BPE BULLETIN OF POLITICAL ECONOMY
14:1 (2020): 39-53

Interest, profit and saving in Arrow-Debreu equilibrium models

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Abstract: The paper aims to point out that the concepts of interest, profit and saving that we come across in the Arrow-Debreu equilibrium models are significantly different from what is usually indicated by these same terms in economic analysis. In fact, in the Arrow-Debreu models, they are not related to the investment of capital. As we shall try to show, the difficulties that the Arrow-Debreu theory encounters with reference to capital and related concepts derive from the hypothesis of markets open in a single moment that characterizes these models.

Keywords: Arrow-Debreu general equilibrium; own-rate of interest; firm profit; saving

JEL classification: D11; D15; D46; D51

INTRODUCTION

The fundamental features of the Arrow-Debreu general equilibrium model are well-known. First, a finite number *L* of commodities are traded. Therefore, since in the neo-Walrasian approach 'a commodity is a good or a service completely specified physically, temporally, and spatially' (Debreu, 1959: p. 32), this means that there is just a finite number of different places and dates of delivery[1]. Second, there is a market and a price for each of the *L* commodities. Third, all the *L* markets are open in only one instant. Since this last characteristic is of central importance for the argument developed within this paper, it is worth examining it more closely.

Some scholars seem to believe that the hypothesis of not reopening markets can be seen as an implication of their completeness. In fact, it is said that even if markets reopened, no further transactions would take place, as agents have already concluded all the necessary trades in order to carry out their optimal consumption and production plans[2]. Actually, this would be true if the reopening of markets were unforeseen. If, instead, the reopening

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had been planned from the beginning, then agents' behaviour would have been much different. If agents know that markets will reopen at some point in the future, they will have expectations about the prices that can be set at that time and, if these expectations deviate from current prices of commodities for future delivery, they will have an incentive to engage in speculative transactions, expecting a gain. Therefore, all the problems related to agents' expectations and speculative trades are avoided in the Arrow-Debreu model, thanks to the assumption that markets do not reopen[3]. It is therefore a crucial assumption.

Moreover, the single moment in which markets are open is also the moment in which decisions are taken. In fact, since agents' decisions depend on the price vector, once an equilibrium is achieved, agents' consumption and production plans are definitively determined, and the contracts needed for their implementation are signed. Using a terminology introduced by Bliss, the moment in which decisions are taken is the 'instruction date' and the moments in which commodities are delivered are the 'action dates', and the former must precede the latter:

> The action date cannot precede the instruction date. To decide upon an action of yesterday is not to decide upon anything; it is either to mentally confirm an action or else it is daydreaming. The instruction date determines for us, therefore, the sequence of action dates that have to be taken into account in our theory and it is necessarily then a matter of central importance to an economic theory which takes account of time. […] Our description of the economy will start at a particular moment of time and this will be our first instruction date. (Bliss, 1975: p. 43)

As a result, markets are open in a moment preceding every possible delivery of commodities – typically the initial instant of the horizon considered. This entails that what is exchanged on the markets is not properly commodities, since all the commodities will be delivered later on, but just the promise of their delivery. In other words, in the Arrow-Debreu model, the distinction between present and future commodities is almost meaningless because the commodities delivered in the first period are also future commodities in the moment in which markets are open.

Once an equilibrium is achieved in the initial moment, commodity prices and agents' plans are fully determined[4]. The actual fulfilment of the contracts and the consequent delivery of commodities in subsequent periods is something that is taken for granted, since it is not particularly relevant for the analysis. Arrow-Debreu equilibrium theory is, accordingly, considered a static theory since it does not address any really dynamic problem. Nonetheless, in introducing his theory of value, Debreu writes:

> By focusing attention on changes of dates one obtains, as a particular case of the general theory of commodities which will be developed below, a theory of saving, investment, capital and interest. (Debreu, 1959: p. 32)

In the present paper, we will attempt to check whether a convincing treatment of savings, capital and interest can really be found in this theory. In particular, we will examine in detail the concepts of interest (section 2), profit (section 3) and saving (section 4) the Arrow-Debreu theory deals with. Some conclusions are drawn in section 5.

