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Temperature Distribution Surrounding a Growing Vapour Bubble Under the Effect of Source and Sink

Mohammadein S. A* & Elgammal M. I**

ABSTRACT

The paper presents the temperature distribution of superheated liquid surrounding a growing of spherical vapour bubble between two-phase temperatures in case of source and sink of heat. The growth rate of a vapour bubble in superheated liquid is derived The heat equation is solved analytically by the modification of similarity parameter method of Screven between two finite boundaries. Under these conditions, the growth of vapour bubble and temperature distribution are obtained in an implicit form. The effect of source and sink is obtained. The growth and temperature field affected by the initial superheating and thermal diffusivity. The Plesset et al and Scriven et al. theories are obtained as special cases from the present model.

1. INTRODUCTION

The heat exchange represents an important operation between the two-phase flow. The growth and collapse of a vapour bubble are affected by heat transfer and the others physical parameters. The bubble radius grows within the superheated liquid has been studied by many authors [1, 3, 4, 5, 7, 8, 9, 10, 12]. The rate of growth of a vapour bubble, once formed, is affected by the surface tension, the liquid inertia, the pressure difference. There are three stages for bubble growth: inertial, thermal, and diffusion stages. In the inertial stage, the bubble nucleus depends strongly on the interfacial mechanical interactions, pressure forces, and surface tension forces. This stage takes a few milliseconds and thermal phenomena are negligible. Therefore, this stage is called isothermal. In thermal stage, the radius of the nucleus increases and the growth becomes mostly dependent on the supply of heat that is consumed to vaporize the liquid on the bubble surface. The rate of expansion of the bubble is much lower than during the isothermal stage. This stage of the bubble growth is called isobaric. It is worth noting that the duration of the isothermal stage is very short compared with the isobaric stage. Most of the bubble growth occurs in a stage that is characterized by an essentially constant bubble pressure. This feature was first noted by Plesst and Zwick [ ], the growth of the bubble is described by mass and momentum equation. The mixture of the vapour and superheated liquid are considered as incompressible and non-viscous. The momentum equation is described by Plesset-Rayleigh and solved under external forces. The effect of gravity and vapour pressure change are neglected.

When the rate of bubble growth becomes appreciable, however, the temperature and, hence, the pressure within the bubble drop and then the rate of growth is decreased. The reduction of the temperature within the bubble is a consequence of the latent heat of evaporation requirement that takes place at the vapour – liquid interface during the growth of the bubble. Temperature distribution surrounding a growing vapour bubble without any effect of heating source and sink is obtained by scriven [10] and Mohammadein et al [6].The effort in this paper is devoted to derive temperature distribution surrounding a growing vapour bubble with the effect of heating source and sink.

* Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt, *E-mail: selimali2009@yahoo.com*

** Department of Preparatory Year, Al-Ghad International Medical Sciences College, Al-Madinah, Kingdom of Saudi Arabia, *E-mail: mahmoud_algammal2002@yahoo.com*

2. ANALYSIS

In the present work, the heat conduction equation is extended for spherical symmetry vapour bubble to establish the effect of radial convection resulting from unequal phase temperatures. The continuity equation for incompressible mixture has the following form:

$$\nabla \cdot \underline{w}(r,t) = 0, \tag{1}$$

where

$$w = \varepsilon \dot{R} \frac{R^2}{r^2}, \quad \text{and} \quad \varepsilon = 1 - \frac{\rho_2}{\rho_1}. \tag{2}$$

The energy equation can be written in the form

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial r} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) + \frac{b\lambda}{\rho_1 c_p r}\frac{\partial T}{\partial r}. \tag{3}$$

on the basis of Scriven assumption

$$T(r,t) = T(s), \tag{4}$$

where

$$r = 2s\sqrt{at}, \quad \text{and} \quad \frac{\partial s}{\partial t} = 0. \tag{5}$$

$$\text{At } r = R, \ s = \beta \quad \text{then} \quad R = 2\beta\sqrt{at}. \tag{6}$$

