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A New 4-D Conservative Chaotic System with Coexistence of Hidden Chaotic Orbits

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Abstract : In this paper a new 4-D conservative chaotic system is reported. When compared with other existing chaotic systems the proposed system has the following unique properties (*i*) hidden chaotic orbits with no equilibria and (*ii*) coexistence of chaotic orbits. The chaotic behaviour of the proposed conservative chaotic system is verified by finding the Lyapunov exponents and plotting the phase portrait, time series plot, Poincare map, frequency spectrum and instantaneous phase. The sum of the Lyapunov exponents of the system is zero which validates the conservative nature of the system.

Keywords: Conservative chaotic system; hidden chaotic orbits; new 4-D chaotic system; coexistence of chaotic orbits.

1. INTRODUCTION

The chaotic systems are observed in many fields like communication, signal processing, economics, robotic, etc. [1], [2]. This leads to the development of new chaotic system with various characteristics [3]. The chaotic systems with higher dimensions are more interesting and effective compared with the low dimensions [4,42-45] ones.

Recently the chaotic systems are categorised into two parts: self-excited attractors and hidden attractors [5]–[9]. The well-known chaotic systems like Lorenz chaotic system [10], Rossler [11], Chen [12], Lu system [13], [14], Qi chaotic attractor [15] belong to the self-excited attractors. The development of hidden attractors was initiated from the pioneer work of [6], [7]. The chaotic systems with (*i*) no equilibria [16], (*ii*) only stable equilibria [17], [18] and (*iii*) plane or line of equilibria, are called the hidden attractors [19]–[22]. The chaotic systems are also grouped as dissipative or conservative chaotic systems [23]. Some conservative chaotic systems with no equilibria are also reported in the literature like in [23], [24]. The reported hidden chaotic systems with no equilibria are classified in Table 1.

It is seen from Table 1 that a 4-D conservative chaotic system with no equilibrium point is not found in the literature. Thus, the motivation behind this paper is to develop a new 4-D chaotic system with no equilibria and reports a new 4-D conservative chaotic system with no equilibria. Various tools are used for analysing the systems and found that the system has coexistence of chaotic orbits.

The rest part of the paper is organised as follows. Section 2 describes the dynamics of the new conservative chaotic system. The basic properties of the system are presented in Section 3. Numerical findings of the proposed system are given in Section 4. Finally, conclusions of the paper are given in Section 5.

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Table 1

Categorisation of 4-D chaotic/hyperchaotic systems with no equilibrium point

Sl. No.	*3-D/4-D System*	*Nature of system*	*References of papers*
Dissipative chaotic system			
2.	4-D Chaotic system	No equilibrium point attractors	[25]
		Multi-scroll or multi-attractor with no equilibrium point	[26], [27]
3.	4-D Hyperchaotic system	No equilibrium point attractors	[28]-[33]
		Multi-scroll or multi-attractor with no equilibrium point	[34], [35]
		Coexistence of attractors with no equilibrium point	[36]
4.	5-D Hyperchaotic system	No equilibrium point attractors	[37]
		Coexistence of attractors with no equilibrium point	[38]
		Multi-scroll or multi-attractor with no equilibrium point	[39]
Conservative chaotic system			
1.	3-D Chaotic system	No equilibrium point chaotic orbits	[23], [24], [46]
2.	4-D Chaotic system	No equilibrium point chaotic orbits	This work
		Coexistence of chaotic orbits with no equilibrium point	This work

2. DYNAMICS OF THE NEW 4-D CONSERVATIVE SYSTEM

The dynamics of the new 4-D conservative chaotic system considered here is described below:

$$\begin{aligned} \dot{x}_1 &= x_2 - x_4 \\ \dot{x}_2 &= -x_1 + a x_2 x_3 \\ \dot{x}_3 &= b - x_2^4 \\ \dot{x}_4 &= -c x_2 \end{aligned} \qquad (1)$$

where a, b and c are positive constant parameters and x_1, x_2, x_3, x_4 are the state variables of system (1). The system is chaotic for $a = 1$, $b = 1$, $c = 0.0037$. All simulations of the proposed system are carried out with initial conditions $x(0) = (0.01, 0.02, 0.02, 0.02)^T$ using ode-45 simulation method in MTALAB environment.

3. BASIC DYNAMICAL PROPERTY OF SYSTEM (1)

Some common basic dynamical properties of the system are described in this section.

