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Performance Evaluation of an Efficient Reduced Complexity Approach for Discrete Hartley Transform Based Optical OFDM Systems

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ABSTRACT

In this paper, a new low complexity transform which combines the fast Walsh Hadamard transform (WHT) and fast Hartley transform (FHT) into a single fast orthonormal unitary transform is proposed for optical orthogonal frequency division multiplexing (O-OFDM) across additive white Gaussian noise (AWGN) channel model. The new transform is developed through the sparse matrices factorization method in OFDM system, which is capable of reducing peak to average power ratio (PAPR) of the transmitted symbols and improving the bit error rate (BER) performance at a reasonable reduced complexity. The system performance is verified via simulations. Compared with the FHT method, the proposed OFDM signal can be generated by FHT via WHT with lower PAPR and also the computational complexity nearly halved. It reveals that the proposed OFDM has the same bit-error-rate (BER) performance as FHT, but proposed OFDM transceiver shows superiority on computational complexity. The proposed scheme is a cost-effective and efficient multicarrier modulation scheme.

Keywords: CCDF, discrete Hartley transform, O-OFDM, PAPR.

1. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) is a multicarrier modulation (MCM) technique in which multiple data streams are modulated with mutually orthogonal subcarriers [1]. The scalability and flexibility to higher order modulation offered by optical OFDM (O-OFDM) is unique compared to single carrier modulation techniques, which require complex and costly coherent schemes. Optical OFDM systems can be used for wide range of applications such as high speed optical LANs, 10Gb/s Ethernet using multimode fiber, interconnects in data centers and high performance computing. These distinctly different techniques provide different degrees of effectiveness, and present different sets of tradeoffs that may include increased system complexity, reduced spectral efficiency, and performance for improved linearity [2]-[4]. Due to the advantage of bandwidth efficiency and the immunity of multipath channel fading, OFDM systems are widely adopted in many digital communication standards. However, the major drawback of O-OFDM transmitted signal exhibits very high peak to average power ratio (PAPR). Several methods have been proposed to address the PAPR problem [5]-[9]. These distinctly different techniques provide different degrees of effectiveness, and present different sets of tradeoffs that may include increased system complexity, reduced spectral efficiency, and performance for improved linearity. An alternative approach to mitigating the PAPR problem is based on signal transformation. Most of OFDM transceivers employ inverse fast Fourier transform (IFFT) and FFT to perform modulation and demodulation in transmitter and receiver,

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respectively. The FFT/IFFT becomes one of the most critical modules in OFDM transceivers. The rapidly increasing demand of OFDM based applications for wireless broadband communications makes processing speed an important major consideration in FFT architecture design [10]. Accordingly, if we would like to design a DFT-OFDM transceiver with lower power consumption or computational complexity, an efficient FFT processor design is important. Once the FFT algorithms have been chosen, the computational complexity of FFT was decided. Even so, nowadays most FFT researches focus on the hardware implementation techniques instead of algorithm study since the FFT algorithms have already been well developed. However, IFFT/FFT is not the only orthogonal basis for OFDM systems. Recently, studies focus on the different trigonometric transforms, such as discrete Hartley transform (DHT) and discrete cosine/sine transform (DCT/DST) have been proposed to be an alternative orthogonal transformation for OFDM systems [18]. The DHT based OFDM is attractive for its intimate relation to DFT.

A cost effective implementation of the O-OFDM is the multicarrier modulation, where the OFDM signal is real signal. To produce a real OFDM signal, the input sequence mapped into a complex constellation is forced to have Hernutian symmetry. In phase and quadrature modulation onto an RF carrier is required in IFFT points. An alternative O-OFDM scheme, which deals with real signals, is based on the DHT [11]. In this transform, the Herrnitian symmetry is not required. Hence all the transform points carry data symbols and the same data rate can be transmitted. The direct and inverse transforms are identical so both functions can be carried out using same function. Fourier transform always implies a complex processing and the phase carries fundamental information, while Hartley transform is a real trigonometric transform. In [12], a DHT based OFDM is proposed to transmit the symbols over the subchannels separately, and the single-tap equalizer is able to compensate the subchannels.

In this paper, a new low complexity transform to combine the Walsh Hadamard transform (WHT) and the inverse fast Hartley transform (IFHT) into a single orthonormal unitary transform is proposed in OFDM system to achieve significant BER performance and considerable PAR reduction. Compared with FFT scheme, it can reduce computational complexity by half.

