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PERISTALTIC TRANSPORT AND HEAT TRANSFER IN A VERTICAL POROUS TUBE

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ABSTRACT

Peristaltic transport of a Newtonian fluid, with heat transfer, in a vertical porous axisymmetric tube under long wave length approximation is considered. A closed form solution is obtained as an asymptotic expansion in terms of free convection and porosity parameters. Expressions for temperature, the pressure flow relationship, and the heat transfer coefficient at the tube wall are derived. It is observed that pressure drop increases as the amplitude ratio increases. Further, it is noticed that the mean flux increases by about 8 to 10 percent as the free convection parameter increases from 1 to 2 for given values of other parameters.

Key words: *Perstalsis, long wave length, porous tube*

Subject classification code: *76 S05*

INTRODUCTION

Peristalsis is a mechanism for fluid transport by the passage of area contraction and expansion waves along the length of the distensible tube. The need for peristaltic pumping may arise to avoid using internal moving parts such as a piston. It is one of the main mechanisms for fluid transport in physiological systems, in particular, urine passage in ureter and food mixing and chyme movement in the intestines. A blood pump in dialysis is designed on this principle to prevent contamination.

Because of its importance, theoretical and experimental studies of peristaltic transport have been carried out by various authors [Shapiro *et al.* (1969); Zien and Ostrach (1970); Radhakrishnamacharya (1982); Takabatake *et al.* (1988); Rao and Usha (1995); Usha and Rao (1997); Vajravelu *et al.* (2005 a, b)]. Radhakrishnamacharya [1982] investigated peristaltic pumping of power-law fluid

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in an axisymmetric tube under long wave approximation. Takabatake *et al.* [1988] developed complete numerical solutions for peristaltic pumping and its efficiency in cartesian and axisymmetric geometries. The thermodynamical aspects of peristalsis have received attention [Bestmen (1979); Radhakrishnamacharya and Radhakrishnamurthy (1993, 1995)] as it might be relevant in certain processes like oxygenation and hemodialysis. Translocation of water in trees involves flow through a matrix of tubes, and some authors [Cany and Phillips (1993); Aikman and Anderson (1971); Radhakrishnamurthy *et al.* (1995)] have investigated peristalsis with particular reference to water transport in trees.

In view of this, we study heat transfer for the motion of a Newtonian fluid in a vertical axisymmetric porous tube, under the action of peristalsis. Assuming long wave length approximation, a perturbation solution in terms of free convection (G_m) and porosity (σ^2) parameters is obtained and analytical expressions are derived for temperature, pressure drop, and the heat transfer coefficient. It is observed that temperature increases as the amplitude of the peristaltic wave increases. Further, it is noticed that the mean flux increases by about 8 to 10 per cent as the free convection parameter increases from 1 to 2 and for given values of all other parameters.

MATHEMATICAL FORMULATION

The flow of a Newtonian, incompressible fluid through an axisymmetric vertical tube, filled with porous material is considered. Peristaltic waves of very large wave length are assumed to travel down the wall of the tube. Cylindrical polar coordinate system (X, R) is chosen such that X and R are the axial and radial coordinates respectively.

The simplified, zeroth order equations, under long wave approximation, governing the flow [Radhakrishnamurthy *et al.* (1995)] are the following:

Momentum equation:

$$0 = -\frac{\partial p}{\partial x} + \frac{\mu}{R}\frac{\partial}{\partial R}\left(R\frac{\partial W}{\partial R}\right) - \frac{\mu}{k_0} W + \rho\, g\, \beta\, (\mathrm{T} - \mathrm{T}_0) \tag{1}$$

Continuity equation:

$$0 = \frac{\partial W}{\partial X} + \frac{U}{R} + \frac{\partial U}{\partial R} \tag{2}$$

Energy equation:

$$0 = \frac{K}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) + \mu\left(\frac{\partial W}{\partial R}\right)^2 + \frac{\mu}{k_0} w^2 \tag{3}$$