3.1. Symmetry and invariance

The system is invariant under the coordinate transformation $(x_1, x_2, x_3, x_4) \rightarrow (-x_1, -x_2, -x_3, -x_4)$.

3.2. Equilibrium point

The equilibrium points of the system can be determined by equating each state equation of (1) to zero. It is determined that the system has no particular solution with the chosen parameters set. Thus, the system has no equilibrium point and hence system (1) has hidden chaotic orbits.

4. NUMERICAL SIMULATION

This section describes some dynamical properties of system (1) using numerical simulation.

4.1. Lyapunov exponents and Lyapunov dimension

System (1) is chaotic for parameter $a = b = 1$, $c = 0.0037$ where the Lyapunov exponents are

$$L_i = (0.0828, 0, 0, -0.0828) \tag{2}$$

The Lyapunov exponents plot of the system are shown in Fig. 1. It is seen from Fig. 1 that the system has one positive, one negative and two zero Lyapunov exponents. The Lyapunov exponents of the system are calculated by using Wolf algorithm [40] with sampling size $\Delta t = 0.01$, $x(0) = (0.01,0.02,0.02,0.02^{T}$. The sum of the Lyapunov exponents is zero. Thus, the system is a conservative chaotic system. The Lyapunov (Kaplan Yorke) dimension of the system is calculated as:

$$D_{KY} = 3 + \frac{L_1 L_2 L_3}{|L_4|} = 3 + 1 = 4.0 \tag{3}$$

The Lyapunov dimension also validate the conservative nature of the proposed conservative system.

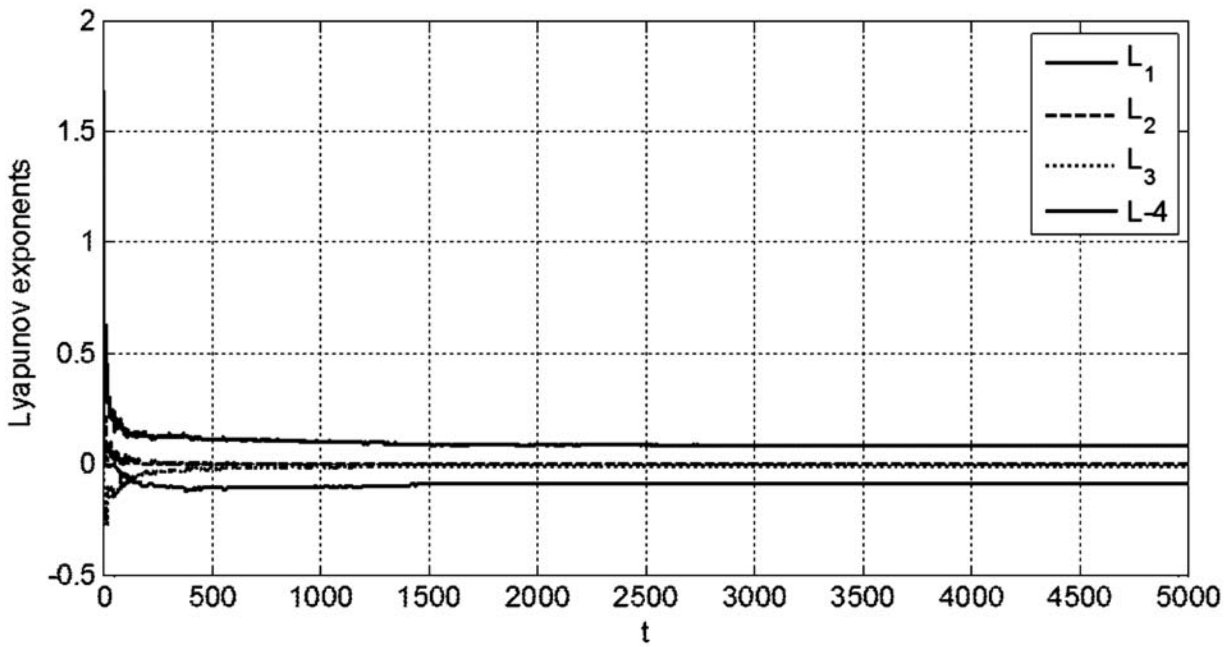
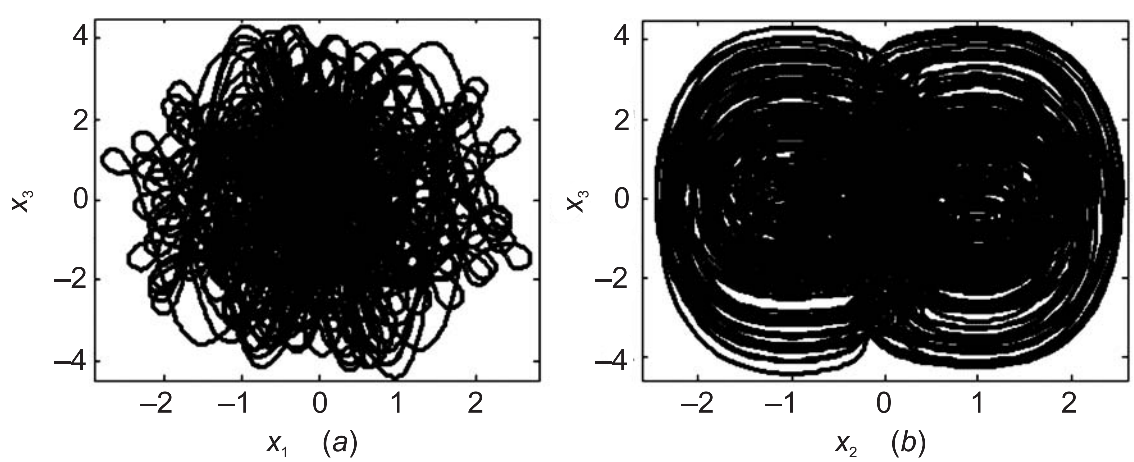


