International Review of Pure and Applied Mathematics
(January-June) 2017, Volume 13, No. 1, pp. 95–100

Square Difference Prime Labeling for Some Path Related Graphs

Sunoj B.S. and Mathew Varkey T.K.

Abstract: A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The graph for which every edge (uv), the labels assigned to u and v are whole numbers and for each vertex of degree at least 2, the *gcd* of the labels of the incident edges is 1. Here we characterize some path related graphs for square difference prime labeling.

Keywords: Graph labeling, prime labeling, prime graphs, path graph.

1. Introduction

All graphs in this paper are finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p, q)-graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1], [2], [3] and [4]. Some basic concepts are taken from Frank Harary [1]. In this paper we investigated square difference prime labeling of some path related graphs.

2. Main Results

Definition 2.1: Let G = (V(G), E(G)) be a graph with p vertices and q edges. Define a bijection f : V(G) $\rightarrow$ {0, 1, 2, ..., $p - 1$} by $f(v_i) = i - 1$, for every i from 1 to p and define *a* 1 – 1 mapping f^*_{sdp} : E(G) $\rightarrow$ set of natural numbers N by $f^*_{sdp}(uv) = |f(u)^2 - f(v)^2|$. The induced function f^*_{sdp} is said to be a square difference prime labeling, if for each vertex of degree at least 2, the $g\ c\ d$ of the labels of the incident edges is 1.

Definition 2.2: A graph which admits square difference prime labeling is called a square difference prime graph.

Definition 2.3: Let V(G) and X(G) denote the vertex set and the edge set of G, respectively. The middle graph M(G) of G whose vertex set is V(G) union X(G) where two vertices are adjacent if and only if

(i) They are adjacent edges of G or

(ii) One is a vertex and other is an edge incident with it.

Definition 2.4: The total graph T(G) of G is the graph whose vertex set is V(G) union X(G) where two vertices are adjacent if and only if

(i) They are adjacent edges of G or

(ii) One is a vertex and other is an edge incident with it.

(iii) They are adjacent vertices of G.

Definition 2.5: Let $G = (V, E)$ be a simple graph and $G' = (V', E')$ be another copy of graph G. Join each vertex v of G to the corresponding vertex v' of G' by an edge. The new graph thus obtained is the 2-tuple graph of G. 2-tuple graph of G is denoted by $T^2(G)$. Further if $G = (p; q)$ then $V\{T^2(G)\} = 2p$ and $E\{T^2(G)\} = 2q + p$.

Definition 2.6: Let G(V, E) be a simple graph. A duplicate graph of G is $DG = (V_1, E_1)$, where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \varphi$ and $f : V \to V'$ is bijective and the edge set E_1 of DG is defined as: The edge ab is in E if and only if both ab' and $a'b$ are edges in E_1.

Definition 2.7: In a pair of path P_n, i^{th} vertex of a path P' is joined with $(i + 1)^{th}$ vertex of a path P''. The resulting graph is denoted by $[Z - (P_n)]$.

Definition 2.8: The distance $d(u, v)$ between two vertices u and v in G is the length of a shortest path joining them if any, otherwise $d(u, v) = \infty$.

Definition 2.9: The square G^2 of a graph G has $V(G^2) = V(G)$ with u, v adjacent in G^2 whenever $d(u, v) \leq 2$ in G.

Definition 2.10: The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G_1 and G_2 join each vertex v in G_1 to the neighbors of the corresponding vertex u in G_2.

Definition 2.11: Strong double graph SD(G) is a graph obtained by taking two copies of G and joining each vertex v_i in one copy with the closed neighborhood $N[v_i] = N(v_i) \cup \{v_i\}$ of corresponding vertices in another copy.

Theorem 2.1: The 2-tuple graph of path P_n admits square difference prime labeling.

Proof: Let $G = T^2(P_n)$ and let $v_1, v_2, ..., v_{2n}$ are the vertices of G.

Here $|V(G)| = 2n$ and $|E(G)| = 3n - 2$

Define a function $f : V \to \{0, 1, 2, ..., 2n - 1\}$ by

$$f(v_i) = i - 1, i = 1, 2, ..., 2n$$

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows

$$f^*_{sdp}(v_i\, v_{i+1}) = 2i - 1, \quad i = 1, 2, ..., 2n - 1$$

$$f^*_{sdp}(v_i\, v_{2n-i+1}) = (2n - 1)(2n - 2i + 1), \quad i = 1, 2, ..., n - 1$$

According to this pattern $T^2(P_n)$, admits square difference prime labeling.