RATE OF INTEREST AND INTERTEMPORAL PRICES

In the marginalist theory of distribution, the rate of interest is the price firms pay to households for the use of capital. In its simplest and standard version, the working of the marginalist theory can be schematically represented as follows. The factors of production – namely labour, land and capital – are available in given quantities and owned by households. Households sell to firms the use of the factors of production and, in so doing, get their income – wages, rents and interest. By the use of the factors of production, firms produce goods and services that they sell to households[5].

The Cambridge controversy about the theory of capital proved that this representation of the economic system has important flaws[6]. In particular, the investment of capital cannot be considered as the quantity of a factor of production and, accordingly, the rate of interest cannot be understood as the price for its use.

The neo-Walrasian theory is utterly in line with these results. The idea of capital as a factor of production is completely absent in this approach: Arrow-Debreu commodities – i.e., goods and services with specific dates and places of delivery – are produced by means of Arrow-Debreu commodities. As a result, the rate of interest is not understood as the price firms pay for the use of a factor of production. Actually, the rate of interest does not seem to be a central concept within this approach. As Bliss wrote in his book[7]:

> capital theory should be liberated from the concept of the rate of interest, meaning by that one rate. In its place we will enthrone not the old king, capital; there can be no going back to days when his rule found unquestioned acceptance. Instead, we will find the

> concept of intertemporal prices to be fundamental and will see that working with the rate of interest is a clumsy groping for that concept. (Bliss, 1975: p. 10)

And again:

> the rate of interest must be dethroned from its unquestioned place as one of the central concepts of the theory. In its place must be substituted an intertemporal price system. (Bliss, 1975: p. 346)

Besides, as Bliss hints in the first quotation, in the neo-Walrasian approach, there is not just one rate of interest, but there is one 'own-rate of interest' for each good or service and each pair of dates[8]. Let $p_{n,t}$ and $p_{n,t+1}$ be the prices of commodity *n* (with *n* = 1, 2, …, *N*) delivered in period *t* and *t*+1, respectively. The own-rate of interest of commodity *n* between the two dates $r_{n,t}$ is defined by the following equation:

$$\frac{p_{n,t}}{p_{n,t+1}} \equiv 1 + r_{n,t} \tag{1}$$

As is clear from equation (1), this rate (or factor) of interest, unlike the one studied by the marginalist economists, is neither the price of a factor of production nor a source of income. It is just a relative price.

The exchange between quantities of the same good delivered on different dates can be seen as a loan: an agent gives one unit of commodity *n* in period *t* and receive a quantity $(1 + r_{n,t})$ of the same commodity in period *t*+1. However, there is no difference between this sort of trade and the exchange of any other pair of Arrow-Debreu commodities. The interest factor $(1 + r_{n,t})$ is just another name for the relative price $p_{n,t}/p_{n,t+1}$; it is the quantity of commodity *n* delivered in period *t*+1 that an agent must pay in order to have a unit of commodity *n* delivered in period *t*. Since this quantity can be less than 1 (but not negative), the own-rate of interest can be negative (but not smaller than-1).

As a result, it is clear that – as Bliss writes in the passages quoted above – the whole theory of Arrow-Debreu equilibrium can be developed without any reference to these own-rates of interest. The own-rates of interest are not fundamental variables, but derived variables. Only relative prices are actually relevant to agents' decisions.

This point is illustrated by Koopmans (1957: pp. 120-121) by means of the following example. Let us imagine 'a world of fantasy in which all commodities were continuously changing their physical and hedonistic characteristic'; in other words, the commodities delivered in *t* are always physically heterogeneous with the commodities delivered in *t*+1. In this world of fantasy, the own-rates of interest cannot be defined because each kind of commodity exists in one period only, but nonetheless, an Arrow-Debreu equilibrium can be defined and determined for this economy.

Discounted prices?

In his definition of a commodity price, Debreu stresses it is an amount paid now:

> With each commodity, say the *h*th one, is associated a real number, its *price*, p_h. This price can be interpreted as the amount paid *now* by (resp. to) an agent for every unit of the *h*th commodity which will be made available to (resp. by) him. […]
>
> Consider as an example the commodity No. 2 Red Winter Wheat available in Chicago a year from now. Its price is the amount which the buyer must pay *now* in order to have one bushel of that grade of wheat delivered to him at that location and at that date. (Debreu, 1959: p. 32; emphasis in the original)

This passage is characterized by a certain ambiguity due to the fact that, in Debreu's view, both prices and payments are real numbers. They are neither amounts of money, as in the real world, nor quantities of a commodity adopted as numéraire, as in economic theory. Debreu stresses, however, that this number must be paid now. This surely means that payments must take place in the moment in which markets are open[9].

