Position Control of Two-link Flexible Manipulator using Low Chattering SMC Techniques

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Abstract: In this paper an attempt is made to control a angular position of the two-link flexible manipulator with slowly varying parameter uncertainties.The lumped parameter model is used throughout the work for obtaining the manipulator dynamics. The problem of controlling the positrion is achieved by using two low chattering sliding mode control (SMC) techniques. The techniques are: Proportional integral (PI) SMC and asymptotic SMC. The importance of these techniques is that chattering is attenuated even in first order SMC and in presence of bounded unknown disturbances including uncertainties associated with the plant. The proposed technique are examined under bounded unknown disturbances and slowly time varying uncertainties in system parameters. Comparison of the performances of the two control techniques are also highlighted.

Keywords: Sliding Mode Control; PI SMC; Asymptotic SMC; Two-link flexible manipulator.

1. INTRODUCTION

Flexible manipulator (FM) has many advantages over rigid manipulators [1] though it has many inherent complexities and nonlinearities. Modeling and control of such manipulators is the main focal area of research. The objective of this paper is to control the FM in presence of disturbances and uncertainties.

Finite element method (FEM) [2], assumed modes method (AMM) [3], lumped parameter method [4] are the most widely used modeling techniques of FM. Position control [5] and trajectory tracking [6] are the main control problem for controlling the manipulators. Many control techniques are available for FM like PID [7], feedback control [8], observer based control [9], SMC [10], H$_\infty$ [11], intelligent control [12], etc.

SMC is the most useful control method because of many inherent advantages like finite time convergence and insensitive to parameter uncertainties and/or disturbances [13]. However, SMC technique has its own main drawback which is the phenomena of chattering. Several SMC techniques are proposed in literature for the reduction in chattering like Adaptive SMC [14], observer-based [15], state dependent gain method [16], use of hysteresis loop [17], using low pass filter [18], second order SMC [19], backstepping smc [22], terminal SMC [23, 24] etc. SMC techniques have been in use by some researchers for different control problems of two-link flexible manipulators (TLFM) [6, 10, 20, 23]. But controlling the position of TLFM in the presence of unknown disturbances and uncertainties with time varying parameters is a challenging task.

In this paper, two different SMC techniques are considered for the position control of two-link flexible manipulator. The proposed techniques worked satisfactorily in presence of unknown disturbances and uncertainties resulting in low chattering. PI and asymptotic sliding surfaces are designed in PI SMC and

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asymptotic SMC, respectively. The Lyapunov stability theory is used to prove the stability of the sliding surfaces. Comparison on the working performances of the techniques are highlighted in terms of settling time, links deflection, reaching time of the sliding surface, Integral square error and control input energies. The manipulator dynamics are designed by using the lumped parameter method. Controlling the position of the TLFM in the presence of bounded unknown disturbance and the time varying parameters using low chattering SMC techniques is the novelty of this paper.

The paper is organised as follows. Firstly, the dynamic modeling of the TLFM is presented in Section II. The description of PI SMC and asymptotic SMC are given in Section III and IV respectively. Section V deals with the results and discussions. Finally, conclusions are presented in Section VI.

2. SYSTEM DYNAMICS

Consider a planar two-link flexible manipulator with bending deformation in the plane of motion. Fig. 1 shows the two-link flexible manipulator. The coordinates frames are given as $(\hat{X}_0, \hat{Y}_0)$, the moving rigid body frame for link i as (X_i, Y_i) and the moving flexible body frame of ith link as $(\hat{X}_i, \hat{Y}_i)$. The hub is attached to the motor shaft which drives the link to reach the desired position.

The dynamic equations of the system is found by the Euler-Lagrange formulation is first obtained. The kinetic energy (KE) and potential energy (PE) of each link and hub are calculated from the given system. The total KE is given as the combination of the links KE and hubs KE.

$$\text{KE}_{total} = \text{KE}_{L1} + \text{KE}_{L2} + \text{KE}_{H1} + \text{KE}_{H2} = \frac{1}{2}J_{eq}\,\dot{\theta}_1^2 + \frac{1}{2}J_{eq}\,\dot{\theta}_2^2 + \frac{1}{2}J_{L1}(\dot{\theta}_1 + \dot{\alpha}_1)^2 + \frac{1}{2}J_{L2}(\dot{\theta}_2 + \dot{\alpha}_2)^2 \quad (1)$$

$$\text{PE}_{total} = \frac{1}{2}k_s\alpha^2 \quad (2)$$

The Euler-Lagrange equation is given as

$$\frac{d}{dt}\left(\frac{\delta L}{\delta\dot{\theta}}\right) - \left(\frac{\delta L}{\delta\theta}\right) = T - B_{eq}\dot{\theta} \quad (3)$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta\dot{\alpha}}\right) - \left(\frac{\delta L}{\delta\alpha}\right) = 0 \quad (4)$$

