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Robust Reliable Control for Uncertain Neutral Systems with Time-varying Delays and IQC Performance

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ABSTRACT

A robust reliable control with integral quadratic constraint (IQC) performance for a class of uncertain neutral systems with state and input time-varying delays is considered in this paper. Two classes of failure situations for sensor or actuator are studied. In the first class, a delay-dependent criterion for time-delay system without perturbations is proposed to design the reliable control with IQC performance. Next, a criterion for uncertain time-delay systems with parameter uncertainties is obtained via simple derivations. The linear matrix inequality (LMI) approach is used to design a robust reliable state feedback control with IQC performance. In the second class, a reliable control with IQC performance is also provided from the previous method. A numerical example is given to illustrate the effectiveness of the procedure.

Keywords: Robust reliable control, Delay-dependent criterion, Linear matrix inequality approach, IQC performance

1. INTRODUCTION

In practical systems, the failures of sensor and actuator will destroy the stability and system performance. Hence reliable controls have been introduced to tolerate some failures for sensors/actuators and maintains the system stability and performance. Many approaches had been proposed to design reliable controls for the outages of sensors and actuators [1]-[5]. In [6]-[10], the reliable control of time-delay systems has been studied. In [1], the Hamilton-Jacobi equation approach is used to design a reliable control for nonlinear systems. In [3], an algebraic Riccati equation approach is presented to guarantee the closed-loop stability and H_∞ performance in some admissible component failures. In [5], an LMI approach is provided to guarantee the H_∞ performance for nonlinear system in some admissible component outages. On the other hand, the system models contain always some uncertain elements and nonlinearities; these uncertainties and nonlinearities may be due to unknown additive noise, environmental influence, poor plant knowledge, and limitations of actuators or sensors. Hence a robust reliable control technology is to be developed to stabilize the uncertain linear or nonlinear systems with sensor and actuator failures [1], [5]-[10]. In Ref. 6, matrix and linear matrix inequalities have been proposed to design reliable controls with IQC performance for uncertain systems with input delay. In [7], reliable control problem for uncertain nonlinear systems with multiple time delays has been solved by proposing some matrix and linear matrix inequalities. In [9], [10], reliable controls for uncertain time-delay systems have been designed via a modified Riccati equation approach. In [11], a reliable control for uncertain fuzzy dynamic systems with time-varying delays has been designed via the LMI approach.

Over the past few decades, the H_∞ control problem for uncertain systems with disturbance inputs has been an active topic in control system theory and application [1], [4], [5], [9], [12], [13]. The H_∞ control is

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proposed to reduce the effect of the disturbance input on the regulated output to within a prescribed level. There are many approaches to dealing the H_∞ control problem. Riccati equation approach was proposed to design H_∞ control for time-delay system [12]. The Riccati equation and Hamilton-Jacobi equation approaches have difficulties in finding feasible solutions and minimizing the H_∞-norm bound (γ). In [4], the LMI approach has been used to design a reliable H_∞ control for a given H_∞-norm bound γ. Reliable control with IQC performance is a generalization of the H_∞ control problem [6], [8]. In [6], a reliable control for uncertain systems with input delay has been considered. In [8], a robust reliable control for uncertain systems with input and state delays has been considered. In this paper, the LMI approach will be used to design reliable control with IQC performance for uncertain neutral systems with both state and input time-varying delays. The useful nonnegative inequalities and the Leibniz-Newton formula will be used to improve the conservativeness and find delay-dependent stabilization results. Two types of faulty are considered in this paper. A numerical example is given to show the main results of this paper.

2. RELIABLE CONTROL FOR UNPERTURBED NEUTRAL SYSTEMS WITH IQC PERFORMANCE

Consider the following neutral system with state and input time-varying delays:

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-h(t)) + A_2 \dot{x}(t-\tau) + B_u u^f(t-\eta(t))$$
$$+ B_w w(t) + f(x(t), x(t-h(t)), \dot{x}(t-\tau)), \; t \ge 0, \tag{1a}$$

$$z(t) = C_0 x(t) + C_1 x(t-h(t)) + C_2 \dot{x}(t-\tau) + D_u u^f(t-\eta(t)) + D_w w(t), \; t \ge 0, \tag{1b}$$

$$x(t) = \phi(t), \quad t \in [-H, 0], \tag{1c}$$

where $x \in \Re^n$, x_t is state at time t defined by $x_t(\theta) := x(t+\theta), \; \forall \theta \in [-H, 0]$, $u^f \in \Re^m$ is the control input of actuator or sensor fault, $w \in \Re^l$ is the disturbance input, $z \in \Re^q$ is the regulated output. $A_i \in \Re^{n\times n}$, $i = 0,1,2$, $B_u \in \Re^{n\times m}$, $B_w \in \Re^{n\times l}$, $C_i \in \Re^{q\times n}$, $i = 0, 1, 2$, $D_u \in \Re^{q\times m}$, and $D_w \in \Re^{q\times l}$ are some given constant matrices. The time-varying delays satisfy $0 \le h(t) \le \bar{h}$, $0 \le \eta(t) \le \bar{\eta}$, $\bar{h}$, $\bar{\eta}$, and τ are nonnegative constants with $H = \max\{\bar{h}, \bar{\eta}, \tau\}$. The initial vector ϕ is a differentiable function on [–H, 0]. At first, the fault of control input for actuator (or sensor) is described as follows:

$$u^f(t) = Ru(t), \tag{2a}$$

where R is the actuator fault matrix with

$$R = \mathrm{diag}[r_1, r_2, \cdots, r_m], \; 0 \le \underline{r}_i \le r_i \le \bar{r}_i, \; \bar{r}_i \ge 1, \; i = 1, 2, \cdots, m, \tag{2b}$$

$\underline{r}_i$ and $\bar{r}_i$, $i = 1, 2, \cdots, m$, are some given constants. $r_i = 0$ means that ith actuator or sensor completely fails, $r_i = 1$ means that ith actuator or sensor is normal.

Define

$$R_0 = \mathrm{diag}[r_{10}, r_{20}, \cdots, r_{m0}], \; r_{i0} = \frac{\bar{r}_i + \underline{r}_i}{2}, \tag{2c}$$

$$R_1 = \mathrm{diag}[r_{11}, r_{21}, \cdots, r_{m1}], \; r_{i1} = \frac{\bar{r}_i - \underline{r}_i}{2}. \tag{2d}$$

Hence the matrix R can be rewritten as

$$R = R_0 + R_1 \cdot \Delta J, \tag{2e}$$

where

$$\Delta J = \mathrm{diag}\left[j_1, j_2, \cdots, j_m\right], \ -1 \le j_i \le 1.$$

The function $f\left(x(t), x(t-h(t)), \dot{x}(t-\tau)\right)$ satisfies the following condition:

$$f^T\left(x(t), x(t-h(t)), \dot{x}(t-\tau)\right) f\left(x(t), x(t-h(t)), \dot{x}(t-\tau)\right)$$
$$\le x^T(t)\Gamma^T\Gamma x(t) + x^T(t-h(t))\Lambda^T\Lambda x(t-h(t)) + \dot{x}^T(t-\tau)\theta^T\theta\dot{x}(t-\tau), \quad (2f)$$

where Γ, Λ, and θ are some given matrices.

Definition 2.1. [8] Consider the system (1), with (2) and $u(t) = -Kx(t)$; let the following conditions be satisfied:

(i) With $w(t) = 0$, the closed-loop system (1), with (2) and $u(t) = -Kx(t)$ is globally asymptotically stable.

(ii) With the zero initial condition (i.e. $\phi = 0$), the signals $w(t)$ and $z(t)$ are bounded by

$$\int_0^\infty \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}^T \cdot \Pi \cdot \begin{bmatrix} z(t) \\ w(t) \end{bmatrix} dt = \int_0^\infty \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^T & \Pi_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix} dt \le 0, \ \forall\, w \in L_2[0,\infty), w \ne 0, \quad (3)$$

where Π is a given matrix, with $\Pi_{11} > 0$ and $\Pi_{22} < 0$. In this condition, the system (1) with (2) is said to satisfy the IQC performance defined by Π and the control law $u(t) = -Kx(t)$ is said to be a reliable control with IQC performance.

Remark 2.1. If we choose $\Pi_{11} = I$, $\Pi_{12} = 0$, $\Pi_{22} = -\gamma^2 \cdot \mathrm{I}$, the condition (3) can be written as

$$\int_0^\infty \|z(t)\|^2 dt \le \gamma^2 \cdot \int_0^\infty \|w(t)\|^2 dt.$$

This yields a standard H_∞ control problem. The parameter $\gamma > 0$ is the H_∞-norm bound for the reliable H_∞ state feedback control $u(t) = -Kx(t)$ (see Ref. 5). Hence the reliable control with IQC performance can be seen as a generalized reliable H_∞ control.