Figure 1: Lyapunov exponents of the system with *a* = *b* = 1, *c* = 0.0037.

4.2. Chaotic orbits and Poincare map



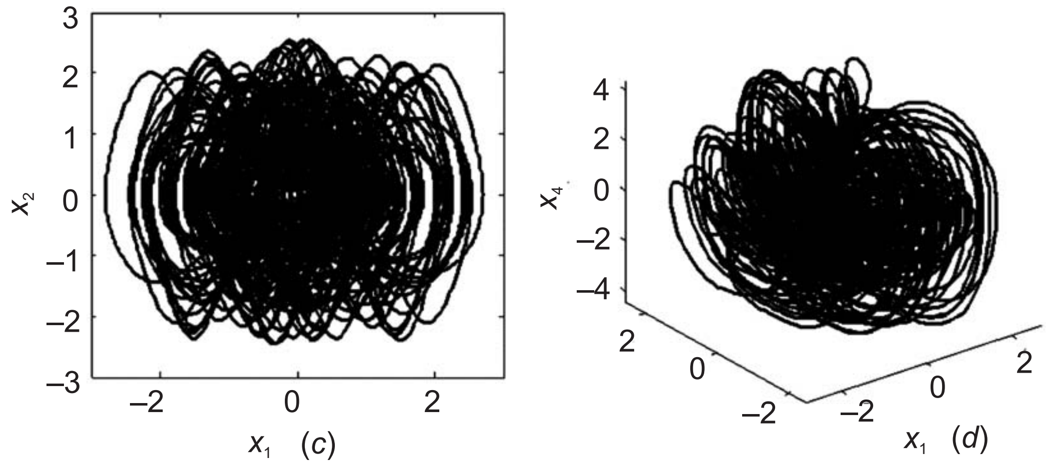


Figure 2: Chaotic orbits of the system with $a = b = 1$, $c = 0.0037$ and $x(0) = (0.01,0.02,0.02,0.02)^T$.

The chaotic orbits of the system with $a = b = 1$, $c = 0.0037$ and $x(0) = (0.01,0.02,0.02,0.02)^T$ are shown in Fig. 2. It is seen from the Fig. 2 that phase portrait of the system has chaotic behaviours. The Poincare maps of the system for different section of the planes are shown in Fig. 3. The random location of dots in the Poincare map indicates the chaotic behaviour of the system.