The remainder of this paper is organized as follows. In section II, the proposed OFDM system model is presented. Then we discuss a low complexity combined WHT-DHT transform structure in section III. The system performance over AWGN channel is simulated in section IV. Finally, conclusion is presented in section V.

2. PROPOSED OFDM SYSTEM MODEL

The block diagram of OFDM system based on fast Hartley transform (FHT) via WHT is depicted in Figure 1. The FHT via WHT scheme combining DHT and WHT is proposed in this work. The N point inverse FHT and FHT are defined by [13],

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} H_k cas\left(2\pi nk / N\right), \quad n = 0,1,\ldots,N-1 \tag{1}$$

$$H_k = \frac{1}{N}\sum_{n=0}^{N-1} x_n cas\left(2\pi nk / N\right), \quad k = 0,1,\ldots,N-1 \tag{2}$$

Where, $cas(.) = cos(.) + sin(.)$

x_n is the IDHT sequence

H_k is the DHT sequence

In proposed OFDM system, the modulating bits are modulated on BPSK, which are fed into the IDHT via WHT to compute the discrete time real baseband signal as

$$X_n = T' x_n \tag{3}$$

Where,

$T' = \frac{1}{N} WH'$ is conversion matrix, W and H' are WHT matrix and the normalized $N \times N$ IDHT matrix rearranged by column reverse order.

In proposed OFDM transmitter, N point IDHT via WHT is performed, instead of applying N point IDHT. The resulting symbols are appended with cyclic prefix (CP) and convolved with channel impulse response, and then corrupted by the AWGN.

Consequently, the received signal after removing the CP can be written as

$$y = h \otimes x + n \tag{4}$$

Where,

At the receiver section, the received signal vector $\{r_n\}$, $n \in [0, N-1]^T$ is fed into the proposed transform block to signal as

$$R = Ty \tag{5}$$

3. DHT VIA WHT TRANSFORM STRUCTURE

In this paper, the proposed transform, which is used in the receiver of Figure 2 is evaluated as

$$T = \frac{1}{N} HW \tag{6}$$

H is the normalized $N \times N$ DHT matrix rearranged by row reverse order, W is the Walsh-Hadamard matrix.

Consequently, the H matrix in row reverse order can be computed as

$$H_N = \begin{bmatrix} A_{\frac{N}{2}} & A_{\frac{N}{2}} \\ B_{\frac{N}{2}} & -B_{\frac{N}{2}} \end{bmatrix} \tag{7}$$

Where, A and B are sub matrices of H.

On the other hand, the WHT of the product of two sequences is equivalent to the dyadic convolution of their WHT,

It means[14]-[15],

$$W\left[X_1 . X_2\right] = WX_1 \bar{\otimes} \; WX_2 \tag{9}$$

Where,

X_1 and X_2 are discrete data vectors in the frequency domain $\bar{\otimes}$ represents dyadic convolution[16], [17].

In general, the W matrix can be written as a function of lower order matrices as

$$W_N = \begin{bmatrix} W_{\frac{N}{2}} & W_{\frac{N}{2}} \\ W_{\frac{N}{2}} & -W_{\frac{N}{2}} \end{bmatrix} \tag{10}$$

