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A MARKOVIAN SINGLE SERVER WORKING VACATION QUEUE WITH SERVER STATE DEPENDENT ARRIVAL RATE

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Abstract: A single server Markovian queueing system with the system alternates between regular busy state and working vacation state has been considered. The system is busy; it functions as a single server Markovian queue. When it is on vacation, again it functions as a single server Markovian queue but with different arrival and service rates. The vacation policy is multiple vacation policy and the vacation period follows negative exponential. The steady state probability vector of number of customers in the queue and the stability condition are obtained using Matrix-Geometric method. Some illustrative examples are also provided.

Keywords: Working vacation, State dependent arrival rate, Matrix-Geometric method.

I. INTRODUCTION

Recent decads have seen an increasing interest in queueing systems with server working vacation, due to their applications in telecommunication systems, manufacturing systems and computer systems. In many real life queueing situations, it can be seen that the server works during his rest period, if the necessity occur. Queueing systems with server vacations have been investigated by many researchers, the readers may refer the survey paper by Doshi (1986) and the monograph of Takagi (1991).

In the working vacation queues, the server works with variable service rate, in particular reduced service rate, rather than completely stops service during vacation period. Servi and Finn(2002) have first analyzed an $M/M/1$ queue with multiple working vacation, in which the vacation times are exponentially distributed. Wu and Takagi (2006) extend the work for $M/G/1$ queue. Kim *et al.* (2003) analyzed the queue length distribution of the $M/G/1$ queue with working vacations. Liu *et al.* (2007), examined stochastic decomposition structure of the queue length and waiting time in an $M/M/1$ working vacation queue. Xu *et al.* (2009) extended the $M/M/1$ working vacation queue to an $M^X/M/1$ working vacation queue. Li *et al.* (2009) used the matrix analytic method to analyze an $M/G/1$ queue with exponential working vacation under a specific assumption. Lin and Ke (2009) consider a multi server queue with single working vacation. Jain and Jain (2010) investigated a single working vacation model with server break down. Ke *et al.* (2010) have given a short survey on vacation models in recent years.

Yechiali and Naor (1971) have considered a single-server exponential queueing model with arrival state depending on operational state or breakdown state of the server. Fond and Ross (1977) analyzed the same model with the assumption that any arrival finding the server busy is lost, and they obtained the steady-state proportion of customer's lost. Shogan (1979) has deals with a single server queueing model with arrival rate dependent on server state. Shanthikumar (1982) has analyzed a single server Poisson queue with arrival rate dependent on the state of the server. Jayaraman *et al.* (1994) analyzed a general bulk service queue with arrival rate dependent on server breakdowns. Tian and Yue (2002) discussed the queueing system with variable arrival rate. The author studied the model by using the principle of quasi-birth and death process (QBD) and matrix-geometric method. Furthermore, the calculated some performance measures, such as the number of customers in the system in steady-state. Matrix-geometric method approach is a useful tool for solving the more complex queueing problems. Matrix-geometric method has been applied by many researchers to solve various queueing problems in different frameworks. Neuts(1981) explained various matrix geometric solutions of stochastic models. Matrix-geometric approach is utilized to develop the computable explicit formula for the probability distributions of the queue length and other system characteristic.

To the best of our knowledge, in the study of working vacation queue, the existing literatures focus mainly on queueing system with server state independent arrival rates, in this work we deviate from these work by assuming server state dependent arrival rates.

In this paper, we consider an $M/M/1$ queue with multiple working vacation. The arrival rate dependents on the server states. The model has been analyzed using matrix geometric method. The rest of this paper is organized as follows: In section 2, we give the model description, establish its quasi-birth-death process. In section 3, we present the steady state solution using matrix geometric method. In section 4, we present some system performance measures. Section 5 gives some particular models. and In section 6, we carried out a numerical study.

II. THE MODEL

We consider a single-server queueing system with the following characteristics:

1. The system alternate between two states, the regular state and the working vacation state.
2. Arrival process follows Poisson with parameter λ during regular state.

