Y)Z + B(Y)R(X, W)Z$$
$$+ D(Z)R(X, Y)W + g(R(X, Y)Z, W)(\rho), \tag{1.3}$$

for all vector fields $X, Y, Z, W \in \chi(M)$. Where A, B and D are 1-forms (not simultaneously zero) and ∇ is the operator of covariant differentiation with respect to the Riemannian metric g. Also ρ is the vector field associated to the 1-form D such that $D(Z) = g(Z, \rho)$.

Definition 1.2 [38]: A non-flat Riemannian manifold M^n $(n > 2)$ is said to be weakly Ricci symmetric if its Ricci tensor S satisfies the condition

$$(\nabla_X S)(Y, Z) = A(X)S(Y, Z) + B(Y)S(X, Z) + D(Z)S(Y, X), \tag{1.4}$$

for all vector fields $X, Y, Z \in \chi(M)$, where A, B and D are 1-forms (not simultaneously zero). The equation (1.4) can also be written as

$$(\nabla_X Q)(Y) = A(X)Q(Y) + B(Y)Q(X) + S(Y, X)\rho, \tag{1.5}$$

where, ρ is a vector field associated to 1-form D such that $D(Z) = g(Z, \rho)$ and Q is the Ricci operator defined as $g(QX, Y) = S(X, Y)$

Definition 1.3 [9]: A non-flat Riemannian manifold $M^n (n > 2)$ is called weakly concircular Ricci symmetric if its concircular Ricci tensor P of type $(0, 2)$ given by

$$P(Y, Z) = \sum C(Y, ei, ei, Z) = S(Y, Z) - \frac{r}{n} g(Y, Z), \tag{1.6}$$

is not identically zero and satisfies the condition

$$(\nabla_X P)(Y, Z) = A(X)P(Y, Z) + B(Y)P(X, Z) + D(Z)P(Y, X), \tag{1.7}$$

where, A, B and D are associated 1-forms(not simultaneously zero), C is the concircular curvature tensor defined by [39]

$$C(Y, U, V, Z) = R(Y, U, V, Z) - \frac{r}{n(n-1)} [g(U, V)g(Y, Z) - g(Y, V)g(U, Z)], \tag{1.8}$$

where, r is the scalar curvature of the manifold.

Motivated by the above studies, the present paper deals with the study of some symmetric properties of Kenmotsu manifold admitting a quarter-symmetric metric connection. The paper is organized as follows: Section 2 is concerned with preliminaries. In section 3, we discuss weakly ϕ-symmetric Kenmotsu manifold with respect to quarter-symmetric metric connection and prove that the manifold is η-Einstein with respect to Levi-Civita connection. Section 4 is devoted to the study of weakly ϕ-Ricci symmetric Kenmotsu manifold with respect to quarter-symmetric metric connection and here we get an expression for Ricci tensor and scalar curvature. The last section deals with the study of weakly ϕ-concircular Ricci symmetric Kenmotsu manifold with respect to quarter-symmetric metric connection.

2. PRELIMINARIES

An almost contact metric manifold [6] is a differentiable manifold M^n endowed with a structure (ϕ, ξ, η, g), where ϕ a tensor field of type (1,1), ξ is a vector field and η is a 1-form satisfying

$$\phi^2 = -I + \eta \, o \, \xi, \eta(\xi) = 1, \tag{2.1}$$

and a Riemannian metric g such that $g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y)$ for any vector fields X and Y . The fundamental 2-form Φ is defined by $\Phi(X,Y) = g(X, \phi Y)$ for any vector fields X and Y. It is well known that contact metric manifolds are almost contact metric manifolds such that $\Phi = d\eta$. Thus a manifold M^n equipped with this structure is called an almost contact manifold and is denoted by (M^n, ϕ, ξ, η). If g is a Riemannian metric on an almost contact manifold M^n such that,

$$(\nabla_X \phi)Y = -\eta(Y)\phi X - g(X, \phi Y)\xi, \tag{2.2}$$

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.3}$$

holds, then (M^n, ϕ, ξ, η) is called a Kenmotsu manifold. Here ∇ denotes the operator of covariant differentiation with respect to g.