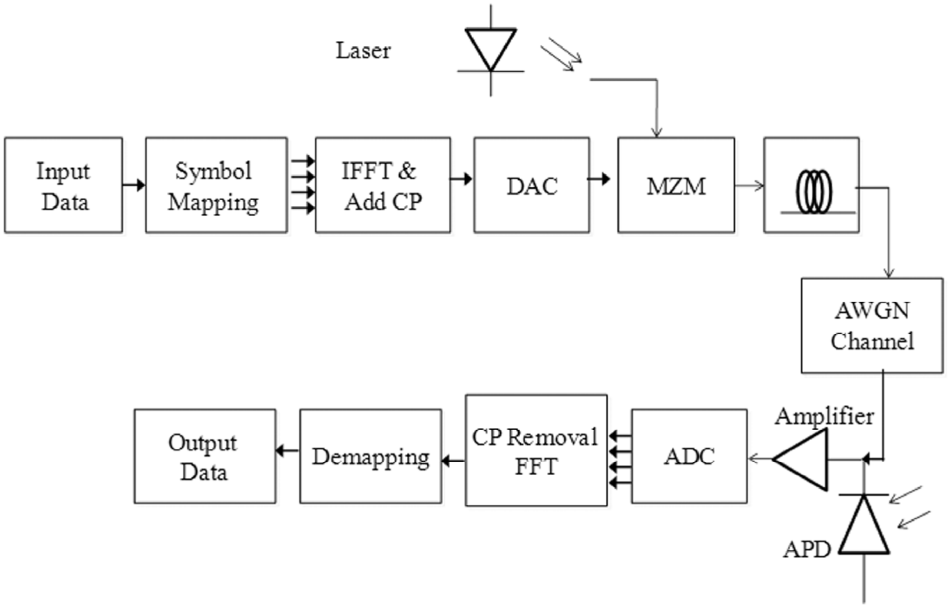

Figure 1: Block diagram of Proposed OFDM system

Upon substituting (7) and (9) into (6), we get

$$T_N = \frac{1}{N}\begin{bmatrix} 2A_{\frac{N}{2}}W_{\frac{N}{2}} & 0 \\ 0 & 2B_{\frac{N}{2}}W_{\frac{N}{2}} \end{bmatrix} \tag{11}$$

To show the block diagonal structure of T_N,

The normalized $N \times N$ Hartley matrix can be written as,

$$H_N = \frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & cas(1) & cas(2) & \cdots & cas(N-1) \\ 1 & cas(2) & cas(4) & \cdots & cas(2(N-1)) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & cas(N-1) & cas(2(N-2)) & \cdots & cas((N-1)(N-1)) \end{bmatrix} \tag{12}$$

For instance, the case $N = 8$.

$$H_8 = \frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & cas(1) & cas(2) & cas(3) & cas(4) & cas(5) & cas(6) & cas(7) \\ 1 & cas(2) & cas(4) & cas(6) & cas(8) & cas(10) & cas(12) & cas(14) \\ 1 & cas(3) & cas(6) & cas(9) & cas(12) & cas(15) & cas(18) & cas(21) \\ 1 & cas(4) & cas(8) & cas(12) & cas(16) & cas(20) & cas(24) & cas(28) \\ 1 & cas(5) & cas(10) & cas(15) & cas(20) & cas(25) & cas(30) & cas(35) \\ 1 & cas(6) & cas(12) & cas(18) & cas(24) & cas(30) & cas(36) & cas(42) \\ 1 & cas(7) & cas(14) & cas(21) & cas(28) & cas(35) & cas(42) & cas(49) \end{bmatrix} \tag{13}$$

The DHT matrix can be rewritten as,

$$H_8 = \frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1.4142 & 1 & 0 & -1 & -1.4142 & -1 & 0 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 0 & -1 & 1.4142 & -1 & 0 & 1 & -1.4142 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -1.4142 & 1 & 0 & -1 & 1.4142 & -1 & 0 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 0 & -1 & -1.4142 & -1 & 0 & 1 & 1.4142 \end{bmatrix} \tag{14}$$

When rearranging the DHT matrix H_8 by row reversed order, can be obtained as

$$H_8 = \frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1.4142 & 1 & 0 & -1 & -1.4142 & -1 & 0 \\ 1 & -1.4142 & 1 & 0 & -1 & 1.4142 & -1 & 0 \\ 1 & 0 & -1 & -\ 1.4142 & -1 & 0 & 1 & 1.4142 \\ 1 & 0 & -1 & 1.4142 & -1 & 0 & 1 & -1.4142 \end{bmatrix} \tag{15}$$

Consequently, the above matrix can be written in terms of sub matrices as

$$H_8 = \frac{1}{\sqrt{8}}\begin{bmatrix} A_4 & A_4 \\ B_4 & -B_4 \end{bmatrix}$$

$$A_2' = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$B_2'' = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \qquad B_2' = \begin{bmatrix} 1 & -1.4142 \\ 1 & 1.4142 \end{bmatrix} \tag{16}$$

Where, A, B are submatrices of H, which can be expressed as

$$A_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \tag{17}$$

$$B_4 = \begin{bmatrix} 1 & 1.4142 & 1 & 0 \\ 1 & -1.4142 & 1 & 0 \\ 1 & 0 & -1 & -1.4142 \\ 1 & 0 & -1 & 1.4142 \end{bmatrix} \tag{18}$$