Since agents' decisions depend on relative prices[10], the price vector is typically normalized. This means that, notwithstanding what Debreu writes, prices are typically expressed in terms of a (single or composite) numéraire commodity. Hence, Debreu's insistence on the idea that prices should be paid now could lead someone[11] to believe that the commodity adopted as a numéraire – i.e., as a unit of measurement of value and therefore of prices – should be a commodity delivered in the first period[12]. This fact is at the origin of the belief that Arrow-Debreu equilibrium theory deals with 'discounted prices' or 'present-value prices', as Bliss names them (1975: p. 51).

Let us denote by $p_{n,2}$ the value of one unit of commodity n (with $n \neq 1$) delivered in period 2 as a quantity of a numéraire commodity, let us say commodity 1 delivered in 1. We can then denote by $p_{n,2}$ the price of the same commodity in terms of commodity 1 delivered in 2. Let $r_{1,1}$ be the own-rate of interest of commodity 1 between periods 1 and 2, we have:

$$p_{n,2} = \frac{1}{1+r_{1,1}} P_{n,2} \tag{2}$$

Equation (2) seems to corroborate the idea that $p_{n,2}$ is obtained by discounting $p_{n,2}$ at a rate corresponding to the own-rate of interest of the numéraire commodity[13]. On the contrary, this is a sort of optical illusion due to: i) the symbols used; and ii) the adoption of a commodity delivered in period 1 as numéraire.

In order to make the point clear, let us start by stressing that the possibility of expressing all the prices in terms of a numéraire commodity is linked to the assumption that there is no gain from triangular exchange: the quantity of a generic commodity α an agent has to pay in order to buy 1 unit of a commodity β is exactly the same whether it trades α directly with β, or it trades α with a third commodity γ and then γ with β[14]:

$$\frac{p_\beta}{p_\alpha} = \frac{p_\gamma}{p_\alpha} \cdot \frac{p_\beta}{p_\gamma} \tag{3}$$

If we say that commodity α is commodity 1 delivered in 1, β is commodity n delivered in 2 and γ is commodity 1 delivered in 2, equation (3) becomes:

$$\frac{p_{n,2}}{p_{1,1}} = \frac{p_{1,2}}{p_{1,1}} \cdot \frac{p_{n,2}}{p_{1,2}} \tag{3'}$$

Now – recalling that: $p_{1,2}/p_{1,1} \equiv 1/(1 + r_{1,1})$; ii) $p_{n,2}/p_{1,2} \equiv P_{n,2}$; and iii) $p_{1,1} = 1$ – equation (3') is nothing but equation (2), written with different symbols. Equation (3') unveils the real meaning of equation (2): like equation (3), it simply expresses the fact there is neither gain nor loss from triangular exchange.

Finally, we can also stress that the absence of gains or losses from triangular exchange does not depend on which commodity (single or composite) is adopted as numéraire. In particular, equation (3) as well as equation (3') perfectly hold even when the numéraire is a composite commodity that includes commodities delivered in different periods. In this case, the own-rate of interest of the numéraire commodity cannot be defined for the very simple reason that it does not have a definite date of delivery[15]. In fact, in this framework, the expression 'discounted price' becomes essentially meaningless.

PROFIT IN A PRIVATE OWNERSHIP ECONOMY

In the classical/Marxian theory of distribution, profit is what capitalists earn from their investment of capital. In a very schematic representation of the capitalistic circuit, at the beginning of the production process, capitalists invest a certain amount of value M in order to advance the costs of production; at a later stage, outputs are obtained and sold on the market so that an amount of revenues *M*' is collected. The difference *M*' – *M* is the profit earned by capitalists[16].

With the advent of the marginalist theory, the meaning of capital and profit was drastically changed. On the one hand, capital was understood as a factor of production – something of the same nature as labour and land – and, accordingly, interest on the capital borrowed by firms was included among the costs. On the other hand, profit – i.e. the difference between revenues and costs – became the income earned by firms for the organization of the production processes. As a result, in their decisions about the production plan, firms want to maximize their profits.

