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SPOT AND CONTRACT MARKETS FOR ELECTRICITY IN BRAZIL

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Abstract: *In this paper we model the interaction between a forward and a spot market whose features are borrowed from the Brazilian electricity market. The spot market is modeled as a random mechanism that yields spot prices of electricity. The forward contract market is comprised of a large number of price-taking suppliers and consumers. We obtain the equilibrium price and quantities of the forward market and discuss how they depend on market participants' estimates of the expected value and variability of the spot price. We also discuss the implications of our results for the Brazilian electricity market.*

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INTRODUCTION

In 2001, the Brazilian economy was rocked by an energy crisis that forced the federal government to take drastic measures to curtail consumption. As a result GDP growth came to a halt, averaging less than 2% in 2001-02. One of the consequences of the crisis was a redesign of the Brazilian electricity system, which led to the institution of the "New Electricity Sector Model" in 2004. One of the most interesting features of the new model is the coexistence of two contract markets where electricity can be traded.

The first contract market, called the Regulated Contracting Environment (RCE)[1], brings together electric utilities, independent power producers, self-generators and power marketers, on the supply side, and distribution companies, who are required to contract their entire forecast demand for captive consumers, on the demand side. Contracts with different delivery dates are auctioned off over time for "new" and "existing" electricity[2]. When the actual demand faced by distribution companies differs from their forecasts, they can purchase additional electricity in annual "adjustment" auctions.

The second contract market is called the Free Contracting Environment (FCE)[3], also known as the "free market" in Brazilian electricity sector parlance. Consumers whose load is greater than or equal to 3MW, typically industrial and commercial

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firms, may opt out of the RCE and instead enter bilateral contracts with generators or other suppliers of electricity in the FCE. They are called free consumers (as opposed to captive consumers) because they do not need to buy power from distribution companies. The contracts signed in the FCE stipulate the price per MWh, the quantity of electricity to be delivered by the seller (in average MW), the submarket[4] where it is to be delivered, the duration of the contract, and possibly other features agreed upon by the parties.

In addition to those two contract markets, there is a "market for differences" where discrepancies between energy contracted and energy effectively produced or consumed by market participants are settled. No purchases or sales of electricity take place in this market. As we will explain later, however, the price used in this market to settle the differences plays an important role in the free contract market.

It is our purpose in this paper to study the Brazilian free contract market and its interaction with the market for differences. We will only be interested in the regulated contract market (RCE) insofar as it places constraints on the amount of energy generators can supply in the free market. To the best of our knowledge, this is the first paper in the literature that models this segment of the Brazilian electricity sector. Our model is also of interest in its own right, for it sheds some light on the interaction between a forward market (the free contract market) and a spot market (the market for differences), when the latter is not a marketplace but a settlement environment based on a random price mechanism.

The paper is organized as follows. Section 2 provides a brief literature review, while section 3 describes the main features of the Brazilian electricity market and how we model them. Section 4 presents the main findings and a discussion of the implications for the Brazilian market. Section 5 concludes and the Appendix presents the main proofs.

LITERATURE REVIEW

There is an abundance of papers in the literature that study the interaction between spot and forward markets. They go from theoretical papers that investigate the general features of this interaction to applications of the theoretical findings to different product markets like that of electricity.

The seminal paper in the theoretical literature is Allaz and Vila (1993). They develop a general model to show that forward markets can emerge even in the absence of uncertainty. More precisely, they solve a two-period game where (duopoly) producers first buy or sell forward (binding and observable) contracts and then, in the second period, play a Cournot game in quantities in a spot market. They show that a single producer has strong incentives to sell forward part of his production, but that to trade in the forward market is a prisoners' dilemma for the two producers, since they end up worse off if they both decide to trade forward.

They also show that Cournot spot markets with forward markets approach efficiency as the number of trading periods goes to infinity.

Later papers challenge the results obtained by Allaz and Vila (1993), arguing that they rely heavily on their assumptions. Mahenc and Salanié (2004), for instance, work with price-setting (instead of Cournot) duopolists in a differentiated products model, and show that in this case producers buy forward their own production. The resulting equilibrium prices are higher than those that would be observed without forward trading. Green and Le Coq (2010) are interested in a different issue, namely the possibility of collusion in a repeated price-setting game. Their conclusion is that, regardless of their discount factor, firms can always achieve some collusive price above marginal cost if they sell the right number of contracts. Nevertheless, collusion is more difficult to sustain the longer the duration of the contract.