In a Kenmotsu manifold M^n, the following relations hold;

$$\eta(R(X,Y)Z) = [g(X,Z)\eta(Y) - g(Y,Z)\eta(X)], \tag{2.4}$$

$$\text{(a) } R(\xi, X)Y = [\eta(Y)X - g(X,Y)\xi], \text{ (b) } R(X,Y)\xi = [\eta(X)Y - \eta(Y)X], \tag{2.5}$$

$$\text{(a) } S(X,Y) = -(n-1)g(X,Y), \text{ (b) } QX = -(n-1)X, \tag{2.6}$$

$$\text{(a) } S(X,\xi) = -(n-1)\eta(X), \text{ (b) } S(\xi,\xi) = -(n-1),$$

$$\text{(c) } Q\xi = -(n-1)\xi, \tag{2.7}$$

$$(\nabla_W R)(X,Y)\xi = g(W,X)Y - g(W,Y)X - R(X,Y)W, \tag{2.8}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y). \tag{2.9}$$

In a Kenmotsu manifold M^n, the quarter-symmetric metric connection $\overline{\nabla}$ and the Levi-Civita connection ∇ are related by

$$\overline{\nabla}_X Y = \nabla_X Y - \eta(X)\phi(Y), \tag{2.10}$$

for all vector fields X, Y on M^n. Let R and $\bar{R}$ be the Riemannian curvature tensors of a Kenmotsu manifold with respect to Levi-Civita connection ∇ and quarter-symmetric metric connection $\overline{\nabla}$ respectively. Then R and $\bar{R}$ are related by [36]

$$\bar{R}(X,Y)Z = R(X,Y)Z + \eta(X)g(\phi(Y),Z)\xi - \eta(Y)g(\phi(X),Z)\xi$$
$$-\eta(X)\eta(Z)\phi(Y) + \eta(Y)\eta(Z)\phi(X) \tag{2.11}$$

Contracting (2.11), we get

$$\bar{S}(Y,Z) = S(Y,Z) + g(\phi(Y), Z), \tag{2.12}$$

where, $\bar{S}$ and S are the Ricci tensor with respect to quarter-symmetric metric connection and Levi-Civita connection respectively. Moreover, for a Kenmotsu manifold with respect to quarter-symmetric metric connection the following relations hold:

$$\bar{R}(X,Y)\xi = \eta(X)[Y-\phi(Y)] - \eta(Y)[X-\phi(X)], \tag{2.13}$$

$$\bar{R}(X,\xi)Y = [g(X,Y) - g(\phi(X),Y)]\xi - \eta(Y)[X-\phi(X)], \tag{2.14}$$

$$\bar{R}(\xi,X)\xi = X - \eta(X)\xi - \phi(X), \tag{2.15}$$

$$\bar{S}(Y,\xi) = S(X,\xi) = -(n-1)\eta(X). \tag{2.16}$$

From (2.1),(2.2),(2.8),(2.10),(2.11) and (2.13), we have

$$\begin{aligned}(\bar{\nabla}_W \bar{R})(X,Y)\xi) &= g(X,W)Y - g(Y,W)X - R(X,Y)W \\ &\quad + [\eta(Y)g(\phi(W),X) - \eta(X)g(\phi(W),Y)]\xi \\ &\quad -\eta(W)[\eta(X)Y - \eta(Y)X + \eta(X)\phi(Y) - \eta(Y)\phi(X)]. \end{aligned}\tag{2.17}$$

Again from (2.10) and (2.11), we have

$$g(\bar{\nabla}_W \bar{R}(X,Y)Z,U) = -g(\bar{\nabla}_W \bar{R}(X,Y)U,Z). \tag{2.18}$$

