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On Pasinetti's 'natural' rate of interest

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Abstract: This paper provides a concise exposition of the notion of 'natural rate of interest' introduced by Luigi Pasinetti in his structural change analysis. Its normative nature is suitably stressed: it is that rate of interest that prevents that debt and credit relations among individuals alter a distribution of national income based on the 'labour principle', that is, a situation where each individual perceives income in proportion to the quantity of labour provided.

Keywords: rate of interest, technical progress, structural change, physical numéraires, nominal numéraires.

JEL, classification: B51, D20, D33, E40, E43, E49.

INTRODUCTION

Luigi Pasinetti's multi-sectoral framework of structural change provides a description of the evolution of modern economic systems as well as a normative frame useful for identifying the ideal conditions that allow an evolving economic system to accomplish its potential as regards employment and the satisfaction of economic human wants. This investigation has been carried out by Pasinetti throughout his academic career, from the times of his doctoral dissertation, and it remains at the center of his research interests. The findings of this investigation have been published by Pasinetti in several successive steps. A first part of his work, containing the essentials of the structural change model, came out in Pasinetti (1965). The problems concerning the choice of a suitable numéraire of the price system in connection to the inflation problems were initially dealt with in an Italian booklet (Pasinetti, 1981a). The entire framework was then published in Pasinetti (1981b) and (1993).

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In this paper I will present a concise exposition of the notion of 'natural' rate of interest worked out by Pasinetti. To understand this notion, it is necessary to start from what Pasinetti considers the ideal configuration of income distribution. It is a situation where the entire national income is devoted to workers in proportion to the quantity of labour provided by each individual worker. He calls it the 'labour principle of income distribution', and it is conceived in a situation where there are no debt or credit relations. As soon as we take into account these relations, a deviations from the labour principle can occur. The natural rate of interest will be defined in such a way as to sterilize these deviations, and keep income distribution pegged to the labour principle. The natural rate of interest will thus configure as a reference level (a 'norm'): there are no reasons, in fact, to expect that this rate takes place autonomously. It is the reference level that prevents people from becoming enriched or impoverished through the establishment of debt or credit relations. It is to be added that, since loans are normally stipulated in terms of a numéraire, a detour concerning the nontrivial problem of the choice of a suitable numéraire of the price system is necessary before turning to the analysis of the determinants of the natural rate of interest.

A SIMPLIFIED ECONOMIC SYSTEM

Consider an economic system where a community of individuals carries out a production activity and a consumption activity. In this system, C commodities are produced by employing only labour.1 Let $c = 1, 2, \cdots, C$ be the index corresponding to each commodity. Each individual produces or participates to the production of a single, specific commodity. Due to this marked division of labour, each individual can achieve very high productivity levels for the specific commodity; all other commodities will be obtained through exchange. We represent this economy by means of a closed Leontief system; given the assumptions on technology, the quantity and the price systems reduce to:

$$q_c = c_c q_N, \quad c = 1, 2, \cdots, C \tag{1a}$$

$$\ell_1 q_1 + \ell_2 q_2 + \cdots + \ell_C q_C = q_N, \tag{1b}$$

and

$$p_c = w\ell_c, \quad c = 1, 2, \cdots, C \tag{2a}$$

$$c_1 p_1 + c_2 p_2 + \cdots + c_C p_C = w, \tag{2a}$$

where c_c and ℓ_c are, respectively, the per capita consumption and the labour coefficient of commodity c, and q_c and p_c are, respectively, the quantity produced and the price of commodity c; finally q_N is the total quantity of labour and w is the wage rate. The necessary and sufficient condition for systems (1) and (2) to have a non-trivial solution is:

$$\sum_{c=1}^{C} c_c \ell_c = 1. \tag{3}$$

This condition, when satisfied, introduces a degree of freedom for both the systems, that are closed by choosing a single quantity and a single price from outside.

For the quantity system, it is reasonable to fix the quantity of labour from outside the system. Let $\bar{q}_N$ be the amount of the available labour force which, for the moment, is supposed to coincide with the population size. Here we set

$$q_N = \bar{q}_N, \tag{4}$$

that is, we will consider the solution corresponding to full employment. For the price system, we can choose for the moment any commodity (single or composite), or the labour unit as the numéraire of the price system, that is, we can set

$$p_h = 1, \tag{5}$$

or

$$\sum_{c=1}^{C} p_c b_c = 1, \tag{6}$$

or

$$w = 1 \tag{7}$$

(we will return to this choice in the next Section).

Condition (3) represents a crucial relation of this framework. Observe, first of all, that even in this system, where all industrial interdependencies have been ruled out at the beginning, all sectors are linked together thanks to this condition: it reflects the interdependencies due to the necessity that each worker has to address other sectors to obtain the final commodities he needs. This gives this relation the character of an actual macro-economic condition. To fully appreciate its economic meaning, note that it can be written in two alternative ways; in fact, thanks to (1a) and (4), the generic addendum of (3) may be written as $c_c \ell_c = \ell_c q_c / \bar{q}_N$, and it represents the *proportion* of employment required by the productive process of commodity c. Hence, (3) can be written as

$$\sum_{c=1}^{C} \ell_c q_c = \bar{q}_N. \tag{3`}$$

On the other hand, thanks to (2a) the generic addendum of (3) may be written as $c_c \ell_c = p_c c_c / w$, representing the proportion of potential income generated in each sector by the expenditure allocated to that sector by the consumer's effective demand. Hence, (3) can also be written as