where α and θ are the two generalized coordinates and

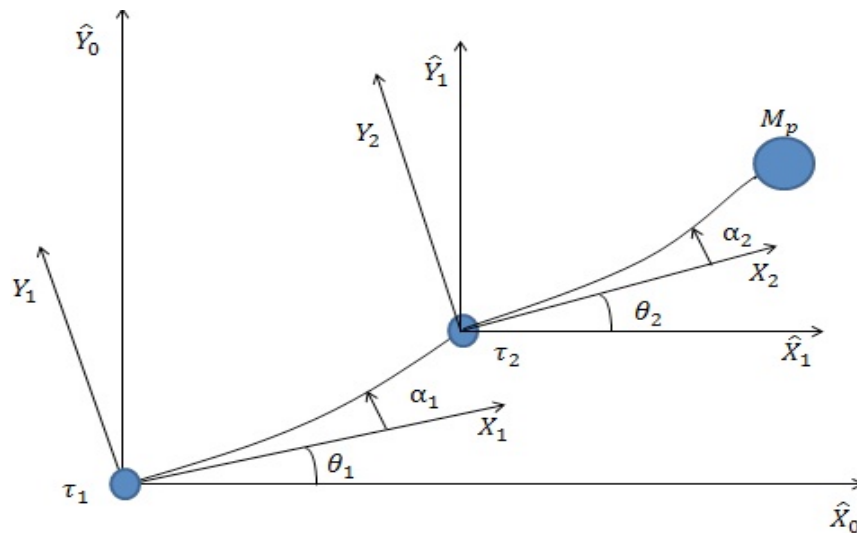$$L = KE_{total} - PE \quad (5)$$

Figure 1: Schematic diagram of two-link flexible manipulator.

Solving the equations (4) and (5), we get

$$\ddot{\theta}_1 = -p_1\dot{\theta}_1 + p_2\alpha_1 + p_3V_m \tag{6}$$

$$\ddot{\theta}_2 = -p_4\dot{\theta}_2 + p_5\alpha_2 + p_6V_m \tag{7}$$

$$\ddot{\alpha}_1 = p_1\dot{\theta}_1 - p_7\alpha_1 - p_3V_m \tag{8}$$

$$\ddot{\alpha}_2 = p_4\dot{\theta}_2 - p_8\alpha_2 - p_6V_m \tag{9}$$

where

$$p_1 = \frac{\eta_m\eta_g k_t k_g^2 k_m + B_{eq} R_m}{R_m J_{eq1}}, \quad p_2 = \frac{k_s}{J_{eq1}}, \quad p_3 = \frac{\eta_m\eta_g k_t k_g}{R_m J_{eq1}}, p_4 = \frac{\eta_m\eta_g k_t k_g^2 k_m + B_{eq} R_m}{R_m J_{eq2}}, \quad p_5 = \frac{k_s}{J_{eq2}}, \quad p_6 = \frac{\eta_m\eta_g k_t k_g}{R_m J_{eq2}} \quad p_7 = k_s\left\{\frac{1}{J_{eq1}} + \frac{1}{J_{arm1}}\right\}, \quad p_8 = k_s\left\{\frac{1}{J_{eq2}} + \frac{1}{J_{arm2}}\right\} \tag{10}$$

Table 1

Parameters of The System (3, 4) [21].

Mass of link 1, $m_1 = 0.065\ Kg$	Coefficient of viscous damping, $B_{eq} = 1.99$	Link M. I, $J_{arm1} = 0.00195\ Kgm^2$
Mass of link 2, $m_2 = 0.070\ Kg$	Efficiency of gear box, $\eta_g = 0.9$	Link M. I, $J_{arm2} = 0.000933\ Kgm^2$
Length of link 1, $L_1 = 0.3\ m$	Efficiency of motor, $\eta_m = 0.69$	Load Torque = T
Length of link 2, $L_2 = 0.2\ m$	Constant of back e. m. f, $K_m = 0.00767$	Motor voltage= V_m
Resistance of Armature, $R_m = 2.6\ \Omega$	Gear ratio, $K_g = 70$	
Equivalent M. I at load, $J_{eq1} = 0.099\ Kgm^2$	Natural frequency, $f_c = 3.2\ H_z$	
Equivalent M. I at load, $J_{eq2} = 0.092\ Kgm^2$	Stiffness of the link, $K_s = 2\pi f_c$	