Furthermore, the sub matrices in (16) and (17) can be subdivided into their internal submarines as

$$A_4 = \begin{bmatrix} A_2' & A_2' \\ A_2' & -A_2' \end{bmatrix} \tag{19}$$

$$B_4 = \begin{bmatrix} B_2' & B_2'' \\ B_2'' & -B_2' \end{bmatrix} \tag{20}$$

Where

$$A_2' = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$B_2'' = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \qquad B_2' = \begin{bmatrix} 1 & -1.4142 \\ 1 & 1.4142 \end{bmatrix} \tag{21}$$

For WHT matrix, $N = 8$

$$W_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} \tag{22}$$

Consequently, the above equation (22) can be written in terms of sub matrices as

$$W_8 = \begin{bmatrix} W_4 & W_4 \\ W_4 & -W_4 \end{bmatrix} \tag{23}$$

Upon substituting (14) & (22) in (6), we get

$$T_8 = \frac{1}{8}\begin{bmatrix} 2A_4W_4 & 0 \\ 0 & 2B_4W_4 \end{bmatrix} \tag{24}$$

$$T_8 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8536 & 0.1464 & 0.3536 & -0.3536 \\ 0 & 0 & 0 & 0 & 0.1464 & 0.8536 & -0.3536 & 0.3536 \\ 0 & 0 & 0 & 0 & 0.3536 & -0.3536 & 0.1464 & 0.8536 \\ 0 & 0 & 0 & 0 & -0.3536 & 0.3536 & 0.8536 & 0.1464 \end{bmatrix} \tag{25}$$

Hence, the proposed transform based on the block diagonal structure reduces the superposition of the subchannels, as seen in (25). The fast proposed transform requires $\frac{\left[N\log_2 N-2\left(N-1\right)\right]}{2}$ of real multiplications. Transform algorithms are compared based on the total number of arithmetic operations for 64 subcarriers as shown in Table 1. Clearly, the results show that the proposed transform involves less computational complexity than the direct computation of DHT.

Table 1
A Comparison of real multiplications for various transforms for 64 subcarriers

Fast Transforms	*Real Multiplications*	*Example (Real Multiplications)*
N point IFFT	$N\log_2 N$	384
N point IFHT	$\frac{N\log_2 N}{2}$	192
Proposed Transform	$\frac{\left[N\log_2 N-2(N-1)\right]}{2}$	129

4. SIMULATION RESULTS

To study the performance of the discrete Hartley based O-OFDM system, this section illustrates the performance analysis of the proposed OFDM, DHT-OFDM and the DFT-OFDM systems across AWGN channel model. Further, the simulation parameters required to investigate the performances of the proposed OFDM illustrated in Table 2.

In order to verify the validity of our analytically derived technique, MATLAB simulation program is performed. The simulation is carried out for the following metrics:

- CCDF performance
- BER performance

Table 2
Simulation parameters for proposed OFDM System

Simulation Parameters	*Values*
FFT/FHT	64
CP	16
Modulation	16-QAM
Channel	AWGN

Figure 2 shows a power spectral density for 64 subcarriers DHT and DFT based OFDM systems. To analyze the performance of the OFDM system, amplitude of DFT-OFDM signal and DHT-OFDM scheme is obtained and plotted in Figure.3(a) and Figure.3(b), respectively. The proposed scheme can enlarge small signals and change the signal peaks, which leads to a higher peak power level of output. This scheme provides reduction of PAPR by extra enlargement of amplitudes.

Figure 3 illustrates the CCDF performance of DFT-OFDM, DHT-OFDM and proposed OFDM signal with 64 subcarriers. In Figure 3, it can be seen that the PAPR of proposed OFDM achieves 6dB which is less than that of conventional DHT-OFDM system by range of 0.8 dB. Figure.4 illustrates the BER performance of DFT-OFDM, DHT-OFDM and proposed OFDM over AWGN channel model. It can be