3. WEAKLY ϕ-SYMMETRIC KENMOTSU MANIFOLD WITH RESPECT TO QUARTER SYMMETRIC METRIC CONNECTION

Definition 3.4: A Kenmotsu manifold M^n is said to be weakly ϕ-symmetric with respect to quarter symmetric metric connection if the curvature tensor $\bar{R}$ satisfies

$$\begin{aligned}\phi^2((\bar{\nabla}_W \bar{R})(X,Y)Z) &= A(W)\phi^2(\bar{R}(X,Y)Z) + B(X)\phi^2(\bar{R}(W,Y)Z) \\ &\quad + B(Y)\phi^2(\bar{R}(X,W)Z) + D(Z)\phi^2(\bar{R}(X,Y)W) \\ &\quad + g(\bar{R}(X,Y)Z,W)\phi^2(\rho), \end{aligned}\tag{3.1}$$

for all vector fields X,Y,Z,W . Here A,B and D are 1-forms (not simultaneously zero) and in particular if $A=B=D=0$, then manifold is ϕ-symmetric with respect to quarter symmetric metric connection [26].

Now by virtue of (2.1), we have from (3.1) that

$$\begin{aligned}&-(\bar{\nabla}_W \bar{R})(X,Y)Z) + \eta((\bar{\nabla}_W \bar{R})(X,Y)Z)\xi \\ &= A(W)[-(\bar{R}(X,Y)Z) + \eta(\bar{R}(X,Y)Z)\xi] \\ &\quad + B(X)[-(\bar{R}(W,Y)Z) + \eta(\bar{R}(W,Y)Z)\xi] \\ &\quad + B(Y)[-(\bar{R}(X,W)Z) + \eta(\bar{R}(X,W)Z)\xi] \\ &\quad + D(Z)[-(\bar{R}(X,Y)W) + \eta(\bar{R}(X,Y)W)\xi] \\ &\quad + g(\bar{R}(X,Y)Z,W)[-\rho + \eta(\rho)\xi]. \end{aligned}\tag{3.2}$$

Putting $Z = \xi$ and then using (2.17) in (3.2), one can find

$$
\begin{aligned}
&[1 + D(\xi)]R(X,Y)W \\
&\quad = g(X,W)Y - g(Y,W)X + A(W)[\eta(Y)(X - \phi X) - \eta(X)(Y - \phi Y)] \\
&\quad + B(X)[\eta(Y)(W - \phi W) - \eta(W)(Y - \phi Y)] \\
&\quad + B(Y)[\eta(W)(X - \phi X) - \eta(X)(W - \phi W)] \\
&\quad + D(\xi)\big[\eta(W)(\eta(X)\phi Y - \eta(Y)\phi X) - \big(\eta(X)g(\phi Y,W) - \eta(Y)g(\phi X,W)\big)\xi\big] \\
&\quad + [\eta(Y)g(X,W) - \eta(X)g(Y,W) + \eta(X)g(\phi Y,W) - \eta(Y)g(\phi X,W)]\rho \\
&\quad - \eta(W)[\eta(X)Y - \eta(Y)X + \eta(X)\phi Y - \eta(Y)\phi X]. \qquad (3.3)
\end{aligned}
$$

Taking an orthonormal frame field and then contracting the above equation over X, we get

$$
\begin{aligned}
[1 + D(\xi)]S(Y,W) = &-[(n-1) + D(\xi)]g(Y,W) + (n-1)A(W)\eta(Y) \\
&+(n-2)B(Y)\eta(W) + (n-1)\eta(Y)\eta(W) \\
&+[B(W) + D(W) + D(\phi W)]\eta(Y) \qquad (3.4)
\end{aligned}
$$

Replacing Y by ϕY and W by ϕW in the above equation, we get

$$[1 + D(\xi)]S(\phi Y,\phi W) = -[(n-1) + D(\xi)]g(\phi Y,\phi W). \qquad (3.5)$$

By virtue of (2.1) and (2.9), we have from (3.5) that

$$S(Y,W) = ag(Y,W) + b\eta(Y)\eta(W), \qquad (3.6)$$

where, $a = -\frac{(n-1)+D(\xi)}{1+D(\xi)}$ and $b = -\frac{(n-2)D(\xi)}{1+D(\xi)}$

Therefore we can state:

Theorem 3.1: A weakly ϕ-symmetric Kenmotsu manifold with respect to quarter symmetric metric connection is η-Einstein, provided $D(\xi) \neq -1$.