Defining, $x_1 = \theta_1, x_2 = \theta_2, x_3 = \alpha_1, \ x_4 = \alpha_2 \ \ x_5 = \dot{\theta}_1, x_6 = \dot{\theta}_2, x_7 = \dot{\alpha}_1, \ x_8 = \dot{\alpha}_2, \ u = V_m$ and using the above equations (6) to (9), manipulator dynamics can be represented as [21]:

$$\dot{x}_1 = x_5$$
$$\dot{x}_2 = x_6$$
$$\dot{x}_3 = x_7$$
$$\dot{x}_4 = x_8$$
$$\dot{x}_5 = -b_1 x_5 + b_2 x_3 + b_3 u + d \tag{11}$$
$$\dot{x}_6 = -b_4 x_6 - b_5 x_4 + b_6 u$$
$$\dot{x}_7 = b_1 x_5 - b_7 x_3 - b_3 u$$
$$\dot{x}_8 = b_4 x_6 - b_8 x_4 - b_6 u$$

where $b_1(t) = p_1 + d_1(t), b_2(t) = p_2 + d_2(t), b_4(t) = p_4 + d_4(t), b_5(t) = p_5 + d_5(t), b_7(t) = p_7 + d_7(t), b_8(t) = p_8 + d_8(t)$, and $d_1(t), \ d_2(t), d_4(t), d_5(t), d_7(t), d_8(t)$ are uncertainties in the plant parameters $p_1, \ p_2, p_4, \ p_5, \ p_7, \ p_8$, respectively, i.e. slowly time varying parameters are considered. d is the plant disturbance. Numerical values of these parameters are given in the results and discussions section. Position control using PI SMC is discussed in the next section.

3. POSITION CONTROL OF TLFM USING PI SMC

This section describes the use of PI SMC technique for the position control of TLFM. The objective is to design a controller in such a manner that it causes the manipulator dynamics to follow the desired position. Thus, by considering the desired position x_d for the links, the error is defined as;

$$e_1 = x_1 - x_{d1}, \; e_2 = x_2 - x_{d2} \tag{12}$$

Here, a unit step angular position is considered as desired for x_{d1} and x_{d2}. Designing of a SMC technique involves two steps. First is the design of sliding surface and in second step, we design the control law [13]. The objective of the first step is to bring the dynamics on the sliding surface and the second step is to maintain the trajectory on the sliding surface [13]. The PI sliding surface is considered as:

$$
\begin{aligned}
s_1 &= \dot{e}_1 + c_1 \int_0^t e_1(\tau) \, d\tau \\
s_2 &= \dot{e}_2 + c_2 \int_0^t e_2(\tau) \, d\tau \\
s_3 &= \dot{x}_3 + c_3 \int_0^t x_3(\tau) \, d\tau \\
s_4 &= \dot{x}_4 + c_4 \int_0^t x_4(\tau) \, d\tau
\end{aligned} \tag{13}
$$

Now, the equivalent sliding surface is considered as

$$s_p = s_1 + a_1 s_2 + a_2 s_3 + a_3 s_4 \tag{14}$$

where $a_i (i = 1, 2, 3), c_i (i = 1, 2, 3, 4)$ are positive constants whose values depend on the choice of designer. The system operates on the sliding surface when it satisfies:

$$s_p = 0, \dot{s}_p = 0 \tag{15}$$

To ensure the occurrence of sliding mode, control law (using (11), (12) and (15)) is proposed as;

$$
\begin{aligned}
u_p = \left(-\frac{1}{k}\right) & [x_3(-b_7 a_2 + b_2 + c_3 a_2) + x_5(b_1 a_2 - b_1) + x_6(-b_4 a_1 - b_4 a_3) + x_4(-b_8 a_3 + c_4 a_3 + \\
& + c_5 a_1) + c_1 e_1 + a_1 c_2 e_2] - \rho \, sgn(s_p) \quad\quad (16)
\end{aligned}
$$

where $k = b_6 a_1 - a_3 b_6 - b_3 a_2 + a_3, \rho$ are positive constants.

Remark 1: The control law u_p in (16) causes the system trajectories (11) to follow the desired position in presence of unknown disturbance and slow time varying parameters and to maintain on the sliding surface $s_p(t)$.

In the next section we shall discuss the performance of the system using asymptotic SMC.

