International Review of Pure and Applied Mathematics
Volume 15 • Number 1 • Jan-June 2019

A STUDY OF *N*EAR-RINGS AND IT'S INVOLVING THEOREMS

[1]*Ms. Jeba Presitha B.*& [2] *Ms. Ramya S.*

Abstract: The purpose of the present paper is to prove some theorems for near-rings with unity. *N*ear-ring is an algebraic structure similar to a ring but satisfying fewer axioms. Every ring is a near-ring, hence near-rings are generalized rings. Two ring axioms are missing: The commutative of addition and other distributive law. The mapping of a group into itself under pointwise addition and composition of maps. The study and research on near-rings is systematic and also it's continuous.

1. INTRODUCTION

Let us start with a brief introduction to what *N*ear-Ring is. It is a generalized ring: addition has not to be commutative only one distributive law is postulated. In Mathematics *N*ear-Ring is algebraic structure similar to the ring but satisfying some particular properties. Sets and binary operations (functions) are the fundamental and main ingredients of algebraic systems. Its arise from functions on groups.

A Near-Ring is a set N together with two binary operations addition and multiplication such that

i) N is a Group under addition (not necessarily abelian)

ii) N is a Semi group under multiplication

iii) $\forall n_1, n_2, n_3 \in$ N

$(n_1 + n_2).\, n_3 = n_1.\, n_3 + n_2.\, n_3$ Right Distributive law.

If (N,+) is abelian we call N is an abelian near-ring. If (N,.) is commutative we call N itself a commutative near-ring.

If N contains a multiplicative semi group M whose elements generate N^+ and satisfy

iv) $(n_1 + n_2).\, m = n_1.\, m + n_2.\, m \quad \forall$ m $\in M$. We say that R is a distributively generated (d.g.) near-ring.

Example

1. The polynomials R[x] C R commutative ring with identity under addition and substitution.

2. An arbitrary additively written group with zero multiplication.

2. BASIC DEFINITION

2.1 Definition:

$N_0 = \{n \in N/\ n0 = 0\}$is called the *Zero-symmetric* part of *N* and $N_c = \{n \in N/n0 = n\}$

Or $N_c = \{n \in N/ \forall n' \in N : nn' = n\}$is called the *Constant part* of *N.*

2.2 Definition:

Let *N,* N' be ∈ n

h*: N*→ N' is called a (near-ring) *homomorphism* if ∀ m, n ∈ *N* :

h(m+n)=h(m)+h(n) Λ h(mn)=h(m).h(n).

2.3 Definition:

A normal subgroup I of (*N*,+) is called *ideal* of *N. (I* ⊴ *N*)

If a) *IN*⊆ *I*

b) $\forall n, n' \in N\ \forall i \in I : n(n' + i)\text{-}nn' \in I$.

2.4 Definition:

A near-ring N is said to be weak commutative if $n_1n_2n_3 = n_1n_3n_2$ for every n_1, n_2, n_3 in N.

2.5 Definition:

N is sub directly irreducible if and only if the intersection of non-zero ideals of *N* is non-zero.

2.6 Definition:

N is integral if *N* has no non-zero divisor of zero. If $(N^* = N\{0\},\ .\ )$ is a group, *N* is called a *near-field*. The set of all non-zero elements of *N* is a group under multiplication. A near-ring with the property that N_d generates (*N*,+) is called *distributively generated near-ring.(d.g)*

2.6 Definition:

Let *N* be a near-ring and a,b ∈ *N*. a ≡ b ⇔∀ n ∈ *N* :na = nb. The number of equivalence classes with respect to this equivalence relation will be denoted as | *N* |. If *N* has an identity element, then clearly a ≡ b ⇔ a = b and consequently, for every non-zero element n ∈ *N* we have n ≢ 0 that is, *Nn*≠ {0}. As usual, (0 : z) :={x∈ *N* | xz =0 } is the (left) annihilator of an element z ∈ *N.*

2.7 Definition:

Let L := {n ∈ *N* | *N*n ≠ *N*}. A near-ring *N* with identity is said to be local if L is an *N-subgroup.*

2.8 Definition:

A sub near-ring *M* of *N* is called *invariant* if *MN* ⊆ *M* and *NM* ⊆ *M*.

3. MAIN SECTION:

Lemma 3.1.

Let *N* a d.g. near-ring such that (*N*,+) is abelian.

Then *N* is a ring. S.ligh in [2] show that:

Lemma 3.2.

Let *N* be a finite near-ring with identity 1 such that (-1) n = n implies n = 0. Then (*N*, +) is abelian.

Theorem 3.3

Let *N* be a d.g. near-ring with identity 1 such that (*N*, +) is abelian and $(n_1\ n_2\ )^2$ $=n_1{}^2.n_2{}^2$,. $\forall n_1, n_2 \in N$ Then *N* is commutative.