$$\sum_{c=1}^{C} p_c c_c = w. \tag{3``}$$

Observe that (3`) coincides with the last equation of system (1) once (4) is substituted into (1b). Moreover, (3``) coincides with the last equation of system (2). The particular form of systems (1) and (2) entails that the first C equations of the quantity system as well as the first C equations of the price system are autonomous from the last equation of the respective system and, therefore, can be satisfied regardless of whether the last equation of each of the two systems is met or not. This means that the first C equations of the quantity system and the first C equations of the price system can coexist with any of the following situations:

$$\sum_{c=1}^{C} c_c \ell_c \lesseqgtr 1 \quad \text{that is,} \quad \sum_{c=1}^{C} \ell_c q_c \lesseqgtr \bar{q}_N \quad \text{that is,} \quad \sum_{c=1}^{C} p_c c_c \lesseqgtr w.$$

Thus, when the '<' symbol prevails in macroeconomic condition (3), labour requirements are *less than* the existing labour force—that is, we have unemployment—and the individual expenditure for consumption is *less than* income per worker. The contrary holds when in (3) the '>' symbol prevails. Thus, in order to achieve full employment, *it is necessary* that national income expenditure be complete, so that effective demand can reach a level which generates production equal to the entire potential income of the economic system. This is the well-known Keynesian principle of effective demand.

So much for the *static* version of the model. Nonetheless, there is no reason to suppose that the data set of this model, that is population, $\bar{q}_N$, labour coefficients, ℓ_c, and consumption coefficients, c_c, remain constant as time passes. Let us suppose that population grows at a proportional rate, g, that is

$$\bar{q}_N(t) = \bar{q}_N(0) e^{gt}. \tag{8}$$

Moreover, let us suppose that labour productivity increases in each sector such that the labour coefficient of each commodity decreases at a rate, ρ_c, specific to the corresponding sector:

$$\ell(t) = \ell(0)e^{-\rho_c t}, \quad c = 1, 2, \ldots, C. \tag{9}$$

Finally, let us suppose that the per capita consumption of each commodity increases at a rate, r_c, specific to each commodity:

$$c_c(t) = c(0)e^{r_c t}, \quad c = 1, 2, \ldots, C. \tag{10}$$

While it is reasonable to expect that the labour coefficients always decrease, the time profile of consumption coefficients is more complex: they normally increase and, from a certain point onwards, remain constant; yet, there are commodities whose demand decreases from a certain point onwards (for example, VHS recorders, which have been replaced by DVD recorders; typewriters by PC, etc.).

By substituting (4), (8), (9) and (10) into the equations of the model, (1) and (2), we obtain a solution that emphasizes a *structural dynamics of quantities*:

$$q_c(t) = c_c(t)\bar{q}_N(t) = c(0)\bar{q}_N(0)e^{(r_c+g)t}, \quad c = 1, 2, \ldots, C$$

(the total output of commodity c increases at the rate of $r_c + g$, different from commodity to commodity), and a *structural dynamics of prices*:

$$p_c(t) = w(t)\ell_c(t) = w(t)e^{-\rho_c t}, \quad c = 1, 2, \ldots, C; \tag{11}$$

(for further details see Pasinetti (1993, ch. IV, §§ 6 and 7); we will return to the structural dynamics of prices in the following sections).

The introduction of dynamics also entails a series of consequences involving employment. Given the multisectoral nature of this framework, we can study the problem either at the sectoral level or at the macroeconomic level. Let us begin at a sectoral level. To isolate this aspect, let us suppose that at the macroeconomic level condition (3) is kept satisfied. At the sectoral level, we can calculate that the *proportion* of employment required in each sector c changes continuously according to equation $\ell_c(t)q_c(t)/\bar{q}_N(t) = \ell_c(t)c_c(t) = \ell(0)c(0)e^{(r_c-\rho_c)t}$: the share of total labourers employed in sector c increases (decreases) if the rate of change in per capita demand for commodity c is higher (lower) than the rate of change in the labour productivity of that sector. Thus structural change requires a continuous *intersectoral mobility* of labour. This phenomenon is less pronounced if we observe the *absolute level* of employment required in each sector c, $E_c(t) = \ell_c(t)q_c(t) = \ell_c(0)c_c(0)\bar{q}_N(0)e^{(r_c+g-\rho_c)t}$: the level of employment in sector c increases (decreases) if the rate of change in total demand for commodity c, that is, $r_c + g$, is higher (lower) than the rate of change in labour productivity, ρ_c. This result describes 'one of the most alarming phenomena of modern industrial systems: the inevitable decline of employment in certain production sectors, as a result of the process of economic development' (Pasinetti, 1993, p. 53).

All this is obtained under the assumption that macroeconomic condition (3) is satisfied. However, this cannot be taken for granted. On the contrary, the main effect of technical change is to reduce the coefficients ℓ_c and, by consequence, the addenda of the sum on its l.h.s.. Hence, consumption coefficients *have to increase* in order to keep condition (3) satisfied. The increase in consumption is thus not only a *possibility*, but it represents a *necessity* in order to preserve full employment. But we can go further. From Engel's law, we know that when income increases, the consumption of the various commodities increases within a given interval of income (except for inferior commodities, whose consumption decreases with respect to income), but after a certain level we observe saturation. Therefore, we cannot go on supposing (as in equation (10)) that the r_cs remain constant as time goes by (they typically will decrease; sometimes they can even become negative). Similar considerations can be made for the rates of change of productivity, ρ_c, and for the rate of growth of population, g. Here, for the sake of simplicity, we will continue to treat the r_cs, ρ_cs and g as if they were constant; but we must keep in mind the provisional character of such a simplification.[2] For example, with regard to the macroeconomic aspect of unemployment, the decrease of the rates of change of per capita consumption is normally not compensated by those of labour coefficients. Hence, Pasinetti argues that the movements of c_c sooner or later will not be sufficient to counter the decreases in labour coefficients; thus there is a *tendency* for the macroeconomic condition to become satisfied with the '<' symbol. Hence, according to this framework, a growing process characterized by structural change *tends to generate unemployment*. Even if at a certain date we have full employment, there is no guarantee that it will be maintained in subsequent periods.