4. POSITION CONTROL OF TLFM USING ASYMPTOTIC SMC

In this section, asymptotic SMC is designed for describing the position control of manipulator dynamics in equation (17). The sliding surface is also designed in terms of control function derivative. In this case control input is the integral of high frequency switching function. So, the manipulator dynamics is modified and is described as

$$
\begin{aligned}
\dot{x}_1 &= x_5 \\
\dot{x}_2 &= x_6 \\
\dot{x}_3 &= x_7 \\
\dot{x}_4 &= x_8 \\
\dot{x}_5 &= -b_1 x_5 + b_2 x_3 + b_3 u \\
\dot{x}_6 &= -b_4 x_6 - b_5 x_4 + b_6 u \\
\dot{x}_7 &= b_1 x_5 - b_7 x_3 - b_3 u \\
\dot{x}_8 &= b_4 x_6 - b_8 x_4 - b_6 u \\
\dot{u} &= v
\end{aligned} \tag{17}
$$

Now, the sliding variables are described as

$$s_1 = \dot{e}_1 + c_{1a}e_1$$
$$s_2 = \dot{e}_2 + c_{2a}e_2$$
$$s_3 = \dot{x}_3 + c_{3a}x_3 \tag{18}$$
$$s_4 = \dot{x}_4 + c_{4a}x_4$$

Here, the equivalent sliding surface is considered as

$$s_A = s_1 + a_{1a}s_2 + a_{2a}s_3 + a_{3a}s_4 \tag{19}$$

where $a_{ia}(i = 1, 2, 3)$, $c_{ia}(i = 1, 2, 3, 4)$ are positive constants whose values are defined by the user. For low chattering, the auxiliary sliding variable is defined as

$$\sigma = \dot{s}_A + \bar{c}\, s_A \tag{20}$$

Now our aim is to design v that provides finite time convergence of $\sigma \to 0$, and an ideal sliding mode to occur in the sliding surface $\sigma = 0$ as

$$\sigma = \dot{s}_A + \bar{c}\, s_A = 0 \tag{21}$$

Control law v that derives the σ to zero in finite time is defined as

$$v = \left(-\frac{1}{G}\right)[Ax_7 + Bx_8 + CC_1 + DD_1 + EE_1 + FF_1 + \bar{c}\,(c_1x_5 + a_1c_2x_6)] - \rho\, sgn(\sigma) \tag{22}$$

where $A = -b_7a_2 + b_2 + c_3a_2\bar{c}$, $B = -b_8a_3 + c_4a_3\bar{c} + +c_5a_1 + c_4a_3\bar{c}$, $C = b_1a_2 - b_1 + c_1 + \bar{c}$, $D = -a_1b_4 + a_1c_2 + a_3b_4 + a_1\bar{c}$, $E = a_2c_3 + a_2\bar{c}$, $F = a_3c_4 + a_3\bar{c}$, $G = b_3 + a_1b_6 - a_7b_3 - a_3b_6$, $C_1 = -b_1x_5 + b_2x_3 + b_3u$, $D_1 = -b_4x_6 + b_5x_4 + b_6u$, $E_1 = b_1x_5 - b_7x_3 - b_3u$, $F_1 = b_4x_6 - b_8x_4 - b_6u$

Remark 2: The control law v in (22) of asymptotic sliding mode makes both $\sigma\ and\ \dot{\sigma} \to 0$ together with convergence of states to desired position as time increases, even in presence of unknown disturbance and slowly time varying parameters.

5. RESULTS AND DISCUSSIONS

We used the fourth order Runga Kutta method for solving the dynamics of (11), (17) and simulating the results with step time h=10^{-3} in MATLAB. The parameter values are: $p_1 = 2.0796$, $p_2 = 7.955$, $p_3 = 1.29$, $p_4 = 2.237$, $p_5 = 4.09$, $p_6 = 1.39$, $p_7 = 412$, $p_8 = 408$. The values of parameter uncertainties are: $d_1 = 0.2\sin(5t)$ $d_2 = 0.7\sin(5t)$, $d_4 = 0.2(5t)$, $d_5 = 0.4\sin(5t)$, $d_7 = 41.2\sin(5t)$, $d_8 = 40.8\sin(5t)$ and
d = $0.1\sin(x_1)\,\sin(x_2)\sin(x_3)\sin(x_4)\sin(x_5)\sin(x_6)\sin(x_7)\sin(x_8)$. The initial conditions for the analysis with PI SMC are chosen as $x(0) = (0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0, 0)^T$, and the value of different constants are: $c_1 = 7$, $c_2 = 2$, $\rho = 5$, $c_3 = 3$, $c_4 = 0.5$, $a_1 = 0.6$, $a_2 = 0.3$, $a_3 = 0.25$. The initial conditions for the analysis with asymptotic SMC are chosen as $x(0) = (0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0)^T$ and the value of different constants are: $c_{1a} = 7$, $c_{2a} = 2$, $\rho = 7$, $c_{3a} = 3$, $c_{4a} = 3$, $a_{1a} = 0.6$, $a_{2a} = 0.3$, $a_{3a} = 0.25$, $\bar{c} = 2.21$.

