IJCTA, 9(1), 2016, pp. 279-297

A Highly Chaotic System with Four Quadratic Nonlinearities, its Analysis, Control and Synchronization via Integral Sliding Mode Control

Sundarapandian Vaidyanathan*

Abstract: First, this paper announces a nine-term novel 3-D chaotic system with four quadratic nonlinearities. The phase portraits of the novel chaotic system are displayed and the mathematical properties are discussed. We show that the novel chaotic system has three unstable equilibrium points. We show that the equilibrium point at the origin is a saddle-point, while the other two equilibrium points are saddle-foci. The Lyapunov exponents of the novel 3-D chaotic system are obtained as $L_1 = 10.45056$, $L_2 = 0$, and $L_3 = -42.36623$. The maximal Lyapunov exponent (MLE) for the novel chaotic system is obtained as $L_1 = 10.45056$. The large value of L_1 shows that the novel chaotic system is highly chaotic and exhibits highly complex behaviour. Also, the Lyapunov dimension of the novel chaotic system is derived as $D_L = 2.2467$. Next, we derive new results for the global chaos control of the novel highly chaotic system via integral sliding mode control (ISMC). We also derive new results for the global chaos synchronization design of the identical novel highly chaotic systems via integral sliding mode control (ISMC). The global chaos control and synchronization results for the novel highly chaotic system have been established using Lyapunov stability theory. In contrast with conventional sliding mode control (SMC), the system motion under integral sliding mode has a dimension equal to that of the state space. In ISMC, the system trajectory always starts from the sliding surface. Accordingly, the reaching phase is eliminated and robustness in the whole state space is promised. Numerical simulations with MATLAB have been shown to validate and demonstrate all the new results derived in this paper for the novel highly chaotic system using integral sliding mode control.

Keywords: Chaos, chaotic systems, chaos control, chaos synchronization, sliding manifold, integral sliding mode control, stability.

1. INTRODUCTION

A *chaotic system* is commonly defined as a nonlinear dissipative dynamical system that is highly sensitive to even small perturbations in its initial conditions [1]. In other words, a chaotic system is a nonlinear dynamical system with at least one positive Lyapunov exponent. Some paradigms of chaotic systems can be listed as Arneodo system [4], Sprott systems [5], Chen system [6], Lü-Chen system [7], Liu system [8], Cai system [9], Tigan system [10], etc.

In the last two decades, many new chaotic systems have been also discovered like Li system [11], Sundarapandian systems [12-13], Vaidyanathan systems [14-33], Pehlivan systems [34-35], Pham systems [36-37], Jafari system [38], etc.

Hyperchaotic systems are the chaotic systems with more than one positive Lyapunov exponent. They have important applications in control and communication engineering. Some recently discovered 4-D hyperchaotic systems are hyperchaotic Vaidyanathan systems [39-40], hyperchaotic Vaidyanathan-Azar system [41], etc. A 5-D hyperchaotic system with three positive Lyapunov exponents was also recently found [42].

Chaos theory has several applications in a variety of fields such as oscillators [43-44], chemical reactors [45-58], biology [59-80], ecology [81-82], neural networks [83-84], robotics [85-86], memristors [87-89], fuzzy systems [90-91], etc.

* Research and Development Centre, Vel Tech University, Avadi, Chennai-600062, INDIA

The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [92-93]. Some popular methods for chaos control are active control [94-98], adaptive control [99-100], sliding mode control [101-103], etc.

Chaos synchronization problem can be stated as follows. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

The synchronization of chaotic systems has applications in secure communications [104-107], cryptosystems [108-109], encryption [110-111], etc.

The chaos synchronization problem has been paid great attention in the literature and a variety of impressive approaches have been proposed. Since the pioneering work by Pecora and Carroll [112-113] for the chaos synchronization problem, many different methods have been proposed in the control literature such as active control method [114-132], adaptive control method [133-149], sampled-data feedback control method [150-151], time-delay feedback approach [152], backstepping method [153-164], sliding mode control method [165-173], etc.