Besides showing the forces that induce unemployment, macroeconomic condition (3) permits us to envisage, at the same time, the forces that can act, and the remedies that can be introduced against unemployment. To see this it is convenient to reformulate the model by taking into account that there is a fraction of the working population that does not contribute for various reasons to productive activity, even if it contributes to consumption. Therefore, let us indicate by $\mu(t)$ the fraction of active population over total population. Similarly, we suppose that workers devote only a fraction $\nu(t)$ of their available time to work. The ensuing relation between the amount of labour force, $q_N(t)$, and population size, $N(t)$, will be

$$q_N(t) = \mu(t)\nu(t)N(t), \quad \text{that is,} \quad N(t) = \frac{q_N(t)}{\mu(t)\nu(t)}. \tag{12}$$

The quantity and the price systems must be reformulated accordingly. In the quantity system, final demand is carried out by the entire population; hence equations (1a) become

$$q_c(t) = c_c(t)N(t) \quad c = 1, 2, \cdots, C,$$

while equation (1b) remains unchanged. Thanks to (12), the quantity system can be rewritten as

$$\begin{aligned} q_c(t) &= \frac{c_c(t)}{\mu(t)\nu(t)} q_N(t) \quad c = 1, 2, \cdots, C, \\ \ell_1(t)q_1(t) &+ \ell_2(t)q_2(t) + \cdots + \ell_C(t)q_C(t) = q_N(t). \end{aligned}$$

In the price system, equations (2a) remain unchanged, while equation (2b) becomes

$$p_1(t)c_1(t)N(t) + p_2(t)c_2(t)N(t) + \cdots p_C(t)c_C(t)N(t) = wq_N(t):$$

consumption is proportional to the entire population, while wages are proportional to the labour force. The price system, thanks to (12), may thus be rewritten as

$$\begin{aligned} &p_c(t) = w(t)\ell_c(t), \quad c = 1, 2, \cdots, C \\ &p_1(t)\frac{c_1(t)}{\mu(t)\nu(t)} + p_2(t)\frac{c_2(t)}{\mu(t)\nu(t)} + \cdots p_C(t)\frac{c_C(t)}{\mu(t)\nu(t)} = w. \end{aligned}$$

Consequently, the condition to exclude the trivial solution is

$$\frac{1}{\mu(t)\nu(t)} \sum_{c=1}^{C} c_c(0)\ell_c(0)e^{(r_c - \rho_c)t} = 1. \tag{13}$$

The macroeconomic condition, written in form (13), gives us the possibility of evaluating alternative ways to counter unemployment. Alongside the increase of individual consumptions, exports are a good way of providing an economy with new markets for its products, but we should consider, at the same time, the opposite effects of imports. Another way to counter unemployment is the increase in the number of new commodities. This might be easily inserted into the present framework by supposing that the number of commodities is a function of time, $C(t)$: even if each addendum in the l.h.s. tends to decrease, the sum can be increased thanks to the larger number of addenda. Other remedies to offset unemployment include a reduction of parameters $\mu(t)$ or í(t), by increasing the number of part-time workers, or of retirements, or by reducing the working time. It should be said that these elements are not mutually exclusive: on the contrary, they must be considered together. The task of the institutions is that of choosing the appropriate mix, a duty that arises in a different way, from time to time, according to the changes undergone by the economic system. Exactly the opposite of *laissez faire*!

STRUCTURAL DYNAMICS OF PRICES

Physical numéraire

Adoption of a physical numéraire

After this digression on the basic structure of the model, and on the normative environment where we are operating, we now focus our attention on the possibilities that open up on the choice of the numéraire of the price system. The solution adopted in the previous Section closes the price system by adopting a *physical* numéraire. For example, with equation (5), the prices of commodities and the wage rate are thus expressed in terms of commodity h. From the formal point of view, the price system could be closed by adopting *any* of the commodities of the system, as well as a bundle of commodities, as in equation (6), or 1 unit of labour, as in equation (7). We say that in this case we adopt a *physical* numéraire, because the unit of account is represented by one of the commodities (or service) produced (or supplied) in the system. Physical numéraires have often been adopted in the past, when the unit of account was represented by a precious metal (for example: gold). More recently, for various reasons, the tendency to adopt 'nominal' numerarires, that is, goods that have a conventional but not an intrinsic value, and are no longer connected with any real basis, has become more and more prevalent. In this section, we will explore briefly the consequences of both alternative choices. Let us start from physical numéraires.