The seminal paper in the literature on electricity markets is Green and Newbery (1992). They apply the concept of supply function equilibrium developed by Klemperer and Meyer (1989) to electricity markets, in the sense that generators submit to the spot market a supply schedule of prices for generation and receive the market-clearing price. They show that the Nash equilibrium in supply schedules yields a high markup on marginal cost and substantial deadweight losses, and use their findings to explain the early outcomes observed in the British electricity spot market.

Another important paper is Powell (1993), whose model of the contract market in Britain features financial contracts known as "contracts for differences" (CfDs). Demand for electricity comes mostly from distribution companies with mean-variance utility. Generators are price setters in the contract market and quantity setters in the spot market. His main conclusions are the following: When generators are non-cooperative in both markets, the competitive result (marginal cost pricing and contract price equal to expected spot price) may emerge; when generators collude in both markets, spot prices are above marginal costs, future prices are above expected spot prices, and hedging is only partial; when generators collude only in the contract market, hedging may be lower still (when risk aversion is sufficiently low).

Green (1999) improves upon the earlier literature by combining some of its modeling tools. He models the electricity market in the UK as a two-stage game of a spot market and a hedging contract (CfDs) market, just like Powell (1993), but the strategies available to generators in the spot market are of a different sort. They simultaneously submit supply functions[5] and the Pool (market operator) considers bids in ascending order. The contract price is determined by an arbitrage condition, which states that it must equal the expected spot price, given the number of contracts sold. The main conclusions of the paper are: (a) A firm with "Bertrand" conjectures will cover all of its expected output in the contract market and will sell at marginal costs in both markets; (b) A firm with "Cournot" conjectures will sell no contracts in equilibrium (in the linear model case); more generally, a risk-neutral firm will not want to use the contract market unless this will affect its rivals' strategies; (c) Generators may cover most of their output in the contract market and still raise prices above their marginal costs; (d) If buyers are risk averse, the contract price may exceed the expected spot price, increasing the generators' incentive to sell in the contract market.

Other contributions to the study of the UK electricity market are von der Fehr and Harbord (1993) and Wolfram (1998). More recent contributions to the literature are Bushnell (2007) (US market), Ciarreta and Espinosa (2010) (Spanish market), and Adilov (2010).

The literature on the Brazilian electricity market is not very well developed. Most papers are written in Portuguese and either provide historical accounts of the Brazilian electricity sector or describe the workings of the current system. There are exceptions, though. Dutra and Menezes (2005), for instance, study the properties and outcomes of the auctions carried out in the regulated part of the Brazilian contract market. And Wolak (2008) discusses a proposal for short-term price determination in the wholesale market.

THE MODEL OF THE BRAZILIAN MARKET

As we mentioned before, we are mostly interested in the segment of the Brazilian electricity market called the free market (FCE), for it is there that forward contracts are traded. The free market has been growing steadily in the past few years. It accounted for approximately 28% of total consumption in the Brazilian electricity system in 2011 (ABRACEEL (2011)).

As explained earlier, the free market is closely linked to the regulated contract market (RCE), so our model needs to take that into account. We first point out that free consumers do not buy in the regulated market, and so we don't need to model any interaction between the two markets as far as demand is concerned. As for the supply side, we make the assumption that generators sell first in the regulated and then in the free market. In other words, they take their commitments in the regulated market as given when making decisions in the free market. This assumption makes sense, since generators submit bids in auctions carried out within the RCE and then enter long term contracts with distribution companies if their bids are successful.

An important feature of the Brazilian model is that generators do not decide or bid the amount of energy to produce. They are required to generate as much electricity as determined by the system operator. It is therefore likely that there will be differences between the energy contracted and the energy effectively produced by generators. These differences, as well as those between actual and contracted energy consumption, are liquidated at the so-called "Liquidation of Differences Price." There are no actual short-term energy trades in this "market for differences," but we call it a spot market nonetheless because of its resemblance to other spot markets around the world and because that's how it's called in Brazil.