5.1 POSITION CONTROL WITH PI SMC

Fig.2 (a, b) shows the position of states (x_1, x_2) i.e angle of the first joint and second joint. Fig. 2 (c, d) shows the position of states (x_3, x_4) i.e deflection of the first and second link. The velocities of the joints angle and links deflection are shown in Fig. 3 (a, b) and Fig. 3 (c, d), respectively. The time responses of the sliding surface and control input are shown in Fig. 4 (a) and Fig. 4 (b), respectively. It is observed from Fig. 2 (a, b) that first and second joint angles reached the desired position in 1.33 s and 1.28 s, respectively. The deflection of the links are very small as shown in Fig. 2 (c, d). It is also apparent from Fig. 3 that velocities of the joints angle and links deflection tend to zero.
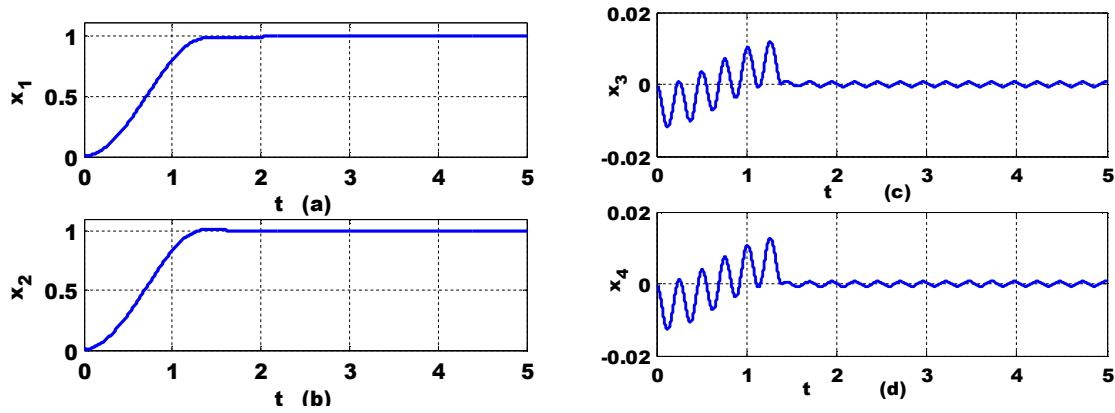
Figure 2: Link position with PI SMC (a) of first link (b) of second link. Link deflection (c) of the first link (d) of the second link with PI SMC.
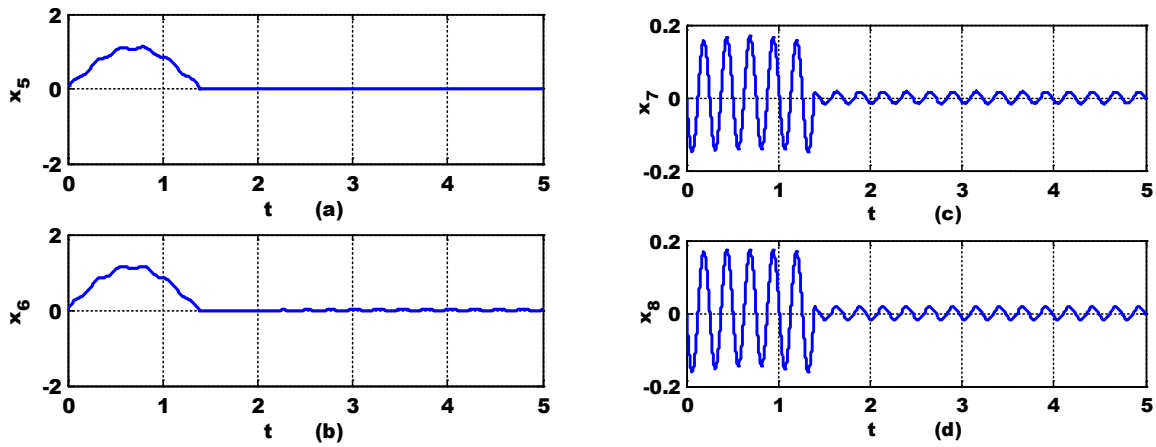