Using (2.18) in (3.2), we have

$$
\begin{aligned}
(\bar{\nabla}_W \bar{R})(X,Y)Z = &-g\big((\bar{\nabla}_W \bar{R})(X,Y)\xi, Z\big)\xi + A(W)[\bar{R}(X,Y)Z) \\
&+g(\bar{R}(X,Y)\xi,Z)\xi + B(X)[(\bar{R}(W,Y)Z) + g(\bar{R}(W,Y)\xi,Z)\xi] \\
&+B(Y)[(\bar{R}(X,W)Z) + g(\bar{R}(X,W)\xi,Z)\xi] \\
&+ D(Z)[(\bar{R}(X,Y)W) + g(\bar{R}(X,Y)\xi,W)\xi] \\
&+g(\bar{R}(X,Y)Z,W)[-\rho + \eta(\rho)\xi]. \qquad (3.7)
\end{aligned}
$$

In view of (2.11) and (2.17), it follows from (3.7) that

$$
\begin{aligned}
(\bar{\nabla}_W \bar{R})(X,Y)Z = {} & [R(X,Y,Z,W) + g(X,Z)g(Y,W) - g(X,W)g(Y,Z) \\
& +\eta(X)\eta(Z)g(\phi W,Y) - \eta(Y)\eta(Z)g(\phi W,X) \\
& +(g(Y,Z) + g(\phi Y,Z))\eta(W)\eta(X) - (g(X,Z) \\
& +g(\phi X,Z))\eta(W)\eta(Y)]\xi + A(W)[R(X,Y)Z \\
& +(2\eta(X)g(\phi Y,Z) - 2\eta(Y)g(\phi X,Z) + \eta(X)g(Y,Z) \\
& -\eta(Y)g(X,Z))\xi + \eta(Y)\eta(Z)\phi X - \eta(X)\eta(Z)\phi Y] \\
& +B(X)[R(W,Y)Z + (\eta(W)g(Y,Z) - \eta(Y)g(W,Z))\xi \\
& + (\eta(Y)\phi W - \eta(W)\phi Y)\eta(Z)] + B(Y)[R(X,W)Z \\
& +(\eta(X)g(W,Z) - \eta(W)g(X,Z))\xi + (\eta(W)\phi X \\
& -\eta(X)\phi W)\eta(Z)] + D(Z)[R(X,Y)W \\
& +\big(\eta(X)g(Y,W) - \eta(Y)g(X,W)\big)\xi + (\eta(Y)\phi X \\
& -\eta(X)\phi Y)\eta(W)] + [R(X,Y,Z,W) + (\eta(X)g(\phi Y,Z) \\
& -\eta(Y)g(\phi X,Z))\eta(W) + (\eta(Y)g(\phi X,W) \\
& -\eta(X)g(\phi Y,W))\eta(Z)](-\rho + \eta(\rho)\xi).
\end{aligned} \tag{3.8}
$$

Now consider a weakly ϕ-symmetric Kenmotsu manifold with respect to Levi-civita connec-tion. Then by virtue of (2.1),(2.8) and (2.18), it follows from (1.3) that

$$
\begin{aligned}
(\nabla_W R)(X,Y)Z = {} & -[g(X,W)g(Y,Z) - g(Y,W)g(X,Z) - g(R(X,Y)W,Z)]\xi \\
& + A(W)[R(X,Y)Z + (\eta(X)g(Y,Z) - \eta(Y)g(X,Z))\xi] \\
& + B(X)[R(W,Y)Z + (\eta(W)g(Y,Z) - \eta(Y)g(W,Z))\xi] \\
& + B(Y)\big[R(X,W)Z + \big(\eta(X)g(W,Z) - \eta(W)g(X,Z)\big)\xi\big] \\
& + D(Z)[R(X,Y)W + (\eta(X)g(Y,W) - \eta(Y)g(X,W))\xi] \\
& + R(X,Y,Z,W)[-\rho + \eta(\rho)\xi].
\end{aligned} \tag{3.9}
$$