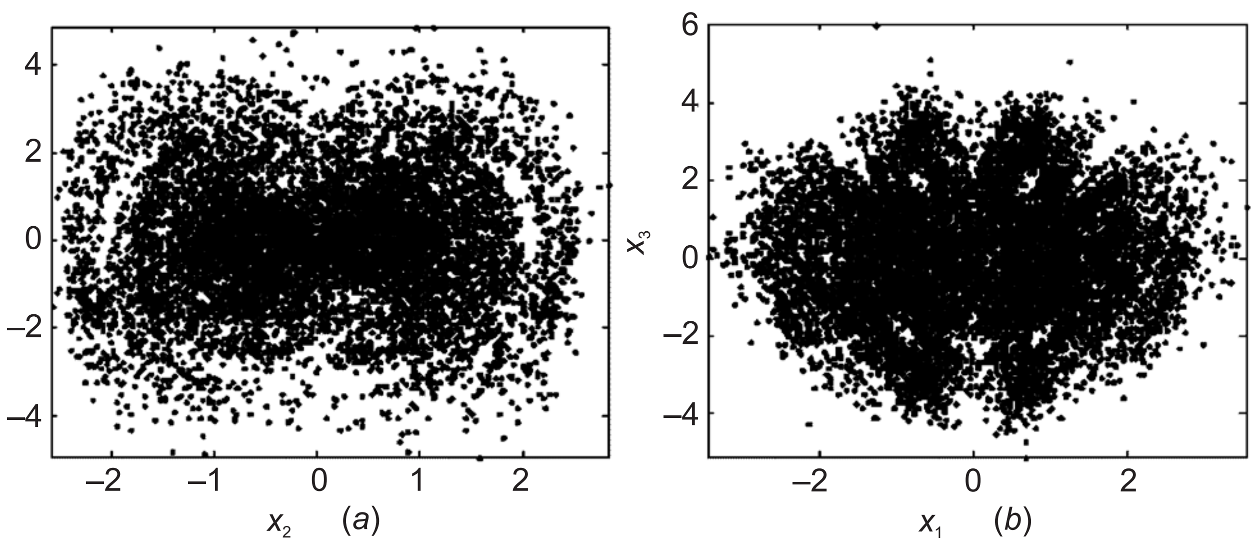


Figure 3: Poincare map of the system with $a = b = 1$, $c = 0.0037$: (*a*) across $x_2 - x_3$ plane for $x_1 = 0$ and (*b*) across $x_1 - x_3$ plane for $x_2 = 0$.

4.3. Frequency spectrum and instantaneous phase plot

The frequency spectra of the signal of system (1) with $a = b = 1$, $c = 0.0037$ are shown in Fig. 4. The random location of peaks in Fig. 5 indicates that the system has chaotic behaviour. The instantaneous phase of x_2 and x_3 signal of the system are shown in Fig. 5. The instantaneous phase of a chaotic system increases monotonically with time, whereas for a periodic signal it remains constant [41]. Thus, system (1) has chaotic behaviour. The Hilbert transformation method is used for the generation of instantaneous phase ($\varnothing$). Here, Hilbert transformation is calculated using the technique given in [41].

4.4. Coexistence of chaotic orbits

The system has coexistence of chaotic orbits for the bifurcation parameter c with changes in the sign of initial conditions. The coexistences of chaotic orbits of the system with $a = b = 1$, $c = 0.0037$ and $x(0) = (\pm 0.01, \pm 0.02, \pm 0.02, \pm 0.02)^T$ are shown in Fig. 6. It is seen from Fig. 6 that different shapes of chaotic orbits coexist with the changes in the sign of initial conditions.