The wall deformation due to the propagation of an infinite train of peristaltic waves is represented by

$$R = H(X, t) = a + b \operatorname{Sin}\left(\frac{2\pi}{\lambda}\right)(X - c\,t) \tag{4}$$

where p is the pressure, μ is the coefficient of viscosity, ρ is the density, g is the acceleration due to gravity, β is the coefficient of expansion, K is the thermal conductivity of fluid, W and U are the velocity components of the fluid in X and R directions respectively, T_0 is the temperature on the boundary, T is the temperature of the fluid, k_0 is the permeability of the medium, 'a' is the mean radius of the tube, b is the amplitude, λ is the wave length, and c is the wave speed.

The boundary conditions are

$$\left.\begin{array}{l} W = 0 \\ T = T_0 \end{array}\right| \quad \text{at } R = H(X, t) \tag{5}$$

Let us introduce the wave frame of reference which moves with a constant speed c relative to the laboratory frame. The variables x and r measured in the wave frame are defined by

$$x = X - c\,t,\ r = R. \tag{6}$$

The corresponding velocity components of the fluid are

$$w = W - c,\ u = U. \tag{7}$$

The system governing the flow in the wave frame of reference is the following:

$$0 = -\frac{\partial p}{\partial x} + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) - \frac{\mu}{k_0}(w + c) + \rho\, g\, \beta (T - T_0) \tag{8}$$

$$0 = \frac{\partial w}{\partial x} + \frac{u}{r} + \frac{\partial u}{\partial r} \tag{9}$$

$$0 = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \mu\left(\frac{\partial w}{\partial r}\right)^2 + \frac{\mu}{k_0}(w + c)^2 \tag{10}$$

The boundary conditions are:

$$\left.\begin{array}{l} w = -c \\ T = T_0 \end{array}\right| \, at\, r = h(x) = a + b \sin(2\pi/\lambda)x \tag{11}$$

Introducing the following non-dimensional quantities,

$$x' = \frac{x}{\lambda},\ \ r' = \frac{r}{a},\ \ w' = \frac{w}{c},\ \ u' = \frac{\lambda u}{ac}\ \ ,\ \ \theta = \frac{T - T_0}{T_0},\ \ p' = \frac{p}{\dfrac{\mu c \lambda}{a^2}},$$

$$\eta(x) = \frac{h(x)}{a} \tag{12}$$

into the equation (8)–(11), we get (after dropping the primes),

$$0 = -\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) - \sigma^2 (w+1) + G_m \theta, \tag{13}$$

$$0 = \frac{\partial w}{\partial x} + \frac{u}{r} + \frac{\partial u}{\partial r}, \tag{14}$$

$$0 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) + E_m\left(\frac{\partial w}{\partial r}\right)^2 + \sigma^2 E_m (w+1)^2. \tag{15}$$

The non-dimensional boundary conditions are

$$\left.\begin{array}{l} w = -1 \\ \theta = 0 \end{array}\right| \text{at } r = \eta(x) \tag{16}$$

where

$$\eta(x) = 1 + \varepsilon \sin 2\pi x, \tag{17}$$

$\sigma^2 = \dfrac{a^2}{k_0}$ (porosity parameter), $G_m = \dfrac{g\beta T_0 a^3}{\nu^2}$ (Grashof number),

$E_m = \dfrac{\mu}{K}\dfrac{c^2}{T_o}$ (Eckert number) and $\varepsilon = \dfrac{b}{a}$ (amplitude ratio).

ANALYSIS

Equations (13) and (15) are non-linear and it is not possible to get an exact solution for arbitrary value of all the parameters. Hence, we seek a perturbation solution in the form of a series:

$$F = (F_{00} + G_m F_{01} + \ldots..) + \sigma^2 (F_{10} + \ldots\ldots) + \ldots\ldots\ldots \tag{18}$$

where F is any flow variable.