In this paper, we derive a nine-term novel 3-D chaotic system with four quadratic nonlinearities. We show that the novel chaotic system has three unstable equilibrium points. We show that the equilibrium point at the origin is a saddle-point, while the other two equilibrium points are saddle-foci. The Lyapunov exponents of the novel 3-D chaotic system are obtained as $L_1 = 10.45056$, $L_2 = 0$, and $L_3 = -42.36623$. The maximal Lyapunov exponent (MLE) for the novel chaotic system is obtained as $L_1 = 10.45056$. The large value of L_1 shows that the novel chaotic system is highly chaotic and exhibits highly complex behaviour. Also, the Lyapunov dimension of the novel chaotic system is derived as $D_L = 2.2467$.

Next, we derive new results for the global chaos control of the novel highly chaotic system via integral sliding mode control (ISMC). We also derive new results for the global chaos synchronization design of the identical novel highly chaotic systems via integral sliding mode control (ISMC).

The global chaos control and synchronization results for the novel highly chaotic system have been established using Lyapunov stability theory. In contrast with conventional sliding mode control (SMC), the system motion under integral sliding mode has a dimension equal to that of the state space. In ISMC, the system trajectory always starts from the sliding surface. Accordingly, the reaching phase is eliminated and robustness in the whole state space is promised. Numerical simulations with MATLAB have been shown to validate and demonstrate all the new results derived in this paper for the novel highly chaotic system using integral sliding mode control.

2. A NOVEL HIGHLY CHAOTIC SYSTEM

In this section, we propose a novel highly chaotic system modelled by the dynamics

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) - x_2 x_3 \\ \dot{x}_2 = b x_1 - x_2 + c x_1 x_3 \\ \dot{x}_3 = -x_1 x_2 - x_2^2 - x_3 \end{cases} \tag{1}$$

where x_1, x_2, x_3 are the states and a, b, c are constant, positive parameters of the system.

The system (1) is a nine-term polynomial chaotic system with four quadratic nonlinearities.

The system (1) describes a *strange chaotic attractor* for the parameter values

$$a = 30,\ b = 1600,\ c = 50 \tag{2}$$

For numerical simulations, we take the initial values of the system (1) as

$$x_1(0) = 0.6, \quad x_2(0) = 1.8, \quad x_3(0) = 1.2 \tag{3}$$

The Lyapunov exponents of the novel system (1) are numerically obtained as

$$L_1 = 10.45056, \quad L_2 = 0, \quad L_3 = -42.36623 \tag{4}$$

Thus, the maximum Lyapunov exponent (MLE) of the novel chaotic system is obtained as $L_1 = 10.45056$, which is a very large value. This shows that the novel chaotic system (1) is highly chaotic and it exhibits complex chaotic behaviour.

Since the sum of the Lyapunov exponents in (4) is negative, it follows that the highly chaotic system (1) is a dissipative system.

Figure 1 shows the strange chaotic attractor of the highly chaotic system (1).

Figures 2-4 show the 2-D view of the strange attractor of the highly chaotic system (1) in (x_1, x_2), (x_2, x_3), and (x_1, x_3) planes respectively.