The equation (3) is solved under the following initial and boundary conditions

$$\text{I.C.}: T(R, t_o) = T_o, \tag{7}$$

$$\text{B.C.}: T(R_o, t_o) = T_s, \tag{8}$$

and

$$\left(\frac{\partial T}{\partial r}\right)_{r=R} = \frac{\rho_2 \dot{R}}{a\rho_1}\left[L + (C_{PL} - C_{PV})\Delta\theta_o\right]. \tag{9}$$

In the present work, the modified assumption (6) in the following form:

$$r = \frac{s}{\beta} f(t), \quad \text{and} \quad \frac{\partial s}{\partial t} = 0. \tag{10}$$

$$\text{At } r = R, \ s = \beta \quad \text{then} \quad R = f(t). \tag{11}$$

By the mathematical roles and when $t = t_0$ in the initial moment of bubble growth stage, $R = R_0$, the above equation can be rewritten in the form:

$$R = \sqrt{2C(t - t_o) + R_o^2}, \tag{12}$$

where C is constant. The above equation represents the growth rate of vapour bubble in superheated liquid incompressible and non-viscous.

By the mathematical roles, the initial condition (7) and the boundary conditions (8) and (9) the solution of equation (3) has the form

$$T(r,t_i)-T_o=-\frac{CM}{a}R^{\left(1+\frac{b\lambda}{\rho_1 aC_p}\right)}(t_i)\int_r^{R_m}\frac{1}{r^{\left(2+\frac{b\lambda}{\rho_1 aC_p}\right)}}\exp\left\{-\frac{C}{a}\left(\frac{r^2}{2R^2(t_i)}+\frac{\varepsilon R(t_i)}{r}-\frac{3}{2}\right)\right\}dr. \qquad (13)$$

The above equation represents the liquid temperature surrounded a growing vapour bubble in superheated liquid incompressible and non-viscous.

Where the constants M and ψ equal:

$$M=\frac{\Delta\theta_o}{J_a}\left[1+\frac{(C_{PL}-C_{PV})\Delta\theta_o}{L}\right]. \qquad (14)$$

And

$$\psi=\frac{b\lambda}{a\rho_1 c_p}. \qquad (15)$$

The liquid temperature distribution at the wall of vapour bubble becomes

$$T\{R(t_i),t_i\}-T_o=-\frac{CM}{a}\left\{\left(\frac{R(t_i)}{R_m}\right)^{\psi}\left\{\sqrt{\frac{\pi a}{6C}}\,erf\left[\sqrt{\frac{3C}{2a}}\left(1-\frac{R(t_i)}{R_m}\right)\right]+\frac{\psi}{\psi^{-1}}\left(\frac{R_m}{R(t_i)}\right)-1\right\}-\frac{\psi}{\psi^{-1}}\right\}. \qquad (16)$$

Also, equation (3) rewritten as follows:

2.1 Estimation of the Constant *C*

By taking and then.

$$t_i=t_o \quad \text{amd} \quad R(t_o)=\text{Ro} \quad \text{then} \quad T(R_o,t_o)=T_s. \qquad (17)$$

Applying the above initial conditions into equation (17), then we have

$$C=\frac{a}{B}\left\{\sqrt{\frac{J_a}{\left[1+\frac{(C_{PL}-C_{PV})\Delta\theta_\circ}{L}\right]}+\left(\varphi_\circ^{\frac{1}{3}}\right)^{2\psi}\frac{\pi}{24B}}-\left(\varphi_\circ^{\frac{1}{3}}\right)^{\psi}\sqrt{\frac{\pi}{24B}}\right\}^2, \qquad (18)$$

and the constant B becomes

$$B=\frac{\psi}{\psi^{-1}}\left(\varphi_o^{-\frac{1}{3}}\right)^{\psi+1}-\left(\varphi_o^{\frac{1}{3}}\right)^{\psi}-\frac{1}{\psi^{-1}}. \qquad (19)$$

Substituting from equation (18) into equation (12) then

$$R=\sqrt{R_\circ^2+\frac{2a}{B}\left\{\sqrt{\frac{J_a}{\left[1+\frac{(C_{PL}-C_{PV})\Delta\theta_\circ}{L}\right]}+\left(\varphi_o^{\frac{1}{3}}\right)^{2\psi}\frac{\pi}{24\ B}}-\left(\varphi_o^{\frac{1}{3}}\right)^{\psi}\sqrt{\frac{\pi}{24B}}\right\}^2(t-t_\circ)}. \qquad (20)$$

The above equation describes the growth of vapour bubble between two-phase flow in the mixture of vapour and superheated liquid.