In the Arrow-Debreu equilibrium theory – notwithstanding deep differences in the conception of the production process – the idea of profit is the same as the marginalist one. In other words, in the Arrow-Debreu theory, profit is understood as an income earned by firms and, therefore, unlike the classical/Marxian approach, it is not related to the investment of capital.

Since, as said, production does not employ the use of factors of production but Arrow-Debreu commodities, a production plan is a list of quantities of the latter. It is a vector of net-supplies, so that it has negative entries for inputs and strictly positive entries for outputs. Hence, let $\mathbf{p} \in \mathbb{R}^L_+$ be a price vector and $\mathbf{y}^f \in \mathbb{R}^L$ the production plan of firm *f*,[17] then $\pi^f = \mathbf{p} \cdot \mathbf{y}^f$ is the firm profit[18]. Given a price vector, the firm adopts that production plan that maximizes its profit within the set of feasible plans $Y \subset \mathbb{R}^L$.

Let $\pi^f(\mathbf{p})$ be the amount of profit of firm *f* associated with the optimal plan at the price vector **p**, in a 'private ownership economy' (Debreu 1959: 78-80) this profit – which can be gains or losses – is divided amongst households and enters into their budget constraints, in accordance with some exogenously given shares[19]. These shares are not assumed to be traded because stock markets are necessarily inactive in an Arrow-Debreu model. As Geanakoplos writes:

> Let us note first of all that in Arrow-Debreu equilibrium there is no trade in shares of firms. A stock certificate is not an Arrow-Debreu commodity, for its possession entitles the owner to additional commodities which he need not obtain through exchange. Note also that in Arrow-Debreu equilibrium, the hypothesis that all prices will remain the same, no matter how an individual firm changes its production plan, guarantees that firm owners unanimously agree on the firm objective, to maximize profit. If there were a market for firm shares, there would not be any trade anyway, since

> ownership of the firm and the income necessary to purchase it would be perfect substitutes. (Geanakoplos, 1987: p. 121)

In other words, since there is no room for expectations in an Arrow-Debreu model, households cannot have different views about the amount of profit realized by each firm. Accordingly, no gain can ever be obtained from trading in shares of firms.

Equilibrium profit

Looking at the analysis we have reconstructed up to this point, firm profit seems to play an important role within Arrow-Debreu equilibrium theory. On the one hand, it is the income firms earn for the organization of the production processes. On the other hand, it goes into the budget households can spend. However, it is not really so since, in equilibrium, the profit of firms disappears.

This fact can be very easily proven. Let us assume there are H households and F firms. Let $\mathbf{z}(\mathbf{p}) \equiv \sum_{h=1}^{H} \mathbf{z}^h(\mathbf{p})$ be the aggregate households' net demand for commodities, expressed as a function of the price vector **p**, and let $\mathbf{y} = \sum_{f=1}^{F} \mathbf{y}^f$ be the aggregate firms' production plan (net supply). Assuming constant returns to scale, if $\mathbf{p}^*$ and $\mathbf{y}^*$ are an equilibrium price vector and an equilibrium aggregate production plan, respectively, then: i) $\mathbf{p}^* \cdot \mathbf{y} \leq 0 \; \forall \; \mathbf{y} \in Y$; ii) $\mathbf{z}(\mathbf{p}^*) = \mathbf{y}^*$ (see Mas-Colell, Whinston and Green, 1995: p. 607). Now, since inactivity is a feasible production plan – i.e. the null vector belongs to Y – and the equilibrium production plan is profit-maximizing, then $\mathbf{p}^* \cdot \mathbf{y}^* = 0$.[20]

Therefore, if constant returns to scale are assumed, once an equilibrium is achieved, firm profit vanishes. As a result, on the one hand, there is no remuneration for the organization of the production process. The equilibrium production plan brings about the same amount of profit as inactivity, namely a production plan corresponding to the null vector, and it is not clear why firms decide to carry on the former instead of the latter[21]. On the other hand, households' purchasing capacity depends only on the value of their endowments of commodities because there is no income coming from their shares of firms. This also means that the initial distribution of shares among households does not affect their wealth in equilibrium.

CONSUMPTION, CONSUMPTION EXPENDITURE AND SAVING

According to Böhm-Bawerk (1891: pp. 36-37), the greatest source of controversies in economic theory is the use of the same word with different meanings. Therefore, with the aim of avoiding possible misunderstandings, it is extremely important to clarify what we intend here for 'saving'[22].

