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Application of ESPRIT Algorithm for Seismic Signal Processing

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Abstract: The estimation of signal parameters like frequency of higher Resolution is an important task in signal processing. In several applications of signal processing like time series analysis, ESPRIT algorithm is used for the perfect frequency estimation with less computational efforts. In this paper, Total Least Squares Estimation version of ESPRIT is implemented for finding the frequency estimates of the seismic signals. ESPRIT employs a basic rotational invariance in the subspaces of the signal. This technique exhibits some computational advantages above earlier algorithms like MUSIC, MEM etc.

Keywords: Seismology, Power Spectral Density, Least Square Estimation, Digital Signal Processing, Applied statistics.

1. INTRODUCTION

1.1. Seismology

Seismology is the study of earth quakes, according to some researches in olden times the earthquakes are caused by the volcanic explosions that take place under the earth's crust and the waves travel to the earth's surface causing the tremors which causes lots of destruction to the mankind and according to some researches the earth quakes are caused [1] by the drifting of continents which causes the landmass to move and create mass earth quakes. These waves are of two types, one is the transverse waves and the other is the longitudinal waves, the transverse waves travel parallel to the epicenter of the earthquake while the longitudinal waves travel perpendicular to the epicenter of the earth quake. Generally, these earthquakes are detected by a device known as seismograph, it simply records the data of the earthquake like duration, magnitude etc. [12] this data is further converted, simplified etc. or simply it is called as processing of seismic signal which is briefly explained below.

1.2. Seismic Signal Processing

Basically, seismic signal processing is a digital signal process in which a seismic signal is taken as input signal and also it is a processing of seismic signals which enhances the signal to noise ratio and provides a clearer response of the earthquake [3]. In several applied signal processing problems, the main aim is to estimate a set of constant parameters from the measurements on which the received signals rely on. For instance, direction of arrival (DOA) of higher resolution estimation is significant in several sensor systems like electronic surveillance, seismic exploration, radar and sonar. Frequency estimation of higher resolution is significant in many applications, present day examples of which involve the control and design of robots and huge space structures. In those tasks, the workable structure of the underlying signals are usually assumed to be known [11]. There are several methods involved in the processing of the seismic signals, algorithms are used to process the seismic data for a better-quality response. Some of the algorithms are Burg's method, maximum entropy method, modified co-variance method, minimum norm method, MUSIC algorithm, ESPRIT algorithm, yule walker's method, Welch's method etc., these methods are classified into parametric methods, non-parametric methods and frequency estimation methods, so basically these methods are classified into three types, the main task for every digital processing technique is to reduce noise and increase the signal strength, so that our data remains as best as possible, the seismic signals are of two types, this type of classification of seismic signals differs only with the source where these signals are being emitted from [13]. The two types of seismic signals are, real time signal and synthetic signal. Real time signals are the signals of earthquakes appeared in real life while the synthetic signals are the signals that are produced by controlled explosions executed under some expert guidance. These synthetic signals could be from the roots of a tree which is blown by the wind on a windy day or from a tunnel in which a metro train passes etc. [14]. Generally, the work of this seismic signal processing starts by taking a seismic signal as input. By applying ESPRIT algorithm the PSD (power spectral density) of the signal is found out. PSD is nothing but representation of the power of the signal by using frequency functions. After this the Raw seismic signal is taken and detrended. Detrending is nothing but removing the aspects of the signal which are causing distortion. This detrended raw seismic signal is then applied to a FIR Bandpass filter, which further converts the signal from frequency domain to time domain, later FFT is applied to the FIR Bandpass filtered seismic signal in order to convert it back to frequency domain.

1.3. ESPRIT Algorithm

ESPRIT stands for estimation of signal parameters via rotational invariance techniques is developed on the similar values just like the other subspace procedures but additionally exploits a deterministic connection among subspaces [6]. It is a frequency estimation technique. This method differs from the other subspace methods in that the signal subspace is estimated from the data matrix **A** rather than the estimated correlation matrix. The essence of ESPRIT lies in the rotational property between staggered subspaces that is invoked to produce the frequency estimates. In the case of a discrete-time signal or time series, this property relies on observations of the signal over two identical intervals staggered in time [9]. This condition arises naturally for discrete-time signals, provided that the sampling is performed uniformly in time. Extensions of the ESPRIT method to a spatial array of sensors, the application for which it was originally proposed, the original, least-squares version of the algorithm is described in first place and then the derivation to total least-squares ESPRIT was extended [8], which is the preferred method for use. Since the derivation of the algorithm requires an extensive amount of formulation and matrix manipulations.