Using (18) in equations (13), (15) and (16) and solving the resultant equations under the relevant boundary conditions, the solutions for the velocity component w and temperature θ can be obtained. However, for brevity the expression for velocity component is not presented here, and the expression for temperature θ is

$$\theta = (\theta_{00} + G_m \theta_{01} + \ldots..) + \sigma^2 (\theta_{10} + \ldots..) + \ldots\ldots \tag{19}$$

where

$$\theta_{00} = -\frac{1}{64} E_m \alpha^2 (r^4 - \eta^4),$$

$$\theta_{01} = -\frac{1}{32} E_m \left[\alpha\tau (r^4 - \eta^4) + \frac{E_m}{768} (r^8 - 12 r^4 \eta^4 + 11 \eta^8) \right],$$

$$\theta_{10} = -\frac{1}{32} E_m \left[\alpha\gamma (r^4 - \eta^4) + \frac{\alpha^2}{36} (2r^6 - 9 r^4 \eta^2 + 7\eta^6) \right],$$

$$- \frac{\alpha^2}{72} (2r^6 - 9 r^4 \eta^2 + 18 r^2 \eta^4 - 20 \eta^6),$$

$$\alpha = \frac{8(\overline{Q}_{00} + \pi)}{\pi (1 - 2\eta^2)},$$

$$\tau = \frac{1}{\pi (1 - 2\eta^2)} \left[32 \overline{Q}_{01} - \frac{1}{9} E_m \alpha^2 (140 \eta^6 - 18 \eta^4 + 1) \right],$$

$$\gamma = \frac{1}{(1 - 2\eta^2)} \left[\frac{8}{\pi} \overline{Q}_{10} - \frac{\alpha}{24} (1 - 21 \eta^4 - 6\eta^2) \right].$$

The pressure drop over one wave length is defined by

$$\Delta p_\lambda = \int_0^\lambda \frac{\partial p}{\partial x} \, dx. \tag{20}$$

Substituting the relation for $\dfrac{\partial p}{\partial x}$ from equation (13) in (20) and using expressions for velocity (which are not presented) and temperature from equation (19), the non-dimensional pressure drop can finally be obtained as

$$\Delta p = \frac{\Delta p_{\lambda}}{\mu c \lambda / a^2} = (\Delta P_{00} + G_m \Delta P_{01} + ...) + \sigma^2 (\Delta P_{10} + ...) + ... \qquad (21)$$

where

$$\Delta P_{00} = \frac{32\varepsilon^2(1-\varepsilon^2/16) - 8\overline{Q}_{00}(1+\frac{3}{2}\varepsilon^2)}{\left(1-\varepsilon^2\right)^{7/2}},$$

$$\Delta P_{01} = \frac{5}{6} E_m \, 1 + \left[\overline{Q}_{00} - (1+\frac{\varepsilon^2}{2})\right]^2 \frac{\left(1+\frac{3}{2}\varepsilon^2\right)}{(1-\varepsilon^2)^{7/2}} + \frac{\overline{Q}_{00} - (1+\frac{\varepsilon^2}{2})}{(1-\varepsilon^2)^{9/2}} \Bigg] - 8\overline{Q}_{01} \frac{(1+\frac{3}{2}\varepsilon^2)}{(1-\varepsilon^2)^{7/2}},$$

$$\Delta P_{10} = -6\overline{Q}_{10} \left[\frac{2+\varepsilon^2}{2(1-\varepsilon^2)^{5/2}}\right] - \frac{7}{20}\left[1 + \frac{(\overline{Q}_{00} - 1)}{(1-\varepsilon^2)^{9/2}}\right],$$

Here $\overline{Q}$ is the non dimensional mean flux given by

$$\overline{Q} = \int_0^1 Q \, \mathrm{dt}$$

where

$$Q = \frac{Q'}{\pi c a^2} = \int_0^{\eta} 2rw \, \mathrm{dr}$$

is the non dimensional flux.