Figure 3: Velocity with PI SMC (a) of the first joint (b) of the second joint. Velocity of the (c) first link deflection (d) second link deflection with PI SMC.
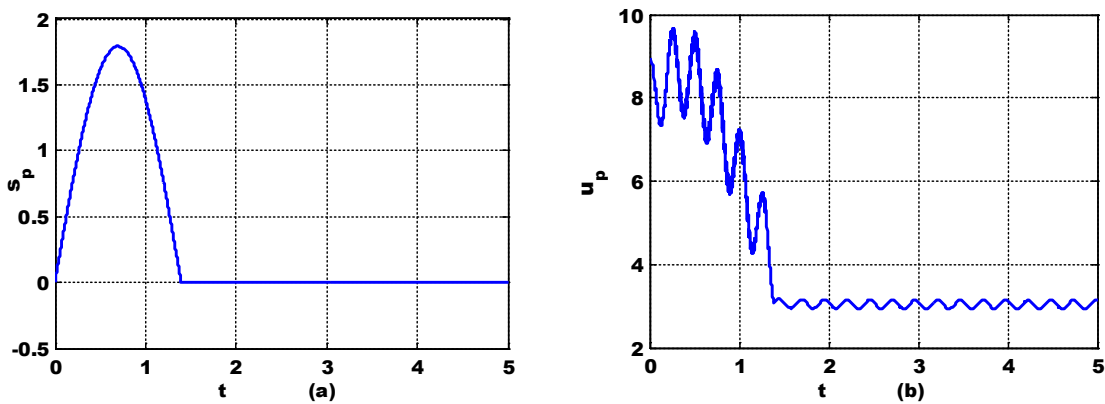

Figure 4: Time response of sliding surface with PI SMC. (b) Time response of control input required with PI SMC.

Sliding surface in Fig. 4 (a) reaches zero (converge) in s but the required control input in this case (PI SMC) is comparatively high. The next subsection deals with the results and discussions of position control using asymptotic SMC.

5.2 Position Control With Asymptotic SMC

The angular position of the first link and second link with settling time 3.05 s with their deflections are given in Fig. 5 (a, b) and Fig. 5 (c, d), respectively, which tends to zero. Their respective velocities are also shown in Fig. 6 (a,b) and Fig. 6 (c, d). Fig. 7 (a) and Fig. 7 (b) gives the sliding surface and control input of the controller with asymptotic SMC.

5.3 Comparisons between PI-SMC and Asymptotic SMC

It is clear that the joints with PI SMC reaches the desired position in less time as compared to the asymptotic SMC. Similarly, comparison of the states i.e. deflection of both the links showed that the deflection of the links with asymptotic SMC is small as compared to PI SMC. The performance indices of the flexible manipulator are given in Table 2. Moreover, to check the control input required, the energies are also calculated and compared.
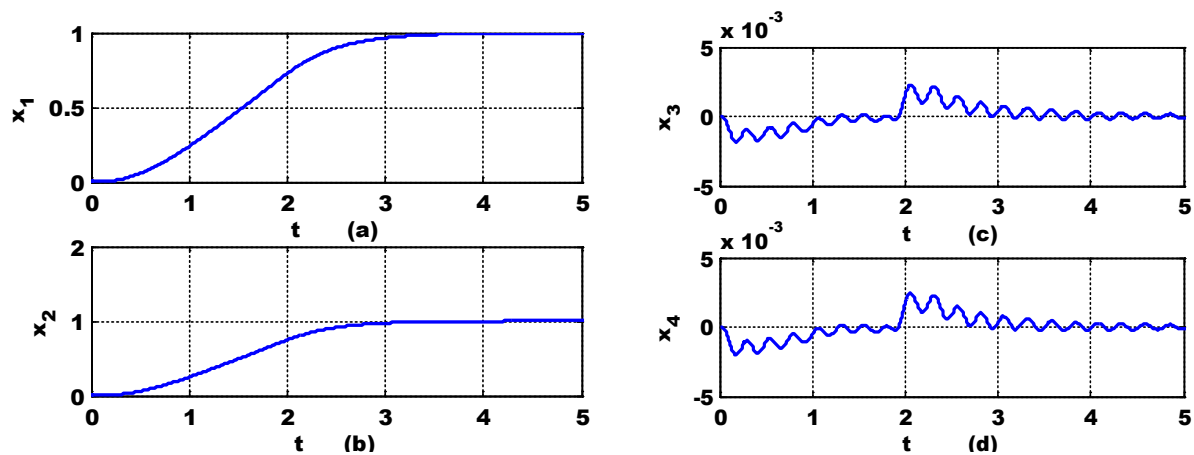