The following two lemmas will be used to design a reliable state feedback control with IQC performance.

Lemma 2.1. [6], [8] Let U, V, W, X be real matrices of appropriate dimensions with X satisfying $X = X^T$. Then

$$X + UVW + W^TV^TU^T < 0, \text{ for all } V^TV \le I,$$

if and only if there exists a scalar $\varepsilon > 0$ such that

$$X + \varepsilon \cdot UU^T + \varepsilon^{-1} \cdot W^TW = X + \varepsilon^{-1} \cdot (\varepsilon \cdot U)(\varepsilon \cdot U)^T + \varepsilon^{-1} \cdot (W)^T(W) < 0.$$

Lemma 2.2. (Schur Complement of [15]). For a given symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}$, with $S_{11} = S_{11}^T$, $S_{22} = S_{22}^T$, the following conditions are equivalent:

(i) $S < 0$,

(ii) $S_{22} < 0$, $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$.

Now, the reliable state feedback control $u(t) = -Kx(t)$ will be designed from the following result.

Theorem 2.1. For a constant $\rho > 0$, let there exist two constants $\varepsilon > 0$, $\sigma > 0$, some $n\times n$ positive-definite symmetric matrices $\bar{P}$, $\bar{Q}_i$, $i = 1, 2, \cdots, 5$, $\bar{R}_{22}$, $\bar{\hat{R}}_{22}$, $\bar{S}_{22}$, $\bar{\hat{S}}_{22}$, some $5n\times 5n$ positive-definite symmetric matrices $\bar{R}_{11}$, $\bar{\hat{R}}_{11}$, $\bar{S}_{11}$, $\bar{\hat{S}}_{11}$, and some matrices $\bar{R}_{12}$, $\bar{\hat{R}}_{12}$, $\bar{S}_{12}$, $\bar{\hat{S}}_{12} \in \Re^{5n\times n}$, $\hat{K} \in \Re^{m\times n}$, such that following LMI conditions are satisfied:

$$\begin{bmatrix} \bar{R}_{11} & \bar{R}_{12} \\ * & \bar{R}_{22} \end{bmatrix} > 0, \begin{bmatrix} \bar{\hat{R}}_{11} & \bar{\hat{R}}_{12} \\ * & \bar{\hat{R}}_{22} \end{bmatrix} > 0, \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} \\ * & \bar{S}_{22} \end{bmatrix} > 0, \begin{bmatrix} \bar{\hat{S}}_{11} & \bar{\hat{S}}_{12} \\ * & \bar{\hat{S}}_{22} \end{bmatrix} > 0, \tag{4a}$$

$$\bar{\hat{R}}_{11} > \bar{R}_{11},\ \bar{\hat{S}}_{11} > \bar{S}_{11},\ \bar{Q}_2 > \bar{R}_{22},\ \bar{Q}_2 > \bar{\hat{R}}_{22},\ \bar{Q}_4 > \bar{S}_{22},\ \bar{Q}_4 > \bar{\hat{S}}_{22}, \tag{4b}$$

$$\Omega(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w) =$$

$$\begin{bmatrix}
\Omega_{11} & \Omega_{12} & 0 & \Omega_{14} & 0 & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} & \Omega_{110} & \Omega_{111} & 0 & \Omega_{113} & 0 & 0 \\
* & 0 & 0 & 0 & 0 & \Omega_{26} & 0 & \Omega_{28} & 0 & \Omega_{210} & 0 & 0 & 0 & \Omega_{214} & 0 \\
* & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & \Omega_{46} & 0 & \Omega_{48} & 0 & \Omega_{410} & 0 & \Omega_{412} & 0 & 0 & 0 \\
* & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Omega_{66} & \Omega_{67} & \Omega_{68} & \Omega_{69} & 0 & \Omega_{611} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Omega_{77} & \Omega_{78} & 0 & \Omega_{710} & 0 & 0 & 0 & 0 & \Omega_{715} \\
* & * & * & * & * & * & * & \Omega_{88} & 0 & \Omega_{810} & \Omega_{811} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Omega_{99} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Omega_{1010} & \Omega_{1011} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & \Omega_{1111} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & \Omega_{1212} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & \Omega_{1313} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & \Omega_{1414} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & \Omega_{1515}
\end{bmatrix}$$

$$+\begin{bmatrix}
\bar{\Delta}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} < 0, \tag{4c}$$

where

$$\overline{\Delta}_{11} = \overline{R}_{12}\overline{\Delta}_1 + \overline{\Delta}_1^T\overline{R}_{12}^T + \hat{\overline{R}}_{12}\hat{\overline{\Delta}}_1 + \hat{\overline{\Delta}}_1^T\hat{\overline{R}}_{12}^T + \overline{S}_{12}\overline{\Delta}_2 + \overline{\Delta}_2^T\overline{S}_{12}^T + \hat{\overline{S}}_{12}\hat{\overline{\Delta}}_2 + \hat{\overline{\Delta}}_2^T\hat{\overline{S}}_{12}^T + \overline{\eta}\cdot\hat{\overline{S}}_{11} + \overline{h}\cdot\hat{\overline{R}}_{11},$$

$$\overline{\Delta}_1 = [I \quad -I \quad 0 \quad 0 \quad 0],\ \hat{\overline{\Delta}}_1 = [0 \quad I \quad -I \quad 0 \quad 0],$$

$$\overline{\Delta}_2 = [I \quad 0 \quad 0 \quad -I \quad 0],\ \hat{\overline{\Delta}}_2 = [0 \quad 0 \quad 0 \quad I \quad -I],$$

$$\Omega_{11} = \overline{P}A_0^T + A_0\overline{P} + \overline{Q}_1 + \overline{Q}_3,\ \Omega_{12} = A_1\overline{P},\ \Omega_{14} = -B_uR_0\hat{K},\ \Omega_{16} = \rho\cdot\overline{P}A_0^T,\ \Omega_{17} = A_2\overline{P},$$

$$\Omega_{18} = B_w + \overline{P}C_0^T\Pi_{12},\ \Omega_{19} = \varepsilon\cdot I,\ \Omega_{110} = \overline{P}C_0^T,\ \Omega_{111} = \sigma\cdot B_uR_1,\ \Omega_{113} = \overline{P}\Gamma^T,$$

$$\Omega_{26} = \rho\cdot\overline{P}A_1^T,\ \Omega_{28} = \overline{P}C_1^T\Pi_{12},\ \Omega_{210} = \overline{P}C_1^T,\ \Omega_{214} = \overline{P}\Lambda^T,\ \Omega_{33} = -\overline{Q}_1,$$

$$\Omega_{46} = -\rho\cdot\hat{K}^TR_0^TB_u^T,\ \Omega_{48} = -\hat{K}^TR_0^TD_u^T\Pi_{12},\ \Omega_{410} = -\hat{K}^TR_0^TD_u^T,\ \Omega_{412} = -\hat{K}^T,$$

$$\Omega_{55} = -\overline{Q}_3,\ \Omega_{66} = \overline{h}\cdot\overline{Q}_2 + \overline{\eta}\cdot\overline{Q}_4 + \overline{Q}_5 - 2\rho\cdot\overline{P},\ \Omega_{67} = \rho\cdot A_2\overline{P},\ \Omega_{68} = \rho\cdot B_w,$$

$$\Omega_{69} = \rho\cdot\varepsilon\cdot I,\ \Omega_{611} = \sigma\cdot\rho\cdot B_uR_1,\ \Omega_{77} = -\overline{Q}_5,\ \Omega_{78} = \overline{P}C_2^T\Pi_{12},$$

$$\Omega_{710} = \overline{P}C_2^T,\ \Omega_{715} = \overline{P}\theta^T,\ \Omega_{88} = \Pi_{22} + D_w^T\Pi_{12} + \Pi_{12}^TD_w,\ \Omega_{810} = D_w^T,$$

$$\Omega_{811} = \sigma\cdot\Pi_{12}^TD_uR_1,\ \Omega_{1010} = -\Pi_{11}^{-1},\ \Omega_{1011} = \sigma\cdot D_uR_1,$$

$$\Omega_{99} = \Omega_{1313} = \Omega_{1414} = \Omega_{1515} = -\varepsilon\cdot I,\ \Omega_{1111} = \Omega_{1212} = -\sigma\cdot I.$$

Then, the system (1) with (2) is asymptotically stabilizable via the reliable control $u(t) = -Kx(t) = -\hat{K}\overline{P}^{-1}x(t)$ with IQC performance.

