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Application of LQG Regulator to Improve The Performance of DC-DC Power Converter

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ABSTRACT

This paper compares the time response specification performance between two different controllers for analysis of DC-DC power converter to achieve constant voltage at the output side. The application of PI controller reduces the settling time with a small overshoot. The Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG) controller are generally used in Optimal Control theory where the basic function is to minimize the cost function. The LQG regulator mainly involves modelling of system in state space, designing an observer to estimate the system states, and figuring out a feedback gain matrix that multiplies the observer state matrix to obtain control. The solution of linear quadratic regulator stabilizes the system as well as improves the stability margin. In this paper the simulation results are obtained using Matlab/Simulink software.

Keyword: DC-DC power converter, State space Averaging Technique, PI controller, LQG controller.

1. INTRODUCTION

In this paper PI controller and LQG controller are designed for non isolated LCLC power converter. The objective of this paper is to achieve constant output voltage irrespective of duty ratio variation. State space averaging (SSA) technique is deployed to describe the averaged model of the non linear system. The medium power application systems are assessed and analyzed where overshoot, settling time and steady state error remained prime focus.

The non isolated LCLC power converter is a special type of DC-DC converter or it is a step up converter followed by a step down converter with a capacitive energy transfer. The output voltage is either high or low that depends upon the duty ratio. The basic structure of the converter is illustrated in Fig 1. The power converter is composed of two states namely “switch on” and “switch off state”. The

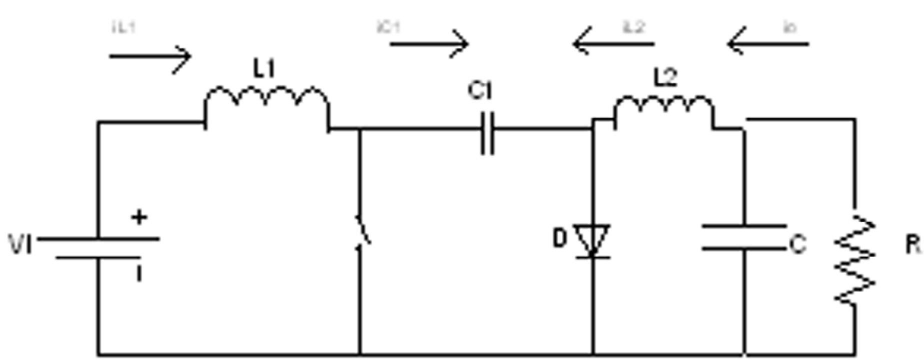

Figure 1: Non-isolated LCLC dc-dc converter

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inductor at the supply side acts as a filter to prevent the large harmonic content. The energy transfer depends on the capacitor C_1.

2. DYNAMIC MODEL OF DC-DC CONVERTER

When the switch "S" is on, the current through L_1 and L_2 increases, at the same time the voltage of capacitor C_1 reverse biases diode D and turns it off. The capacitor C_1 discharges its energy to the circuit formed by C_1, L_2, and C. The differential equation model for switch on state is given in equation (1), (2), (3) and (4).

$$V_i - L_1 \frac{di_{L1}}{dt} = 0 \tag{1}$$

$$v_{C1} - L_2 \frac{di_{L2}}{dt} - v_{C2} = 0 \tag{2}$$

$$C_1 \frac{dv_{C1}}{dt} + i_{L2} = 0 \tag{3}$$

$$C_2 \frac{dv_{C2}}{dt} - i_{L2} - \frac{v_{C2}}{R} = 0 \tag{4}$$

In state space model the above equations can be written as

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{v}_{C1} \\ \dot{v}_{C2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1}{L_2} \\ 0 & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_{C1} \\ v_{C2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} V_i$$

When the switch "S" is off, at the same time the diode D is forward biased and capacitor C_1 is charged through L_1, D and the input supply V_i. The differential equations for switch off state is given in equation (5), (6), (7) and (8).