Theorem 2.2: The middle graph of path P_n admits square difference prime labeling.

Proof: Let $G = M(P_n)$ and let $v_1, v_2, ..., v_{2n-1}$ are the vertices of G.

Here $|V(G)| = 2n - 1$ and $|E(G)| = 3n - 4$

Define a function $f: V \rightarrow \{0, 1, 2, ..., 2n - 2\}$ by

$$f(v_i) = i - 1, i = 1, 2, ..., 2n - 1$$

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows:

$$f^*_{sdp}(v_i\, v_{i+1}) = 2i - 1, \quad i = 1, 2, ..., 2n - 2$$

$$f^*_{sdp}(v_{2i}\, v_{2i+2}) = 8i, \quad i = 1, 2, ..., n - 2$$

According to this pattern $M(P_n)$, admits square difference prime labeling.

Theorem 2.3: The total graph of path P_n admits square difference prime labeling.

Proof: Let $G = T(P_n)$ and let $v_1, v_2, ..., v_{2n-1}$ are the vertices of G.

Here $|V(G)| = 2n - 1$ and $|E(G)| = 4n - 5$

Define a function $f: V \rightarrow \{0, 1, 2, ..., 2n - 2\}$ by

$$f(v_i) = i - 1, i = 1, 2, ..., 2n - 1$$

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows

$$f^*_{sdp}(v_i\, v_{i+1}) = 2i - 1, \quad i = 1, 2, ..., 2n - 2$$

$$f^*_{sdp}(v_{2i}\, v_{2i+2}) = 8i, \quad i = 1, 2, ..., n - 2$$

$$f^*_{sdp}(v_{2i+1}\, v_{2i+1}) = 8i - 4, \quad i = 1, 2, ..., n - 1$$

According to this pattern $T(P_n)$, admits square difference prime labeling.

Theorem 2.4: The duplicate graph of path P_n admits square difference prime labeling.

Proof: Let $G = D(P_n)$ and let $v_1, v_2, ..., v_{2n}$ are the vertices of G.

Here $|V(G)| = 2n$ and $|E(G)| = 2n - 2$

Define a function $f: V \rightarrow \{0, 1, 2, ..., 2n - 1\}$ by

$$f(v_i) = i - 1, i = 1, 2, ..., 2n$$

For the vertex labeling f, the induced edge labeling f_{sdp}^{*} is defined as follows

$$f_{sdp}^{*}(v_i\, v_{i+1}) = 2i - 1, \quad i = 1, 2, ..., n - 1$$

$$f_{sdp}^{*}(v_{n+i}\, v_{n+i+1}) = 2n + 2i - 1, \quad i = 1, 2, ..., n - 1$$

According to this pattern $D(P_n)$, admits square difference prime labeling.

Theorem 2.5: The graph P_n^2 admits square difference prime labeling.

Proof: Let $G = P_n^2$ and let $v_1, v_2, ..., v_n$ are the vertices of G.

Here $V(G) = n$ and $E(G) = 2n - 3$

Define a function $f : V \to \{0, 1, 2, ..., n - 1\}$ by

$$f(v_i) = i - 1, \quad i = 1, 2, ..., n$$

For the vertex labeling f, the induced edge labeling f_{sdp}^{*} is defined as follows

$$f_{sdp}^{*}(v_i\, v_{i+1}) = 2i - 1, \quad i = 1, 2, ..., n - 1$$

$$f_{sdp}^{*}(v_i\, v_{i+2}) = 4i, \quad i = 1, 2, ..., n - 2$$

According to this pattern $(P_n)^2$, admits square difference prime labeling.

Theorem 2.6: The graph $[Z - (P_n)]$ admits square difference prime labeling when $n \equiv 2 (\text{mod } 3)$.

Proof: Let $G = [Z - (P_n)]$ and let $v_1, v_2, ..., v_{2n}$ are the vertices of G.