3. When the system is in regular state it serve customers based on exponential distribution with rate μ.
4. The server takes vacation, if there are no customers in the queue at a service completion point.
5. During vacation, the arrival follows Poisson with rate λ_1 ($\lambda_1 < \lambda$).
6. Vacation period follows negative exponential with rate ? and the vacation policy is multiple vacation policy, that is, the server takes vacation until the server finds at least one customer at a vacation completion point.
7. When the server is in vacation, if customer arrives, the server serve the customers using exponential distribution with rate μ_1 ($\mu_1 < \mu$). As this vacation period ends, the server instantaneously switches over to the normal service rate μ, if there is at least one customer waiting for service. Upon completion of a service at a vacation period, the server will (i) Continue the current vacation if it is not finished and no customer is waiting for service; (ii) Continue the service with rate μ_1 if the vacation has not expired and if there is at least one customer waiting for service.
8. The first come first served (FCFS) service rule is followed to select the customer for service.

A. The quasi-birth-and-death (QBD) process

The model defined in this article can be studied as a QBD process. The following notations are necessary for the analysis:

Let $L(t)$ be the number of customers in the queue at time t and let

$$J(t) = \begin{cases} 0, \textit{ if the server is on working vacation} \\ 1, \textit{if the server is busy} \end{cases}$$

be the server state at time t.

Let $X(t) = (L(t), J(t))$, then $\{(X(t)): t \geq 0\}$ is a Continuous time Markov chain (CTMC) with state space $S = \{(i, j): i = 0, 1; j \geq 0\}$, where j denotes the number of customer in the queue and i denotes the server state.

Using the lexicographical sequence for the states, the rate matrix Q, is the infinitesimal generator of the Markov chain and is given by

$$Q = \begin{bmatrix} B_0 A_0 & & & & \\ A_2 A_1 A_0 & & & & \\ & A_2 A_1 A_0 & & & \\ & & A_2 A_1 A_0 & & \\ & & \cdot\cdot & \cdot & \\ & & & \cdot & \cdot\ \ \cdot \\ & & & \cdot & \cdot\ \ \cdot \end{bmatrix}$$

where the sub-matrices A_0, A_1 and A_2 are of order 2×2 and are appearing as

$$A_0 = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -(\lambda_1 + \mu_1 + \theta) & 0 \\ 0 & -(\lambda + \mu) \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu \end{bmatrix}$$

and the boundary matrix is defined by

$$B_0 = \begin{bmatrix} -(\lambda_1 + \theta) & \theta \\ \mu & -(\lambda + \mu) \end{bmatrix}$$

We define the matrix $A = A_0 + A_1 + A_2$. This matrix A is a 2×2 matrix and it can be written as

$$A = \begin{bmatrix} -\theta & \theta \\ 0 & 0 \end{bmatrix}$$

III. THE STEADY STATE SOLUTION

Let $P = (p_0, p_1, p_2, \ldots)$ be the stationary probability vector associated with Q, such that $PQ = 0$ and $Pe = 1$, where e is a column vector of 1's of appropriate dimension.

Let $p_0 = (p_{00}, p_{10})$ and $p_i = (p_{0i}, p_{1i})$ for $i \geq 1$.

If the steady state condition is satisfied, then the sub vectors p_i are given by the following equations:

$$p_0 B_0 + p_1 A_2 = 0 \tag{1}$$

$$p_i A_0 + p_{i+1} A_1 + p_{i+2} A_2 = 0,\ i \geq 0 \tag{2}$$

$$p_i = p_0 R^i;\ i \geq 1 \tag{3}$$

where R is the rate matrix, is the minimal non-negative solution of the matrix quadratic equation (see Neuts(1981)).

$$R^2 A_2 + RA_1 + A_0 = 0, \tag{4}$$

the matrices A_0, A_1 and A_2 are upper triangular matrices of order 2.