2. MATHEMATICAL MODELLING

Let us take a complex exponential $S_0 = e^{j2\pi fn}$ which has a complex amplitude α and a frequency f. The property of the signal which we have taken is shown below [2].

$$S_0(n+1) = \alpha e^{j2\pi f(n+1)} = S_0(n)e^{j2\pi f} \tag{1}$$

Hence, the phase-shifted version of the present value is the succeeding sample value. The rotation on the unit circle $e^{j2\pi f}$ is a representation of this phase shift.

$$x(n) = \sum_{p=1}^{P} \alpha_p V(f_p) e^{j2\pi n f_p} + W(n) = V\varnothing^n \ \alpha + W(n) = S(n) + W(n) \tag{2}$$

where the P columns of matrix U are length-N interval frequency vectors of the complex exponentials.

$$U = [U(f_1), U(f_2), ..., U(f_p)]. \tag{3}$$

The complex exponentials α_p amplitudes are present in the vector α. The diagonal matrix of phase shifts among the adjacent time samples of the individual is the matrix ∅ complex exponential elements of S(*n*).

$$\varnothing = \text{diag}\{\varnothing_1, ..., \varnothing_p\} = \begin{bmatrix} e^{j2\pi n f_1} & 0 & \cdots & 0 \\ 0 & e^{j2\pi n f_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & e^{j2\pi n f_p} \end{bmatrix}$$

For, $p = 1, 2, \ldots, P$, $\varnothing_p = e^{j2\pi n f_p}$ (4)

This rotation matrix is entirely expressed by the complex exponential frequencies f_p. If ∅ can be acquired, then frequency estimates can be acquired. Take two overlaying sub-windows of length N – 1 with the length N time window vector and signal which has the sum of complex exponentials.

$$S(n) = \begin{bmatrix} S_{N-1}(n) \\ S(n+N-1) \end{bmatrix} = \begin{bmatrix} S(n) \\ S_{N-1}(n+1) \end{bmatrix} \tag{5}$$

where, $S_{N-1}(n)$ is the length (N – 1) subwindow of S(*n*), hence $S_{N-1}(n) = U_{N-1}\varnothing^n\alpha$.

Matrix U_{N-1} is built in the equivalent way as U other than its time-window frequency vectors are of length N – 1, represented as $U_{N-1}(f)$.

$$U_{N-1} = [U_{N-1}(f_1), U_{N-1}(f_2), ..., U_{N-1}(f_p)] \tag{6}$$

Remember that S(*n*), a scalar signal which is shaped up of the sum of complex exponentials at time *n*.

$$U_1 = U_{N-1}\varnothing^n \text{ and } U_2 = U_{N-1}\varnothing^{n+1} \tag{7}$$

U_1 and U_2 relate to the unstaggered and staggered windows, which is

$$U\varnothing = \begin{bmatrix} U_1 \\ * \ * \ldots * \end{bmatrix} = \begin{bmatrix} * \ * \ldots\ldots * \\ U_2 \end{bmatrix} \tag{8}$$

The two matrices with vectors having intervals are expressed as

$$U_2 = U_1\varnothing \tag{9}$$

Observe that both matrices spaces a distinct, however related, (N – 1) dimensional subspace. Assume that we possess a data matrix A with M data records of the length N interval vector signal *x*(*n*). By singular value decomposition (SVD), data matrix is

$$A = L\,\Sigma\,V^H \tag{10}$$

L is a M × M matrix of left singular vectors and *v* is a N × N matrix of right singular vectors. Each of these matrices are unit matrices, hence $L^HL = I$ and $V^HV = I$. Dimensions of the matrix Σ are M × N which contains singular quantities on the main diagonal which is ordered in a magnitude of decreasing value. The singular valued magnitudes are squared and are equivalent to the eigen values of $\hat{R}$ scaled with a M factor and the V's columns are their related eigen vectors. Hence, *v* shapes an orthogonal and normalized foundation for the underlying N-dimensional vector space. The signal and noise subspaces are formed by dividing this subspace as $V = [V_s/V_n]$