From (3.8) and (3.9), we can state the following

Theorem 3.2: A weakly ϕ-symmetric Kenmotsu manifold M^n is invariant under quarter symmetric metric connection if and only if the relation

$$\big[\big(\eta(X)g(\phi W,Y)-\eta(Y)g(\phi W,X)\big)\eta(Z)+\big(g(Y,Z)+g(\phi Y,Z)\big)\eta(W)\eta(X)\big]\xi$$
$$+A(W)[(\eta(Y)\phi X-\eta(X)\phi Y)\eta(Z)]+B(X)[(\eta(Y)\phi W-\eta(W)\phi Y)\eta(Z)]$$
$$+B(Y)[(\eta(W)\phi X-\eta(X)\phi W)\eta(Z)]+D(Z)[(\eta(Y)\phi X-\eta(X)\phi Y)\eta(W)]$$
$$+[(\eta(X)g(\phi Y,Z)-\eta(Y)g(\phi X,Z))\eta(W)+(\eta(Y)g(\phi X,W)$$
$$-\eta(X)g(\phi Y,W))\eta(Z)](-\rho+\eta(\rho)\xi)=0, \tag{3.10}$$

holds for arbitrary vector fields X, Y, Z and W.

4. WEAKLY ϕ -RICCI SYMMETRIC KENMOTSU MANIFOLD WITH RESPECT TO QUARTER SYMMETRIC METRIC CONNECTION

Definition 4.5: A Kenmotsu manifold M^n is said to be weakly ϕ-Ricci symmetric with respect to quarter symmetric metric connection if there are non-zero 1-forms A, B and D such that

$$\phi^2((\bar{\nabla}_W\bar{Q})(Y))=A(X)\phi^2(\bar{Q}(Y))+B(Y)\phi^2(\bar{Q}(X))+\bar{S}(Y,X)\phi^2(\rho) \tag{4.1}$$

If $A=B=D=0$, then the manifold becomes ϕ-Ricci symmetric [35].

Suppose a Kenmotsu manifold M^n is weakly ϕ-Ricci symmetric with respect to quarter symmetric metric connection. Then by virtue of (2.1), it follows from (4.1) that

$$-(\bar{\nabla}_X\bar{Q})(Y)+\eta((\bar{\nabla}_X\bar{Q})(Y))\xi=A(X)[-\bar{Q}(Y)+\eta(\bar{Q}Y)\xi]$$
$$+B(Y)[-\bar{Q}(X)+\eta(\bar{Q}X)\xi]+\bar{S}(Y,X)[-\rho+\eta(\rho)\xi]. \tag{4.2}$$

From (4.2), it follows that

$$-g((\bar{\nabla}_X\bar{Q})(Y),Z)+\bar{S}(\bar{\nabla}_XY,Z)+\eta((\bar{\nabla}_X\bar{Q})(Y))\eta(Z)$$
$$=A(X)[-\bar{S}(Y,Z)+\bar{S}(Y,\xi)\eta(Z)]+B(Y)[-\bar{S}(X,Z)+\bar{S}(X,\xi)\eta(Z)]$$
$$+\bar{S}(Y,X)[-D(Z)+\eta(\rho)\eta(Z)]. \tag{4.3}$$

Put $Y=\xi$ in (4.3). Then by using (2.12) and (2.16), we get

$$[1+B(\xi)]S(X,Z)=-(n-1)g(X,Z)-[1+B(\xi)]g(\phi X,Z)$$
$$+(n-1)[B(\xi)-\eta(\rho)]\eta(X)\eta(Z)+(n-1)\eta(X)D(Z). \tag{4.4}$$

Contracting (4.4) over X and Z, we get

$$[1+B(\xi)]r=(n-1)[B(\xi)-n]. \tag{4.5}$$

This leads to the following:

Theorem 4.3: In a weakly ϕ-Ricci symmetric Kenmotsu manifold with respect to quarter symmetric metric connection, the Ricci tensor and the scalar curvature are respectively given by (4.4) and (4.5).