Figure 5: Links position (a) of first link (b) of second link with asymptotic SMC. (c) deflection of first link (d) deflection of second link with asymptotic SMC.
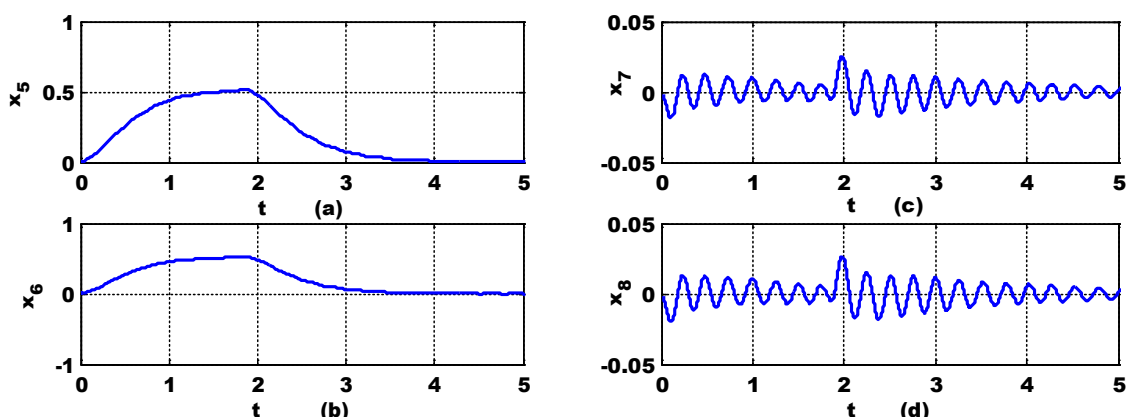

Figure 6: Velocity of the joint angle (a) the first joint (b) the second joint with asymptotic SMC. Velocity (c) of the first link (d) of the second link with asymptotic SMC.
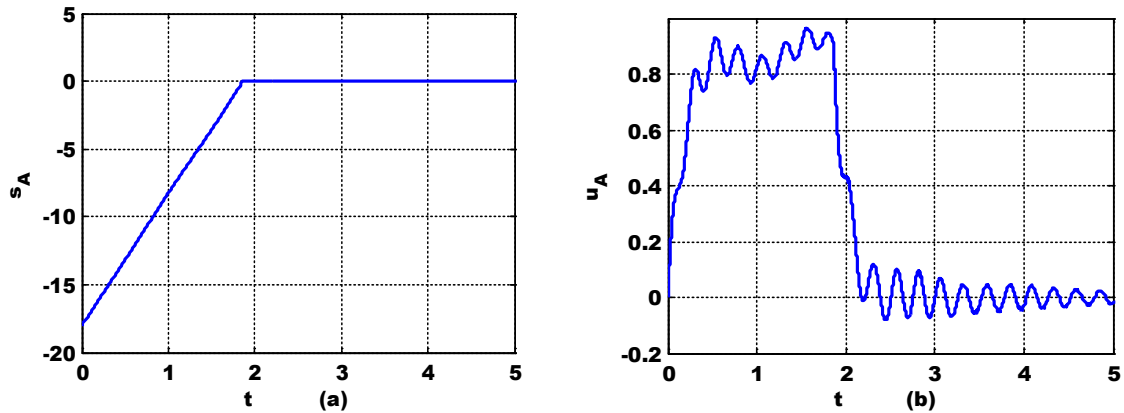
Figure 7: Time response of sliding surface with asymptotic SMC. (b) The time response of control input required for asymptotic SMC.

Table 2

Comparisons on The Performance of PI SMC and Asymtotic SMC.

S. L.	Performance Indices	PISMC	Asymptotic SMC	S.L	Performance Indices	PISMC	Asymptotic SMC
1.	Settling time of θ_1	1.33s	3.15s	3.	Sliding surface reaching time	1.3s	1.8s
	Settling time of θ_2	1.28s	3.05s	4.	Integral Square Error (ISE)	0.531	1.1732
2.	α_1	High deflection magnitudes	Small deflection magnitudes	5.	Control input energy	1.083×10^5	1.29×10^3
	α_2	Large deflection magnitudes	Small deflection magnitudes				

6.   CONCLUSIONS

Two low chattering SMC control techniques for position tracking and tip deflection control of a two-link flexible manipulator are presented. The control techniques are been developed on the basis of low chattering in sliding surfaces and in control input. The techniques are implemented within the simulation environment with bounded unknown disturbances and slowly time varying uncertainties in parameter of TLFM. The performances of the control techniques are evaluated. A comparative analysis of the two control techniques reveal that PI SMC technique results in better performance than asymptotic SMC in respect of settling time of hub angular position, sliding surface reaching time and in intergral square error. However, the links deflection and control input energy are less in case as of asymptotic SMC.

References

[1]   M. O. Tokhi and A. K. M. Azad, "Flexible Robot Manipulators: Modelling, simulation and control," *IET control engineering series, Institution of Electrical Engineers*, Vol. 68, ISSN pp. 0262–1797, 2008.

[2]   M. Farid and W. L. Cleghorn, "Dynamic Modeling of Multi-Flexible-Link Planar Manipulators Using Curvature-Based Finite Element Method," *Journal Of Vibration and Control,* Vol. 20, No. 11, pp. 1682-1696, August 2014.