The spot price is computed weekly (by submarket) and is based on the marginal operational cost of the system, with lower and upper bounds set by the regulator. Since the Brazilian system is preponderantly hydroelectric, the spot price is computed by a stochastic dynamic programming algorithm that seeks to find the optimal balance between using water today and storing it for future use. To use as much water as possible today to produce electricity is the best short term solution, but that would increase the likelihood of electricity shortfalls in the future. On the other hand, to conserve water today by keeping reservoirs full is the most reliable solution, but it requires higher fossil-fuel generation and, thus, higher electricity costs and prices.

Inputs to the algorithm include several stochastic variables such as forecast demand, level of water reservoirs, precipitation, evaporation, and other uses of water (irrigation, water supply etc.)[6] Accordingly, we model the spot market as a mechanism yielding a random spot price p that follows a normal distribution. Each market participant, supplier or consumer, might have a different estimate of the expected value and standard deviation of the spot price.

There are two periods in our model. In period 0, a competitive forward contract market with n electricity suppliers (indexed by k) and m consumers (indexed by i) opens. This forward market is a representation of the Brazilian free market (FCE). In period 1, differences between observed and contracted quantities of electricity are settled at the spot price and forward contracts are settled at the forward equilibrium price.

Suppliers and consumers are risk averse and have negative exponential utility functions given by $u(\pi) = -e^{-a\pi}$, where π is profit.

Consumer i wants to maximize her profit

$$\pi_i^c = Z_i - p\left(R_i - y_i^c\right) - qy_i^c \tag{1}$$

where Z_i is the profit she earns in her product market (keep in mind that a free consumer in the forward electricity market is a producer/seller in her product market), R_i is the given amount of electricity she needs to buy in either the forward or spot market, q is the forward price, and y_i^c is the quantity she buys forward. We assume that the consumer's profit in her product market is proportional to her consumption of electricity, i.e. $Z_i = \alpha_i R_i$.

It is well known that maximization of expected utility when the utility function is negative exponential boils down to maximizing the certainty equivalent measure. Therefore, the problem consumer *i* faces is to

$$\max E\left(\pi_i^c\right) - \frac{a_i^c}{2} Var\left(\pi_i^c\right) \tag{2}$$

where a_i^c is her coefficient of risk aversion.

Since R_i is given, we can easily compute

$$\begin{aligned} E\left(\pi_i^c\right) &= \alpha_i R_i - \overline{p}_i^c (R_i - y_i^c) - q y_i^c \\ Var\left(\pi_i^c\right) &= Var\left(\alpha_i R_i - p(R_i - y_i^c) - q y_i^c\right) = \left(R_i - y_i^c\right)^2 \left(\sigma_i^c\right)^2 \end{aligned} \tag{3}$$

where $\overline{p}_i^c$ and σ_i^c are consumer *i*'s expected value and standard deviation of the spot price, respectively.

The solution to problem (2) can now be calculated:

$$y_i^c = \frac{\overline{p}_i^c - q}{a_i^c \left(\sigma_i^c\right)^2} + R_i = \left(R_i + \frac{\overline{p}_i^c}{a_i^c \left(\sigma_i^c\right)^2}\right) - \frac{q}{a_i^c \left(\sigma_i^c\right)^2} = A_i - B_i q\,, \tag{4}$$

where $A_i = R_i + \overline{p}_i^c \Big/ \left(a_i^c \left(\sigma_i^c\right)^2\right)$ and $B_i = 1 \Big/ \left(a_i^c \left(\sigma_i^c\right)^2\right)$. Notice that $A_i > 0$ and $B_i > 0$, as required of a demand function.

Now let's look at the supplier's decision. His profit function is given by:

$$\pi_k^g = p\left(F_k - y_k^g\right) + q y_k^g - v_k F_k \tag{5}$$

where y_k^g is his quantity of output sold forward, F_k is his actual electricity output net of his sales in the regulated market, and v_k is his constant marginal (and average) cost of producing electricity.

We make the simplifying assumption that suppliers can perfectly forecast the amount of power they will be required to generate by the system operator. This implies that F_k is a given quantity to supplier *k*.