Proof. Define the Lyapunov function as

$$V(x_t) = x^T(t)Px(t) + \int_{t-\overline{h}}^{t} x^T(s)Q_1x(s)ds + \int_{t-\overline{h}}^{t}\left(s-\left(t-\overline{h}\right)\right)\dot{x}^T(s)Q_2\dot{x}(s)ds$$
$$+\int_{t-\overline{\eta}}^{t} x^T(s)Q_3x(s)ds + \int_{t-\overline{\eta}}^{t}\left(s-\left(t-\overline{\eta}\right)\right)\dot{x}^T(s)Q_4\dot{x}(s)ds + \int_{t-\tau}^{t}\dot{x}^T(s)Q_5\dot{x}(s)ds, \qquad (5)$$

where $P = \overline{P}^{-1} > 0$, $Q_i = \overline{P}^{-1}\overline{Q}_i\overline{P}^{-1} > 0$, $i = 1, 2, \cdots, 5$. The time derivative of $V(x_t)$ in (5), along the trajectories of the system (1), with (2) and $u(t) = -Kx(t)$, is given by

$$\dot{V}(x_t) = \left[A_0x(t) + A_1x(t-h(t)) + A_2\dot{x}(t-\tau) + B_uu^f(t-\eta(t)) + B_ww(t) + f\right]^T Px(t)$$
$$+ x^T(t)P\left[A_0x(t) + A_1x(t-h(t)) + A_2\dot{x}(t-\tau) + B_uu^f(t-\eta(t)) + B_ww(t) + f\right]$$
$$+ x^T(t)Q_1x(t) - x^T\left(t-\overline{h}\right)Q_1x\left(t-\overline{h}\right) + \overline{h}\cdot\dot{x}^T(t)Q_2\dot{x}(t) - \int_{t-\overline{h}}^{t}\dot{x}^T(s)Q_2\dot{x}(s)ds$$
$$+ x^T(t)Q_3x(t) - x^T(t-\overline{\eta})Q_3x(t-\overline{\eta}) + \overline{\eta}\cdot\dot{x}^T(t)Q_4\dot{x}(t) - \int_{t-\overline{\eta}}^{t}\dot{x}^T(s)Q_4\dot{x}(s)ds$$
$$+ \dot{x}^T(t)Q_5\dot{x}(t) - \dot{x}^T(t-\tau)Q_5\dot{x}(t-\tau), \qquad (6)$$

where f is the abbreviation of $f(x(t), x(t-h(t)), \dot{x}(t-\tau))$. By the Leibniz-Newton formulas, we have

$$\int_{t-\bar{h}}^{t} \dot{x}(s)ds = x(t) - x(t-\bar{h}), \tag{7a}$$

$$\int_{t-\bar{\eta}}^{t} \dot{x}(s)ds = x(t) - x(t-\bar{\eta}), \tag{7b}$$

$$\int_{t-\tau}^{t} \dot{x}(s)ds = x(t) - x(t-\tau). \tag{7c}$$

Define

$$X^T = \begin{bmatrix} x^T(t) & x^T(t-h(t)) & x^T(t-\bar{h}) & x^T(t-\eta(t)) & x^T(t-\bar{\eta}) \end{bmatrix}.$$

From (4a), we have

$$\int_{t-h(t)}^{t} \begin{bmatrix} X \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} \begin{bmatrix} X \\ \dot{x}(s) \end{bmatrix} ds = h(t)\cdot X^T R_{11} X + 2X^T R_{12}[x(t) - x(t-h(t))] + \int_{t-h(t)}^{t} \dot{x}^T(s) R_{22} \dot{x}(s) ds \ge 0, \tag{8a}$$

$$\int_{t-\bar{h}}^{t-h(t)} \begin{bmatrix} X \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} \hat{R}_{11} & \hat{R}_{12} \\ * & \hat{R}_{22} \end{bmatrix} \begin{bmatrix} X \\ \dot{x}(s) \end{bmatrix} ds = (\bar{h} - h(t))\cdot X^T \hat{R}_{11} X + 2X^T \hat{R}_{12}[x(t-h(t)) - x(t-\bar{h})] + \int_{t-\bar{h}}^{t-h(t)} \dot{x}^T(s) \hat{R}_{22} \dot{x}(s) ds \ge 0, \tag{8b}$$

$$\int_{t-\eta(t)}^{t} \begin{bmatrix} X \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} \begin{bmatrix} X \\ \dot{x}(s) \end{bmatrix} ds = \eta(t)\cdot X^T S_{11} X + 2X^T S_{12}[x(t) - x(t-\eta(t))] + \int_{t-\eta(t)}^{t} \dot{x}^T(s) S_{22} \dot{x}(s) ds \ge 0, \tag{8c}$$

$$\int_{t-\bar{\eta}}^{t-\eta(t)} \begin{bmatrix} X \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} \hat{S}_{11} & \hat{S}_{12} \\ * & \hat{S}_{22} \end{bmatrix} \begin{bmatrix} X \\ \dot{x}(s) \end{bmatrix} ds = (\bar{\eta} - \eta(t))\cdot X^T \hat{S}_{11} X + 2X^T \hat{S}_{12}[x(t-\eta(t)) - x(t-\bar{\eta})] + \int_{t-\eta}^{t-\eta(t)} \dot{x}^T(s) \hat{S}_{22} \dot{x}(s) ds \ge 0, \tag{8d}$$

where $\hat{P} = diag\begin{bmatrix} \bar{P}^{-1} & \bar{P}^{-1} & \bar{P}^{-1} & \bar{P}^{-1} & \bar{P}^{-1} & \bar{P}^{-1} \end{bmatrix}$,

$$\begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} = \hat{P} \begin{bmatrix} \bar{R}_{11} & \bar{R}_{12} \\ * & \bar{R}_{22} \end{bmatrix} \hat{P} > 0, \quad \begin{bmatrix} \hat{R}_{11} & \hat{R}_{12} \\ * & \hat{R}_{22} \end{bmatrix} = \hat{P} \begin{bmatrix} \bar{\hat{R}}_{11} & \bar{\hat{R}}_{12} \\ * & \bar{\hat{R}}_{22} \end{bmatrix} \hat{P} > 0,$$

$$\begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} = \hat{P} \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} \\ * & \bar{S}_{22} \end{bmatrix} \hat{P} > 0, \quad \begin{bmatrix} \hat{S}_{11} & \hat{S}_{12} \\ * & \hat{S}_{22} \end{bmatrix} = \hat{P} \begin{bmatrix} \bar{\hat{S}}_{11} & \bar{\hat{S}}_{12} \\ * & \bar{\hat{S}}_{22} \end{bmatrix} \hat{P} > 0.$$

From condition (2f), we have

$$x^T(t)\Gamma^T\Gamma x(t) + x^T(t-h(t))\Lambda^T\Lambda x(t-h(t)) + \dot{x}^T(t-\tau)\theta^T\theta\dot{x}(t-\tau) - f^T f \ge 0. \tag{9}$$

From the system (1) with (2a) and $u(t) = -Kx(t)$, we have

$$\dot{V}(x_t) + \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}^T \cdot \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^T & \Pi_{22} \end{bmatrix} \cdot \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}$$

$$\le [A_0 x(t) + A_1 x(t-h(t)) + A_2 \dot{x}(t-\tau) - B_u RKx(t-\eta(t)) + B_w w(t) + f]^T Px(t)$$

$$+ x^T(t)P[A_0 x(t) + A_1 x(t-h(t)) + A_2 \dot{x}(t-\tau) - B_u RKx(t-\eta(t)) + B_w w(t) + f]$$

$$+ x^T(t)Q_1 x(t) - x^T(t-\bar{h})Q_1 x(t-\bar{h}) + \bar{h}\cdot\dot{x}^T(t)Q_2\dot{x}(t) - \int_{t-\bar{h}}^{t} \dot{x}^T(s)Q_2\dot{x}(s)ds$$