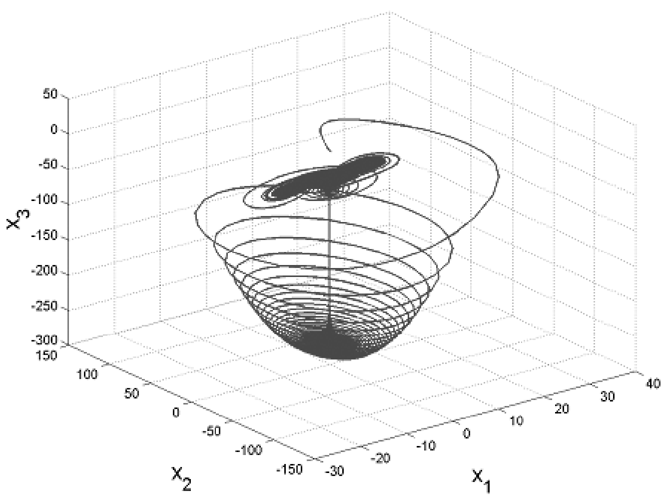

Figure 1: Strange attractor of the novel highly chaotic system

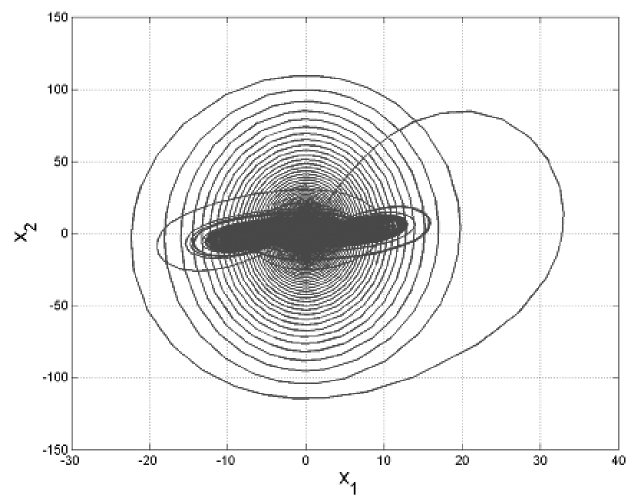

Figure 2: 2-D view of the novel highly chaotic system in (x_1, x_2) plane

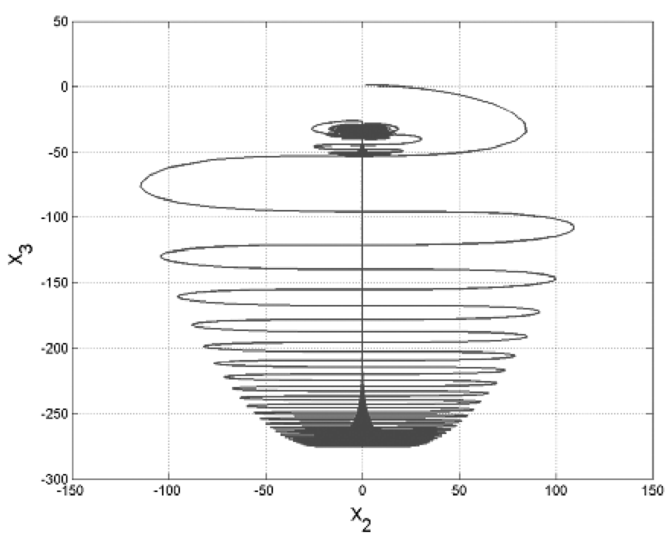

Figure 3: 2-D view of the novel highly chaotic system in (x_2, x_3) plane

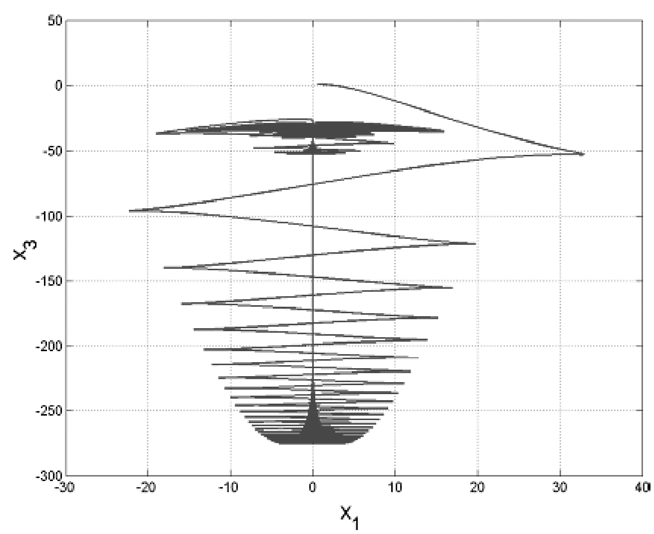

Figure 4: 2-D view of the novel highly chaotic system in (x_1, x_3) plane

3. PROPERTIES OF THE NOVEL 3-D CHAOTIC SYSTEM

In this section, we detail the qualitative properties of the novel highly chaotic system (1), which is described in Section 2.