$$V_i - L_1 \frac{di_{L1}}{dt} - v_{C1} = 0 \tag{5}$$

$$L_2 \frac{di_{L2}}{dt} + v_{C2} = 0 \tag{6}$$

$$i_{L1} = C_1 \frac{dv_{C1}}{dt} \tag{7}$$

$$C_2 \frac{dv_{C2}}{dt} = i_{L2} - \frac{v_{C2}}{R} \tag{8}$$

In state space model the above equations can be written as

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{v}_{C1} \\ \dot{v}_{C2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1}{L_1} & 0 \\ 0 & 0 & 0 & -\frac{1}{L_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_{C1} \\ v_{C2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} V_i$$

The output equation is same for switching ON and OFF state and is given in equation (9).

$$v_0 = v_C \text{..........(9)} \quad v_0 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_{C1} \\ v_{C2} \end{bmatrix}$$

In this converter the input voltage, output voltage and the duty ratio are related by the formula as in (10).

$$\frac{V_0}{V_i} = \frac{D}{1-D} \tag{10}$$

In this paper the output voltage is regulated by controlling the duty ratio of the switch. The converter described here is operating in continuous conduction mode with an operating frequency of 25 KHz and duty ratio of 0.8. The design data for the converter is specified in Table-1.

Table 1
(converter parameter specification)

V_i	25V
L_1	1mH
L_2	1mH
C_1	100µF
C_2	450 µF
R	100Ω
$V_{O(desired)}$	100V
Duty ratio	0.8

3. STATE SPACE AVERAGING (SSA) TECHNIQUE

The objective of this technique is to convert a non linear model to a linear one and to derive a small signal transfer function $V_o(s)/d(s)$ where V_o is the output voltage and d is the duty ratio. The converter is operating in continuous conduction mode. In continuous conduction mode there are two states "on state" and "off state". During each circuit state the linear circuit is described by means of the state variable vector 'x'. Here inductor current and capacitor voltage are considered as two state variables of the system.

To derive an average model of the circuit over a switching period the equation corresponding to two states are time weighted and averaged and the resulting equations are given by equation (11) and (12)

$$\dot{x} = [A_1 d + A_2(1-d)]x + [B_1 d + B_2(1-d)]v_{in} \tag{11}$$

$$v_o = [C_1 d + C_2(1-d)]x \tag{12}$$

The steady state dc voltage transfer function and the ac transfer function are given by the equation (13) and (14) respectively.

$$\frac{v_o(s)}{v_{in}(s)} = C(SI - A)^{-1}B \tag{13}$$

$$\frac{v_0(s)}{d(s)} = C[SI - A]^{-1}[(A_1 - A_2)x + (B_1 - B_2)v_{in}] + (C_1 - C_2)x \tag{14}$$

The state-space average model of the converter is formed by taking a weighted average of the "on state" and "off state" equations.

Where

$$A = A_1 d + A_2(1-d)$$
$$B = B_1 d + B_2(1-d)$$
$$C = C_1 d + C_2(1-d)$$

A_1, A_2 are the on state matrix and off state matrix respectively. B_1, B_2 are the on state and off state input matrix respectively. C_1, C_2 are the on state and off state output matrix respectively.

Using State space averaging technique the small signal model with respect to duty ratio is obtain as in (15)

$$\frac{V_o(s)}{d(s)} = \frac{\begin{array}{l}2.778\times10^{8}s^{10} - 7.654\times10^{10}s^{9} + 5.564\times10^{15}s^{8} - 1.384\times10^{18}s^{7} + 3.307\times10^{22}s^{6} - 6.235\times10^{24}s^{5} + 5.043\times10^{28}s^{4} + 1.836\times10^{29}s^{3} + 9.12\times10^{33}s^{2}\\ +7.901\times10^{34}s + 4.39\times10^{38}\end{array}}{\begin{array}{l}s^{12} + 66.67s^{11} + 2.707\times10^{7}s^{10} + 1.656\times10^{9}s^{9} + 2.469\times10^{14}s^{8} + 1.373\times10^{16}s^{7} + 7.828\times10^{20}s^{6} + 3.878\times10^{22}s^{5} + 2.201\times10^{26}s^{4}\\ +7.327\times10^{27}s^{3} + 2.145\times10^{31}s^{2} + 3.582\times10^{32}s + 7.023\times10^{35}\end{array}} \tag{15}$$