The possible ambiguity related to the notion of saving actually concerns its complement: consumption. Consumption can, in fact, be understood as either the activity of consuming goods and services, or the expenditure made for the purchase of consumption goods and services. The adoption of one or the other idea of consumption brings about different notions of saving.

Saving and demand for securities in a temporary equilibrium model

In order to introduce the analysis of the concept of saving, we shall start from a temporary equilibrium model in which the implementation of agents' plans requires that they trade securities so as to move their purchasing power from one date to another.

Let us consider a temporary equilibrium model in which, at each date, agents can trade N commodities delivered in that period and a security that is a title to receive (or pay) 1 unit of numéraire delivered in the following period. Accordingly, in a generic period t, N+1 markets are open and N+1 prices are determined: N commodity prices listed in the vector and the security price v_t[23]. As for the numéraire in which these prices are expressed and securities denominated, let $\mathbf{e}$ be a vector of 1s, we may posit: $\mathbf{p}_t \cdot \mathbf{e} = 1$.

Given the current prices and given their expectations about the prices that will be determined in future periods, agents take their decisions on the quantities of commodities and securities traded. In particular, focusing on a household h, its budget constraint in period t can be written as follows:

$$a_{t-1}^h + \mathbf{p}_t \cdot \boldsymbol{\omega}_t^h = \mathbf{p}_t \cdot \mathbf{x}_t^h + v_t a_t^h \qquad (4)$$

On the LHS of equation (4) there is the wealth of the household – i.e., its purchasing capacity – in the moment in which markets open in period t. Precisely, a_{t-1}^h is the quantity of the numéraire commodity delivered in t that the household receives (pay, if $a_{t-1}^h < 0$) as a result of the securities purchased (sold) in period t–1, and $\mathbf{p}_t \cdot \boldsymbol{\omega}_t^h$ is the value of its endowment of commodities delivered in t. As far as the RHS is concerned, $\mathbf{p}_t \cdot \mathbf{x}_t^h$ is the expenditure for commodities delivered in t and $v_t a_t^h$ is the amount of numéraire paid for the purchase (received from the sale, if $a_t^h < 0$) of securities in t. Because of the latter trade, the household will receive (pay, if $a_t^h < 0$) a quantity of numéraire a_t^h in period t+1.

In this framework, the definition of saving is quite natural. On the one hand, the wealth of the household in period t – i.e., the value of its initial endowment of securities and commodities – is $m_t^h = a_{t-1}^h + \mathbf{p}_t \cdot \boldsymbol{\omega}_t^h$. On the other hand, its expenditure for consumption is $c_t^h = \mathbf{p}_t \cdot \mathbf{x}_t^h$, with $\mathbf{x}_t^h$ being the bundle of consumption goods and services household *h* buys in *t*. Saving is the difference $s_t^h = m_t^h - c_t^h$ and, because of the budget constraint (3), it is equal to the amount of numéraire spent for securities $v_t a_t^h$.

In the temporary equilibrium model, the reason why households save is clear. They save in order to move wealth from the markets open in *t* to the those open in *t*+1. Saving in *t* will allow households to spend more in *t*+1, although this does not necessarily mean that they will consume greater quantities of goods and services. This transfer of purchasing power is made possible by the existence of a form of store of value: securities.

Expenditure and consumption in an Arrow-Debreu model

In the temporary equilibrium framework, the time sequence of expenditure and that of consumption correspond. Markets are open on each date and the households purchase the commodities consumed in that period. The consumption of commodities in period *t* brings about an expenditure on the markets open in *t*.

In the Arrow-Debreu model, it is not so. Markets are open in a single instant, and this is the only moment in which households can spend. Household wealth exists and is entirely spent in that moment. Therefore, on the one hand, trading securities makes no sense in this framework[1] and, on the other, consumption goods and services delivered in different periods are traded simultaneously.