The supplier's problem can now be expressed in terms of the certainty equivalent measure:

$$\max \overline{p}_k^g \left(F_k - y_k^g\right) + q y_k^g - v_k F_k - \frac{a_k^g}{2} Var\left(p\left(F_k - y_k^g\right) + q y_k^g - v_k F_k\right), \tag{6}$$

where a_k^g and $\overline{p}_k^g$ are supplier k's coefficient of risk aversion and expected spot price, respectively.

The solution to this problem is

$$y_k^g = \frac{q - \overline{p}_k^g}{a_k^g \left(\sigma_k^g\right)^2} + F_k = \left(F_k - \frac{\overline{p}_k^g}{a_k^g \left(\sigma_k^g\right)^2} \right) + \frac{q}{a_k^g \left(\sigma_k^g\right)^2} = C_k + D_k q, \tag{7}$$

where σ_k^g is supplier k's estimate of the standard deviation of the spot price, $C_k = F_k - \overline{p}_k^g \Big/ \left(a_k^g \left(\sigma_k^g\right)^2 \right)$ and $D_k = 1 \Big/ \left(a_k^g \left(\sigma_k^g\right)^2 \right)$. It can be easily seen that $D_k > 0$, as required of a supply function.

An important point to make is that even though, in theory, it is possible for suppliers to sell short ($y_k^g > F_k$) and for consumers to go long ($y_i^c > R_i$), this is not something they will want to do in the Brazilian free electricity market. Since there are no actual electricity trades in the Brazilian spot market, a supplier will not be able to purchase electricity in the spot market to honor his short sales. Similarly, a consumer cannot profit from sales of surplus electricity in the spot market. In what follows, we will first present the results of our analysis ignoring this constraint, and then talk about the implications for the Brazilian market.

FINDINGS

Given the demand functions (4) and the supply functions (7), we look for the equilibrium of a competitive forward market. We model the forward market as competitive because it is a good approximation to a Brazilian market where there are very large numbers of suppliers and consumers (there were 1527 consumers and 507 generators in the Brazilian market as of October 2012[7]). Accordingly, suppliers and consumers are price-takers in this market. Our first proposition below describes the equilibrium.

Proposition 1: The equilibrium price and quantities in a competitive forward market are the following:

$$\begin{aligned} q^* &= \left(\sum_{k=1}^{n} \overline{p}_k^g D_k + \sum_{i=1}^{m} \overline{p}_i^c B_i \right) \Big/ \left(\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i \right) \\ y_i^c &= R_i + B_i \left(\overline{p}_i^c - q^* \right) \\ y_k^g &= F_k - D_k \left(\overline{p}_k^g - q^* \right) \end{aligned} \tag{8}$$

In particular, if suppliers and consumers share the same expected value of the spot price ($\overline{p}_k^g = \overline{p}_i^c = \overline{p}\ \forall k = 1,\ldots,n\ \forall i = 1,\ldots,m$)*, then* $q^* = \overline{p}, y_k^g = F_k$ *and* $y_i^c = R_i$.

Proof: See Appendix.

Notice that the equilibrium forward price q^* is equal to the weighted average of the expected spot prices of suppliers and consumers, where the weight of each market participant (supplier or consumer) is equal to the reciprocal of the product of that participant's coefficient of risk aversion and variance of the spot price. It comes as no surprise then that the equilibrium forward price would be equal to the expected spot price if suppliers and consumers shared the same expected spot price. Notice that in this case the power traded forward by a consumer would equal her actual consumption, and the power traded forward by a supplier would equal his actual (net) supply of electricity. In other words, there would be full hedging.

When expected spot prices vary by market participant, the amount of hedging depends on the difference between that participant's expected spot price and the weighted average of all expected spot prices. More precisely:

$$y_i^c \begin{cases} \leq R_i \text{ if } \overline{p}_i^c \leq q^* \\ \geq R_i \text{ if } \overline{p}_i^c \geq q^* \end{cases}$$

$$y_k^g \begin{cases} \leq F_k \text{ if } \overline{p}_k^g \geq q^* \\ \geq F_k \text{ if } \overline{p}_k^g \leq q^* \end{cases}$$

We can see that a supplier will sell forward part of his (net) supply of electricity (partial hedging) if his expected spot price is greater than the (weighted) average expected spot price, and will short sell power in hopes of buying it cheaper in the spot market for delivery if his expected spot price is lower than the average. A consumer, on the other hand, will buy forward less than her actual consumption (partial hedging) if her expected spot price is less than the average expected spot price, and will go long when her expected spot price is higher than the average.