Relating to the *p* largest magnitudes of the singular values V_s is a matrix of right hand singular vectors [4]. All of these frequency vectors for $f = f_1, f_2, ..., f_p$ should lie in the signal subspace since the sum of complex exponentials formed as time-window frequency vectors $U(f)$ are contained in the signal portion. Hence, U and V_s matrices occupy the identical subspace. Hence, there lies an invertible transformation T that draws V_s into $U = V_s T$

In this derivation, T transformation is never elucidated, in the other way it is only constructed just like a mapping inside the subspace of the signal among these two matrices. Divide the subspace of the signal as two tiny subspaces of dimensions(N – 1).

$$V_s = \begin{bmatrix} V_1 \\ * \; * \ldots\ldots * \end{bmatrix} = \begin{bmatrix} * \; * \ldots\ldots * \\ V_2 \end{bmatrix} \tag{11}$$

where, V_1 and V_2 relate to the staggered and unstaggered subspaces because U_1 and U_2 related to the equivalent subspaces.

$$U_1 = V_1 T,\ U_2 = V_2 T \tag{12}$$

The rotation Ø subspaces are being corresponded by the matrix U's staggered and un-staggered elements. A same, though un-like, rotation should be present that associates V_1 to V_2 because the matrices V_1 and V_2 also spaces these subspaces as $V_2 = V_1 \Psi$

Where, Ψ is the matrix of rotation. Remember that the estimation of frequency arrives below for summarizing the rotation matrix $\varnothing$ subspace. Rotations among the subspaces of staggered signal and the relations altogether combined can be made use of the estimation of $\varnothing$. From the data matrix A's SVD the matrices V_1 and V_2 are known from the procedure. Primarily solve Ψ by utilizing the technique of least squares.

$$\Psi = (V_1^H V_1)^{-1} V_1^H V_2 \tag{13}$$

Substituting $V_2 = V_1\Psi$, $U_2 = V_2 T = V_1 \Psi T$ is aquired. In the same way, solve U_2, utilizing the relation $U_2 = U_1 \varnothing$ and substituting $U_1 = V_1 T$ and $U_2 = V_2 T$ for U_1, $U_2 = U_1 \varnothing = V_1 T \varnothing$. Hence, by equating both the right-hand sides of $U_2 = V_2 T = V_1 \Psi T$ and $U_2 = U_1 \varnothing = V_1 T \varnothing$. The relation among the two subspaces rotations is

$$\Psi T = T \varnothing \text{ or}$$

Similarly,

$$\Psi = T \varnothing T^{-1} \tag{12}$$

Equations $\Psi T = T\varnothing$ and $\Psi = T\varnothing T^{-1}$ must be realized as the association among the matrix Ψ's eigen vectors and eigen values. Hence, elements of the diagonal of $\varnothing$, $\varnothing_p$ for $p = 1, 2, 3, \ldots, P$ are commonly the Ψ's eigenvalues. Finally, the frequency estimates are

$$\widehat{f_p} = \frac{\angle \varnothing_p}{2\pi} \tag{13}$$

where the phase of $\varnothing_p$ is $\angle \varnothing_p$. Even though the utilization of rotational subspaces is the property of the ESPRIT algorithm is very easy. Pay heed that only matrix simple relationships are utilized by us. Primarily, provide an algorithm which has a version of total least squares, a best technique to utilize. Pay heed that V_1 and V_2 subspaces are the original subspace's only estimates that relate to U_1 and U_2, naturally acquired through the data matrix A. The subspace rotation's estimate was acquired by solving $V_2 = V_1\Psi$ utilizing the least square criterion.

$$\Psi_{ls} = (V_1^H V_1)^{-1} V_1^H V_2 \tag{14}$$

This least square result is acquired by reducing the errors in least square perception from the formulation as given below.