Special Cases

1. Scriven theory [10] is obtained as a special case when we take

$$k \to 0, \quad R_o \to 0 \quad \text{and} \quad t_o \to 0. \tag{21}$$

Substituting from equation (21) in equation (20), then we have

$$R = \frac{J_a}{1 + \frac{(C_{PL} - C_{PV})\Delta\theta_\circ}{L}} \sqrt{\frac{12a}{\pi} t}\,. \tag{22}$$

2. Plesset and Zwick theory [8] is obtained as a special case when we take

$k \to 0, \quad R_o \to 0 \quad$ and $\quad t_o \to 0$ and by putting $\Delta\theta_o \to 0$, then

$$R = J_a \sqrt{\frac{12a}{\pi} t}\,. \tag{23}$$

3. Mohammadein *et al*., theory [12] is obtained as a special case when we take $b = 0$.

The temperature distribution and growth of vapour bubble are derived under the effect of heating source ($b = 1$) and heating sink ($b = -1$).

3. DISCUSSION OF THE RESULTS

The heat exchange problem is solved by analytical way. The solution is obtained in terms of temperature distribution (13) and the growth of bubble radius in terms of time (20). The numerical calculation for the temperature and bubble radius are calculated by Haar [2] and Wagner [11] at saturation pressure (P_{sat} = 101.3 kPa and T_{sat} = 373.15 K) in superheated liquid for some physical parameters

$a = 9.9 \times 10^{-10}\,\text{m}^2/\text{s}$, $\rho_1 = 958.3\,\text{Kgm}^{-3}$, $pi = 3.14$, $\rho_2 = 0.597\,\text{Kgm}^{-3}$, $L = 419100\,(\text{W/m}^2)/(\text{K/m})$,

$C_{pl} = 4220\,\text{KJ/(Kg K)}$, $C_{pv} = 2030\,\text{KJ/(Kg K)}$, $R_0 = 0.0001\,\text{m}$, $R_m = 0.001\,\text{m}$, $\lambda = 0.687\,(\text{W/mK})$).

The growth of vapour bubble in terms of time for two different values of thermal diffusivity **a** is shown by Fig. 1. It is observed that, the growth is proportional with thermal diffusivity coefficient a values. The

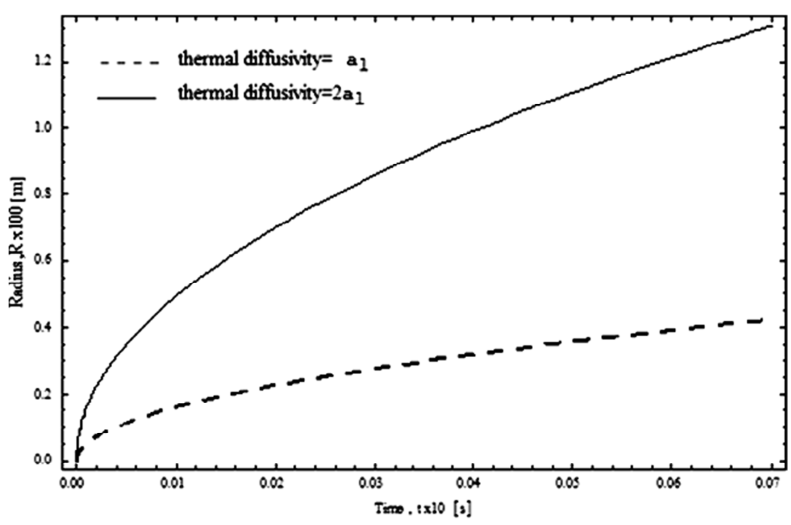

Figure 1: The Growth of Vapour Bubble in Terms of Time for Two Different Values of Liquid Thermal Diffusivity a_1 in Case of Source