The next proposition tells us the results of some elementary comparative statics exercises.

Proposition 2:

(i) If the expected spot price of a supplier is greater than the average expected spot price, then an increase in the supplier's coefficient of risk aversion or estimated standard deviation of the spot price leads to an increase in the quantity of electricity he sells in the forward market, and vice-versa.

(ii) If the expected spot price of a supplier is smaller than the average expected spot price, then an increase in the supplier's coefficient of risk aversion or estimated standard deviation of the spot price leads to a decrease in the quantity of electricity he sells in the forward market, and vice-versa.

(iii) If the expected spot price of a consumer is greater than the average expected spot price, then an increase in the consumer's coefficient of risk aversion or estimated standard deviation of the spot price leads to a decrease in the quantity of electricity she buys in the forward market, and vice-versa.

(iv) If the expected spot price of a consumer is smaller than the average expected spot price, then an increase in the consumer's coefficient of risk aversion or estimated standard deviation of the spot price leads to an increase in the quantity of electricity she buys in the forward market, and vice-versa.

Proof: See Appendix.

This proposition yields some interesting results. First, recall that when a supplier expects the spot price to be greater than the forward price there is partial hedging, that is, he sells less than his total (net) supply in the forward market. In this case, his sales of electricity in the forward market increase with his degree of risk aversion or estimate of spot price variability. This is the standard result that the amount of hedging is proportional to factors that increase the risk of being exposed to the spot market or make the agent more risk-averse. Analogously, when a consumer expects the spot price to be smaller than the forward price, she will hedge more when she is more risk averse or when her risk of exposure increases.

Results (ii) and (iii), however, seem to be counterintuitive, for suppliers sell less and consumers buy less in the forward market as their risk aversion or estimated spot market variability increase. Let's look at the supply side first. Recall that when suppliers expect the spot price to be smaller than the forward price, they sell short. This is due to the profit motive, that is, they expect to be able to buy energy in the spot market and make a profit per MWh equal to the difference between the spot price and the forward price. But suppliers cannot be sure of what the spot price is actually going to be, and so when they become more risk averse or their perception of the variability of the spot price increases, they reduce their short sales.

Similarly, we can make sense of the behavior of consumers if we recall that when consumers expect the spot price to be greater than the forward price, they go long. When they become more risk averse or estimate a higher variability of the spot price, however, the impact of the profit motive lessens and they reduce their purchases in the forward market.

Another analysis worth pursuing is that of the amount of aggregate hedging. Notice that

$$\sum_{i=1}^{m} y_i^c = \sum_{i=1}^{m}\left(R_i + B_i\left(\overline{p}_i^c - q^*\right)\right) = \sum_{i=1}^{m} R_i + \sum_{i=1}^{m} B_i\left(\overline{p}_i^c - q^*\right)$$
$$\sum_{k=1}^{n} y_k^g = \sum_{k=1}^{n}\left(F_k - D_k\left(\overline{p}_k^g - q^*\right)\right) = \sum_{k=1}^{n} F_k - \sum_{k=1}^{n} D_k\left(\overline{p}_k^g - q^*\right)$$

Therefore, if enough consumers expect the spot price to be smaller than the forward price, aggregate purchases of electricity in the forward market will be less than free consumers' total consumption of electricity. In particular, if consumers share the same expected spot price, i.e. $\overline{p}_i^c = \overline{p}^c$ for all i, then $\overline{p}^c < q^*$ implies $\sum_{i=1}^{m} y_i^c < \sum_{i=1}^{m} R_i$, and vice-versa.

Similarly, if enough suppliers expect the spot price to be greater than the forward price, aggregate sales of electricity in the forward market will be less than total (net) production of electricity. In particular, if suppliers share the same expected spot price, i.e. $\overline{p}_k^g = \overline{p}^g$ for all k, then $\overline{p}^g > q^*$ implies $\sum_{k=1}^{n} y_k^g < \sum_{i=k}^{n} F_k$, and vice-versa.