$$V_2 + E_2 = V_1\Psi \tag{15}$$

Since E_2 is a matrix which has errors among V_2 and the original subspace relating to U_2. The least square formulation presumes that errors exist especially on V_2 estimation and on the other side it presumes that there exist no errors among V_1 and the original subspace that it is trying to estimate relating to U_1 [5]. Hence, V_1 is an estimated subspace too, an extremely accurate formulation is

$$V_2 + E_2 = (V_1 + E_1)\Psi \tag{16}$$

Errors among V_1 and the original subspace relating to U_1 is expressed by the matrix E_1. By minimizing the frobenius norm of the two error matrices can acquire the result to this problem, which is known as total least squares(TLS).

$$\| E_1 \quad E_2 \| F \tag{17}$$

As the properties of TLS are far away from the expectation, normally lend the process to acquire the TLS solution of Ψ. Primarily, prepare a matrix constructed by the staggered signal subspace matrices V_1 and V_2 located adjacent to each other and execute an SVD.

$$[V_1 \quad V_2] = \tilde{L}\tilde{\Sigma}\tilde{V}^H \tag{18}$$

Later we work on $2P \times 2P$ matrix $\tilde{V}$ of right singular vectors which are divided as $P \times P$ quadrants.

$$\tilde{V} = \begin{bmatrix} \tilde{V}_{11} & \tilde{V}_{12} \\ \tilde{V}_{21} & \tilde{V}_{22} \end{bmatrix} \tag{19}$$

The subspace rotation matrix Ψ Total least square solution is $\Psi_{tls} = -\tilde{V}_{12}\tilde{V}_{22}^{-1}$. The estimation of frequencies is then acquired by $\Psi = T\varnothing T^{-1}$ and $\tilde{f}_p = \dfrac{\angle\varnothing_p}{2\pi}$ by utilizing Ψ_{tls} from $\Psi_{tls} = -\tilde{V}_{12}\tilde{V}_{22}^{-1}$.

3. SIMULATION AND RESULTS

Step 1: The data utilized for the observation is acquired from Book_Seismic_Data.mat of east Texas [7] land line is the file name. We have taken the source as a dynamite blast which took place at a depth of around 100ft, one trace has 1501 samples of 0.002s sampling interval.

Step 2: The algorithm's functioning is assessed with known synthetic signal and then ESPRIT algorithm is applied to calculate the seismic signal's tonal.

Step 3: The normalized frequencies are 0.2π and 0.7π. The signal generated is shown in Figure 1.

Step 4: In Figure 2, PSD of the synthetic signal is shown. The Figure shows peaks are at 0.2 and 0.7 normalized frequencies. That means ESPRIT algorithm is working fine.

Step 5: In Figure 3, the seismic signal data is shown.

Step 6: In Figure 4 the detrended seismic signal data is shown.

Step 7: ESPRIT algorithm is applied on detrended seismic signal and the PSD obtained is shown in Figure 5. The max peak is at 0.958 normalized frequency.

$$w = \frac{2\pi f}{fs} = 0.0958\pi$$

$$= \frac{2\pi f}{500}$$

$$= \frac{2\pi}{fs} f = 0.\ 0958\pi$$

$$f = \frac{500}{2} \times 0.0958$$

$$= 250 \times 0.0958$$

$$= 25 \times 0.0958$$

$$= 23.950 \text{ Hz}$$

Step 8: In the reference book, it is written that the data is band pass filtered in the range [15Hz, 60Hz]. For ensuring purpose, a BP filter with FIR order 8 is realized. The transfer function of the same is shown in Figure 6.

Step 9: The detrended seismic signal is convolved with FIR BPF and the output is shown in Figure 6.

Step 10: The same PSD, as shown in Figure 7. Is obtained. So, the seismic signal tonal is 23.95Hz.