Given the dynamic context, it will be convenient to re-express equation (5) in an equivalent, but apparently more complicated, way (this will allow us to analyse and compare two types of numéraires—physical and nominal—within a unique framework). After substituting (9) into (2a) we obtain

$$p_c(t) = w(t)\ell(0)e^{-\rho_c t}. \qquad \text{(2a`)}$$

Choosing commodity *h* as numéraire, that is, setting $p_h(t) = w(t)\ell_h(0)e^{-\rho_h t} = 1$ leads to identifying a *univocal* path for the wage rate,

$$w(t) = \frac{1}{\ell_h(0)} e^{\rho_h t} : \tag{14}$$

the wage rate increases at the rate of change of productivity in industry *h*. The wage rate follows thus an exponential path,

$$w(t) = w(0) e^{\sigma_w t}, \tag{15a}$$

where

$$w(0) = \frac{1}{\ell_h(0)} \equiv \bar{w}_h(0) \tag{15b}$$

$$\sigma_w = \rho_h; \tag{15c}$$

and $w_h(0)$ is the wage rate expressed in terms of commodity *h* at $t = 0$.

The price of generic commodity *c*, expressed in terms of commodity *h*, varies according to the following rule:

$$p_c(t) = \frac{p_c(t)}{p_h(t)} = \frac{w(t)\ell_c(0)e^{-\rho_c t}}{w(t)\ell_h(0)e^{-\rho_h t}} = \frac{\ell_c(0)}{\ell_h(0)} e^{(\rho_h - \rho_c)t}, \quad c = 1, \ldots, C. \tag{16}$$

Commodity *c* rises (falls) with respect to commodity *h*, if the rate of growth of productivity in its industry is lower (higher) than the rate of growth of productivity in the industry which produces the commodity adopted as numéraire. This is quite an obvious result given the simple 'pure-labour' structure here considered: changes in prices reflect the change of the labour content of each commodity with respect to the labour content of the commodity adopted as numéraire.

Among physical numéraires, one particular choice is to express the prices of commodities in terms of commanded labour: this is the case of normalization (7). In this case, the dynamic path followed by the wage rate is described by a 'degenerate' exponential function,

$$w(t) = w(0) e^{\sigma_w t}, \tag{17a}$$

where

$$w(0) = 1 \tag{17b}$$

$$\sigma_w = 0, \tag{17c}$$

that is, a constant function. If prices are normalized by equation (7) or, equivalently, (17), the price of each commodity *decreases* at the rate of change of productivity of the corresponding industry:

$$p_c(t) = \ell_c(t) = \ell_c(0) e^{-\rho_c t}, \quad c = 1, \ldots, C.$$ [3]

The following analytical remark can be deduced from what has been seen thus far: the adoption of *any* (physical) numéraire can always be specified by imposing a specific dynamic path to the wage rate as in (15), when the numéraire is any commodity *h*, or like in (17), when the numéraire is commanded labour.

Stability of the average price level: the 'dynamic Standard commodity'

From equation (16) we see that the widespread and variegated technical progress entails a set of variations of *all* relative prices, as the labour content of each commodity decreases for each commodity, in a differentiated way from commodity to commodity. But the *directions* of these changes are not univocal. For example, let us rank sectors with respect to the rate of change of productivity, that is, let us number sectors in such a way that

$$\rho_1 > \rho_2 > \cdots > \rho_c > \ldots > \rho_C. \tag{18}$$

Thus, commodity '1' is produced in the most innovative (vertically integrated) sector, while commodity *C* is produced in the least (v. i.) innovative sector. If all prices were expressed in terms of commodity '1', *all relative prices would increase*, as the labour content of the numéraire decreases more rapidly than that of the commodities produced by all the other sectors. On the contrary, if all prices were expressed in terms of commodity *C*, *all relative prices would decrease*, for the opposite reason. Any 'intermediate' numéraire, *h*, between 1 and *C*, will make commodities $c < h$ fall and commodities $c > h$ rise.

Hence, in general, the adoption of a physical numéraire does *not* achieve the goal of price stability. This is a fairly obvious result for a system which is undergoing such differentiated technical progress. However, we could look for a suitable 'intermediate' numéraire which allows for the average of prices to remain *constant*. The latter reasoning suggests that as long as we remain at the 'extremes' of this ranking, we obtain the maximum variability in prices. But as we move from the extremes towards the center, the degree of variability is reduced. If we could find a commodity with a rate of change of productivity exactly equal to the 'mean between the extremes',[4] it would solve our problem: it would reduce the average price variability to zero; it would guarantee, in other words, the average stability of prices. Let us call this average rate 'Standard rate of change of productivity'. Obviously, a commodity having such a rate of change of productivity cannot be found in the real world. And even if it were to be found in a given period of time, the continuous change of technical coefficients and of the proportions among sectors would almost immediately make it lose its characteristic of ensuring average price stability.

But if such a commodity does not exist, we can build it analytically. In particular, as we are looking for a commodity to be adopted as a numéraire, we could take advantage of the analytical device for adopting a particular commodity as numéraire presented at the end of the previous subsection: if we impose that the wage rate changes at the Standard rate of change of productivity, prices are immediately expressed in terms of this composite commodity which, in analogy with Sraffa's Standard commodity, has been called by Pasinetti the 'dynamic Standard commodity'. We have thus to calculate the average rate of change of productivity, that is, we have to identify the weights to apply to the sectoral rates. By following Pasinetti, these weights are constituted by the addenda of macroeconomic condition (13). Hence, the Standard rate of change of productivity is given by

$$\sum_{c=1}^{C} \rho_c \cdot \frac{c_c(t)\ell_c(t)}{\mu(t)\nu(t)} = \rho^*(t). \tag{19}$$