Let's now think about the implications of these results for the Brazilian electricity market. First, let's consider the effect of the constraint we ignored before, namely that there are no sales or purchases of electricity in the spot market. This means that suppliers will never sell short and consumers will never go long in the forward market. When a supplier, for instance, expects the spot price to be smaller than the forward price, he sells all of his electricity in the forward market, just as he would if he expected it to equal the forward price. Likewise, a consumer buys all her electricity consumption in the forward market when she expects the spot price to be greater than the forward price, just as she would if she expected them to be the same.

Second, it is clear from our results that if all free market participants receive a signal that the spot price will increase, like information that the level of hydroelectric plants' reservoirs has dropped, the forward price will also go up, other things being equal. Since the lower price of electricity in the free market relative to that of the regulated market is one of the main reasons consumers migrate from the RCE to the FCE, higher expected spot prices can prove a challenge to the growth of the free market, a concern not only for market participants but also for the Brazilian electricity regulator. In other words, when the expected spot price goes up there is an incentive for consumers who buy their electricity in the RCE to stay put, and for those who participate in the FCE to migrate back to the RCE. Given the existence of significant costs to shift between markets, an eventual later reduction of the expected spot price might not lead to an immediate return of those consumers to the FCE.

Finally, it is worth pointing out the effects of increased spot price variability on the free market. Suppose the share of electricity generation coming from fossil fuel plants increases over time, leading to more unstable spot prices. Suppliers and consumers will likely update their estimates of the standard deviation of the spot price, increasing them. As we proved earlier, the equilibrium price in the forward market is equal to the weighted average of the expected spot prices of suppliers and consumers, where the weight of each market participant is equal to the reciprocal of the product of that participant's coefficient of risk aversion and estimated variance of the spot price. Therefore, if the weights of those with a higher than average expected spot price increase relative to those of participants with a lower than average expected spot price, the forward price increases and migration from the FCE to the RCE is likely. This occurs if members of the first group increase their estimated values of the standard deviation of the spot price by less than those of the second group. Evidently, the forward price would decrease and there would likely be migration from the RCE to the FCE if the opposite was true.

CONCLUSIONS

In this paper, we studied the equilibrium of a competitive forward market comprised of suppliers and consumers who cannot adjust their energy portfolios by submitting bids to the spot market. The main features of our model come from the Brazilian electricity market, where the spot price is not the result of the direct interaction of demand and supply, but instead the output of a dynamic programming algorithm designed to achieve the optimal balance between using water today and storing it for future use.

We were able to show that the equilibrium forward price is a weighted average of the expected spot prices of buyers and sellers, and that the amount of hedging that takes place by an individual market participant depends on how his or her expected spot price compares to that average. In addition, we showed that the amount of aggregate hedging by consumers depends on how the number of consumers who expect the spot price to be smaller than the forward price compares to the number of those who expect it to be higher, and similarly for suppliers.

We also discussed some implications of our theoretical results for the Brazilian market. One of those results entertains the possibility of suppliers selling electricity short and consumers going long in the forward market. We explain that this will never happen in the Brazilian market as it is currently set up, for suppliers cannot purchase electricity to honor their short sales and consumers cannot sell surplus electricity in the spot market.

The impact on the Brazilian contract markets of changes in the expected value and the variability of the spot price can also be evaluated using our theoretical findings. For instance, if market participants receive a signal that leads them to increase their expected spot price, the equilibrium forward price will increase and consumers will migrate from the free market (FCE) to the regulated market (RCE). The assessment of the impact of changes in the variability of the spot price, as perceived by market participants, is a little more complicated, though. It depends on how the weights of those with a higher than average expected spot price (in the calculation of the equilibrium forward price) change relative to those of participants with a lower than average expected spot price.

There are many ways our model can be improved upon, and we look forward to pursuing them. For instance, we envision investigating the strategic behavior of suppliers in a dynamic game where they make sequential decisions about how to split their production between the free (FCE) and regulated (RCE) contract markets. Other possibilities are to study the effects of replacing our constant risk aversion with a relative risk aversion utility function, and to use different types of market structure when modeling the forward market.