The reduced order transfer function with respect to duty ratio is obtained as in (16)

$$\frac{Vo(s)}{d(s)} = \frac{0.0154s^3 + 2.778\times10^8 s^2 - 8.889\times10^{10}s + 5.556\times10^{14}}{s^4 + 22.22s^3 + 9.022\times10^6 s^2 + 1.511\times10^8 s + 8.889\times10^{11}} \tag{16}$$

4. DESIGN OF PI CONTROLLER

The main objective of designing PI controller for converter system is to achieve constant voltage with less steady state error with minimum overshoot and minimum settling time. The controller parameters are obtained based on Zeigler- Nichols method and each parameter has an effect on the error. The transfer function of the PI controller is stated in (17)

$$G_c(s) = K_p + \frac{K_i}{s} \tag{17}$$

The tuning of PI controller is based on Zeigler Nichols method first approach where the plant dynamics is known. Fig 2 show the simulink model of non isolated LCLC converter which is fed from the dc source with a voltage of 25 volt. The output voltage waveform is shown in Fig 3 which is having a very large overshoot at the beginning.

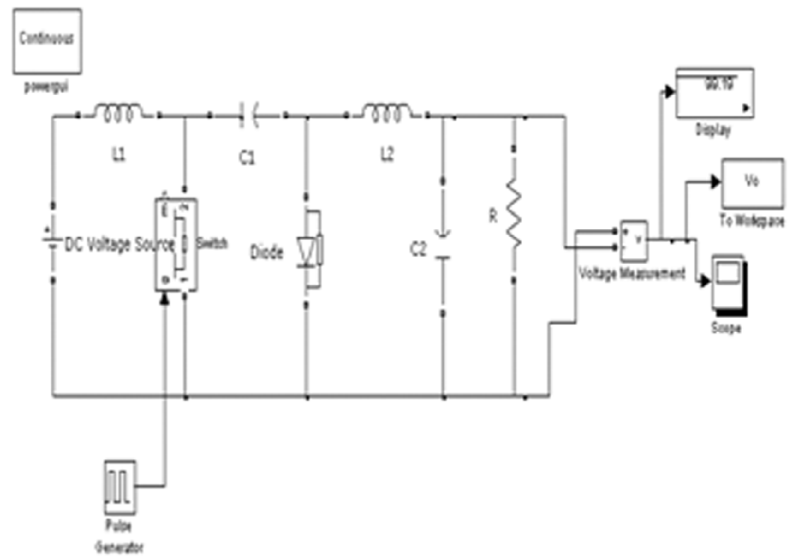

Figure 2: Open loop model of non isolated LCLC converter

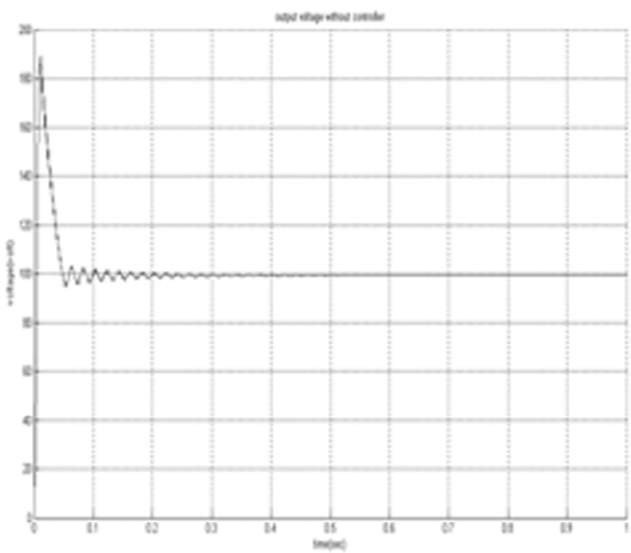

Figure 3: output voltage waveform without controller

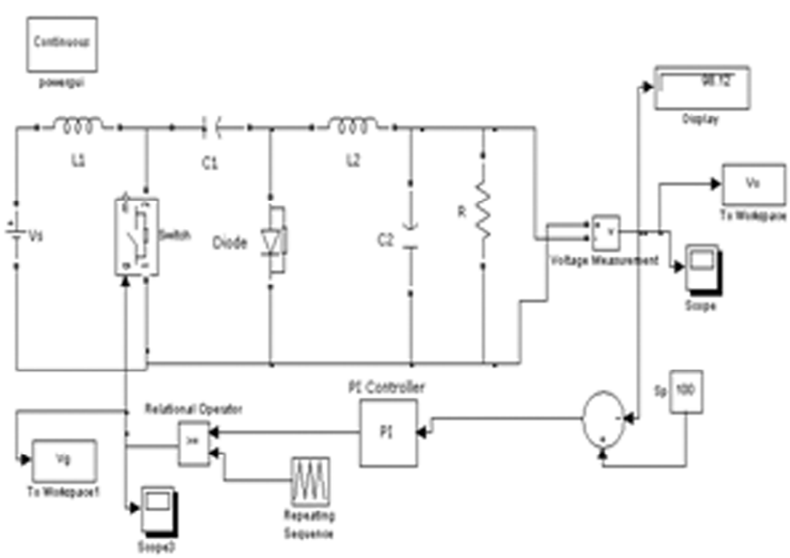

Figure 4: Simulink model of converter with controller

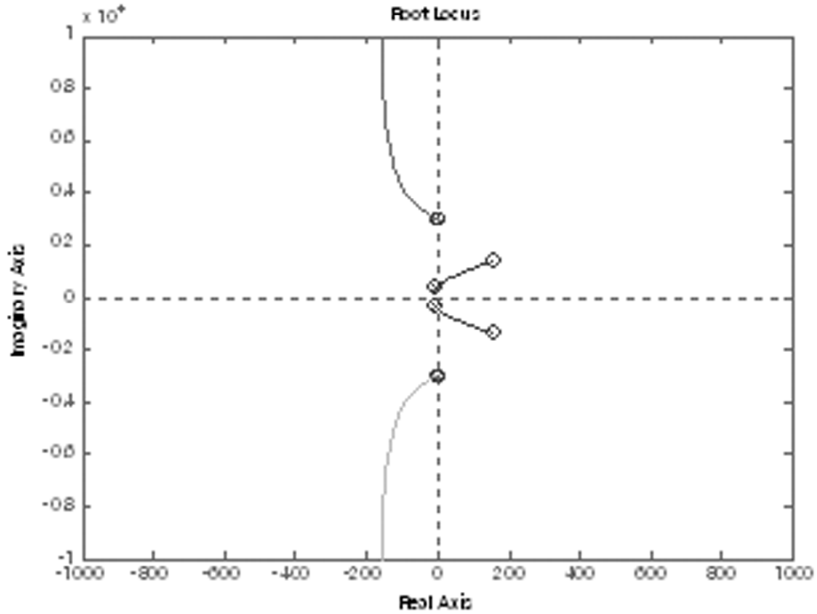

Figure 5: root locus plot

Fig 4 shows the closed loop simulink model where the output voltage of the converter is compared with a set-point value and the error produced is passed through the PI controller. The output of the PI controller is compared with the repeating sequence using the Pulse Width Modulation (PWM) technique to generate the pulse which is fed to the gate of the converter switch.

The critical gain and critical time period are 0.00152 and 0.0145 sec respectively which are calculated from the root locus plot. Fig 5 shows the root locus plot of the converter system. The root locus plot where it cuts the imaginary axis that gain is known as critical gain and corresponding frequency is known as critical frequency.