$$
\begin{aligned}
&+x^T(t)Q_3x(t)-x^T(t-\overline{\eta})Q_3x(t-\overline{\eta})+\overline{\eta}\cdot\dot{x}^T(t)Q_4\dot{x}(t)-\int_{t-\overline{\eta}}^{t}\dot{x}^T(s)Q_4\dot{x}(s)ds \\
&+\dot{x}^T(t)Q_5\dot{x}(t)-\dot{x}^T(t-\tau)Q_5\dot{x}(t-\tau) \\
&+\left[C_0x(t)+C_1x(t-h(t))+C_2\dot{x}(t-\tau)-D_uRKx(t-\eta(t))+D_ww(t)\right]^T\Pi_{11}\cdot \\
&\qquad\qquad \left[C_0x(t)+C_1x(t-h(t))+C_2\dot{x}(t-\tau)-D_uRKx(t-\eta(t))+D_ww(t)\right] \\
&+2w^T(t)\Pi_{12}^T\left[C_0x(t)+C_1x(t-h(t))+C_2\dot{x}(t-\tau)-D_uRKx(t-\eta(t))+D_ww(t)\right] \\
&+w^T(t)\Pi_{22}w(t) \\
&+h(t)\cdot X^TR_{11}X+2X^TR_{12}\left[x(t)-x(t-h(t))\right]+\int_{t-h(t)}^{t}\dot{x}^T(s)R_{22}\dot{x}(s)ds \\
&+\left(\overline{h}-h(t)\right)\cdot X^T\hat{R}_{11}X+2X^T\hat{R}_{12}\left[x(t-h(t))-x(t-\overline{h})\right]+\int_{t-\overline{h}}^{t-h(t)}\dot{x}^T(s)\hat{R}_{22}\dot{x}(s)ds \\
&+\eta(t)\cdot X^TS_{11}X+2X^TS_{12}\left[x(t)-x(t-\eta(t))\right]+\int_{t-\eta(t)}^{t}\dot{x}^T(s)S_{22}\dot{x}(s)ds \\
&+\left(\overline{\eta}-\eta(t)\right)\cdot X^T\hat{S}_{11}X+2X^T\hat{S}_{12}\left[x(t-\eta(t))-x(t-\overline{\eta})\right]+\int_{t-\eta}^{t-\eta(t)}\dot{x}^T(s)\hat{S}_{22}\dot{x}(s)ds \\
&-2\rho\cdot\dot{x}^T(t)P\dot{x}(t) \\
&+\rho\cdot\dot{x}^T(t)P\cdot\left[A_0x(t)+A_1x(t-h(t))+A_2\dot{x}(t-\tau)-B_uRKx(t-\eta(t))+B_ww(t)+f\right] \\
&+\rho\cdot\left[A_0x(t)+A_1x(t-h(t))+A_2\dot{x}(t-\tau)-B_uRKx(t-\eta(t))+B_ww(t)+f\right]^TP\cdot\dot{x}(t) \\
&+\varepsilon^{-1}\cdot\left(x^T(t)\Gamma^T\Gamma x(t)+x^T(t-h(t))\Lambda^T\Lambda x(t-h(t))+\dot{x}^T(t-\tau)\theta^T\theta\dot{x}(t-\tau)-f^Tf\right) \\
&=Z^T\Sigma Z-\int_{t-h(t)}^{t}\dot{x}^T(s)(Q_2-R_{22})\dot{x}(s)ds-\int_{t-\overline{h}}^{t-h(t)}\dot{x}^T(s)\left(Q_2-\hat{R}_{22}\right)\dot{x}(s)ds \\
&-\int_{t-\eta(t)}^{t}\dot{x}^T(s)(Q_4-S_{22})\dot{x}(s)ds-\int_{t-\overline{\eta}}^{t-\eta(t)}\dot{x}^T(s)\left(Q_4-\hat{S}_{22}\right)\dot{x}(s)ds \\
&-h(t)\cdot X^T\left(\hat{R}_{11}-R_{11}\right)X-\eta(t)\cdot X^T\left(\hat{S}_{11}-S_{11}\right)X,
\end{aligned}
\tag{10a}
$$

where

$$
Z^T=\begin{bmatrix}x^T(t) & x^T(t-h(t)) & x^T(t-\overline{h}) & x^T(t-\eta(t)) & x^T(t-\overline{\eta}) & \dot{x}^T(t) & \dot{x}^T(t-\tau) & w^T(t) & f^T\end{bmatrix},
$$

$$
\Sigma=\begin{bmatrix}
\Sigma_{11} & \Sigma_{12} & 0 & \Sigma_{14} & 0 & \Sigma_{16} & \Sigma_{17} & \Sigma_{18} & P \\
* & \Sigma_{22} & 0 & 0 & 0 & \Sigma_{26} & 0 & \Sigma_{28} & 0 \\
* & * & \Sigma_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & \Sigma_{46} & 0 & \Sigma_{48} & 0 \\
* & * & * & * & \Sigma_{55} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Sigma_{66} & \Sigma_{67} & \Sigma_{68} & \rho\cdot P \\
* & * & * & * & * & * & \Sigma_{77} & \Sigma_{78} & 0 \\
* & * & * & * & * & * & * & \Sigma_{88} & 0 \\
* & * & * & * & * & * & * & * & -\varepsilon^{-1}\cdot I
\end{bmatrix}+\begin{bmatrix}
\Delta_{11} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$+\begin{bmatrix} C_0^T \\ C_1^T \\ 0 \\ -K^T R^T D_u^T \\ 0 \\ 0 \\ C_2^T \\ D_w^T \\ 0 \end{bmatrix} \left(\Pi_{11}^{-1}\right)^{-1} \begin{bmatrix} C_0 & C_1 & 0 & -D_u RK & 0 & 0 & C_2 & D_w & 0 \end{bmatrix}, \tag{10b}$$

$\Delta_{11} = R_{12}\Delta_1 + \Delta_1^T R_{12}^T + \hat{R}_{12}\hat{\Delta}_1 + \hat{\Delta}_1^T \hat{R}_{12}^T + S_{12}\Delta_2 + \Delta_2^T S_{12}^T + \hat{S}_{12}\hat{\Delta}_2 + \hat{\Delta}_2^T \hat{S}_{12}^T + \overline{\eta} \cdot \hat{S}_{11} + \overline{h} \cdot \hat{R}_{11}$,

$\Delta_1 = \begin{bmatrix} I & -I & 0 & 0 & 0 \end{bmatrix}$, $\hat{\Delta}_1 = \begin{bmatrix} 0 & I & -I & 0 & 0 \end{bmatrix}$,

$\Delta_2 = \begin{bmatrix} I & 0 & 0 & -I & 0 \end{bmatrix}$, $\hat{\Delta}_2 = \begin{bmatrix} 0 & 0 & 0 & I & -I \end{bmatrix}$,

$\Sigma_{11} = A_0^T P + PA_0 + Q_1 + Q_3 + \varepsilon^{-1} \cdot \Gamma^T \Gamma$, $\Sigma_{12} = PA_1$, $\Sigma_{14} = -PB_u RK$, $\Sigma_{16} = \rho \cdot A_0^T P$,

$\Sigma_{17} = PA_2$, $\Sigma_{18} = PB_w + C_0^T \Pi_{12}$, $\Sigma_{22} = \varepsilon^{-1} \cdot \Lambda^T \Lambda$, $\Sigma_{26} = \rho \cdot A_1^T P$, $\Sigma_{28} = C_1^T \Pi_{12}$,

$\Sigma_{33} = -Q_1$, $\Sigma_{46} = -\rho \cdot K^T R^T B_u^T P$, $\Sigma_{48} = -K^T R^T D_u^T \Pi_{12}$, $\Sigma_{55} = -Q_3$,

$\Sigma_{66} = \overline{h} \cdot Q_2 + \overline{\eta} \cdot Q_4 + Q_5 - 2\rho \cdot P$, $\Sigma_{67} = \rho \cdot PA_2$, $\Sigma_{68} = \rho \cdot PB_w$, $\Sigma_{77} = -Q_5 + \varepsilon^{-1} \cdot \theta^T \theta$,

$\Sigma_{78} = C_2^T \Pi_{12}$, $\Sigma_{88} = \Pi_{22} + D_w^T \Pi_{12} + \Pi_{12}^T D_w$.