Substituting the equation (3) in (1), we have

$$p_0 (B_0 + RA_2) = 0 \tag{5}$$

and the normalizing condition is

$$p_0 (I - R)^{-1} e = 1 \tag{6}$$

Theorem 3.1

The queueing system described in section II is stable if and only if $\rho < 1$, where $\rho = \dfrac{\lambda}{\mu}$

Proof

The matrix

$R = \begin{bmatrix} r_0 & r_{01} \\ 0 & r_1 \end{bmatrix}$ satisfies the quadratic equation (4). In turn it satisfies the two quadratic equations

$$\mu_1 r_0^2 - (\lambda_1 + \mu_1 + \theta) r_0 + \lambda_1 = 0 \tag{7}$$

$$\mu r_1^2 - (\lambda + \mu) r_1 + \lambda = 0 \tag{8}$$

(a) The equation $\mu_1 Z^2 - (\lambda_1 + \mu_1 + \theta)Z + \lambda_1 = 0$ includes the service rate μ_1 of server who is in vacation and the vacation parameter θ. The above quadratic equation has two different real roots $r_0 < r_0^*$ and $0 < r_0 < 1$, $r_0^* > 1$

where

$$r_0 = \frac{1}{2\mu_1}\left[(\lambda_1 + \mu_1 + \theta) - \sqrt{(\lambda_1 + \mu_1 + \theta)^2 - 4\lambda_1\mu_1}\right]$$

$$r_0^* = \frac{1}{2\mu_1}\left[(\lambda_1 + \mu_1 + \theta) + \sqrt{(\lambda_1 + \mu_1 + \theta)^2 - 4\lambda_1\mu_1}\right]$$

(b) The equation $\mu Z^2 - (\lambda + \mu)Z + \lambda = 0$ includes the service rate μ of the system in regular busy period, but it does not include the parameter θ. From this it is clear that the above quadratic is for the case when the server is busy in service and it has two different real roots $r_1 = \rho = \frac{\lambda}{\mu}$ and $r_1^* = 1$.

From the matrix R, we find that spectral radius $sp(R) = \max(r_0, r_1) < 1$ if and only if $\rho < 1$ is the necessary and sufficient condition for that the stability of the process $\{X(t), t \geq 0\}$ is to be positive recurrent (see Neuts(1981)).

Theorem: 3.2

If $\rho < 1$, the matrix equation (4) has the minimal non-negative solution

$$R = \begin{bmatrix} r_0 & r_{01} \\ 0 & r_1 \end{bmatrix}$$

where

$$r_{01} = \frac{-\theta r_0}{[\mu(r_0 + r_1) - (\lambda + \mu)]}$$

$$r_0 = \frac{1}{2\mu_1}\left[(\lambda_1 + \mu_1 + \theta - \sqrt{(\lambda_1 + \mu_1 + \theta)^2 - 4\lambda_1\mu_1}\right]$$

$$r_1 = \frac{\lambda}{\mu}$$

Proof

Since the coefficient matrices of equation (4) are all upper triangular, so let

$$R = \begin{bmatrix} r_0 & r_{01} \\ 0 & r_1 \end{bmatrix} \tag{9}$$

$$R^2 A_2 = \begin{bmatrix} \mu_1 r_0^2 & \mu r_{01}(r_0 + r_1) \\ 0 & \mu r_1^2 \end{bmatrix} \tag{10}$$

$$RA_1 = \begin{bmatrix} -(\lambda_1 + \mu_1 + \theta) r_0 & \theta r_0 - (\lambda + \mu) r_{01}) \\ 0 & -(\lambda + \mu) r_1 \end{bmatrix} \tag{11}$$