[3]   B. Subudhi and A. S. Morris, "Dynamic modelling, simulation and control of a manipulator with flexible links and joints," *Robotics and Autonomous. Syst.,* Vol. 41, pp. 257-270, 2002.

[4] S. Kim, "Lumped Element Modeling of a Flexible Manipulator System*," IEEE/ ASME Transactions on Mechatronics,* Vol. 20, No. 2, pp. 967-974, April 2015.

[5] K. Lochan, S. Suklabaidya and B. K. Roy, "Dynamic Modelling, Simulation of Single-Link Flexible Robot Manipulator and Its Control," *Journal of Control & Instrumentation,* Vol. 5, No. 3, 2014.

[6] W. Yim, "Cartesian Trajectory Control of a Flexible Manipulator Using Sliding Mode," *Mechatronics,* Vol. 4, No. 6, pp. 635-652, 1994.

[7] I. H. Akyuz, E. Yolacan, H. M. Ertunc and Z. Bingul, "PID and State Feedback Control of a Single-Link Flexible Joint Robot Manipulator," *IEEE International Conference on Mechatronics*, pp. 409-414, April 2011.

[8] S. Suklabaidya, K. Lochan, B. K. Roy, "Dynamic Modelling and State-Feedback Control of a Rotational Base Flexible Link Manipulator," *Journal of Electronic Design Technology,* Vol. 5, Issue 3, pp. 1-7, 2014.

[9] L. Zhang and J. Liu, "Observer-based partial differential equation boundary control for a flexible two-link manipulator in task space," *IET Control Theory Appl.,* Vol. 6, Iss. 13, pp. 2120–2133, 2012.

[10] W. Dongmei, "The Design of Terminal Sliding Controller of Two-Link Flexible Manipulators," *IEEE International Conference on Control and Automation*, Guangzhou, China, June 2007.

[11] S. Lee and C. Lee, "Hybrid Control Scheme for Robust Tracking of Two-Link Flexible Manipulator,"*Journal of Intelligent and Robotic Systems*, Vol. 34, pp.431–452, 2002.

[12] B. Subudhi and A. S. Morris, "Soft computing methods applied to the control of a flexible robot Manipulator," *Journal of Applied Soft Computing*, Vol. 9, Issue 1, pp. 149-158, January 2009.

[13] J. E. Slotine and W. Li, "Applied Nonlinear Control, Prentice Hall," New Jersey, pp. 40-97, 276-289, 1991.

[14] I. Boiko, "Analysis of sliding modes in the frequency domain," *International Journal of Control*, Vol. 78, No. 10, pp. 969–981, 2005.

[15] V. I. Utkin, J. Guldner and J. Shi, "Sliding mode control in electromechanical systems", *Taylor and Francis,* London 1999.

[16] H. Lee and V. I. Utkin, "Chattering suppression methods in sliding mode control systems,"*Annual Reviews in Control,* Vol. 31, pp. 179–188, 2007.

[17] A. Aroudi, B. Robert and R. Leyva, "Sliding mode control of a high voltage DC–DC buck converter. Circuit Theory and Design," *Proc. of the European Conference*, Vol. 3, No. 28, pp. III/55–III/59, 2005.

[18] M. Tseng and M. Chen, "Chattering reduction of sliding mode control by low-pass filtering the control signal,"*Asian Journal of Control*, Vol. 12, No. 3, pp. 392 398, May 2010.

[19] A. Levant, "Higher-order sliding modes, differentiation and outputfeedback control," *International Journal of Control,* Vol. 769, No.9, pp. 924–941, 2003.

[20] Y. Wang, Y. Feng and X. Yu, "Fuzzy Terminal Sliding Mode Control of Two-link Flexible Manipulators," *IEEE 34th Annual Conference of Industrial Electronics*, pp. 1620–1625, November 2008.

[21] D. S. Rana, "Mathematical Modelling, Stability Analysis and Control of Flexible Double Link Robotic Manipulator: A Simulation Approach," *IOSR Journal of Engineering*, Vol. 3, No. 4, pp. 29-40, 2013.

[22] N. Meena and B. B. Sharma, "Backstepping Algorithm with Sliding Mode Control for Input-Affine Nonlinear Systems," *2nd International Conference on Emerging Trends in Engineering and Technology,* May 30-31, London (UK), pp. 125-130, 2014.

[23] Y. Wang, Y.Feng and X. Yu, "Fuzzy Terminal Sliding Mode Control of Two-link Flexible Manipulators," *34th Annual Conference of IEEE Industrial Electronics*, pp. 1620–1625, November 2008.

[24] Y. Wang, H. W. Xia and C. Wang, "Hybrid Controllers for Two-Link Flexible Manipulators," Part III, CCIS 226, pp. 409–418, 2011.