In this way, simply by setting

$$w(t) = w(0)e^{\sigma_w t}, \tag{20a}$$

with

$$w(0) = \bar{w}^*(0) \tag{20b}$$

$$\sigma_w = \rho^*(t), \tag{20c}$$

the prices of the various commodities and the wage rate are expressed in terms of the dynamic Standard commodity.[5]

Once prices and the wage rate are expressed in terms of the dynamic Standard commodity, the price of the generic commodity c, $p_c(t) = w(t)\ell_c(t) = \bar{w}^*(0)\ell_c(0)e^{[\rho^*(t)-\rho_c]t}$, changes at the rate of

$$\sigma_c = \rho^*(t) - \rho_c, \quad c = 1, 2, \ldots, C. \tag{21}$$

Equations (20c) and (21) thus identify the rates of change of the wage rate and of the prices of the various commodities when they are expressed in terms of the dynamic Standard commodity. Approximately half of the prices decrease (those referring to commodities with a rate of change of productivity higher than $\rho^*(t)$), and the other half increase (those which refer to commodities with a rate of change of productivity lower than $\rho^*(t)$). Thus, on average, they will remain precisely constant.[6] Yet, one could argue that the economic meaning of this constancy is doubtful: it is ascertained with respect to a numéraire whose physical composition *changes* as time goes by.[7] An attempt to overcome this objection has been put forward by

Garbellini (2010, pp. 123-8), who built the dynamic Standard commodity by keeping constant the nominal value of a consumption basket which is updated from period to period, according to the evolution of coefficients $c_c(t)$. Furthermore, it must be recalled that the entire construction of the dynamic Standard commodity finds its ultimate reason for being in the identification of the Standard rate of increase of productivity, which will be employed, as we will soon see, in separating the real from the nominal component of the inflation rate.

General Instability of the average price level.

What happens to the general level of prices if any other physical numéraire, *h*, is adopted as a numéraire instead of the dynamic Standard commodity? To investigate this point, let us accordingly impose condition (15), that is, the wage rate changes at the rate of ρ_h, which differs from $\rho^*(t)$ which is, by definition, that rate of change of the wage rate which would keep the average price level constant. The general price level changes, on average, at the rate given by the difference,[8]

$$\rho_h - \rho^*(t),$$

(which can be either positive or negative). This difference is the rate of price inflation (or deflation) due to structural change: the movement of all prices caused by the fact that the labour content of the numéraire adopted changes at a rate which is different with respect to the average. The general price level will thus be affected to the extent that the rate of change of productivity of the industry that produces the numéraire is distant from the average one. Pasinetti calls this 'inflation from structural dynamics' (1993, ch. V, § 9). In general, the difference[9] $\sigma_w - \rho^*(t)$ 'precisely quantifies the deviation of the general movement of prices—associated with any physical numéraire *h* [...]—from the norm of price stability' (Pasinetti, 1993, p. 75).

Nominal numéraires

Adoption of a nominal numéraire

The dynamic Standard commodity is the unique physical standard which guarantees a stable general price level. But, as we noticed, its composition changes as time goes by! On the other hand, all the other physical numéraires—we have just seen—do not guarantee the stability of the general level of prices, as we have seen at the end of the previous subsection. It is probably for all these reasons that physical numéraires have gradually been abandoned by actual economic systems, and have been replaced in more recent times by nominal numéraires, as the inconvertible fiduciary currencies (paper money) adopted at present (Euro, Pound, Yen, etc.).[10]

How does one introduce a nominal numéraire into our price equations? To this purpose, it is useful to analyse the consequences of the adoption of a nominal unit of account on the price of any commodity or on the wage rate. Let us begin by a generic commodity c chosen at will. To set at 1 the value of a nominal unit of account in each period entails that in each period the price of commodity c assumes a given specific value (different, in general, from period to period). In other words, the dynamic path of $p_c(t)$ remains determined univocally. The conventionality implicit in the choice of the numéraire reflects on a given specific dynamic path of $p_c(t)$, which is completely unconnected to any other real element of the system. It can be any path. The only simplification introduced here is that it is described by an exponential function.

$$p_c(t) = p_c(0)e^{\sigma_c t}, \tag{22a}$$

where

$$p_c(0) = \bar{p}_c^{(M)}(0) \tag{22b}$$

$$\sigma_c = \bar{\sigma}_c : \tag{22c}$$

$\bar{p}_c^{(M)}(0)$ is the level assumed by the price of commodity c at time 0 when expressed in terms of the nominal unit of account and $\bar{\sigma}_c$ is its rate of change 'induced' by this numéraire. The prices of the other commodities will vary with respect to commodity c in such a way as to reconstruct at each period the structure of relative prices determined by the price equations.

If, instead of looking at the consequences of fixing at 1 the value of a nominal unit of account on the price of commodity h, we look at the consequences on the wage rate, we can deduce that it is the dynamics of the wage rate to be identified univocally as follows:

$$w(t) = w(0)e^{\sigma_w t}, \tag{23a}$$

where

$$w(0) = \bar{w}^{(M)}(0) \tag{23b}$$

$$\sigma_w = \bar{\sigma}_w : \tag{23c}$$

$\bar{w}^{(M)}(0)$ is the level assumed by the wage rate at time 0 when expressed in terms of the nominal unit of account and $\bar{\sigma}_w$ is its rate of change induced by the nominal numéraire chosen.