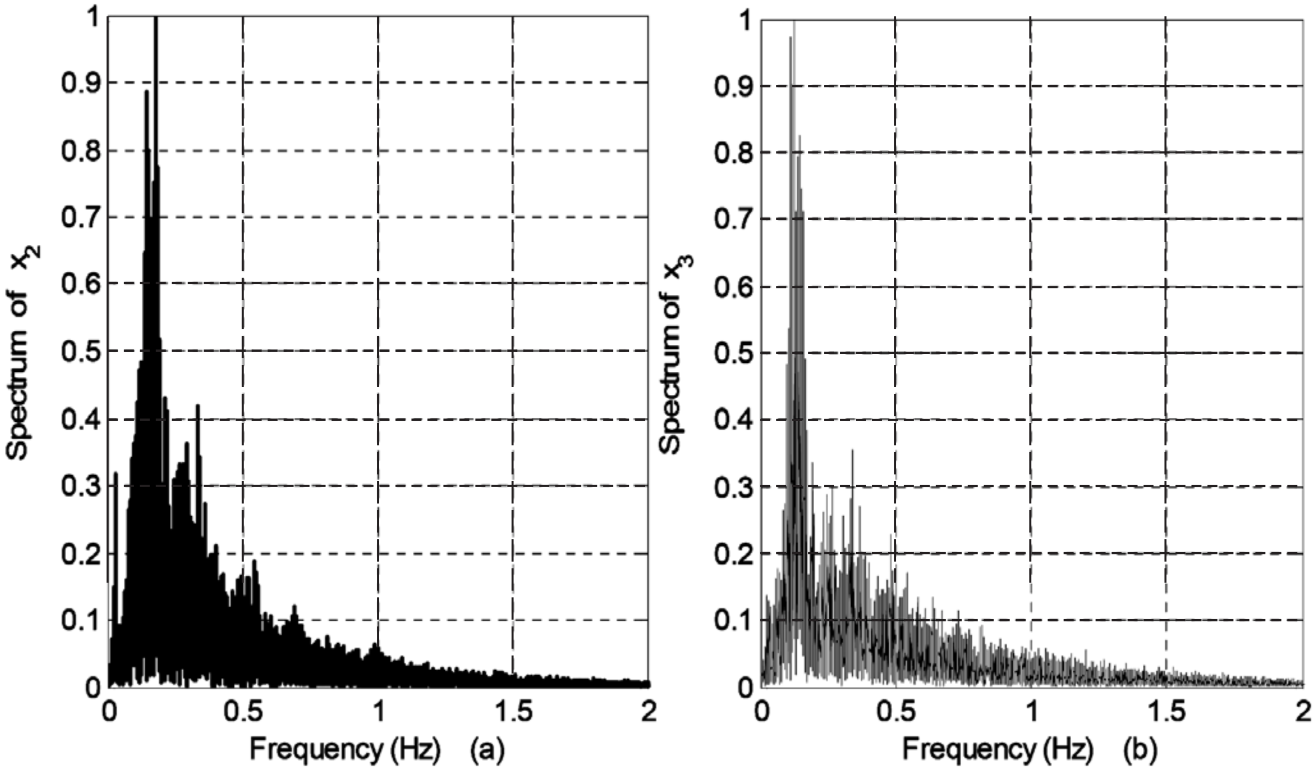


Figure 4: Frequency spectrum of the system with *a* = *b* = 1, *c* = 0.0037 for : (*a*) x_2 signal and (*b*) x_3 plane.