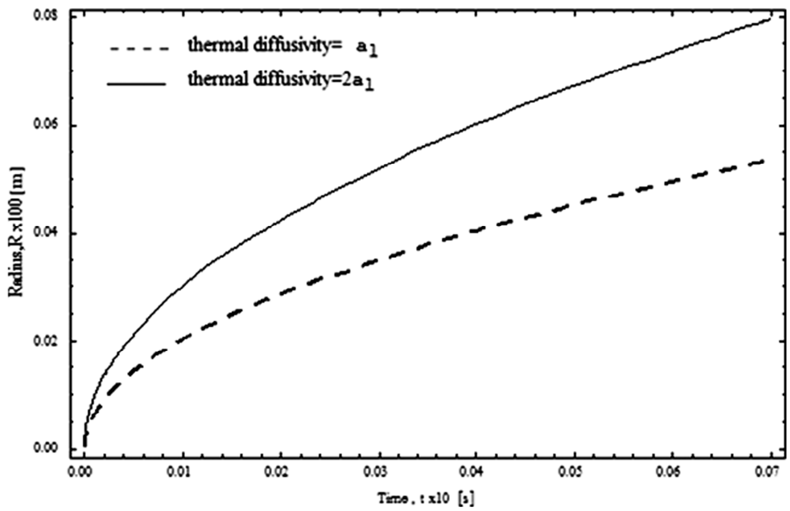

Figure 2: The Growth of Vapour Bubble in Terms of Time for Two Different Values of Liquid Thermal Diffusivity a_1 in Case of Sink

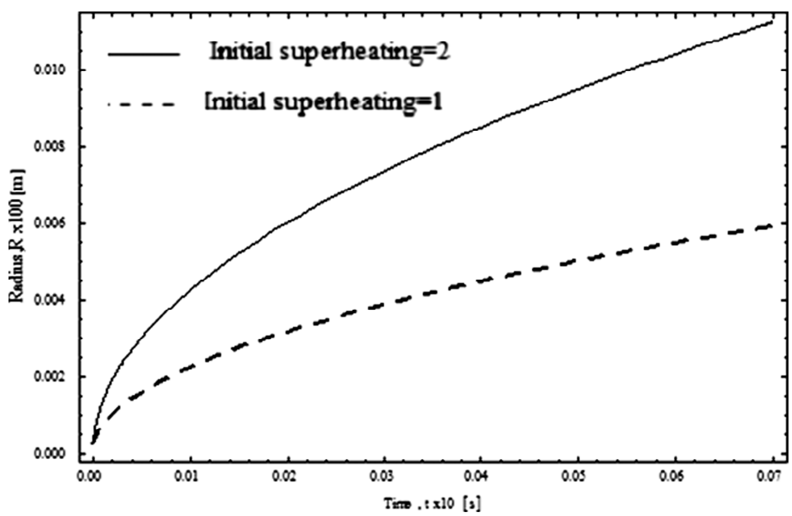

Figure 3: The Growth of Vapour Bubble in Terms of Time for Two Different Values of Initial Superheating $\Delta\theta_o$ in Case of Sink