Substituting (10), (11) and A_0 into (4), gives the following set of equations

$$\mu_1 r_0^2 - (\lambda_1 + \mu_1 + \theta) r_0 + \lambda_1 = 0 \tag{12}$$

$$\mu r_{01}(r_0 + r_1) + \theta r_0 - (\lambda + \mu) r_{01} = 0 \tag{13}$$

$$\mu r_1^2 - (\lambda + \mu) r_1 + \lambda = 0 \tag{14}$$

From equation (13), we get

$$r_{01} = \frac{-\theta r_0}{[\mu(r_0 + r_1) - (\lambda + \mu)]}$$

Substituting the values of r_0, r_{01} and r_1 in (9), we get

$$R = \begin{bmatrix} \dfrac{[(\lambda_1 + \mu_1 + \theta) - \sqrt{(\lambda_1 + \mu_1 + \theta)^2 - 4\lambda_1 \mu_1}}{2\mu_1} & \dfrac{-\theta r_0}{[\mu(r_0 + r_1) - (\lambda + \mu)]} \\ 0 & \dfrac{\lambda}{\mu} \end{bmatrix} \tag{15}$$

It is clear that the above equation have unique non-negative solution. Therefore, this non-negative solution must be the minimal.

Theorem: 3.3

If $\rho < 1$, the stationary probability vectors $p_0 = (p_{00}, p_{10})$ and $p_i = (p_{0i}, p_{1i})$ for $i \geq 1$ are

$$p_{00} = \frac{\mu(1 - r_0)(1 - r_1)}{\mu(1 - r_1 - r_{01}) - [r_0 \mu_1 - (\lambda_1 + \theta)](1 - r_0)}$$

$$p_{10} = \frac{-(1-r_0)(1-r_1)[r_0\mu_1 - (\lambda_1+\theta)]}{\mu(1-r_1-r_{01}) - [r_0\mu_1 - (\lambda_1+\theta)](1-r_0)}$$

and $p_i = p_0 R^i$; $i \geq 1$

Proof

p_{00} and p_{10} follows from the equations (15), (5) and (6).

Remark: 3.1

Even though R in Theorem 3.2 has a nice structure which enables us to make use of the properties like $R^n = \begin{bmatrix} r_0^n & r_{01}\Sigma_{j=0}^{n-1} r_0^j r_1^{n-j-1} \\ 0 & r_1^n \end{bmatrix}$, for $n \geq 1$, due to the form of r_0 & r_{01}, it may not be easy to cary out the computation required to calculate the p_i and the performance measures. Hence, we explore the possibility of algorithmic computation of R. The computation of R can be carried out using a number of well-known methods. We use Theorem 1 of Latouche and Neuts (1980). The matrix R is computed by successive substitutions in the recurrence relation:

$$R(0) = 0 \qquad (16)$$

$$R(n+1) = -A_0A_1^{-1} - [R(n)]^2 A_2 A_1^{-1} \text{ for } n \geq 0 \qquad (17)$$

and is the limit of the monotonically increasing sequence of matrices $\{R(n), n \geq 0\}$.

IV. PERFORMANCE MEASURES

Using straightforward calculations the following performance measures have been obtained:

(i) Mean queue length $E(L) = p_0 R(\mathrm{I} - R)^{-2} e$

(ii) $E(L^2) = p_0 R(I + R)(I - R)^{-3} e$

(iii) Variance of L = var(L)

$$= p_0 R \{(I + R) - p_0 R(I - R)^{-1} e\} (I - R)^{-3} e$$

(iv) Probability that the server is ideal $= p_0 e$

(v) Mean queue length when the server is an vacation period $= \Sigma_{i=0}^{\infty} i p_{0i}$

(vi) Mean queue length when the server is in regular busy period $= \Sigma_{i=0}^{\infty} i p_{1i}$

(vii) Probability that the server is in working vacation period $= pr\{J = 0\} = \Sigma_{i=1}^{\infty} p_{0i}$

(viii) Probability that the server is in regular busy period $= pr\{J = 1\} = \Sigma_{i=1}^{\infty} p_{1i}$