Premultiplying and postmultiplying the matrix Σ in (10b) by

$$\mathrm{diag}\begin{bmatrix} P^{-1} & P^{-1} & P^{-1} & P^{-1} & P^{-1} & P^{-1} & P^{-1} & I & \varepsilon \cdot I \end{bmatrix} > 0$$

with $\overline{P} = P^{-1}$, we can obtain the following matrix with (3e), $\hat{K} = K\overline{P}$, $\overline{Q}_i = \overline{P} Q_i \overline{P}$, $i = 1, \cdots, 5$,

$$\overline{\Sigma} = \begin{bmatrix} \overline{\Sigma}_{11} & \overline{\Sigma}_{12} & 0 & \overline{\Sigma}_{14} & 0 & \overline{\Sigma}_{16} & \overline{\Sigma}_{17} & \overline{\Sigma}_{18} & \varepsilon \cdot I \\ * & \overline{\Sigma}_{22} & 0 & 0 & 0 & \overline{\Sigma}_{26} & 0 & \overline{\Sigma}_{28} & 0 \\ * & * & \overline{\Sigma}_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & \overline{\Sigma}_{46} & 0 & \overline{\Sigma}_{48} & 0 \\ * & * & * & * & \overline{\Sigma}_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \overline{\Sigma}_{66} & \overline{\Sigma}_{67} & \overline{\Sigma}_{68} & \rho \cdot \varepsilon \cdot I \\ * & * & * & * & * & * & \overline{\Sigma}_{77} & \overline{\Sigma}_{78} & 0 \\ * & * & * & * & * & * & * & \overline{\Sigma}_{88} & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon \cdot I \end{bmatrix} + \begin{bmatrix} \overline{\Delta}_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$+\begin{bmatrix} \overline{P}C_0^T \\ \overline{P}C_1^T \\ 0 \\ -\hat{K}^T R^T D_u^T \\ 0 \\ 0 \\ \overline{P}C_2^T \\ D_w^T \\ 0 \end{bmatrix} \left(\Pi_{11}^{-1}\right)^{-1} \begin{bmatrix} C_0\overline{P} & C_1\overline{P} & 0 & -D_u R\hat{K} & 0 & 0 & C_2\overline{P} & D_w & 0 \end{bmatrix}, \tag{11}$$

$$\overline{\Delta}_{11} = \overline{R}_{12}\overline{\Delta}_1 + \overline{\Delta}_1^T\overline{R}_{12}^T + \hat{\overline{R}}_{12}\hat{\overline{\Delta}}_1 + \hat{\overline{\Delta}}_1^T\hat{\overline{R}}_{12}^T + \overline{S}_{12}\overline{\Delta}_2 + \overline{\Delta}_2^T\overline{S}_{12}^T + \hat{\overline{S}}_{12}\hat{\overline{\Delta}}_2 + \hat{\overline{\Delta}}_2^T\hat{\overline{S}}_{12}^T + \overline{\eta}\cdot\hat{\overline{S}}_{11} + \overline{h}\cdot\hat{\overline{R}}_{11},$$

$$\overline{\Delta}_1 = \begin{bmatrix} I & -I & 0 & 0 & 0 \end{bmatrix},\ \hat{\overline{\Delta}}_1 = \begin{bmatrix} 0 & I & -I & 0 & 0 \end{bmatrix},$$

$$\overline{\Delta}_2 = \begin{bmatrix} I & 0 & 0 & -I & 0 \end{bmatrix},\ \hat{\overline{\Delta}}_2 = \begin{bmatrix} 0 & 0 & 0 & I & -I \end{bmatrix},$$

$$\overline{\Sigma}_{11} = \overline{P}A_0^T + A_0\overline{P} + \overline{Q}_1 + \overline{Q}_3 + \varepsilon^{-1}\cdot\overline{P}\Gamma^T\Gamma\overline{P},\ \overline{\Sigma}_{12} = A_1\overline{P},\ \overline{\Sigma}_{14} = -B_u R\hat{K},$$

$$\overline{\Sigma}_{16} = \rho\cdot\overline{P}A_0^T,\ \overline{\Sigma}_{17} = A_2\overline{P},\ \overline{\Sigma}_{18} = B_w + \overline{P}C_0^T\Pi_{12},\ \overline{\Sigma}_{22} = \varepsilon^{-1}\cdot\overline{P}\Lambda^T\Lambda\overline{P},$$

$$\overline{\Sigma}_{26} = \rho\cdot\overline{P}A_1^T,\ \overline{\Sigma}_{28} = \overline{P}C_1^T\Pi_{12},\ \overline{\Sigma}_{33} = -\overline{Q}_1,\ \overline{\Sigma}_{46} = -\rho\cdot\hat{K}^T R^T B_u^T,$$

$$\overline{\Sigma}_{48} = -\hat{K}^T R^T D_u^T\Pi_{12},\ \overline{\Sigma}_{55} = -\overline{Q}_3,$$

$$\overline{\Sigma}_{66} = \overline{h}\cdot\overline{Q}_2 + \overline{\eta}\cdot\overline{Q}_4 + \overline{Q}_5 - 2\rho\cdot\overline{P},\ \overline{\Sigma}_{67} = \rho\cdot A_2\overline{P},\ \overline{\Sigma}_{68} = \rho\cdot B_w,$$

$$\overline{\Sigma}_{77} = -\overline{Q}_5 + \varepsilon^{-1}\cdot\overline{P}\theta^T\theta\overline{P},\ \overline{\Sigma}_{78} = \overline{P}C_2^T\Pi_{12},\ \overline{\Sigma}_{88} = \Pi_{22} + D_w^T\Pi_{12} + \Pi_{12}^T D_w.$$

Define the matrix

$$\hat{\Sigma} = \begin{bmatrix} \overline{\Sigma}_{11} & \overline{\Sigma}_{12} & 0 & \hat{\Sigma}_{14} & 0 & \overline{\Sigma}_{16} & \overline{\Sigma}_{17} & \overline{\Sigma}_{18} & \varepsilon\cdot I & \overline{P}C_0^T \\ * & \overline{\Sigma}_{22} & 0 & 0 & 0 & \overline{\Sigma}_{26} & 0 & \overline{\Sigma}_{28} & 0 & \overline{P}C_1^T \\ * & * & \overline{\Sigma}_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & \hat{\Sigma}_{46} & 0 & \hat{\Sigma}_{48} & 0 & \hat{\Sigma}_{410} \\ * & * & * & * & \overline{\Sigma}_{55} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \overline{\Sigma}_{66} & \overline{\Sigma}_{67} & \overline{\Sigma}_{68} & \rho\cdot\varepsilon\cdot I & 0 \\ * & * & * & * & * & * & \overline{\Sigma}_{77} & \overline{\Sigma}_{78} & 0 & \overline{P}C_2^T \\ * & * & * & * & * & * & * & \overline{\Sigma}_{88} & 0 & D_w^T \\ * & * & * & * & * & * & * & * & -\varepsilon\cdot I & 0 \\ * & * & * & * & * & * & * & * & * & -\Pi_{11}^{-1} \end{bmatrix}$$

$$+\begin{bmatrix} \bar{\Delta}_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} B_u R_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \rho B_u R_1 \\ 0 \\ \Pi_{12}^T D_u R_1 \\ 0 \\ D_u R_1 \end{bmatrix} \Delta J \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\hat{K}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\hat{K}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta J^T \begin{bmatrix} B_u R_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \rho B_u R_1 \\ 0 \\ \Pi_{12}^T D_u R_1 \\ 0 \\ D_u R_1 \end{bmatrix}^T, \tag{12}$$

where

$$\hat{\Sigma}_{14} = -B_u R_0 \hat{K}, \ \hat{\Sigma}_{46} = -\rho \cdot \hat{K}^T R_0^T B_u^T, \ \hat{\Sigma}_{48} = -\hat{K}^T R_0^T D_u^T \Pi_{12}, \ \hat{\Sigma}_{410} = -\hat{K}^T R_0^T D_u^T.$$

By Lemmas 2.1-2.2, we get that $\Omega < 0$ in (4c) is equivalent to $\hat{\Sigma} < 0$ in (12). By Lemma 2.2, $\hat{\Sigma} < 0$ is equivalent to $\bar{\Sigma} < 0$ in (11). Condition $\bar{\Sigma} < 0$ is also equivalent to $\Sigma < 0$ in (10b). From (8), (9), (10a), and $\Sigma < 0$ with $w(t) = 0$, there exists a constant $\lambda > 0$ satisfying

$$\dot{V}(x(t))\Big|_{w(t)=0} + z^T(t)\Pi_{11} z(t) \le -\lambda \cdot \|x(t)\|^2.$$

With $\Pi_{11} > 0$, we obtain the following condition:

$$\dot{V}(x(t))\Big|_{w(t)=0} \le -\lambda \cdot \|x(t)\|^2.$$

Hence, the closed system (1)-(2) with $u(t) = -Kx(t) = -\hat{K}\bar{P}^{-1}x(t)$ and $w(t) = 0$ is asymptotically stable [16], [17].