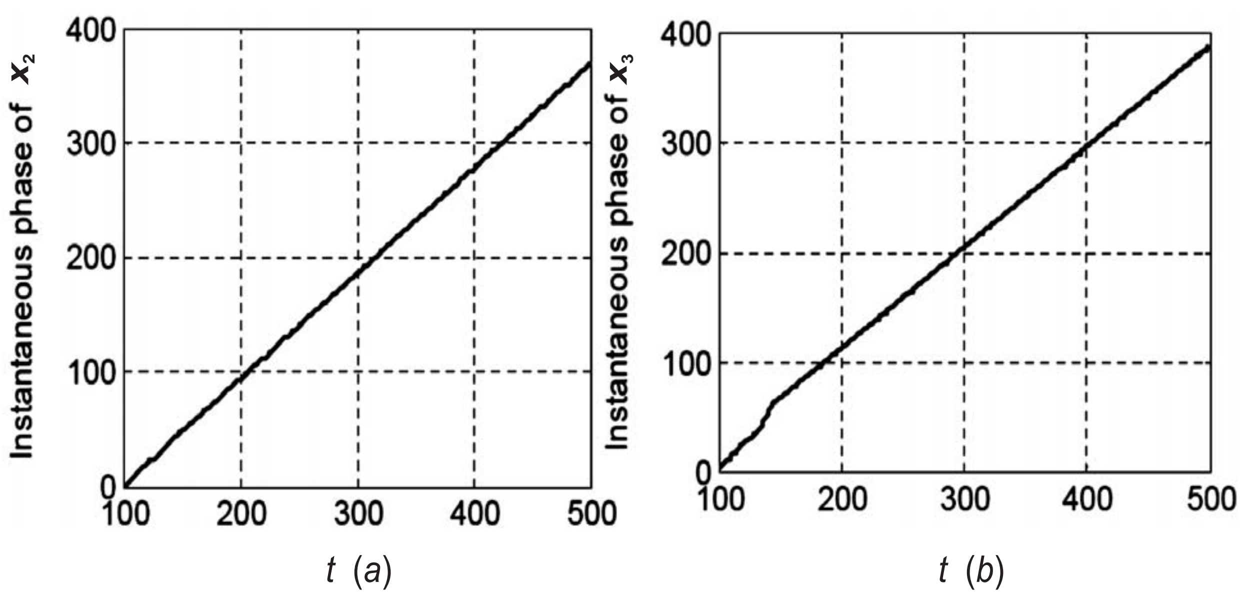


Figure 5: Instantaneous phase of the system with *a* = *b* = 1, *c* = 0.0037 for: (*a*) x_2 signal and (*b*) x_3 plane.

5. CONCLUSIONS

We have proposed a new 4-D conservative chaotic system having hidden chaotic orbits with no equilibria and coexistences of chaotic orbits. The zero sums of the Lyapunov exponents and the integer value of the Lyapunov dimension confirm the conservative nature of the proposed system. Different theoretical and numerical tools are used to validate the chaotic behaviour of the system. Such 4-D chaotic system is reported for the first time to the best of our knowledge.

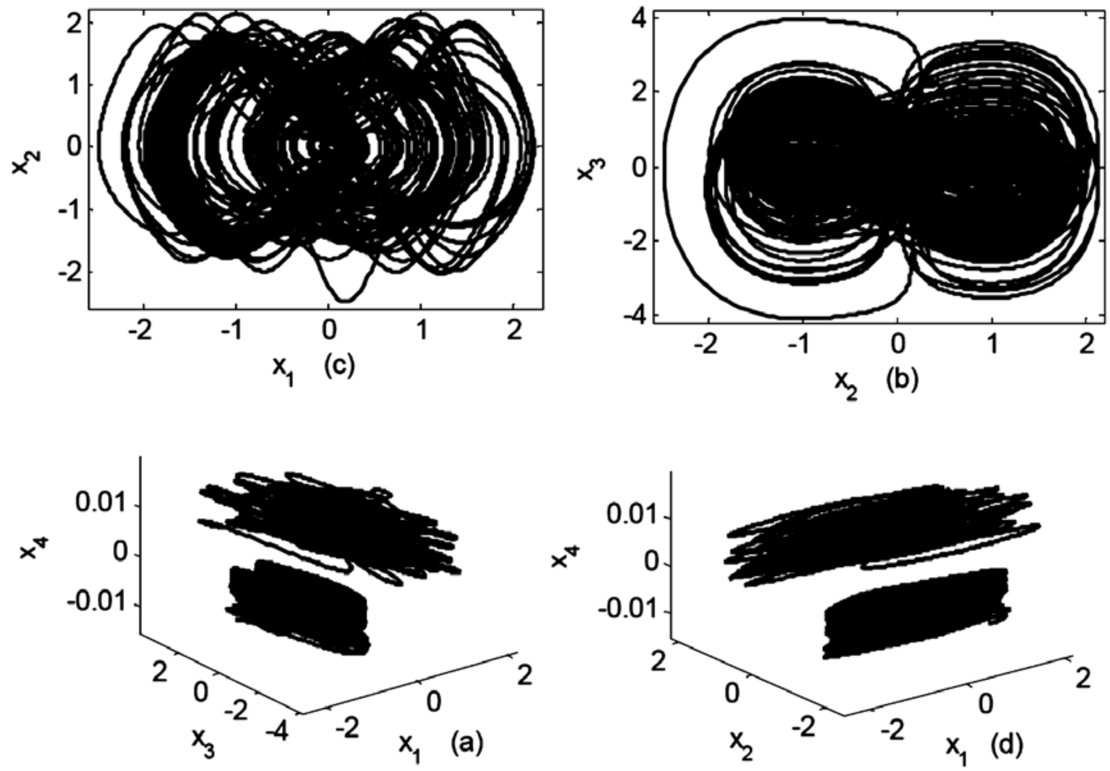


Figure 6: Coexistence of chaotic orbits of the system with $a = b = 1$, $c = 0.0037$ and $x(0) = (\pm 0.01, \pm 0.02, \pm 0.02, \pm 0.02)^T$.

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