3.1. Dissipativity

We write the system (1) in vector notation as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix} \tag{5}$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) = a(x_2 - x_1) - x_2 x_3 \\ f_2(x_1, x_2, x_3) = bx_1 - x_2 + cx_1 x_3 \\ f_3(x_1, x_2, x_3) = -x_1 x_2 - x_2^2 - x_3 \end{cases} \tag{6}$$

We take the parameter values as

$$a = 30, \quad b = 1600, \quad c = 50 \tag{7}$$

The divergence of the vector field f on R^3 is obtained as

$$\text{div} f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -a - 1 - 1 = -\mu, \tag{8}$$

where

$$\mu = a + 2 = 32 > 0 \tag{9}$$

Let Ω be any region in R^3 having a smooth boundary.

Let $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f.

Let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, it follows that

$$\frac{dV}{dt} = \int\limits_{\Omega(t)} (\text{div} f)\, dx_1\, dx_2\, dx_3 = \int\limits_{\Omega(t)} (-\mu)\, dx_1\, dx_2\, dx_3 = -\mu V \tag{10}$$

Integrating the linear differential equation (10), we get the solution as

$$V(t) = V(0)\exp(-\mu t) \tag{11}$$

From Eq. (10), it follows that the volume $V(t)$ shrinks to zero exponentially as $t \to \infty$.

Thus, the novel highly chaotic system (1) is dissipative.

Hence, the asymptotic motion of the system (1) settles exponentially onto a set of measure zero, *i.e.* a strange attractor.

3.2. Symmetry

The novel highly chaotic system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) \mapsto (-x_1, -x_2, x_3) \tag{12}$$

Since the transformation (12) persists for all values of the system parameters, the novel 3-D chaotic system (1) has rotation symmetry about the x^3–axis and that any non-trivial trajectory must have a twin trajectory.

3.3. Invariance

The x^3–axis ($x_1 = 0, x_2 = 0, x_4 = 0$) is invariant for the system (1). Hence, all orbits of the system (1) starting on the x^3–axis stay in the x^3–axis for all values of time.

Also, this invariant motion is governed by the scalar differential equation

$$\dot{x}_3 = -x_3 \tag{13}$$

which is globally exponentially stable.

3.4. Equilibrium Points

The equilibrium points of the novel 3-D chaotic system (1) are obtained by solving the following nonlinear system of equations

$$\begin{cases} f_1(x_1, x_2, x_3) = a(x_2 - x_1) - x_2 x_3 = 0 \\ f_2(x_1, x_2, x_3) = bx_1 - x_2 + cx_1 x_3 = 0 \\ f_3(x_1, x_2, x_3) = -x_1 x_2 - x_2^2 - x_3 = 0 \end{cases} \tag{14}$$

We take the parameter values as in the chaotic case, *viz.*

$$a = 30, \quad b = 1600, \quad c = 50 \tag{15}$$

Solving the equations (14) using the values (15), we obtain three equilibrium points:

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 6.6742 \\ 3.2300 \\ -31.9903 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -6.6742 \\ -3.2300 \\ -31.9903 \end{bmatrix} \tag{16}$$

The Jacobian matrix of the novel chaotic system (1) at any point $X \in R^3$ is obtained as

$$J(x) = \begin{bmatrix} -a & a - x_3 & -x_2 \\ b + cx_3 & -1 & cx_1 \\ -x_2 & -x_1 - 2x_2 & -1 \end{bmatrix} = \begin{bmatrix} -30 & 30 - x_3 & -x_2 \\ 1600 + 50x_3 & -1 & 50x_1 \\ -x_2 & -x_1 - 2x_2 & -1 \end{bmatrix} \tag{17}$$

The Jacobian of the system (1) at E_0 is obtained as

$$J_0 = J(E_0) = \begin{bmatrix} -30 & 30 & 0 \\ 1600 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{18}$$

The eigenvalues of J_0 are numerically obtained as

$$\lambda_1 = -1, \quad \lambda_2 = -235.0683, \quad \lambda_3 = 204.0683 \tag{19}$$

This shows that the equilibrium E_0 is a saddle point, which is unstable.