APPENDIX

Proof of Proposition 1: Aggregate quantity supplied is equal to aggregate quantity demanded in equilibrium, and so

$$\begin{aligned}
&\sum_{i=1}^{m} R_i + \sum_{i=1}^{m} \frac{\overline{p}_i^c - q}{a_i^c\left(\sigma_i^c\right)^2} = \sum_{k=1}^{n} F_k + \sum_{k=1}^{n} \frac{q - \overline{p}_k^g}{a_k^g\left(\sigma_k^g\right)^2} \\
&\Rightarrow \sum_{k=1}^{n} \frac{q - \overline{p}_k^g}{a_k^g\left(\sigma_k^g\right)^2} - \sum_{i=1}^{m} \frac{\overline{p}_i^c - q}{a_i^c\left(\sigma_i^c\right)^2} = 0 \\
&\Rightarrow q\left(\sum_{k=1}^{n} \frac{1}{a_k^g\left(\sigma_k^g\right)^2} + \sum_{i=1}^{m} \frac{1}{a_i^c\left(\sigma_i^c\right)^2}\right) = \sum_{k=1}^{n} \frac{\overline{p}_k^g}{a_k^g\left(\sigma_k^g\right)^2} + \sum_{i=1}^{m} \frac{\overline{p}_i^c}{a_i^c\left(\sigma_i^c\right)^2} \\
&\Rightarrow q = \left(\sum_{k=1}^{n} \overline{p}_k^g D_k + \sum_{i=1}^{m} \overline{p}_i^c B_i\right) \Big/ \left(\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i\right)
\end{aligned}$$

where we used the fact that $\sum_{k=1}^{n} F_k = \sum_{i=1}^{m} R_i$ (by definition).

When $\overline{p}_k^g = \overline{p}_i^c = \overline{p}\ \forall k = 1,\ldots,n\ \ \forall i = 1,\ldots,m$ we have

$$\begin{aligned}
&\sum_{i=1}^{m} y_i^c = \sum_{k=1}^{n} y_k^g \Rightarrow \sum_{i=1}^{m} R_i + \sum_{i=1}^{m} \frac{\overline{p} - q}{a_i^c\left(\sigma_i^c\right)^2} = \sum_{k=1}^{n} F_k + \sum_{k=1}^{n} \frac{q - \overline{p}}{a_k^g\left(\sigma_k^g\right)^2} \\
&\Rightarrow (\overline{p} - q)\left(\sum_{i=1}^{m} B_i + \sum_{k=1}^{n} D_k\right) = 0 \\
&\Rightarrow q = \overline{p}
\end{aligned}$$

It is straightforward to show that then $y_k^g = F_k$ and $y_i^c = R_i$.

Proof of Proposition 2: First notice that

$$\frac{\partial q^*}{\partial B_j} = \frac{\overline{p}_j^c\left(\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i\right) - \left(\sum_{k=1}^{n} \overline{p}_k^g D_k + \sum_{i=1}^{m} \overline{p}_i^c B_i\right)}{\left(\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i\right)^2}$$

$$= \frac{\overline{p}_j^c}{\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i} - \frac{\sum_{k=1}^{n} \overline{p}_k^g D_k + \sum_{i=1}^{m} \overline{p}_i^c B_i}{\left(\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i\right)^2} = \frac{\overline{p}_j^c - q^*}{\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i}$$

This implies $\partial q^* / \partial B_j \geq 0$ if $\overline{p}_j^c \geq q^*$ and $\partial q^* / \partial B_j \leq 0$ if $\overline{p}_j^c \leq q^*$.