Focusing on household *h*, its endowment of commodities is $\boldsymbol{\omega}^h = \{\boldsymbol{\omega}_t^h\}_{t=1}^T$. Given a vector of prices $\mathbf{p} = \{\mathbf{p}_t\}_{t=1}^T$ – expressed in terms of a numéraire, household's wealth is $m^h = \mathbf{p} \cdot \boldsymbol{\omega}^h$. This wealth is entirely employed in order to achieve a consumption plan $\mathbf{x}^h = \{\mathbf{x}_t^h\}_{t=1}^T$, with a consumption expenditure $c^h = \mathbf{p} \cdot \mathbf{x}^h$. Therefore, if we defined household saving as the difference between wealth and consumption expenditure in the moment in which markets are open, then, in an Arrow-Debreu model, its amount is necessarily nil since $\mathbf{p} \cdot \boldsymbol{\omega}^h = \mathbf{p} \cdot \mathbf{x}^h$ because of the budget constraint[2].

Nonetheless, although wealth and expenditure can exist only when markets are open – and hence just in a single moment, goods and services are delivered and consumed in different periods. In this framework, consumption as expenditure and consumption as activity take place at different times. In some sense, it is similar to buying a can of beans today in order to consume it tomorrow.

As a result, one may wonder if other conceptions of saving can be introduced that refer to consumption as activity instead of as expenditure.

For instance, Hahn (1982: 366) defines saving as the difference, in value terms, between the endowment of commodities delivered in t and the quantities of the same commodities demanded for consumption. According to this view, household saving in period t is $\tilde{s}_t^h = \mathbf{p}_t \cdot (\boldsymbol{\omega}_t^h - \mathbf{x}_t^h)$. Is this an acceptable concept of saving?

Answering this question is not an easy task. We shall just draw attention to a couple of points. First, the difference $\boldsymbol{\omega}_t^h - \mathbf{x}_t^h$ is nothing other than household's net supply of commodities delivered in t. Even if we convert this net supply into an amount of value, by its multiplication by the price vector $\mathbf{p}_t$, it is not clear why we should consider the result as saving. Second, this idea of saving seems more plausible in the first period (period 1) than in the last one (period T). In fact, once the optimal consumption stream is determined, it may very well happen that $\mathbf{p}_T \cdot \left(\boldsymbol{\omega}_T^h - \mathbf{x}_T^h\right) > 0$ and this seems to contradict the well-known principle according to which no rational agent wants to have strictly positive saving in its last period of life.

In conclusion, it is clear that saving as a transfer of purchasing power in order to spend it in the future is inconceivable in the Arrow-Debreu framework. Other notions of saving could be introduced – especially referring to consumption as activity, but their meaning and relevance seem rather questionable.

CONCLUSIONS

While Debreu (1959: p. 32) writes that 'a theory of saving, investment, capital and interest' can be deduced from the working of an Arrow-Debreu general equilibrium model, in the present paper we have tried to show that a convincing theory of capital cannot be found within that framework. The words 'interest', 'profit' and 'saving' seem to still be there, but in fact they are just words attached to different concepts. As we have seen, interest – or the own-rate/factor of interest – is the relative price of commodities of the same kind delivered on different dates (section 2). Firm profit is neither linked to the amount of capital invested nor, in equilibrium, it is an income that enters into households' wealth (section 3). Saving is either inconceivable in this model, or it is just another name for households' excess supply of commodities delivered on a given date (section 4).

The reason why the Arrow-Debreu model is not a framework in which the phenomena connected with the investment of capital can find a role lies in the assumption that markets are only open on one date. Although this fact is openly admitted by several authors, its implications are rarely examined. As Starr writes:

> It is precisely because markets reopen over time that agents may find it desirable to carry abstract purchasing power from one date to succeeding date. Typically, this will take the form of transactions on spot markets at a succession of dates with money or other financial assets held over time to reflect the (net) excess value of prior sales over purchases. (Starr, 1987: p. 311)

We believe that with this paper, we have taken a first step in the direction of clarifying this important limit of the Arrow-Debreu general equilibrium theory, a theory that tends to cut out every kind of complication by hypothesis, with serious harm to its explicative power.

ACKOWLEDGMENTS

Thanks are due to Gaetano Bloise and Christian Gehrke. The usual disclaimer applies.