The Jacobian of the system (1) at E_1 is obtained as

$$J_1 = J(E_1) = \begin{bmatrix} -30 & 61.9903 & -3.23 \\ 0.4850 & -1 & 333.71 \\ -3.23 & -13.1342 & -1 \end{bmatrix} \tag{20}$$

The eigenvalues of J_1 are numerically obtained as

$$\lambda_1 = -41.3798, \quad \lambda_{2,3} = 4.6899 \pm 69.0627i \tag{21}$$

This shows that the equilibrium E_1 is a saddle-focus, which is unstable.

The Jacobian of the system (1) at E_2 is obtained as

$$J_2 = J(E_2) = \begin{bmatrix} -30 & 61.9903 & 3.23 \\ 0.4850 & -1 & -333.71 \\ 3.23 & 13.1342 & -1 \end{bmatrix} \tag{22}$$

The eigenvalues of J_3 are numerically obtained as

$$\lambda_1 = -41.3798, \quad \lambda_{2,3} = 4.6899 \pm 69.0627i \tag{23}$$

This shows that the equilibrium E_2 is a saddle-focus, which is unstable.

Thus, all the three equilibrium points of the novel 3-D chaotic system (1) are unstable.

3.5. Lyapunov Exponents and Lyapunov Dimension

We take the parameter values of the novel system (1) as

$$a = 30, \quad b = 1600, \quad c = 50 \tag{24}$$

We take the initial conditions of the novel system (1) as

$$x_1(0) = 0.6, \quad x_2(0) = 1.8, \quad x_3(0) = 1.2 \tag{25}$$

The Lyapunov exponents of the system (1) are numerically obtained with MATLAB as

$$L_1 = 10.45056, \quad L_2 = 0, \quad L_3 = -42.36623 \tag{26}$$

Thus, the system (1) is highly chaotic, since it has a large positive Lyapunov exponent.

Since $L_1 + L_2 + L_3 = -31.9157 < 0$, it is immediate that the system (1) is dissipative.

The Lyapunov dimension of the chaotic system (1) is determined as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.2467 \tag{27}$$

which is fractional.

The MATLAB plot of the Lyapunov exponents of the novel highly chaotic system (1) is depicted in Figure 5.