Integrating the function in (10a) from 0 to ∞ and from $\Sigma < 0$, we have

$$\lim_{t\to\infty} V(x(t)) - V(x_0) + \int_0^\infty \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^T & \Pi_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix} dt \le 0.$$

With the zero initial condition ($x_0 = 0$), we have

$$V(x_0) = 0, \ \lim_{t\to\infty} V(x(t)) \ge 0,$$

and

$$\int_0^\infty \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^T & \Pi_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix} dt \le 0, \ \forall\, w \in L_2[0,\infty), w \ne 0.$$

By the Definition 2.1, the system (1)-(2) satisfies IQC performance.

If the constaints of time-varying delays are given by:

$$0 \le h(t) \le \bar{h}\,,\ 0 \le \eta(t) \le \bar{\eta}\,,\ \dot{h}(t) \le h_D < 1\,,\ \dot{\eta}(t) \le \eta_D < 1\,, \tag{13}$$

where $\bar{h}$, $\bar{\eta}$, h_D, and η_D are some given constants. Then Theorem 2.1 can be rewritten as the following result:

Corollary 2.1. For a constant $\rho > 0$, let there exist two constants $\varepsilon > 0$, $\sigma > 0$, some $n \times n$ positive-definite symmetric matrices $\bar{P}$, $\bar{Q}_i$, $i = 1, 2, \cdots, 7$, $\bar{R}_{22}$, $\hat{\bar{R}}_{22}$, $\bar{S}_{22}$, $\hat{\bar{S}}_{22}$, some $5n \times 5n$ positive-definite symmetric matrices $\bar{R}_{11}$, $\hat{\bar{R}}_{11}$, $\bar{S}_{11}$, $\hat{\bar{S}}_{11}$, and some matrices $\bar{R}_{12}$, $\hat{\bar{R}}_{12}$, $\bar{S}_{12}$, $\hat{\bar{S}}_{12} \in \Re^{5n \times n}$, $\hat{K} \in \Re^{m \times n}$, such that LMI conditions (4a)-(4b) and the following condition are satisfied:

$$\hat{\Omega}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w) =$$

$$\begin{bmatrix}
\hat{\Omega}_{11} & \Omega_{12} & 0 & \Omega_{14} & 0 & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} & \Omega_{110} & \Omega_{111} & 0 & \Omega_{113} & 0 & 0 \\
* & \hat{\Omega}_{22} & 0 & 0 & 0 & \Omega_{26} & 0 & \Omega_{28} & 0 & \Omega_{210} & 0 & 0 & 0 & \Omega_{214} & 0 \\
* & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \hat{\Omega}_{44} & 0 & \Omega_{46} & 0 & \Omega_{48} & 0 & \Omega_{410} & 0 & \Omega_{412} & 0 & 0 & 0 \\
* & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Omega_{66} & \Omega_{67} & \Omega_{68} & \Omega_{69} & 0 & \Omega_{611} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Omega_{77} & \Omega_{78} & 0 & \Omega_{710} & 0 & 0 & 0 & 0 & \Omega_{715} \\
* & * & * & * & * & * & * & \Omega_{88} & 0 & \Omega_{810} & \Omega_{811} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Omega_{99} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Omega_{1010} & \Omega_{1011} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & \Omega_{1111} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & \Omega_{1212} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & \Omega_{1313} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & \Omega_{1414} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & \Omega_{1515}
\end{bmatrix}$$

$$+\begin{bmatrix}
\bar{\Delta}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} < 0\,, \tag{14}$$

where

$$\hat{\Omega}_{11}=\overline{P}A_0^T+A_0\overline{P}+\overline{Q}_1+\overline{Q}_3+\overline{Q}_6+\overline{Q}_7,\ \hat{\Omega}_{22}=-(1-h_D)\cdot\overline{Q}_6,\ \hat{\Omega}_{44}=-(1-\eta_D)\cdot\overline{Q}_7,$$

other matrices Ω_{ij}, $i,\ j=1,2,\cdots,15$, and $\overline{\Delta}_{11}$ are defined in Theorem 2.1. Then, the system (1) with (2) and (13) is asymptotically stabilizable via the reliable control $u(t)=-Kx(t)=-\hat{K}\overline{P}^{-1}x(t)$ with IQC performance.

Proof. Define the Lyapunov functional as follows:

$$\overline{V}(x_t)=V(x_t)+\int_{t-h(t)}^{t}x^T(s)Q_6x(s)ds+\int_{t-\eta(t)}^{t}x^T(s)Q_7x(s)ds,$$

where $V(x_t)$ is defined in (5), $Q_i=\overline{P}^{-1}\overline{Q}_i\overline{P}^{-1}>0$, $i=6,7$. This proof is same as Theorem 2.1.

3. RELIABLE CONTROL FOR UNCERTAIN NEUTRAL SYSTEMS WITH IQC PERFORMANCE

Consider the following uncertain neutral system with state and input time-varying delays:

$$\dot{x}(t)=A_0(t)x(t)+A_1(t)x(t-h(t))+A_2(t)\dot{x}(t-\tau)+B_u(t)u^f(t-\eta(t))+B_w(t)w(t)$$
$$+f(x(t),x(t-h(t)),\dot{x}(t-\tau)),\ t\geq 0, \tag{15a}$$

$$x(t)=\phi(t),\ t\in[-H,0], \tag{15b}$$

$$z(t)=C_0(t)x(t)+C_1(t)x(t-h(t))+C_2(t)\dot{x}(t-\tau)+D_u(t)u^f(t-\eta(t))+D_w(t)w(t),\ t\geq 0, \tag{15c}$$

where

$A_0(t)=A_0+\Delta A_0(t)$, $A_1(t)=A_1+\Delta A_1(t)$, $A_2(t)=A_2+\Delta A_2(t)$, $B_u(t)=B_u+\Delta B_u(t)$,

$B_w(t)=B_w+\Delta B_w(t)$, $C_0(t)=C_0+\Delta C_0(t)$, $C_1(t)=C_1+\Delta C_1(t)$, $C_2(t)=C_2+\Delta C_2(t)$,

$D_u(t)=D_u+\Delta D_u(t)$, $D_w(t)=D_w+\Delta D_w(t)$,

A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w, are some given constant matrices, $\Delta A_0(t)$, $\Delta A_1(t)$, $\Delta A_2(t)$, $\Delta B_u(t)$, $\Delta B_w(t)$, $\Delta C_0(t)$, $\Delta C_1(t)$, $\Delta C_2(t)$, $\Delta D_u(t)$, $\Delta D_w(t)$, are some time-varying functions satisfying

$$\begin{bmatrix}\Delta A_0(t) & \Delta A_1(t) & \Delta A_2(t) & \Delta B_u(t) & \Delta B_w(t)\\ \Delta C_0(t) & \Delta C_1(t) & \Delta C_2(t) & \Delta D_u(t) & \Delta D_w(t)\end{bmatrix}$$

$$=\begin{bmatrix}M_x\\ M_z\end{bmatrix}\cdot F(t)\cdot\begin{bmatrix}N_0 & N_1 & N_2 & N_3 & N_4\end{bmatrix}, \tag{15d}$$

M_x, M_z, N_i, $i=0,1,2,3,4$, are some given constant matrices, $F(t)$ is real time-varying function with appropriate dimensions and bounded as follows:

$$F(t)^T\cdot F(t)\leq I,\ \forall\ t\geq 0.$$

With the result of Theorem 2.1 and by comparing system (1) and system (15), a result to design the robust reliable control $u(t)=-Kx(t)$ with IQC performance for system (15) is presented.

Theorem 3.1. For a constant $\rho > 0$, let there exist some constants $\varepsilon > 0$, $\sigma > 0$, $\mu > 0$, some $n \times n$ positive-definite symmetric matrices $\overline{P}$, $\overline{Q}_i$, $i = 1, 2, \cdots, 5$, $\overline{R}_{22}$, $\hat{\overline{R}}_{22}$, $\overline{S}_{22}$, $\hat{\overline{S}}_{22}$, some $5n \times 5n$ positive-definite symmetric matrices $\overline{R}_{11}$, $\hat{\overline{R}}_{11}$, $\overline{S}_{11}$, $\hat{\overline{S}}_{11}$, and some matrices $\overline{R}_{12}$, $\hat{\overline{R}}_{12}$, $\overline{S}_{12}$, $\hat{\overline{S}}_{12} \in \Re^{5n \times n}$, $\hat{K} \in \Re^{m \times n}$, such that that (4a)-(4b) and following LMI conditions are satisfied:

$$\begin{bmatrix} \Omega(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w) & \Lambda_1 \\ * & \Lambda_2 \end{bmatrix} < 0, \tag{16a}$$

where $\Omega(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w)$ is defined in (4c) and

$$\Lambda_1 = \begin{bmatrix} \mu \cdot M_x & \overline{P} N_0^T \\ 0 & \overline{P} N_1^T \\ 0 & 0 \\ 0 & -\hat{K}^T R_0^T N_3^T \\ 0 & 0 \\ \mu \cdot \rho \cdot M_x & 0 \\ 0 & \overline{P} N_2^T \\ \mu \cdot \Pi_{12}^T M_z & N_4^T \\ 0 & 0 \\ \mu \cdot M_z & 0 \\ 0 & \sigma \cdot R_1^T N_3^T \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{16b}$$

$$\Lambda_2 = \begin{bmatrix} -\mu \cdot I & 0 \\ 0 & -\mu \cdot I \end{bmatrix}. \tag{16c}$$

Then, the system (15) with (2) is asymptotically stabilizable by the reliable control $u(t) = -Kx(t) = -\hat{K}\overline{P}^{-1}x(t)$ with IQC performance.