The K_p and K_i values are found to be 0.00076 and 0.10482 respectively. So the controller transfer function is obtained as in (18)

$$G_c(s) = \frac{0.00076s + 0.1048}{s} \tag{18}$$

The output voltage of the converter with PI controller is shown in Fig 6.

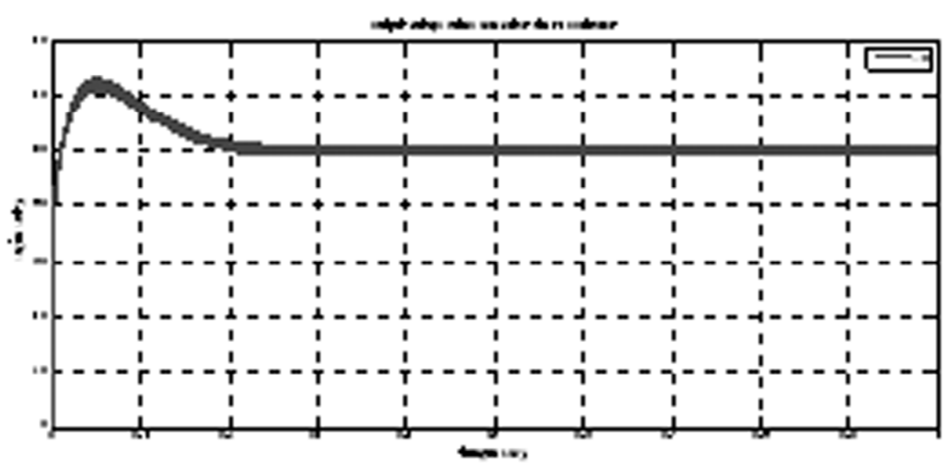

Figure 6: output voltage of the converter with PI controller

5. IMPLEMENTATION OF OPTMAL CONTROLLER

Fig 7 shows the general block diagram of a plant with linear quadratic regulator. Here the plant (dc-dc converter) is a continuous linear time invariant system is in the form of

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \\ z &= Gx + Hu \end{aligned} \tag{19}$$

Here 'y' represents the measured output which is used for control and 'z' is the controlled output which is to be small within shortest possible of time. Sometimes z = y which means that the control objective is simply to make the measured output very small. In this block diagram reference signal is absent and it uses negative feedback. The LQR problem can be defined as in equation (20)

$$J_{LQR} = \int_0^\infty \|z(t)\|^2 dt + \int_0^\infty \rho \|u(t)\|^2 dt \tag{20}$$

The first term corresponds to the energy of the controlled output and the second term to the energy of the control signal. The main objective is to regulate the plant output y around zero. For designing LQG regulator, white Gaussian noise is modelled which drives a low pass filter with cut off frequency of 10rad/sec. The simulink block diagram for designing LQG regulator is shown in Fig 8.

LQG regulator is a combination of linear quadratic regulator and kalman estimator. The optimal control