Here $|V(G)| = 2n$ and $|E(G)| = 3n - 3$

Define a function $f : V \to \{0, 1, 2, ..., 2n - 1\}$ by

$$f(v_i) = i - 1, i = 1, 2, ..., 2n$$

For the vertex labeling f, the induced edge labeling f_{sdp}^{*} is defined as follows:

$$f_{sdp}^{*}(v_i\, v_{i+1}) = 2i - 1, \quad i = 1, 2, ..., n - 1$$

$$f_{sdp}^{*}(v_{n+i}\, v_{n+i+1}) = 2n + 2i - 1, \quad i = 1, 2, ..., n - 1$$

$$f_{sdp}^{*}(v_{i+1}\, v_{n+i}) = (n + 2i - 1)(n - 1), \quad i = 1, 2, ..., n - 1$$

According to this pattern $[Z - (P_n)]$, admits square difference prime labeling.

Theorem 2.7: The shadow graph of path P_n admits square difference prime labeling when $n \equiv 2 (\text{mod } 3)$ and $n \neq 21k - 1 (k = 1, 2, ...)$.

Proof: Let $G = D_2(P_n)$ and let $v_1, v_2, ..., v_{2n}$ are the vertices of G.

Here $|V(G)| = 2n$ and $|E(G)| = 4n - 4$

Define a function $f : V \to \{0, 1, 2, ..., 2n - 1\}$ by $f(v_i) = i - 1, i = 1, 2, ..., 2n$

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows

$$f^*_{sdp}(v_i\, v_{i+1}) = 2i - 1 \quad i = 1, 2, ..., n - 1$$

$$f^*_{sdp}(v_{n+i}\, v_{n+i+1}) = 2n + 2i - 1, \quad i = 1, 2, ..., n - 1$$

$$f^*_{sdp}(v_{i+1}\, v_{n+i}) = (n + 2i - 1)(n - 1), \quad i = 1, 2, ..., n - 1$$

$$f^*_{sdp}(v_i\, v_{n+i+1}) = (n + 2i - 1)(n + 1), \quad i = 1, 2, ..., n - 1$$

According to this pattern $D_2(P_n)$, admits square difference prime labeling.

Theorem 2.8: Strong double graph of path P_n admits square difference prime labeling.

Proof: Let G = SD(P_n) and let v_1, v_2,--------------,v_{2n} are the vertices of G.

Here $|V(G)| = 2n$ and $|E(G)| = 5n - 4$.

Define a mapping $f : V \to \{0, 1, 2, 3, ..., 2n - 1\}$ by

$$f(v_i) = i - 1, \quad i = 1, 2, ..., n \text{ and}$$

$$f(u_i) = 2n - i, i = 1, 2, ..., n.$$

Clearly f is a one – one, onto mapping.

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows:

$$f^*_{sdp}(v_i\, v_{i+1})) = 2i - 1, \quad 1 \le i \le n - 1$$

$$f^*_{sdp}(u_i\, u_{i+1})) = 4n - 2i - 1, \quad 1 \le i \le n - 1$$

$$f^*_{sdp}(u_i\, v_i)) = (2n - 1)(2n - 2i + 1), \quad 1 \le i \le n$$

$$f^*_{sdp}(v_i\, v_{i+1}) = (2n - 2)(2n - 2i), \quad 1 \le i \le n - 1$$

$$f^*_{sdp}(u_i\, v_{i+1}) = 2n(2n - 2i), \quad 1 \le i \le n - 1$$

$$\text{gcd of } (v_1) = \text{gcd of } (1, (2n - 1)^2) = 1$$

$$\text{gcd of } (u_1) = \text{gcd of } (2n - 1, 4n - 3) = \text{gcd of } (2n - 2, 2n - 1) = 1$$

From the definition of edge labeling it is clear that gcd of all other vertices are one and the edge labels are also distinct. So the graph SD(P_n) admits square difference prime labeling.

References

[1] F Harary, Graph Theory, Addison- Wesley, Reading, Mass, (1972).

[2] Joseph A. Gallian, A. Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics (2015), #DS6, Pages 1 – 389.

[3] S. Uma Maheswari, Dr. K Indirani, On Triangular Sum Prime Graphs, International Journal of Computer and Mathematical Sciences, Volume 5, Issue 3 March 2016, pages 20-22.

[4] T.K. Mathew Varkey, Some Graph Theoretic Generations Associated with Graph Labeling, PhD Thesis, University of Kerala 2000.

Sunoj B.S.
Department of Mathematics,
Government Polytechnic College,
Attingal, India.
Email: spalazhi@yahoo.com

Mathew Varkey T.K.
Department of Mathematics,
T.K.M. College of Engineering,
Kollam, India.
Email: mathewvarkeytk@gmail.com