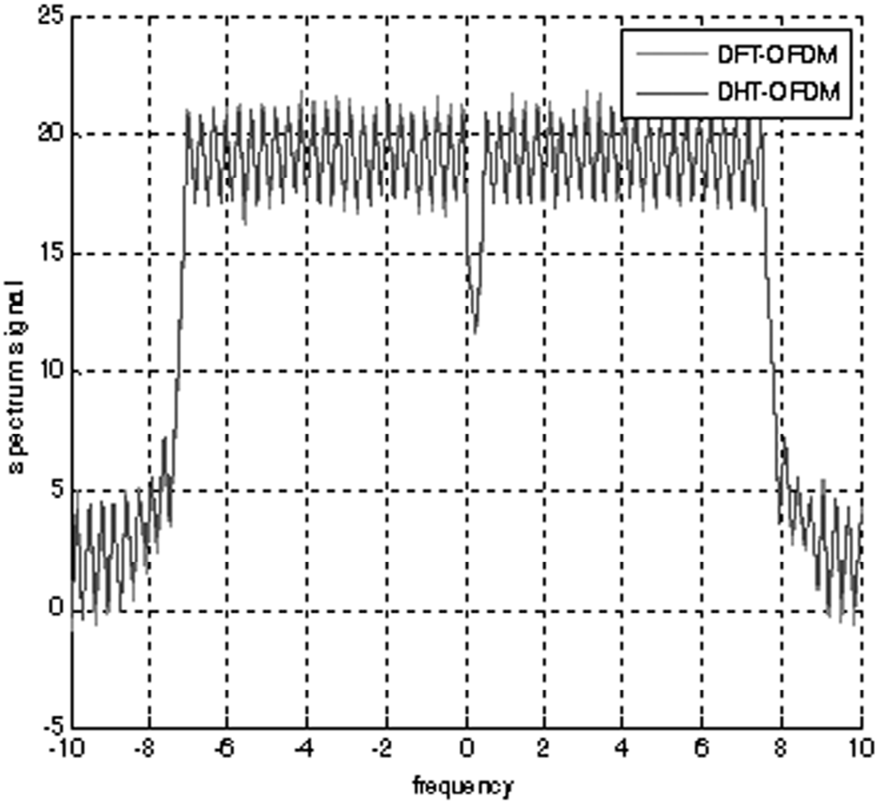

Figure 2: Spectrum representation of DFT OFDM and DHT OFDM

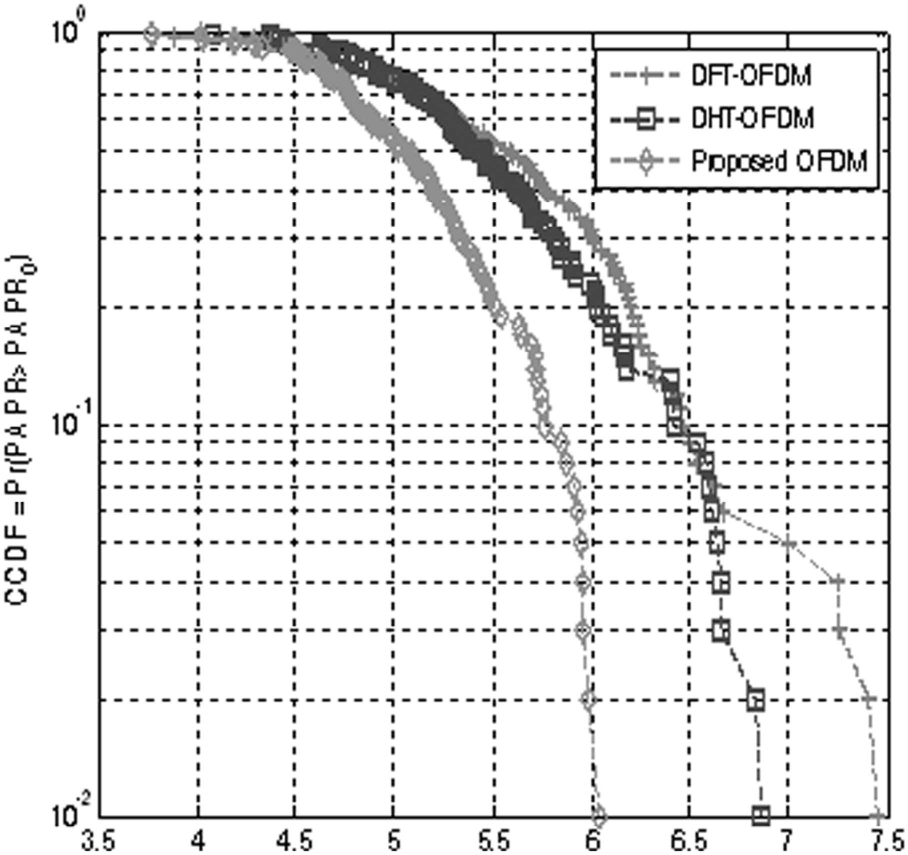

Figure 3: CCDF plot for DFT OFDM, DHT OFDM and proposed OFDM signals with 64 subcarriers