Proof: From the systems (1) and (15) with Theorem 2.1, a sufficient condition to design the reliable control with IQC performance for the system (15) is given by

$$\Omega(A_0(t), A_1(t), A_2(t), B_u(t), B_w(t), C_0(t), C_1(t), C_2(t), D_u(t), D_w(t))$$
$$= \Omega(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w) + \Xi_1 F(t) \Xi_2^T + \Xi_2 F^T(t) \Xi_1^T,$$

where

$$\Xi_1^T = \begin{bmatrix} M_x^T & 0 & 0 & 0 & 0 & \rho \cdot M_x^T & 0 & M_z^T \Pi_{12} & 0 & M_z^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_2^T = \begin{bmatrix} N_0 \overline{P} & N_1 \overline{P} & 0 & -N_3 R_0 \hat{K} & 0 & 0 & N_2 \overline{P} & N_4 & 0 & 0 & \sigma \cdot N_3 R_1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

By Lemmas 2.1 and 2.2, condition (16) is equivalent to

$$\Omega(A_0(t), A_1(t), A_2(t), B_u(t), B_w(t), C_0(t), C_1(t), C_2(t), D_u(t), D_w(t)) < 0 .$$

The proof is completed.

Corollary 3.1. For a constant $\rho > 0$, let there exist some constants $\varepsilon > 0$, $\sigma > 0$, $\mu > 0$, some $n \times n$ positive-definite symmetric matrices $\overline{P}$, $\overline{Q}_i$, $i = 1, 2, \cdots, 7$, $\overline{R}_{22}$, $\hat{\overline{R}}_{22}$, $\overline{S}_{22}$, $\hat{\overline{S}}_{22}$, some $5n \times 5n$ positive-definite symmetric matrices $\overline{R}_{11}$, $\hat{\overline{R}}_{11}$, $\overline{S}_{11}$, $\hat{\overline{S}}_{11}$, and some matrices $\overline{R}_{12}$, $\hat{\overline{R}}_{12}$, $\overline{S}_{12}$, $\hat{\overline{S}}_{12} \in \Re^{5n \times n}$, $\hat{K} \in \Re^{m \times n}$, such that LMI conditions (4a)-(4b) and the following condition are satisfied:

$$\begin{bmatrix} \hat{\Omega}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w) & \Lambda_1 \\ * & \Lambda_2 \end{bmatrix} < 0,$$

where $\hat{\Omega}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w)$ is defined in (14), Λ_1 and Λ_2 are defined in (16b)-(16c). Then, the system (15) with (2) and (13) is asymptotically stabilizable via the reliable control $u(t) = -Kx(t) = -\hat{K}\overline{P}^{-1}x(t)$ with IQC performance.

In many practical systems, the actuator or sensor is working either in normal or completely faulty conditions [8], [10]. When the actuator or sensor has some failures, the possible fault matrices R in (2a) can be rewritten as

$$R_i = \mathrm{diag}[r_{1i}, r_{2i}, \cdots, r_{mi}],\ r_{ji} = 0 \text{ or } r_{ji} = 1,\ j = 1, 2, \cdots, m,\ i = 1, 2, \cdots, N, \tag{17}$$

where r_{ji}, $j = 1, 2, \cdots, m$, $i = 1, 2, \cdots, N$, are some given constants, but $r_{ji} = 0$, for all $j = 1, 2, \cdots, m$ and some $i = 1, 2, \cdots, N$ (no input) are not allowed.

With the results of Theorems 2.1-2.2, the robust reliable control $u(t) = -Kx(t)$ with IQC performance for system (15) with (17) is provided in the following result.

Theorem 3.2. For a constant ρ > 0, let there exist two constants ε > 0, μ > 0, some $n \times n$ positive-definite symmetric matrices $\overline{P}$, $\overline{Q}_i$, $i = 1, 2, \cdots, 5$, $\overline{R}_{22}$, $\hat{\overline{R}}_{22}$, $\overline{S}_{22}$, $\hat{\overline{S}}_{22}$, some $5n \times 5n$ positive-definite symmetric matrices $\overline{R}_{11}$, $\hat{\overline{R}}_{11}$, $\overline{S}_{11}$, $\hat{\overline{S}}_{11}$, and some matrices $\overline{R}_{12}$, $\hat{\overline{R}}_{12}$, $\overline{S}_{12}$, $\hat{\overline{S}}_{12} \in \Re^{5n \times n}$, $\hat{K} \in \Re^{m \times n}$, such that that (4a)-(4b) and following LMI conditions are satisfied:

$$\begin{bmatrix} \tilde{\Omega}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w, R_i) & \tilde{\Lambda}_{1i} \\ * & \Lambda_2 \end{bmatrix} < 0,\ i = 0, 1, 2, \cdots, N, \tag{18a}$$

where Λ_2 is defined in (16c), $R_0 = I$, and

$$\tilde{\Lambda}_{1i} = \begin{bmatrix} \mu \cdot M_x & \overline{P} N_0^T \\ 0 & \overline{P} N_1^T \\ 0 & 0 \\ 0 & -\hat{K}^T R_i^T N_3^T \\ 0 & 0 \\ \mu \cdot \rho \cdot M_x & 0 \\ 0 & \overline{P} N_2^T \\ \mu \cdot \Pi_{12}^T M_z & N_4^T \\ 0 & 0 \\ \mu \cdot M_z & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{18b}$$

$$\tilde{\Omega}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w, R_i) =$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & 0 & \tilde{\Omega}_{14} & 0 & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} & \Omega_{110} & \tilde{\Omega}_{111} & 0 & 0 \\ * & 0 & 0 & 0 & 0 & \Omega_{26} & 0 & \Omega_{28} & 0 & \Omega_{210} & 0 & \tilde{\Omega}_{212} & 0 \\ * & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & \tilde{\Omega}_{46} & 0 & \tilde{\Omega}_{48} & 0 & \tilde{\Omega}_{410} & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & \Omega_{67} & \Omega_{68} & \Omega_{69} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Omega_{77} & \Omega_{78} & 0 & \Omega_{710} & 0 & 0 & \tilde{\Omega}_{713} \\ * & * & * & * & * & * & * & \Omega_{88} & 0 & \Omega_{810} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Omega_{99} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Omega_{1010} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \tilde{\Omega}_{1111} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \tilde{\Omega}_{1212} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & \tilde{\Omega}_{1313} \end{bmatrix}$$

$$+ \begin{bmatrix} \overline{\Delta}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Omega}_{111} = \overline{P}\Gamma^T,\ \tilde{\Omega}_{212} = \overline{P}\Lambda^T,\ \tilde{\Omega}_{713} = \overline{P}\theta^T,\ \tilde{\Omega}_{1111} = \tilde{\Omega}_{1212} = \tilde{\Omega}_{1313} = -\varepsilon \cdot I,$$
$$\tilde{\Omega}_{14} = -B_u R_i \hat{K},\ \tilde{\Omega}_{46} = -\rho \cdot \hat{K}^T R_i^T B_u^T,\ \tilde{\Omega}_{48} = -\hat{K}^T R_i^T D_u^T \Pi_{12},\ \tilde{\Omega}_{410} = -\hat{K}^T R_i^T D_u^T,$$

other matrices Ω_{ij}, $i, j = 1, 2, \cdots, 15$, and $\overline{\Delta}_{11}$ are defined in Theorem 2.1. Then, the system (15) with (2a), (2f), (17) is asymptotically stabilizable by the reliable control $u(t) = -Kx(t) = -\hat{K}\overline{P}^{-1}x(t)$ with IQC performance.