Proof:

By 3.1,

N is a ring. Suppose $n_1 \in$N, y∈ N. Then $[n_1(n_2+1)]^2 = n_1{}^2\ (n_2+\ 1)^2$
$= n_1{}^2(n_2+1)(n_2+1) = n_1{}^2(n_2{}^2+n_2+n_2+1) = n_1{}^2(n_2{}^2+2n_2+1)$
$= n_1{}^2 n_2{}^2 +\ 2n_2\ 2n_2 + n_1{}^2$ (3.1)

But also

$[n_1(n_2+1)]^2 = (n_1 n_2 + n_1)(n_1 n_2 + n_1)$
$=(n_1 n_2)^2 + n_1 n_2 n_1 + n_1(n_1 n_2) + n_1{}^2$
$=(n_1)^2(n_2)^2 +\ n_1 n_2 n_1 + n_1{}^2 n_2 + n_1{}^2$ (3.2)

By (3.1),(3.2) we have $n_1 n_2 n_1 + n_1{}^2 n_2 = 2n_1{}^2 n_2$ this becomes$n_2 n_1 - n_1 n_2$ and the theorem is proved.

Theorem 3.4.

Let *N* be a finite near-ring with identity 1 such that (-1) $n_1 = n_1$ implies $n_1 = 0$ and $(n_1 n_2)^2 = (n_1{}^2 n_2)\ n_1 \quad \forall n_1, n_2 \in$N Then N is commutative.

Proof.

By 3.2,

(*N*;+) is a abelian thus by 3.1, R is a ring.

Suppose $n_1 \in N$; $n_2 \in N$

Now, repeating this argument for n_2+1 instead ofn_2 , we obtain

$[n_1(n_2+1)]^2 = [n_1(n_2+1)^2]\, n_1$ implies $n_1(n_1n_2) = (n_1n_2)n_1$

Now, repeating this argument n_1+1 instead of n_1,we obtain

$[(n_1+1)n_2]^2 = [(n_1+1){n_2}^2]\,(n_1+1)$ implies $n_1(n_1n_2) + n_1n_2 = (n_1n_2)n_1 + n_2n_1$.

By (3.1),(3.2) we have $n_1n_2 - n_2n_1$

And the theorem is proved.

Corollary 3.5.

Let *N* be a finite near-ring with identity 1 such that (-1(-1) $n_1 = n_1$ implies $n_1 = 0$ and $(n_1n_2)^2 = ({n_2}^2 n_1)\, n_1 \quad \forall n_1, n_2 \in N$ Then *N* is commutative.

3. 6 Definition

If n∈*N* is *idempot*ent if n^2 =n..

3.7 Definition

If n∈*N* is *nil*potent if n^r =0.

Lemma 3.8.

Every sub directly irreducible zero symmetric near-ring N without nonzero nilpotent is integral. Every non-zero idempotent is a right identity.

Lemma 3.9.

If *N* is a zero symmetric near-ring then for any ideal *I* of *N, NI* ⊆ *I* and hence *NIN* ⊆ *I.*

Proof:

n(n' + i) - nn' ∈ *I* for any i in *I* and n,n' in *N.*

Substituting n'=0 and using the hypothesis that *N* is zero-symmetric

we get *NI*⊆*I*. Also *I N* ⊆*I.* Hence *N I N* ⊆*IN*⊆*I.*

its proved.

Lemma 3.10.

If *N* is a sub commutative near-ring and $E \neq 0$ then idempotent are central.

Proof:

If *N* is a sub commutative near -ring

Then Na = aN for every a ∈ N.Let e ∈ E

Where E is the set of all idempotent of N as in Lemma 3.8.

Now Ne = eN implies for any n ∈ N ne = em and en = xe for some m, x in N.

ene = e(ne)=e(em)=em = ne (3.3)

and

ene =(en)e =(xe)e = xe = en (3.4)

Now equations (3.3) and (3.4) imply en = ne.

REFERENCE

[1] A. Ali, H.E. Bell, P. Miyan, Generalized derivations in rings, Int.J. Math. Math. Sci. (2013), Article ID 170749, 5 p.

[2] S. Ligh, A generalization of a theorem of Zassenhaus, Canad. Math. Bull, 12 (1069) 677- 678. http://dx.doi.org/10.4153/CMB-1969-089-2

[3] M. Sammana, L. Oukhtitea,1, A. Bouaba A study of near-rings with generalized derivations Journal of Taibah University for Science 9 (2015) 407–413.

[4] S.Uma $\propto_1,\propto_2$ Near-Rings ,Int,J of Algebra, Vol,4,2010,no.2,71-79.

[5] Gerhard Wendt, International Journal of Algebra, Vol. 3, 2009, no. 1, 21 – 32 On Zero Divisors in Near-Rings

[6] Henry E. Heartherly, Regular Near Rings, Journal of Indian Math. Soc. 38 (1974) 345 - 354

[1]**Ms. Jeba Presitha B.**
Assistant Professor,
Parks College, Chinnakarai,
Tirupur, Tamil Nadu.

[2]**Ms. Ramya S.**
Assistant Professor,
Parks College, Chinnakarai,
Tirupur, Tamil Nadu.