Another insignificant tonal is $w_1 = \frac{2\Pi f}{f_s} = 0.119\pi$

$$\frac{2\Pi f}{500} = 0.119$$

$$f = \frac{0.119\pi \times 500}{2\pi}$$

$$\therefore \quad f = 250 \times 0.119$$

$$= 29.75 \text{ Hz}$$

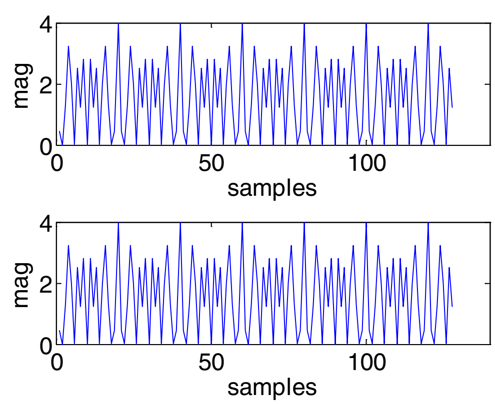

Figure 1: Synthetic signal with and without noise

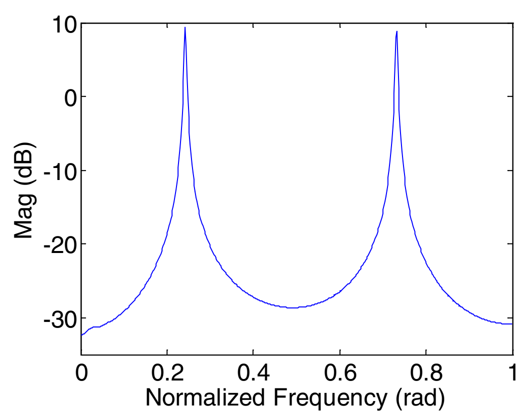

Figure 2: PSD using ESPIRIT TLS Method for synthetic signal

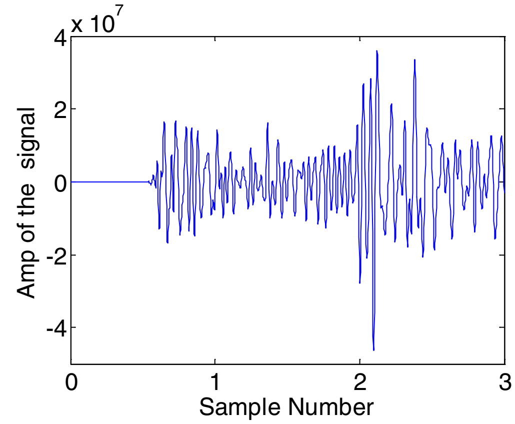

Figure 3: Raw seismic signal

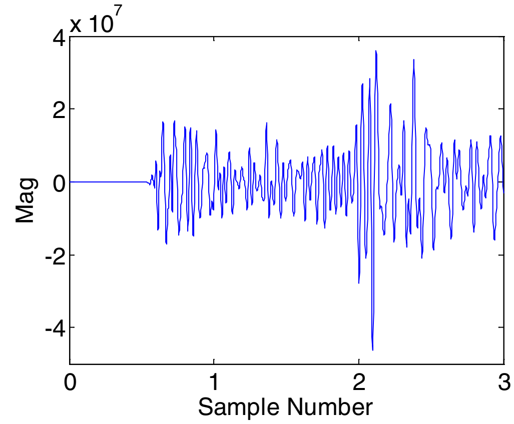

Figure 4: De-trended raw seismic signal

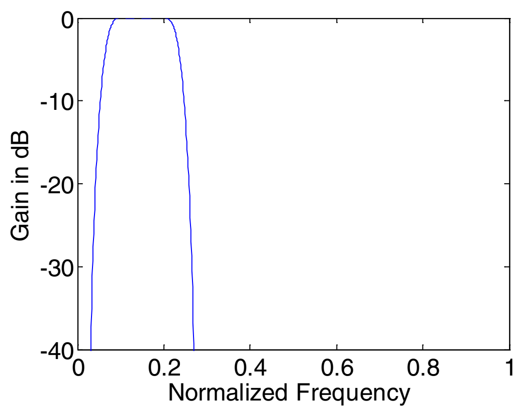

Figure 5: FIR bandpass filtered frequency spectrum

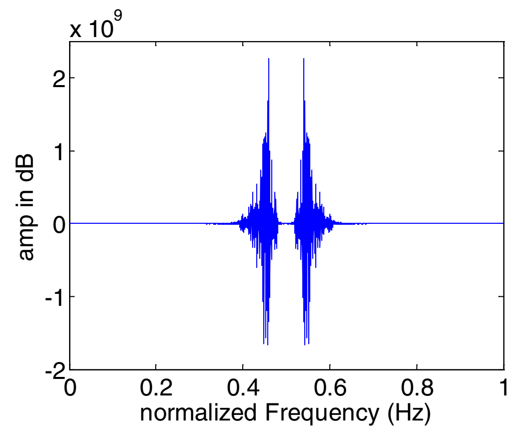

Figure 6: FFT of bandpass filtered seismic signal

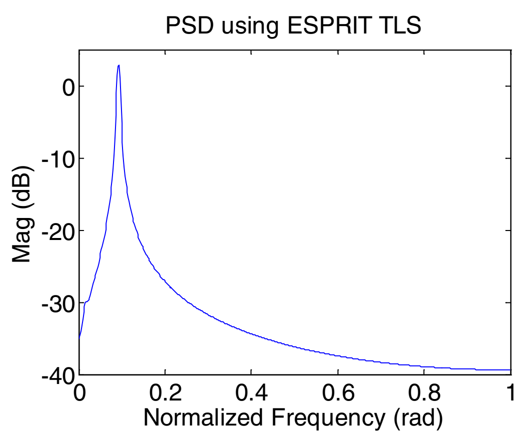

Figure 7: ESPRIT spectrum of BPF de-trended seismic signal

4. CONCLUSION

In this paper Signal parameter estimation with high resolution is obtained using ESPRIT algorithm. Results obtained are accurate since the false peaks are removed and an error free signal can be plotted. From the results obtained it is concluded that the ESPRIT algorithm is the best technique for frequency estimation in seismic signal processing which requires less computation.

REFERENCES

[1] Wail A. Mousa, Abdullatif A. Al-Shuhail, "Processing of Seisemic Reflection data using Matlab", Synthesis Lectures On Signal Processing., Morgon& Claypool Publishers.

[2] Monson H.Hayes, "Statiscal Digital Signal Processing and Modeling", John Wiley & Sons, Inc,1996

[3] Petre Stoica and Randolph Moses, "Spectral Analysis Of Signals", Prentice Hall, Inc, 2005.