Proof: Under the vertex conditions in (17), we have the following convex combination for the matrix R in (2a):

$$R = \sum_{i=0}^{N} \alpha_i \cdot R_i,\ \forall \sum_{i=0}^{N} \alpha_i = 1.$$

From the conditions (18), we have

$$\begin{bmatrix} \tilde{\Omega}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w, R) & \tilde{\Lambda}_1 \\ * & \Lambda_2 \end{bmatrix}$$
$$= \sum_{i=0}^{N} \alpha_i \cdot \begin{bmatrix} \tilde{\Omega}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w, R_i) & \tilde{\Lambda}_1 \\ * & \Lambda_2 \end{bmatrix} < 0.$$

From the spirit of Theorems 2.1 and 3.1, this will imply that the asymptotic stabilization for the reliable control of the system with IQC performance under normal and failure situations has been achieved.

Corollary 3.2: For a constant ρ > 0, let there exist two constants ε > 0, μ > 0, some $n\times n$ positive-definite symmetric matrices $\overline{P}$, $\overline{Q}_i$, $i = 1, 2, \cdots, 7$, $\overline{R}_{22}$, $\hat{\overline{R}}_{22}$, $\overline{S}_{22}$, $\hat{\overline{S}}_{22}$, some $5n\times 5n$ positive-definite symmetric matrices $\overline{R}_{11}$, $\hat{\overline{R}}_{11}$, $\overline{S}_{11}$, $\hat{\overline{S}}_{11}$, and some matrices $\overline{R}_{12}$, $\hat{\overline{R}}_{12}$, $\overline{S}_{12}$, $\hat{\overline{S}}_{12} \in \Re^{5n\times n}$, $\hat{K} \in \Re^{m\times n}$, such that LMI conditions (4a)-(4b) and the following condition are satisfied:

$$\begin{bmatrix} \tilde{\tilde{\Omega}}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, C_2, D_u, D_w, R_i) & \tilde{\Lambda}_1 \\ * & \Lambda_2 \end{bmatrix} < 0,\ i = 0, 1, 2, \cdots, N, \tag{19}$$

where $\tilde{\tilde{\Omega}}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, D_u, D_w, R_i)$ is defined by

$$\tilde{\tilde{\Omega}}(A_0, A_1, A_2, B_u, B_w, C_0, C_1, D_u, D_w, R_i) =$$
$$\begin{bmatrix}
\tilde{\Omega}_{11} & \Omega_{12} & 0 & \tilde{\Omega}_{14} & 0 & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} & \Omega_{110} & \tilde{\Omega}_{111} & 0 & 0 \\
* & \tilde{\Omega}_{22} & 0 & 0 & 0 & \Omega_{26} & 0 & \Omega_{28} & 0 & \Omega_{210} & 0 & \tilde{\Omega}_{212} & 0 \\
* & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \tilde{\Omega}_{44} & 0 & \tilde{\Omega}_{46} & 0 & \tilde{\Omega}_{48} & 0 & \tilde{\Omega}_{410} & 0 & 0 & 0 \\
* & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Omega_{66} & \Omega_{67} & \Omega_{68} & \Omega_{69} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Omega_{77} & \Omega_{78} & 0 & \Omega_{710} & 0 & 0 & \tilde{\Omega}_{713} \\
* & * & * & * & * & * & * & \Omega_{88} & 0 & \Omega_{810} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Omega_{99} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Omega_{1010} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & \tilde{\Omega}_{1111} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & \tilde{\Omega}_{1212} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & \tilde{\Omega}_{1313}
\end{bmatrix}$$

$$
+\begin{bmatrix} \overline{\Delta}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
$$

$$
\tilde{\Omega}_{11} = \overline{P}A_0^T + A_0\overline{P} + \overline{Q}_1 + \overline{Q}_3 + \overline{Q}_6 + \overline{Q}_7,\ \tilde{\Omega}_{22} = -(1-h_D)\cdot\overline{Q}_6,\ \tilde{\Omega}_{44} = -(1-\eta_D)\cdot\overline{Q}_7,
$$

other notations are defined in Theorem 3.1, $\tilde{\Lambda}_1$ and Λ_2 are defined in (18b) and (16c), respectively. Then, the system (15) with (2a), (2f), (13), (17) is asymptotically stabilizable via the reliable control $u(t) = -Kx(t) = -\hat{K}\overline{P}^{-1}x(t)$ with IQC performance.

4. NUMERICAL EXAMPLE

Consider the uncertain neutral system (15) with the following parameters:

$$
A_0 = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & -0.5 & 0 \\ 0 & 0 & 1 & 1 \\ 0.5 & 0 & 0.5 & -0.1 \end{bmatrix},\ A_1 = \begin{bmatrix} -0.3 & 0.2 & 0.3 & 0.1 \\ 0 & 0.2 & 0.3 & 0.3 \\ 0.1 & -0.2 & 0.2 & 0.2 \\ 0.2 & 0.1 & 0.2 & 0.3 \end{bmatrix},
$$

$$
A_2 = \alpha\cdot\begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 \\ 0 & 0.05 & -0.1 & 0.1 \\ 0.1 & -0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix},\ B_u = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 2 & 2 \\ -1 & 1 \end{bmatrix},\ B_w = \begin{bmatrix} 0.4 & 0.3 \\ 0.5 & -0.5 \\ 1 & -0.5 \\ 0.5 & 0.3 \end{bmatrix},
$$

$$
C_0 = \begin{bmatrix} 0 & 0.2 & 0 & 0.8 \\ 0 & 0 & 0 & 0.5 \end{bmatrix},\ C_1 = C_2 = 0,\ D_u = \begin{bmatrix} 0.8 & 0 \\ 1 & 0.7 \end{bmatrix},\ D_w = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix},
$$

$$
M_x = \begin{bmatrix} 0.1 \\ 0 \\ 0.2 \\ 0 \end{bmatrix},\ M_z = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},\ N_0 = \begin{bmatrix} 0.2 & 0.4 & 0.3 & 0.2 \end{bmatrix},\ N_1 = \begin{bmatrix} 0.1 & 0.1 & 0.2 & 0.1 \end{bmatrix},
$$

$$
N_2 = \beta\cdot\begin{bmatrix} 0.1 & 0.2 & 0.1 & 0.2 \end{bmatrix},\ N_3 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix},\ N_4 = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix},\ \Gamma = \Lambda = \theta = 0,
$$

$$
\tau > 0,\ \overline{\eta} = 0.125,\ \eta_D = 0,\ h_D = 0,\ \overline{h} = 2. \tag{20}
$$

The obtained results with α = β = 0.1 in this paper are formulated in the following.

(a) Consider the failure condition in (2), $0.6 \le r_1 \le 1$, $0.9 \le r_2 \le 1.1$, and the IQC performance bound

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^T & \Pi_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0.1 & 0.2 \\ 0 & 1 & 0.2 & 0.1 \\ 0.1 & 0.2 & -2 & 0.1 \\ 0.2 & 0.1 & 0.1 & -2 \end{bmatrix}. \tag{21}$$

From (2c)-(2d), we have

$$R_0 = \begin{bmatrix} 0.8 & 0 \\ 0 & 1 \end{bmatrix}, \ R_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

By using Corollary 3.1 with ρ = 0.22, the system (15) with (2) and (20) is asymptotically stabilizable by the robust reliable control

$$u(t) = -\hat{K}\overline{P}^{-1}x(t) = -\begin{bmatrix} 0.2084 & -0.7022 & 0.3597 & 0.1186 \\ 0.2418 & 0.6306 & 0.7472 & 0.7154 \end{bmatrix} x(t)$$

with IQC performance bound in (21).

(b) Consider the fault matrix $R_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ in (17) (the input $u_1(t)$ may completely fail in the future). From Corollary 3.2 with ρ = 0.27, the system (15) with (17) and (20) is asymptotically stabilizable by the robust reliable control

$$u(t) = -\hat{K}\overline{P}^{-1}x(t) = -\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4247 & -0.6866 & 2.3032 & 0.7418 \end{bmatrix} x(t)$$

with IQC performance bound in (21).

Even when in the special condition α = β = 0, the LMI conditions in our past results in Ref. 8 have no feasible solution for the above two cases. The results in Ref. 8 cannot be used for any τ > 0, Γ, Λ, θ, α, β ≠ 0.

5. CONCLUSION

In this paper, a robust reliable control with IQC performance for a class of uncertain neutral systems with state and input time-varying delays has been considered. The asymptotic stabilization has been guaranteed by our proposed reliable control via the LMI approach. A numerical example has been given to show the use of our results.

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