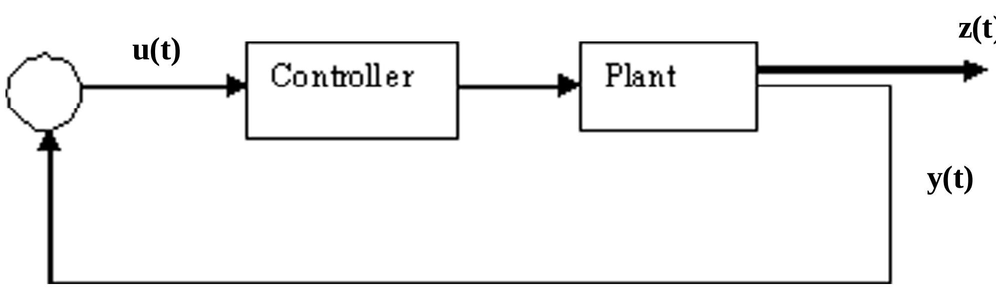

Figure 7: Linear quadratic regulation (LQR) feedback configuration

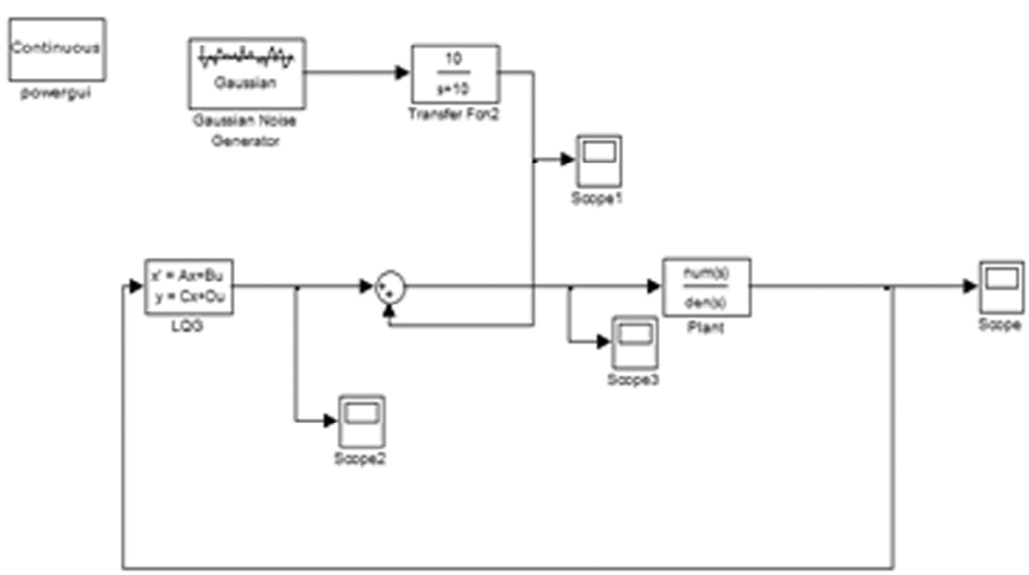

Figure 8: Simulink model of the plant with LQG regulator and low pass filter

law is stated as u = -Kx where K is the optimal controller gain matrix. The optimal controller gain for this system is found to be

$$K = 1.0e+003 \quad *[0.0411 \quad 0.2109 \quad 0.1698 \quad 1.5970]$$

By using the command "kalman", the kalman estimator is calculated and using the command "F = lqgreg (Kest, K)" the state space model of the LQG regulator is formed. The transfer function of the LQG regulator is calculated and is given in equation (21)

$$F(s) = \frac{-0.001s - 1.7850}{16.41} \tag{21}$$

The closed loop transfer function without any filter is calculated and the step response is plotted in Fig 9.

$$\frac{C(s)}{R(s)} = \frac{0.01554s^3 + 4.557\times10^8 s^2 - 1.458\times10^{11} s + 9.115\times10^{14}}{1.641s^4 + 2.778\times10^5 s^3 + 4.217\times10^8 s^2 + 3.972\times10^{11} s + 9.93\times10^{14}} \tag{22}$$