5. WEAKLY ϕ-CONCIRCULAR RICCI SYMMETRIC KENMOTSU MANIFOLD WITH RESPECT TO QUARTER SYMMETRIC METRIC CONNECTION

Definition 5.6: A Kenmotsu manifold M^n is said to be weakly ϕ-concircular Ricci symmet-ric with respect to quarter symmetric metric connection if its concircular Ricci tensor $\bar{P}$ of type (0,2) given by

$$\bar{P}(Y,Z) = \sum \bar{C}(Y,ei,ei,Z) = \bar{S}(Y,Z) - \frac{\bar{r}}{n} g(Y,Z), \tag{5.1}$$

is not identically zero and satisfies the condition

$$\phi^2((\bar{\nabla}_X \bar{P})(Y,Z)) = A(X)\phi^2(\bar{P}(Y,Z)) + B(Y)\phi^2(\bar{P}(X,Z)) + D(Z)\phi^2(\bar{P}(Y,X)), \tag{5.2}$$

where, $\bar{C}$ denotes concircular curvature tensor with respect to the connection $\bar{\nabla}$.

In view of (5.1), (5.2) can also be written as

$$\phi^2\big((\bar{\nabla}_X \bar{Q})(Y)\big) - \frac{d\bar{r}(X)}{n}\phi^2(Y) = A(X)[\phi^2\big(\bar{Q}(Y)\big) - \frac{\bar{r}}{n}\phi^2(Y)] + B(Y)[\phi^2\big(\bar{Q}(X)\big) - \frac{\bar{r}}{n}\phi^2(X)] + \phi^2(\rho)[\bar{S}(X,Y) - \frac{\bar{r}}{n} g(X,Y)] \tag{5.3}$$

By using (2.1) in (5.3), we get

$$-(\bar{\nabla}_X \bar{Q})(Y) + \eta\big((\bar{\nabla}_X \bar{Q})(Y)\big)\xi - \frac{d\bar{r}(X)}{n}[-Y + \eta(Y)\xi] = A(X)[-\bar{Q}(Y) + \eta\big(\bar{Q}(Y)\big)\xi - \frac{\bar{r}}{n}(-Y + \eta(Y)\xi)] + B(Y))[-\bar{Q}(X) + \eta\big(\bar{Q}(X)\big)\xi - \frac{\bar{r}}{n}(-X + \eta(X)\xi)] + (-\rho + \eta(\rho)\xi)[\bar{S}(X,Y) - \frac{\bar{r}}{n} g(X,Y)] \tag{5.4}$$

From (5.4), it follows that

$$-g(\bar{\nabla}_X \bar{Q})(Y),Z) + \bar{S}(\bar{\nabla}_X Y,Z) + \eta\big((\bar{\nabla}_X \bar{Q})(Y)\big)\eta(Z) - \frac{d\bar{r}(X)}{n}[-g(Y,Z) + \eta(Y)\eta(Z)] = A(X)[-\bar{S}(Y,Z) + \bar{S}(Y,\xi)\eta(Z) - \frac{\bar{r}}{n}(-g(Y,Z) + \eta(Y)\eta(Z))] + B(Y))[-\bar{S}(X,Z) + \bar{S}(X,\xi)\eta(Z) - \frac{\bar{r}}{n}(-g(X,Z) + \eta(X)\eta(Z))] + (-D(Z) + \eta(\rho)\eta(Z))[\bar{S}(X,Y) - \frac{\bar{r}}{n} g(X,Y)] \tag{5.5}$$

Putting $Y = \xi$ in (5.5) and then using (2.3), (2.7), (2.10) and (2.12), we obtain

$$[1-B(\xi)]S(X,Z)=\left[\frac{\bar{r}}{n}-(n-1)\right]g(X,Z)+[B(\xi)-1]g(\phi X,Z)$$
$$-[D(\xi)+B(\xi)]\left(n-1+\frac{\bar{r}}{n}\right)\eta(X)\eta(Z)-2(n-1)A(X)\eta(Z)$$
$$+\left(n-1+\frac{\bar{r}}{n}\right)D(Z)\eta(X). \quad (5.6)$$

Thus we have the following:

Theorem 5.4: In a weakly ϕ-concircular Ricci symmetric Kenmotsu manifold with respect to quarter symmetric metric connection, the Ricci tensor is of the form (5.6).

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