V. PARTICULAR MODEL

In the above model, we assume that λ1 = λ, and $\mu_1 = \mu$, then we get

$$R = \begin{bmatrix} \dfrac{[(\lambda + \mu + \theta) - \sqrt{(\lambda + \mu + \theta)^2 - 4\lambda\mu}]}{2\mu} & \dfrac{-\theta r_0}{[\mu(r_0 + r_1) - (\lambda + \mu)]} \\ 0 & \dfrac{\lambda}{\mu} \end{bmatrix}$$

$$p_{00} = \frac{\mu(1 - r_0)(1 - r_1)}{\mu(1 - r_1 - r_{01}) - [r_0\mu - (\lambda + \theta)](1 - r_0)}$$

$$p_{10} = \frac{-(1 - r_0)(1 - r_1)[r_0\mu - (\lambda + \theta)]}{\mu(1 - r_1 - r_{01}) - [r_0\mu - (\lambda + \theta)](1 - r_0)}$$

and $p_i = p_0 R^i$; $i \geq 1$

VI. NUMERICAL STUDY

In this section, some examples are given to show the effect of the parameters λ, λ_1, μ, μ_1 and θ on the performance measures mean queue length, $E(L^2)$, variance of L, probability that the server is idle, mean queue length when the server is an vacation period, mean queue length when the server is regular busy period, probability that the server is in working vacation period and probability that the server is in regular busy period for the model analyzed in this paper. The corresponding results are presented as case(1), case(2) and case(3).

Case(1)

If λ = 0.5, λ_1 = 0.3,μ = 0.6, μ_1 = 0.4 and θ = 10, the matrix R is obtained using the equations (16) & (17)

$$R = \begin{bmatrix} 0.028067 & 0.481284 \\ 0 & 0.833325 \end{bmatrix}$$

and the invariant probability vector is

$P = (p_0, p_1, p_2,\ldots)$

where

$p_0 = (0.009906823, 0.169881761)$

and the remaining vectors p_i's are evaluated using the relation $p_i = p_0 R^i; i \geq 1$

$p_1 = (0.000278054794762, 0.146334722638130)$

$p_2 = (0.000007804163943, 0.122078210115433)$

$p_3 = (0.000000219039478, 0.101734586060047)$

$p_4 = (0.000000006147781, 0.084778085350990)$

$p_5 = (0.000000000172550, 0.070647701621056)$

$p_6 = (0.000000000004843, 0.058872498571873)$

$p_7 = (0.000000000000136, 0.049059927463531)$

$p_8 = (0.000000000000004, 0.040882866829634)$

$p_9 = (0.0000000000000001, 0.034068714827299)$

$p_{10} = (0.00000000000000003, 0.028390312567353)$

$p_{11} = (0.0000000000000000000843510, 0.023658357560634)$

$p_{12} = (0.0000000000000000000023675, 0.019715102389454)$

$p_{13} = (0.0000000000000000000000664, 0.016429089009761)$

$p_{14} = (0.0000000000000000000000019, 0.013690770603716)$

For the chosen parameters $p_{14} \to 0$, and the sum of the steady state probabilities is found to be 0.99045631.

The performance measures are

(i) Mean queue length $E(L) = 5.273624$

(ii) $E(L^2) = 58.00553$

(iii) Variance of L = var(L) = 30.194421

(iv) Probability that the server is ideal = 0.179788

(v) Mean queue length when the server is an vacation period = 0.02796839

(vi) Mean queue length when the server is regular busy period = 3.87358602

(vii) Probability that the server is in working vacation period = pr{J = 0}=0.010195

(viii) Probability that the server is in regular busy period = pr{J=1} = 0.9802203

Case(2)

If $\lambda = 0.4$, $\lambda_1 = 0.2$, $\mu = 0.5$, $\mu_1 = 0.3$ and $\theta = 10$, the matrix R is obtained using the equations (16) & (17)

$$R = \begin{bmatrix} 0.019058 & 0.388564 \\ 0 & 0.799998 \end{bmatrix}$$

and the invariant probability vector is

$$P = (p_0, p_1, p_2, \ldots)$$

Where

$p_0 = (0.009902884, 0.201905593)$

and the remaining vectors $p_i's$ are evaluated using the relation $p_i = p_0 R^i$; $i \geq 1$