Now we compute the derivative of y_j^c with respect to a_j^c:

$$\frac{\partial y_j^c}{\partial a_j^c} = B_j\left(-\frac{\partial q^*}{\partial B_j}\frac{\partial B_j}{\partial a_j^c}\right) + \frac{\partial B_j}{\partial a_j^c}\left(\overline{p}_j^c - q^*\right) = \frac{\partial B_j}{\partial a_j^c}\left[\left(\overline{p}_j^c - q^*\right) - B_j\frac{\partial q^*}{\partial B_j}\right]$$

$$= \frac{\partial B_j}{\partial a_j^c}\left[\left(\overline{p}_j^c - q^*\right) - B_j\frac{\overline{p}_j^c - q^*}{\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i}\right] = \frac{\partial B_j}{\partial a_j^c}\left[\left(\overline{p}_j^c - q^*\right)\left(1 - \frac{B_j}{\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i}\right)\right]$$

$$= \frac{\partial B_j}{\partial a_j^c}\left[\left(\overline{p}_j^c - q^*\right)\left(\frac{\sum_{k=1}^{n} D_k + \sum_{i\neq j}^{m} B_i}{\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i}\right)\right]$$

Since $\partial B_j / \partial a_j^c < 0$, we have

$$\frac{\partial y_j^c}{\partial a_j^c}\begin{cases} \leq 0 \text{ if } \overline{p}_j^c \geq q^* \\ \geq 0 \text{ if } \overline{p}_j^c \leq q^* \end{cases}$$

Similarly,

$$\frac{\partial q^*}{\partial D_l} = \frac{\overline{p}_l^g\left(\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i\right) - \left(\sum_{k=1}^{n} \overline{p}_k^g D_k + \sum_{i=1}^{m} \overline{p}_i^c B_i\right)}{\left(\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i\right)^2}$$

$$= \frac{\overline{p}_l^g}{\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i} - \frac{\sum_{k=1}^{n} \overline{p}_k^g D_k + \sum_{i=1}^{m} \overline{p}_i^c B_i}{\left(\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i\right)^2} = \frac{\overline{p}_l^g - q^*}{\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i}$$

and

$$\frac{\partial y_l^g}{\partial a_l^g} = D_l\left(\frac{\partial q^*}{\partial D_l}\frac{\partial D_l}{\partial a_l^g}\right) + \frac{\partial D_l}{\partial a_l^g}\left(q^* - \overline{p}_l^g\right) = \frac{\partial D_l}{\partial a_l^g}\left[D_l\frac{\partial q^*}{\partial D_l} - \left(\overline{p}_l^g - q^*\right)\right]$$

$$= \frac{\partial D_l}{\partial a_l^g}\left[D_l\frac{\overline{p}_l^g - q^*}{\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i} - \left(\overline{p}_l^g - q^*\right)\right] = \frac{\partial D_l}{\partial a_l^g}\left[\left(\overline{p}_l^g - q^*\right)\left(\frac{D_l}{\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i} - 1\right)\right]$$

Since $D_l \Big/ \left(\sum_{k=1}^{n} D_k + \sum_{i=1}^{m} B_i\right) < 1$ and $\partial D_l / \partial a_l^g < 0$, we have

$$\frac{\partial y_l^g}{\partial a_l^g}\begin{cases} \geq 0 \text{ if } \overline{p}_l^g \geq q^* \\ \leq 0 \text{ if } \overline{p}_l^g \leq q^* \end{cases}$$

Similar reasoning can be used to show that

$$\frac{\partial y_j^c}{\partial \sigma_j^c}\begin{cases} \leq 0 \text{ if } \overline{p}_j^c \geq q^* \\ \geq 0 \text{ if } \overline{p}_j^c \leq q^* \end{cases}$$

and

$$\frac{\partial y_l^g}{\partial \sigma_l^g}\begin{cases} \geq 0 \text{ if } \overline{p}_l^g \geq q^* \\ \leq 0 \text{ if } \overline{p}_l^g \leq q^* \end{cases}$$

Notes

1. "Ambiente de Contratação Regulada" in Portuguese.
2. "New" electricity refers to power to be generated by plants yet to be built, and "existing" electricity refers to power generated by existing plants.
3. "Ambiente de Contratação Livrg" in Portuguese.

4. There are four submarkets in the Brazilian electricity system, whose boundaries are determined by transmission constraints.
5. Green (1999) works with linear supply functions most of the time.
6. For a detailed exposition of the stochastic dual dynamic programming based algorithms used by the Brazilian system operator, see Maceira *et al.* (2008).
7. These numbers refer to agents registered with the Brazilian Electricity Commercialization Clearinghouse (or CCEE by its Portuguese acronym), and were obtained from the CCEE's website *http://www.ccee.org.br* on October 15, 2012.

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