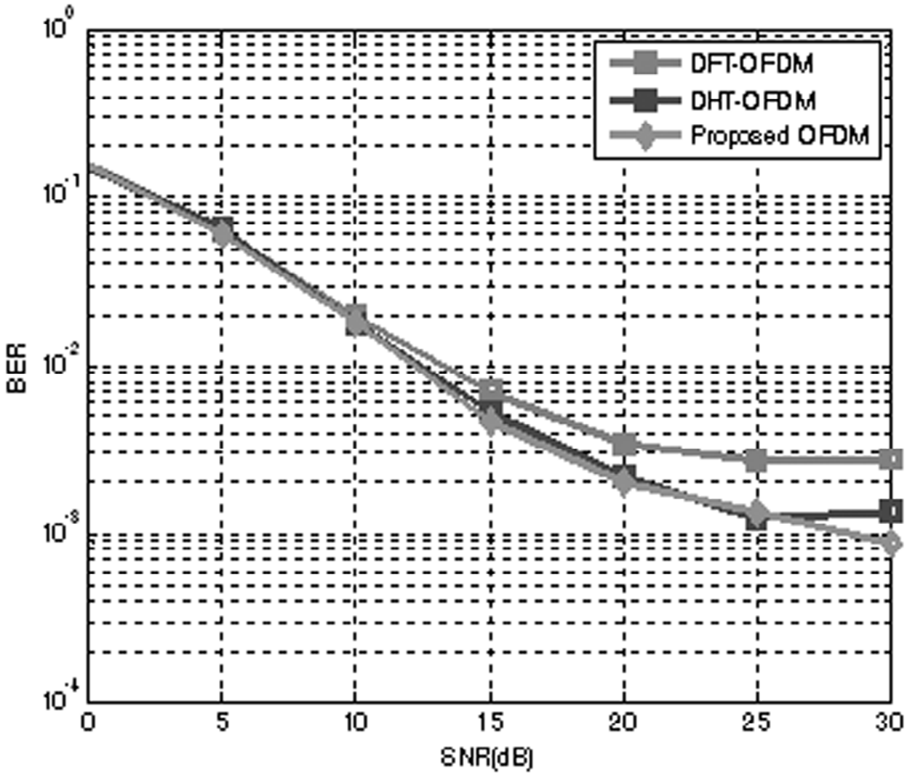

Figure 4: BER performance of DFT OFDM, DHT OFDM and proposed OFDM signals with 64 subcarriers

shown that the proposed OFDM improves the BER performance significantly than the conventional DHT-OFDM system. As can be seen from Figure. 4, the proposed OFDM system achieves SNR about 30dB at 10^{-3} BER level which has better BER performance than DHT-OFDM system. Hence, the proposed OFDM system outperforms the conventional OFDM interms of PAPR reduction, significant complexity reduction and better BER performance.

5. CONCLUSION

In this paper, we have proposed a cost effective and efficient low complexity transform for O-OFDM scheme. Compared with FHT scheme, the fast FHT via WHT can generate the OFDM signal with lower PAPR and reduced computational complexity. Based on the sparse block diagonal transform structure, the proposed OFDM system can achieve PAPR reduction by a range of 0.8dB than that of DHT OFDM system. The simulation results also indicate that the proposed OFDM system has a better BER performance than the DHT-OFDM system. Furthermore, the size of the DHT via WHT scheme was demonstrated to be the same as the number of sub channels, thus there is no data rate losses when utilizing the T-transform with the multicarrier transmission techniques. Simulation results was shown that the proposed T-DCT OFDM system outperforms the OFDM system in the presence of fading channels, and also minimize considerable PAPR. Consequently, a combined DHT via WHT OFDM system will benefit from the reduced PAPR, SNR and no BER performance degradation. For $N = 64$, the proposed OFDM achieves 1.4dB PAPR reduction over the conventional DFT-OFDM. Consequently, the proposed transform based O-OFDM schemes will benefit from the improved BER performance, reduced PAPR level and computational complexity reduction.

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