The heat transfer coefficient Z on the boundary of the tube in non-dimensional form is given by

$$\mathrm{Z} = \left\langle \frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial r} \quad \frac{d\eta}{dx} \right\rangle_{r\,=\,\eta} \qquad (22)$$

which, in view of (18), can be expressed as

$$\mathrm{Z} = (\mathrm{Z}_{00} + \mathrm{G}_m \mathrm{Z}_{01} + \ldots\ldots) + \sigma^2 (\mathrm{Z}_{10} + \ldots\ldots\ldots) + \ldots\ldots. \qquad (23)$$

where

$$Z_{00} = \frac{dn}{dx}\frac{\partial \theta_{00}}{\partial r}, \quad Z_{01} = \frac{dn}{dx}\frac{\partial \theta_{01}}{\partial r}, \quad Z_{10} = \frac{dn}{dx}\frac{\partial \theta_{10}}{\partial r}.$$

RESULTS AND DISCUSSION

Analytical expressions for temperature, pressure drop, and the heat transfer coefficient are given by the equations (19), (21) and (23) respectively. To explicitly study the effects of various parameters on these flow variables, these quantities are numerically evaluated and the results are presented in Figs. 1–8 and Tables 1-4.

Table 1

Heat Transfer variation with E_m

($G_m = 3$, $\sigma^2 = 2$, $\varepsilon = 0.1$)

x	$E_m = 1$	$E_m = 3$	$E_m = 5$
0.0	2.42945	22.0037	61.17046
0.4	2.84512	25.72656	71.150905
0.8	0.39168	3.56543	9.91642

Table 2

Heat Transfer variation with G_m

($E_m = 3$, $\sigma^2 = 2$, $\sigma = 0.1$)

x	$G_m = 1$	$G_m = 3$	$G_m = 5$
0.0	7.30205	22.0037	36.70534
0.4	8.54303	25.72656	42.910105
0.8	1.18121	3.56543	5.94964

Table 3

Heat Transfer variation with σ^2

($E_m = 3$, $G_m = 3$, $\varepsilon = 0.1$)

x	$\sigma^2 = 1$	$\sigma^2 = 2$	$\sigma^2 = 3$
0.0	22.42945	22.0037	21.99219
0.4	25.94013	25.72656	25.451301
0.8	3.76532	3.56543	3.47135

Table 4

Heat Transfer variation with ε

($G_m = 3$, $E_m = 3$, $\sigma^2 = 2$)

x	$\varepsilon = 0$	$\varepsilon = 0.1$	$\varepsilon = 0.2$
0.0	0.0	22.0037	44.0073
0.4	0.0	25.72656	72.89057
0.8	0.0	3.56543	3.763

Figs. 1-4 show the variation of temperature versus x with respect to various parameters. It is observed from Figs. 1–4 that for fixed values of all other parameters, temperature first increases down the tube and then decreases. This may be due to the effect of peristalsis. From Figs. 1 and 2, we can see that for fixed values of all other parameters, the temperature increases as the Eckert number (E_m) or the Grashof number (G_m) increases. Further, the temperature increases as σ^2 or ε increases i.e. the temperature increases as the tube becomes more porous or the peristaltic wave amplitude increases (Figs. 3 and 4).

Fig. 1: Temp. Variation with E_m(G_m= 3, σ^2 = 2, E = 0.1)