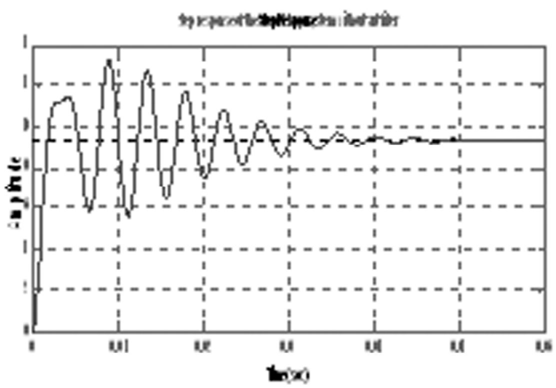

Figure 9: closed loop step response with LQG regulator

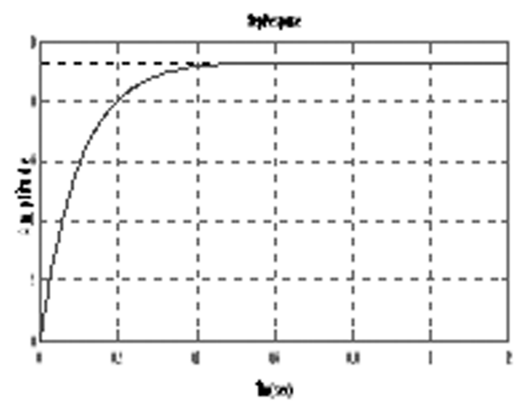

Figure 10: closed loop step response with LQG regulator and low pass filter

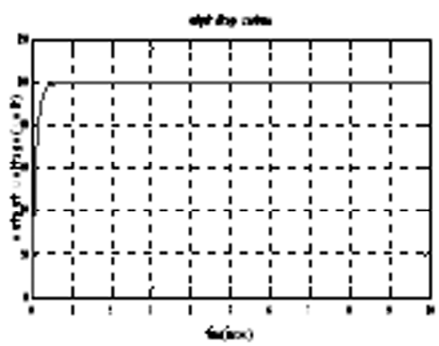

Figure 11: Vo with LQG regulator

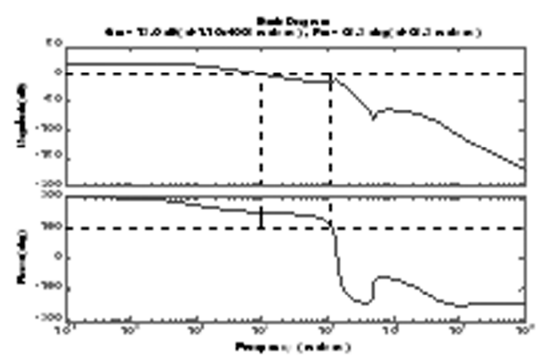

Figure 12: Bode plot without LQG regulator

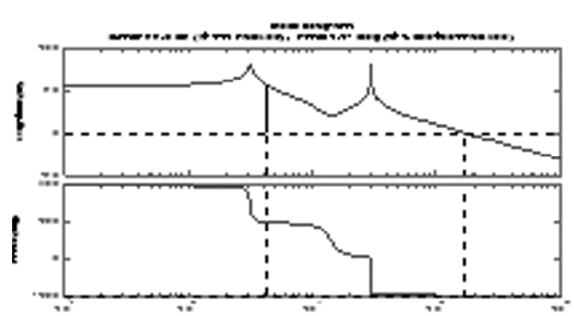

Figure 13: Bode plot with LQG regulator

After designing the LQG regulator the peak overshoot is reduced to some extent but it is not zero percent so when the response passes through the low pass filter the peak overshoot is reduced to zero with less settling time. The step response of the closed loop system after passing through the low pass filter is shown in figure 10. The output voltage of the converter with LQG regulator is shown in figure 11.

6. PERFORMANCE COMPARISON

Table-2 shows the performance comparison of LCLC converter with PI controller and optimal controller for a fixed duty ratio.

Table 2
(Performance specification)

Performance specification	Without controller	With PI controller	With LQG regulator
Output voltage (volt)	Nearly 100	98.83	100
Peak Overshoot (%)	90.49%	25%	0%
Settling time (sec.)	0.52	0.23	0.39
Steady state error	Nearly 0	Nearly 0	0

The frequency response plot without controller and with LQG regulator is shown in Fig12 and Fig 13 respectively. From the above bode plot it is found that the gain margin and phase margin both are positive and the gain crossover frequency is less than the phase crossover frequency. Hence the system is absolutely stable system with all poles lying on the left half of s-plane but without using any controller the gain margin is found to be negative with gain crossover frequency more than the phase crossover frequency.

7. CONCLUSION

The applicability of PI controller to the non-isolated power converter settles down the output voltage very fast at the same with little high overshoots. Overshoot is marginalized with reduction of gain hence increase in settling time is expected. Stability and robustness of the system is expected to improve by using LQG regulator. The output voltage of the converter with LQG controller has less overshoot and less settling time with constant voltage. Hence much stability and robustness can be achieved even in varying system parameters.

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