The analytical difference between physical and nominal numéraires can be grasped if we compare the ensuing dynamics of the wage rate or if we compare the dynamics of a generic price. Let us start from the wage rate and compare equations (15) with equations (23).

Once a physical commodity *h* is adopted as numéraire the wage rate at $t = 0$ is expressed in terms of the labour-value of commodity *h* at $t = 0$ *and* the rate of change of productivity *of the same* industry, ρ_h, regulates the dynamics of the wage rate (see equation (15c)). On the contrary, the adoption of a nominal numéraire entails a wage rate at $t = 0$ fixed at a specific given value and a rate of change of this wage rate, $\bar{\sigma}_w$, that is not pegged to any real element. An analogous difference emerges if we compare the dynamics of the price of a generic commodity *c* ensuing from the adoption of a physical numéraire, *h*, or of a nominal numéraire. The price of a generic commodity, *c*, expressed in terms of commodity *h* (a physical numéraire) is described by the exponential function (16) which can also be written as

$$p_c(t) = p_c(0)e^{\sigma_c t}, \tag{24a}$$

Where

$$p_c(0) = \frac{\ell_c(0)}{\ell_h(0)} \tag{24b}$$

$$\sigma_c = \rho_h - \rho_c. \tag{24c}$$

Compare now equations (24) with (22). We see, again, that a physical numéraire pegs the levels *and* the dynamics of prices to *real* elements: the ratio of labour contents (see (24b)) *and their* temporal evolution (see (24c)). The nominal numéraire sets the initial price level at a conventional level (equation (22b)) and its temporal evolution at a rate of change totally unconnected with the real elements of the corresponding industry: in equation (22c) $\bar{\sigma}_c$ has no links with the rates of change of productivity.

This 'de-coupling' between the initial price level and its temporal evolution is what Pasinetti considers the second degree of freedom provided by the adoption of nominal numéraires, the first being the level to which to peg the level of prices in the initial period (see Pasinetti, 1993, chap. V, in particular § 3). Obviously, this arbitrariness in $\bar{\sigma}_w$ or in $\bar{\sigma}_c$ concerns nominal magnitudes, the wage rate or the price of commodity *c* expressed in terms of the nominal unit of account: it has no real consequences, as the price of the various commodities are determined by the price equations (11).

Again, general instability of the average price level.

By using the same reasoning as in the case of a physical numéraire, when prices and the wage rate are expressed in terms of a nominal numéraire the average price level changes at the rate

$$\sigma_M = \bar{\sigma}_w - \rho^*(t). \tag{25}$$

One of these magnitudes, $\rho^*(t)$, is a real magnitude: as known, it is that rate of change of wages that keeps the general price level constant. On the contrary, σ_w is the rate of change of a magnitude (the wage rate) expressed in a purely *nominal* unit of account. As such it has no limit: it can reach whichever level, in relation to the value characteristics of the chosen nominal unit of account. σ_M is thus the rate of *monetary inflation*. Contrary to the case of a physical numéraire—where the maximum rate of inflation due to structural change can be $\rho_1 - \rho^*(t)$, and the maximum rate of deflation can be $\rho^*(t) - \rho_M$, according to the physical numéraire chosen—the rate of change σ_M is no longer subject to any physical constraint. If, on the one hand, nominal numéraires have been introduced to pursue the goal of price stability, which physical numéraires fail to obtain, they do not prevent in principle the possibility of hyper-inflation or of hyper-deflation.

Structural and monetary inflation

According to the chosen numéraire we have seen that the average price level changes at the rate of $\sigma_w - \underset{*}{\rho}$. More precisely,

$$\sigma_w - \rho = \begin{cases} \rho_h - \rho^*(t) & \text{with a } \textit{physical} \text{ numéraire} \\ \bar{\sigma}_w - \rho^*(t) & \text{with a } \textit{nominal} \text{ numéraire.} \end{cases}$$

On the basis of each price equation (11), the price of each single commodity c varies thus at the rate of $\sigma_c = \sigma_w - \rho_c$. It is however possible to decompose this difference in two addenda:

$$\sigma_c = [\sigma_w - \rho^*(t)] + [\rho^*(t) - \rho_c].$$

In particular, the rate of change of the price of commodity c expressed in terms of commodity h is

$$\sigma_c = \begin{cases} [\rho_h - \rho^*(t)] + [\rho^*(t) - \rho_c], & \text{with a } \textit{physical} \text{ numéraire} \\ [\bar{\sigma}_w - \rho^*(t)] + [\rho^*(t) - \rho_c] & \text{with a } \textit{nominal} \text{ numéraire.} \end{cases}$$

In both these expressions, the first parenthesis corresponds to the rate of inflation of the *general* price level (due to structural or to monetary reasons), while the second parenthesis reflects a *structural* component.

A 'NATURAL RATE OF INTEREST'

We can come now to the central argument of our reconstruction: the introduction of debt/credit relations in the structural change framework, and the identification of a normative reference level for the rate of interest.

Thus, we develop the inquiry within the 'natural' configuration of the system, where the macro-economic condition (13) is satisfied. In principle, this condition excludes that savings or dissavings can occur at the macro-economic level, as condition (13) entails that all incomes are entirely spent. But this does not exclude that debt/credit relations may occurs at the level of single persons. Someone may spend more than his income if someone else is willing to lend him the corresponding purchasing power. Thus, also in a natural configuration a whole set of debt/credit relations may take place: the sole condition is that they cancel out overall. A question may thus arise: at what rate of interest (if any) 'must' they take place? Remember that in the natural system national income is distributed to workers in proportion to the quantity of labour provided by each worker. It holds, as we have said in the Introduction, the 'labour principle' of income distribution. Hence, in coherence with it, we have to find that rate(s) of interest which does not alter this principle.