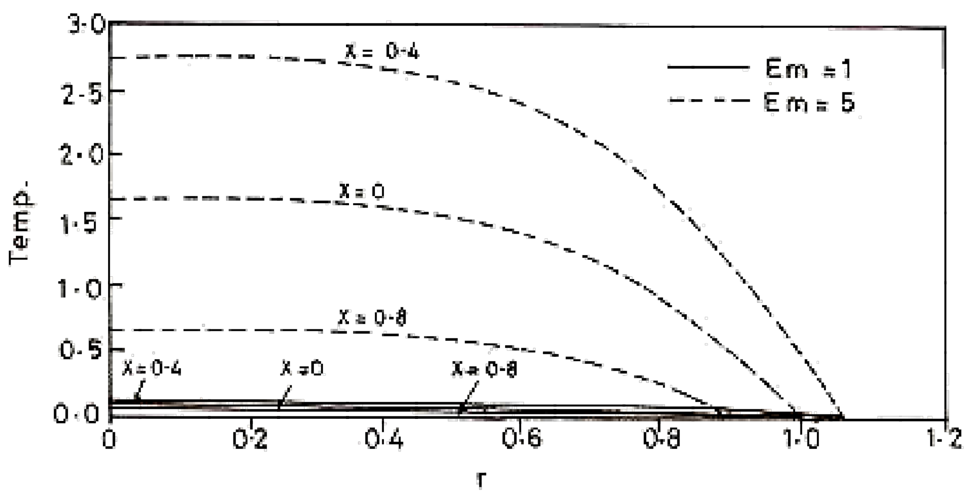


Fig. 2: Temp. Variation with G_m(E_m= 3, σ^2 = 2, E = 0.1)

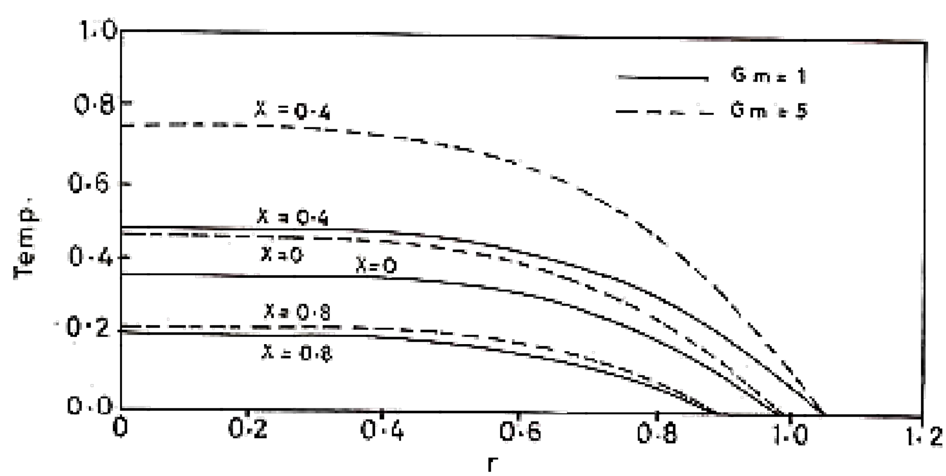


Fig. 3: Temp. Variation with $\in$ ($E_m = 3$, $G_m = 3$, $\sigma^2 = 2$)