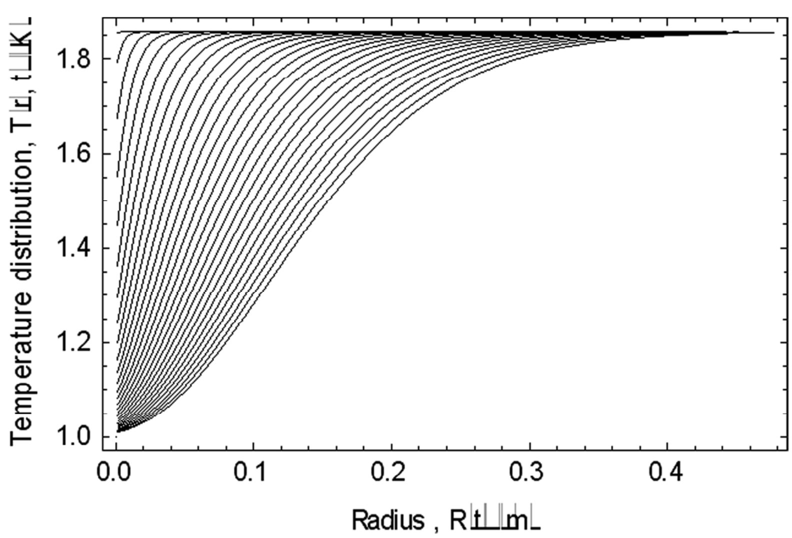

Figure 4: The Temperature Distribution in Superheated Liquid Surround the Growing Vapour Bubble in Case of Source

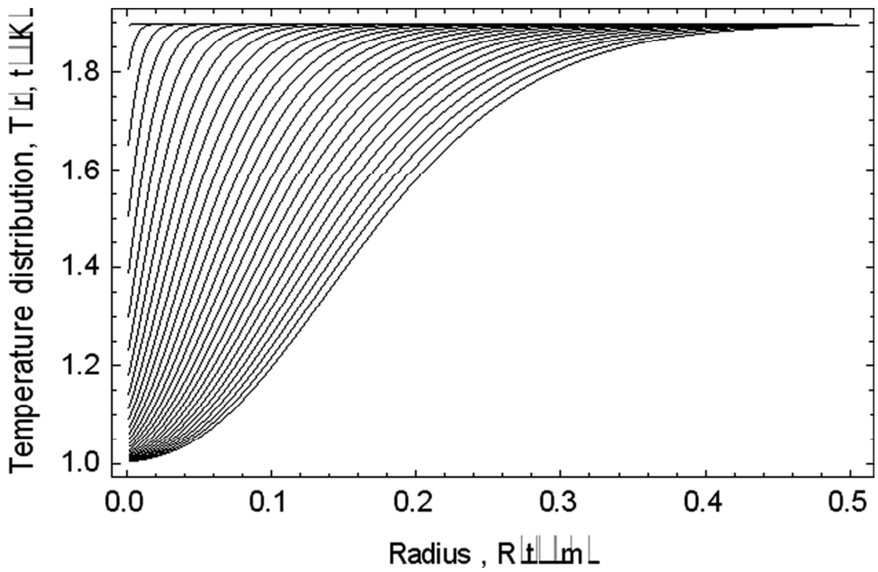

Figure 5: The Temperature Distribution in Superheated Liquid Surround the Growing Vapour Bubble in Case of Sink

growth of vapour bubble in terms of time for two different values of superheating $\Delta\theta_o$ is shown by Fig. 3. It is observed that, the growth is proportional with superheating $\Delta\theta_o$ values. The distribution of temperature around the growing vapour bubble in case of source is shown in Figs. 4. It is observed that the distribution of temperature around the growing vapour bubble increases with the constitutes time. The distribution of temperature around the growing vapour bubble in case of sink is shown in Figs. 5. It is observed that the distribution of temperature around the growing vapour bubble decreases with the constitutes time.

4. CONCLUSION

Heat exchange problem (3) with initial and two boundary conditions is solved by the modification of similarity parameters method of Scriven. Under the effect of heating source and sink , the distribution of temperature and the growth of vapour bubble radius are obtained by relations (13) and (20) respectively. The dissucion of results and figures concluded the following remarks:

1. The distribution of temperature is affected with initial superheating $\Delta\theta_o$. The distribution of temperature $T(r, t)$ around the growing vapour bubble is proportional with the thermal diffusivity a. Moreover the distribution of temperature around the growing vapour bubble proportional with the distance outside the growing vapour bubble and inversely with the time.
2. The growth of vapour bubble in terms of time is proportional with thermal diffusivity coefficient and superheating values.
3. The Scriven theory [10], Plesset and Zwick theory [8] and Mohammadein *et al*., theory [12] are obtained as a special from the present model.

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