Suppose, to begin with, that debts and credits relations are denominated in terms of a specific commodity, say, commodity h. If one person lends (the equivalent of) one unit of commodity h and receives, after one period, (the equivalent of) one unit of commodity h, he receives a commodity that incorporates a quantity of labour *lower* than what was incorporated in the unit of commodity which was lent. This would be as if the lender had lent $\ell_h(t)$ units of labour at time t and received $\ell_h(t+1) < \ell_h(t)$ units of labour one period later. Due to technical progress which occurred in the vertically integrated sector of commodity h, the lender would be paid back by an amount of labour lower than the ρ_c percent with respect to the amount of labour lent. In this case, the loan would entail a 're-distribution' of income, that is, a deviation from the labour principle of income distribution. This principle would be re-established if interest were calculated on the loan exactly at the rate of ρ_c percent. In this case, the 'natural' rate of interest, that is, the rate that re-establishes a distribution of income based on the labour-principle of income distribution, is[11]

$$i_N = \rho_h. \tag{26}$$

Alternatively, if the numéraire adopted to stipulate all debt/credit relations is the dynamic Standard commodity, an analogous reasoning brings us to deduce that the natural rate of interest is

$$i_N = \rho^*(t) \tag{27}$$

the dynamic Standard commodity incorporates a labour content which decreases, on average, at the rate of $\rho^*(t)$; the natural rate of interest exactly offsets this depreciation.

Evidently, only in the case where all prices are expressed in terms of labour commanded, no interest should be charged on credit and debt relations. The ensuing natural rate of interest becomes in this case

$$i_N = 0. \tag{28}$$

Consider, lastly, the situation, typical nowadays, of prices expressed in terms of a nominal unit of account. By (23) we see that wages, expressed in terms of the nominal unit of account, increase at the rate of $\bar{\sigma}_w$. Specularly, the nominal unit of account depreciates at the rate of $\bar{\sigma}_w$ in terms of commanded labour; that is, each unit of nominal account commands a quantity of labour which decreases at the rate of $\bar{\sigma}_w$. Thus, an interest of $\bar{\sigma}_w$ percent must be recognized for each nominal unit of account, lent or borrowed, in order to offset its depreciation. Hence, the natural rate of interest is $\bar{\sigma}_w$ which, thanks to (25), can be written as

$$i_N = \bar{\sigma}_w = \rho^*(t) + \sigma_M. \tag{29}$$

With a nominal unit of account, the natural rate of interest comes to include a monetary component (σ_M) alongside a real component ($\rho^*(t)$).

It is to be recalled that all the expressions found for the natural rate of interest are equivalent among themselves in *real* terms: all have in fact been conceived in such a way as to restore the 'labour principle' of income distribution in each of the debt/credit relations in which they are involved. Another perspective to see this point is to observe that, thanks to equations (15c), (20c) and (17c), all the expressions found for the natural rate of interest (26), (27) and (28) can be re-expressed by the *same* equation, which coincides with (29):

$$i_N = \sigma_w,$$

where σ_w indicates the rate of change of the wage rate expressed in whichever numéraire is chosen to express prices, wages and debt and credit relations.

CONCLUDING REMARKS

Two elements must be highlighted at the end of this presentation of Pasinetti's notion of the natural rate of interest. First, its *normative* nature must be stressed. No automatic mechanism ensures the establishment of such a particular configuration for the rate of interest. It is merely an ideal configuration, which does not allow for financial relations to have real effects on income distribution. In other words, it is the rate that prevents to enrich/impoverish debtors and creditors through financial relations. It constitutes a benchmark level to which compare actual levels of rates of interest. Secondly, the perspective here followed provides a distinction that traditional analysis, both classical and neoclassical, has very rarely caught: the distinction between the rate of interest and the rate of profit. The rate of interest has emerged here in a framework of pure labour, that is, in a system *without capital*! It is thus something that is autonomous and independent of the notions of capital and profit rate. In a capitalist system, there are probably forces that tend to put one of the rates in relation with the other one. Nonetheless, conceptually they are two different variables. The rate of interest is a variable that dates very far back (it is even mentioned in the Bible!): since people first established debt and credit relations. The rate of profit, on the other hand, has appeared on economic scene quite recently: about three centuries ago, with the advent of capitalism. A simple inspection of their formulae shows how their 'natural' determinants are quite different. For the rate of interest, let us take our last definition, obtained for the case of debt and credit relations stipulated in terms of a nominal numéraire:

$$i^* = \rho^*(t) + \sigma_M.$$

Once a numéraire is given, a *unique* natural rate of interest is obtained. On the other hand, the natural rates of profit are defined by Pasinetti, for systems with capital goods, as those rates that allow each vertically integrated sector to expand at the rate of change of the corresponding final demand:

$$\pi_c^* = g + r_c, \quad c = 1, 2, \ldots, C$$

(see Pasinetti, 1981b, chap. VII, §§ 3 and 4). Therefore, we have as many natural rates of profit as there are final commodities. Accordingly, the rate of interest and the rates of profit must be regulated differently, on the basis of the respective purposes. In Pasinetti's view, the natural rate of interest prevents income redistributions arising from debt/credit relations, while the natural rates of profit provide the resources necessary to finance the growth of the means of production of the various sectors according to the evolution of the final demands of each specific commodity. The comparison reveals the different conceptual nature of financial transactions for industries from those among families.