$p_1 = (0.000188729172805, 0.165371969342232)$

$p_2 = (0.000003596800525, 0.132370576262474)$

$p_3 = (0.000000068547827, 0.105897590517998)$

$p_4 = (0.000000001306385, 0.084717884659767)$

$p_5 = (0.000000000024897, 0.067774139344692)$

$p_6 = (0.000000000000474, 0.054219175130129)$

$p_7 = (0.000000000000009, 0.043375231325626)$

$p_8 = (0.0000000000000001, 0.034700099378824)$

$p_9 = (0.00000000000000004, 0.027760010212660)$

$p_{10} = (0.000000000000000001, 0.022207953035831)$

$p_{11} = (0.0000000000000000000003, 0.0177663173526525)$

$p_{12} = (0.0000000000000000000001, 0.014213018119335)$

$p_{13} = (0.0000000000000000000000006, 0.0113703859969)$

For the chosen parameters $p_{13} \rightarrow 0$, and the sum of the steady state probabilities is found to be 0.993745121.

The performance measures are

(i) Mean queue length $E(L) = 4.136662$

(ii) $E(L^2) = 37.228764$

(iii) Variance of $L = \text{var}(L) = 20.116791$

(iv) Probability that the server is ideal = 0.211808

(v) Mean queue length when the server is an vacation period = 0.00019618

(vi) Mean queue length when the server is regular busy period = 2.76400365

(vii) Probability that the server is in working vacation period = pr{J=0}=0.010095

(viii) Probability that the server is in regular busy period = pr{J=1}=0.9836982

Case(3)

If $\lambda = \lambda_1 = 0.5, \mu = \mu_1 = 0.6$ and $\theta = 10$, the matrix R is obtained using the equations (16) & (17)

$$R = \begin{bmatrix} 0.045155 & 0.788170 \\ 0 & 0.833325 \end{bmatrix}$$

and the invariant probability vector is

$P = (p_0, p_1, p_2, \ldots)$
where

$p_0 = (0.009919, 0.173131)$

and the remaining vectors $p_i's$ are evaluated using the relation $p_i = p_0 R^i; i \geq 1$

$p_1 = (0.000447892, 0.152092248)$

$p_2 = (0.000020224582840, 0.127095311880112)$

$p_3 = (0.000000913241024, 0.105927646160126)$

$p_4 = (0.000000041237399, 0.088272877037525)$

$p_5 = (0.000000001862075, 0.073560029268265)$

$p_6 = (0.000000000084082, 0.061299413442612)$

$p_7 = (0.000000000003797, 0.051082335412502)$

$p_8 = (0.000000000000171, 0.042568188160658)$

$p_9 = (0.000000000000008, 0.035473138093948)$

$p_{10} = (0.000000000000001, 0.029560653492808)$

$p_{11} = (0.0000000000000000157, 0.02463363297)$

$p_{12} = (0.0000000000000000007, 0.02052782289)$

For the chosen parameters $p_{12} \to 0$, and the sum of the steady state probabilities is found to be 0.995612406.

The performance measures are

(i) Mean queue length $E(L) = 5.490897$

(ii) $E(L^2) = 60.396572$

(iii) Variance of $L = \text{var}(L) = 30.246622$

(iv) Probability that the server is ideal = 0.183050

(v) Mean queue length when the server is an vacation period = 0.00049125

(vi) Mean queue length when the server is regular busy period = 3.64304497

(vii) Probability that the server is in working vacation period = $\text{pr}\{J = 0\} = 0.010388$

(viii) Probability that the server is in regular busy period = $\text{pr}\{J = 1\} = 0.985204$

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