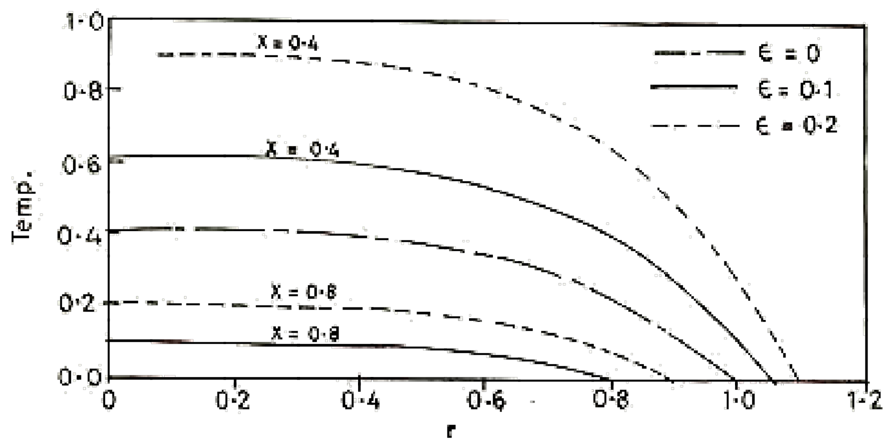
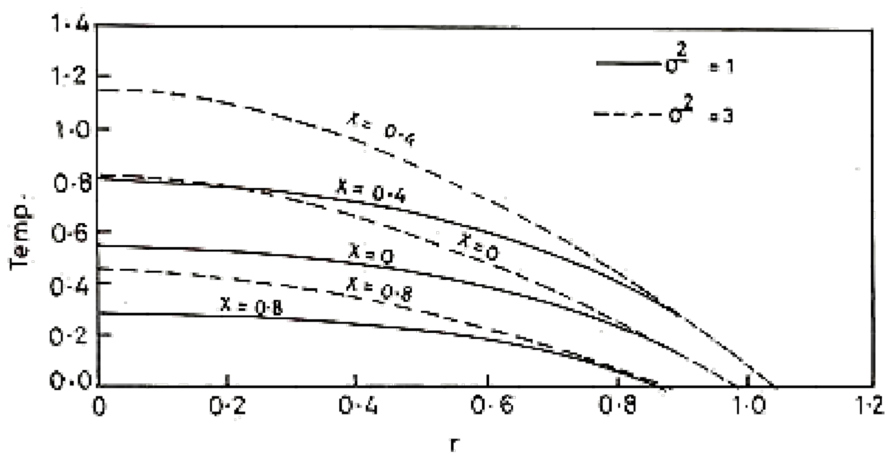


Fig. 4: Temp. Variation with σ^2 ($E_m = 3$, $G_m = 3$, $\in = 0.1$)



Variation of pressure drop (Δp) with mean flux ($\overline{Q}$) is depicted in Figs. 5–8. Notice that for fixed values of all other parameters, pressure drop decreases with mean flux ($\overline{Q}$) and it increases with amplitude ratio (ε). Δp increases as E_m or G_m increases. The effect of wall porosity parameter on Δp is very insignificant.

Observe that the mean flux, $\overline{Q}$, increases by about 8 to 10 percent as the free convection parameter increases from 1 to 2 for given values of other parameters.

The heat transfer coefficient, Z, on the boundary of the tube is evaluated numerically, and the results are presented in Tables 1-4. Z increases down the tube and then decreases, as in the case of temperature, which may be due to peristalsis. From Tables 1 and 2, we can see that, for fixed values of all other parameters, the

Fig. 5: Variation of ΔP with $\bar{Q}$ ($G_m = 1, \sigma^2 = 1, E_m = 0$)

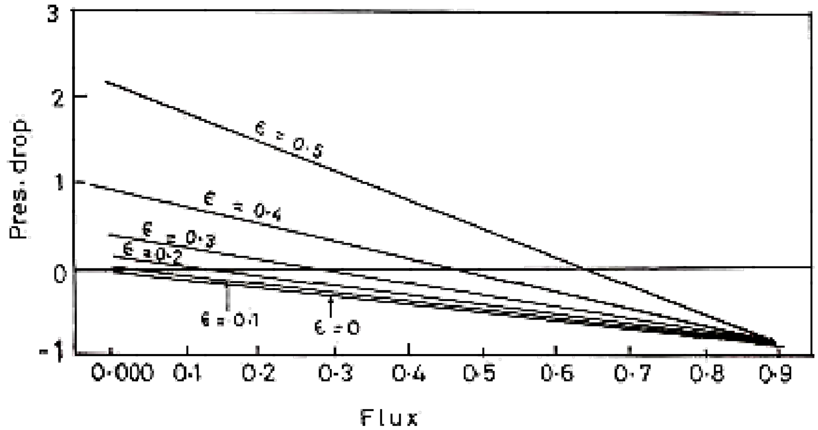


Fig. 6: Variation of ΔP with $\bar{Q}$ ($G_m = 1, \sigma^2 = 1, E_m = 2$)