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NOTES

1 This is a very restrictive assumption. Yet, Pasinetti provides an argument to consider this particular situation as representative of the more general situation where commodities are produced by commodities and labour. The main part of his analysis is, in fact, devoted to studying an ideal configuration of the system, which, if realized, accomplishes the potential of the system concerning growth, employment and the satisfaction of economic human wants. The accomplishment of such a configuration, which he calls the 'natural system', is obviously not automatic in a free competitive system (think of the phenomenon of unemployment which we normally observe, especially in advanced economies). It should be seen a reference point for a normative analysis. Among the conditions that characterize the natural system, Pasinetti identifies an ideal configuration of income distribution, where profits are set exactly at that level that guarantees the expansion of each sector at the rate of change of the final demand of the corresponding commodity, while wages take the residuum. It is possible to prove that in such a situation, the price of each commodity is *proportional* to its vertically 'hyperintegrated' labour coefficient; that is, the quantity of labour necessary to produce the commodity, to replace its means of production and to expand them at the rate of change of the final demand of the commodity. This 'reduction' of prices to labour quantities only allows us to assimilate more general cases to that of a pure labour economy. For details see Pasinetti (1981b, ch. VII, §§ 3 and 4) and (1988).

2 Pasinetti gives the analytics to handle the case of variation of the rates of change of c_c, r_c and g: see Pasinetti (1981b, chap. V, § 1) and (1993, chap.IV, § 4).

3 Correspondingly, the wage rate expressed in terms of *each* commodity increases at the rate of change of productivity in that industry:

$$\frac{w(t)}{p_h(t)} = \frac{1}{\ell_c(t)} = \frac{1}{\ell_c(0)} e^{\rho_c t}, \quad c = 1, \ldots, C.$$

4 The expression in quotes is taken from Ricardo (1923, p. 405).

5 The term $\bar{w}^*(0)$ in eqution (20b) stands for the wage rate expressed in terms of the dynamic Standard commodity. Here, it appears as an arbitrary given number. In fact, similarly to the (static) Standard commodity, also the dynamic Standard commodity is defined by its *proportions*, not by its dimension. Hence, the magnitude of term $\bar{w}^*(0)$ will depend on the scale adopted for the dynamic Standard commodity. Since we do not build the dynamic Standard commodity explicitly here, this term will remain unspecified.

6 The analogy with Sraffa's Standard commodity can be carried over. Technical progress entails a continuous change of the labour content of each commodity, i.e. their value. Hence, when used as a numéraire, the commodity just obtained, whose labour content changes at the average rate of the whole system, allows

us to identify how the labour content (the value) of each commodity changes with respect to what happens to the entire system. In other words, the dynamic Standard commodity constitutes an *invariable measure of value* with respect to changes in technology.

7 I owe this observation to an anonymous referee.

8 Observe, in fact, that when prices are expressed in terms of commodity *h*, their expression is $p_c(t) = \bar{w}_h(0)e^{\rho_h t}\ell_c(t)$, $c = 1, 2, \ldots C$, and the general price level is

$$\sum_{c=1}^{C} p_c(t)c_c(t) = \sum_{c=1}^{C} \bar{w}_h(0)e^{\rho_h t}\ell_c(t)c_c(t) =$$

$$\sum_{c=1}^{C} \bar{w}_h(0)e^{[\rho_h-\rho^*(t)+\rho^*(t)]t}\ell_c(t)c_c(t) = e^{[\rho_h-\rho^*(t)]t}\frac{\bar{w}_h(0)}{\bar{w}^*(0)}\sum_{c=1}^{C}\left[\bar{w}^*(0)e^{\rho^*(t)t}\ell_c(t)\right]c_c(t),$$

where the terms in square brackets are the prices of the various commodities expressed in terms of the dynamic Standard commodity. The sum is thus the average level of prices with prices expressed in terms of the dynamic Standard commodity. Hence, it is constant. So, our original expression, that is, the general level of prices expressed in terms of a generic commodity *h* reduces to

$$\sum_{c=1}^{C} p_c(t)c_c(t) = e^{[\rho_h-\rho^*(t)]t}\frac{\bar{w}_h(0)}{\bar{w}^*(0)}\cdot K,$$

which varies at the rate of $\rho_h - \rho^*(t)$.

9 The reasoning of footnote 8 above can here be repeated with σ_w instead of ρ_h.

10 The passage from metallic currency to inconvertible paper money has gradually taken place for most currencies over the last century. One exception is the Dollar, which remained convertible to gold for a longer period; besides, the majority of other currencies were linked to the Dollar, guaranteing a physical pegging of all those currencies. In 1971, however, the Dollar was declared inconvertible, thus uncoupling all currencies from a real basis.

11 In the example just provided, we have implicitly considered the rate of decrease of labour content as an *annual* rate. If, in analogy with the entire analysis developed in the previous sections, we were to consider *instantaneous* rates, the quantity of labour to produce 1 unit of commodity *m* reduces by $\ell_h(0)e^{-\rho_c t} - \ell_h(0)e^{-\rho_c(t+1)}$ in one period. Thus, the interest factor which would re-constitute the original quantity of labour is $e^{-\rho_c t}/e^{-\rho_c(t+1)} = e^{\rho_c}$ which, for sufficiently small rates ρ_cs, can be approximated by $1 + \rho_c$.

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