[4] G. Manolakis and Vinay k. Ingle "Statistical And Adaptive Signal Processing" McGraw-Hill, 2000.

[5] Richard Roy, Thomas Kailath, "ESPRIT-Estimation of Signal Parameters Via Rotational Invariance Techniques", *IEEE transactions on Acoustucs, speech and signal processing*, vol.37, No.7, 1989, pp.984-995

[6] Wail A.Mousa and Abdullatif A.Al-Shuhail, "Processing of Seismic Reflection data using MATLAB",2011

[7] R. Roy, A. Paulraj, and T. Kailath, "ESPRIT-A subspace rotation approach to estimation of parameters of cisoids in noise,·· IEEE Trans. Acoust., Speech, Signal Processing, Vol. ASSP-34, pp. I 340-1342, Oct. 1986.

[8] A. Paulraj, R. Roy, and T. Kailath, "A subspace rotation approach to signal parameter estimation," Proc. IEEE, pp. 1044-1045, July 1986.

[9] R. Roy, B. Ottersten, L. Swindlehurst, and T. Kailath, "Multiple invariance ESPRIT,'' IEEE Trans. Acoust., Speech, Signal Processining, in preparation.

[10] S.Rouqutte, M.Najim, "Estimation of frequency and damping factors by two dimensional ESPRIT type methods", IEEE transactions on signalprocessing, vol.49, Issue1, Aug 2002

[11] A.N. Lemma; A.-J. van der Veen; E.F. Deprettere, "Analysis of joint angle frequency estimation using ESPRIT" IEEE transactions on signalprocessing, vol.55, Issue5, pp: 1264-1283, April 2003.

[12] RL Kirlin, WJ Done," covariance analysis of seismic signal processing", Seismic library 1999.

[13] N Yuen, B Friedlander,"performance analysis of higher order ESPRIT for localization of near field sources, IEEE transactions on signal processing, Vol. 46, No. 3, march 1998

[14] SV Vaseghi, "Advanced signal processing and noise reduction", John Weley and Sons,2008