NOTES

1. Assuming, for the sake of simplicity, that there are N different goods and services, T different dates of delivery, and just one possible place, then $L = N \times T$.
2. A similar statement is made, for instance, in Geanakoplos (1987, p. 122). See also: Arrow and Hahn (1971, p. 122).
3. As Grandmont (1987, p. 621) writes, in the Arrow-Debreu framework '[t]here is no sequence of markets over time, and no role for expectations, money, financial assets, or stock markets.'
4. We are leaving aside the possibility of equilibrium indeterminacy. After all, as has been proved – see Debreu (1970), Mas-Colell (1975) and Kehoe (1980) – Arrow-Debreu economies with problems of equilibrium indeterminacy are almost impossible: their Lebesgue measure in the set of possible economies is zero.
5. On the marginalist representation of the economic system, see: Fratini (2019) and (2020).
6. For a survey of the debate, the reader can refer to Harcourt (1972).
7. A similar standpoint can also be found in Koopmans (1957, pp. 113-115). He writes that agents' decisions depend on the price vector – one price for each Arrow-Debreu commodity, whereas the rate of interest is irrelevant for the efficient allocation of resources. According to Koopmans, changes in the commodity in which loans are denominated would result in different interest rates for the same pair of dates, without any change in agents' decisions.
8. On the notion of own-rate of interest, see in particular: Debreu (1959, pp. 33-34) and Bliss (1975, pp. 51-55).
9. See also Debreu (1959, p. 28).

10. Both household net demand functions and firm net supply functions (if they are single-valued correspondences) are homogeneous of degree zero.
11. See, for instance, Burmeister (1980, p. 10).
12. Since markets are open in a single moment only, all the transactions take place in that moment: promises of delivery of commodities are exchanged for promises of delivery of other commodities. Accordingly, all the prices are paid now – i.e. in that moment – independently of the commodity adopted as numéraire.
13. For an example of this interpretation of equation (2), see Eatwell (2019, p. 6).
14. By contrast, if gains (or losses) from triangular exchange are possible, then we cannot derive the relative price of b in terms of a from the prices of these two commodities in terms of g. Accordingly, if there are L commodities, the $L-1$ prices in terms of a numéraire commodity would provide just partial information since complete information needs $L(L-1)/2$ relative prices.
15. In the Arrow-Debreu framework, every single commodity has a date of delivery. However, composite commodities might not be referred to a specific period. This is the case with a composite commodity that includes goods and services delivered on different dates. Since the own-rates of interest, as seen, derive from the relative prices of goods and services of the same quality with different dates of delivery, the own-rate of interest of this sort of composite commodity cannot be defined.
16. On the amount of profit as capitalists' income in the classical/Marxian approach, see, in particular, Garegnani (1984). For a comparison of this idea of profit with the marginalist one, see Fratini (2020).
17. In the present paper, prices are row vectors and quantities column vectors.
18. In other words, let $\mathbf{y}^f = [y_1^f, y_2^f, \dots, y_L^f] \in \mathbb{R}^L$ be the production plan of a firm *f*, it is a vector of net supplies of commodities. This means that if $y_n^f < 0$, then it (taken in terms of absolute value) is the quantity of commodity *n* employed as input by firm *f*. If instead $y_n^f > 0$, then it is the quantity of commodity *n* obtained as output by that firm. As a result, the inner product $\mathbf{p} \cdot \mathbf{y}^f$ directly expresses the difference between revenues and costs.
19. See, also, Arrow and Hahn (1971, p. 77).
20. Actually, in the proof of proposition 17.F.1, Mas-Colell, Whinston and Green (1995, p. 607) get the same result with a different argument.
21. It is in some sense puzzling that what firms want to maximize by their activities is tantamount, in equilibrium, to what they would have obtained by inactivity.
22. As for the notions of income, saving and investments, Hicks (1946, p. 171) claims that, '[i]n spite of their familiarity', they are not 'suitable tools for any analysis which aims at logical precision'. He adds that '[t]here is far too much equivocation in their meaning, equivocation which cannot be removed by the most painstaking effort'. As a result, a few pages later, Hicks (1946, p. 177) says that 'we shall be well advised to eschew income and saving in economic

dynamics' since '[t]hey are bad tools, which break in our hands'.

23. Accordingly, the rate of return on 1 unit of numéraire invested in securities in period t is $r_t = 1/v_t - 1$. Although it corresponds to the own-rate of interest of the numéraire commodity, r_t is, in a sense, a real rate of interest and not just a relative price because the security is a real form of store of purchasing power.
24. As Currie and Steedman (1990, p. 147) stress, the idea of transferring purchasing power from one date to another makes no sense in a model in which markets are open on one date only. What a household does not spend in the only moment in which markets are open cannot be spent anymore.
25. For the sake of simplicity, locally non-satiated preferences can be assumed.

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