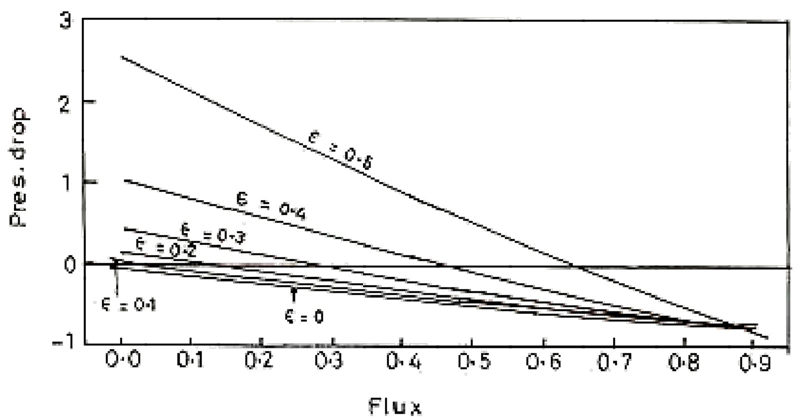


Fig. 7: Variation of ΔP with $\bar{Q}$ ($G_m = 1, \sigma^2 = 2, E_m = 2$)

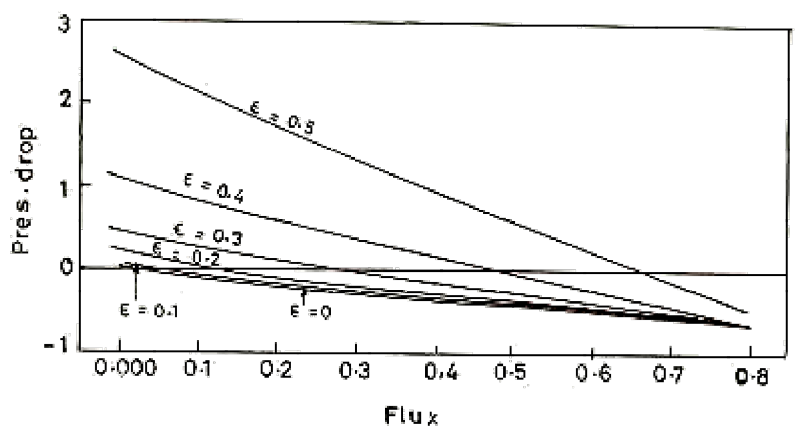
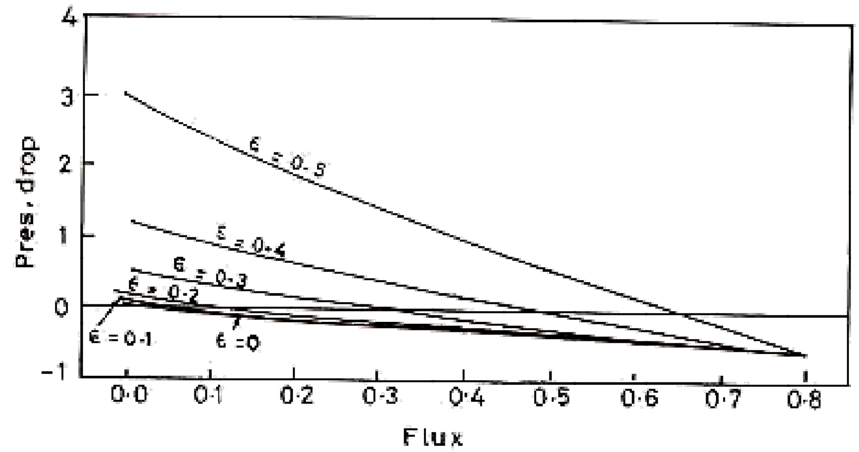


Fig. 8: Variation of ΔP with $\bar{Q}$ ($G_m = 2$, $\sigma^2 = 1$, $E_m = 2$)



heat transfer coefficient increases as the Eckert number or the Grashof number increases. Table 3 shows that heat transfer decreases with porosity (though not very significantly) but increases significantly with amplitude ratio (Table 4).

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