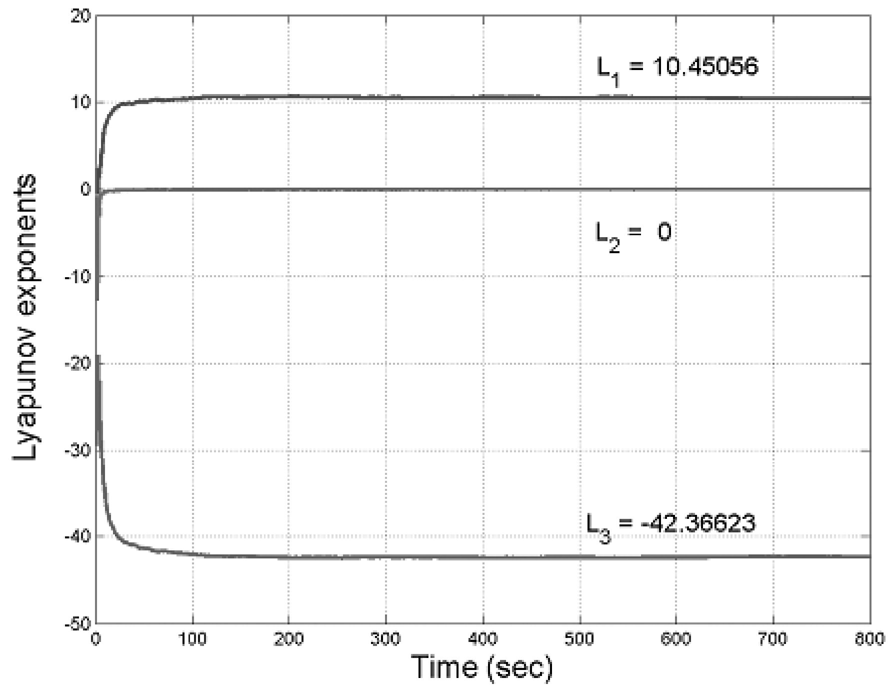

Figure 5: Lyapunov exponents of the novel highly chaotic system

4. GLOBAL CHAOS CONTROL OF THE NOVEL HIGHLY CHAOTIC SYSTEM VIA INTEGRAL SLIDING MODE CONTROL

In this section, we design new results for the global chaos control of the novel highly chaotic system via integral sliding mode control [174]. The main control result in this section is established using Lyapunov stability theory [175].

Thus, we consider the novel highly chaotic system with controls given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) - x_2 x_3 + u_1 \\ \dot{x}_2 = bx_1 - x_2 + cx_1 x_3 + u_2 \\ \dot{x}_3 = -x_1 x_2 - x_2^2 - x_3 + u_3 \end{cases} \tag{28}$$

where x_1, x_2, x_3 are state variables, a, b, c are constant, positive, parameters of the system and u_1, u_2, u_3 are integral sliding mode controls to be designed.

Based on the sliding mode control theory [174], the integral sliding surface of each state variable x_i, (i = 1, 2, 3) is defined as follows:

$$s_i = \left[\frac{d}{dt} + \lambda_i \right] \left[\int_0^t x_i(\tau) d\tau \right] = x_i + \lambda_i \int_0^t x_i(\tau) d\tau, \ \ (i = 1, 2, 3) \tag{29}$$

The derivative of each equation in (29) yields

$$\dot{s}_i = \dot{x}_i + \lambda_i x_i, \quad (i = 1, 2, 3) \tag{30}$$

The Hurwitz condition is satisfied if $\lambda_i > 0$ for i = 1, 2, 3.

Based on the exponential reaching law [217], we set

$$\dot{s}_i = -\eta_i \operatorname{sgn}(s_i) - k_i s_i, \quad (i = 1, 2, 3) \tag{31}$$

where sgn(·) is the sign function and η_i, k_i, (i = 1, 2, 3) are positive constants.

Comparing the equations (30) and (31), we get

$$\begin{cases} \dot{x}_1 + \lambda_1 x_1 = -\eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ \dot{x}_2 + \lambda_2 x_2 = -\eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ \dot{x}_3 + \lambda_3 x_3 = -\eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \end{cases} \tag{32}$$

Using (28), we can rewrite the equations (32) as follows. S_1, S_2, S_3

$$\begin{cases} a(x_2 - x_1) - x_2 x_3 + u_1 + \lambda_1 x_1 = -\eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ bx_1 - x_2 + cx_1 x_3 + u_2 + \lambda_2 x_2 = -\eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ -x_1 x_2 - x_2^2 - x_3 + u_3 + \lambda_3 x_3 = -\eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \end{cases} \tag{33}$$

From (33), the control laws are obtained as follows.

$$\begin{cases} u_1 = -a(x_2 - x_1) + x_2 x_3 - \lambda_1 x_1 - \eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ u_2 = -bx_1 + x_2 - cx_1 x_3 - \lambda_2 x_2 - \eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ u_3 = x_1 x_2 + x_2^2 + x_3 - \lambda_3 x_3 - \eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \end{cases} \tag{34}$$

Next, we state and prove the main result of this section.

Theorem 1. The novel highly chaotic system (28) with constant system parameters is globally and asymptotically stabilized for all initial conditions $x(0) \in R^3$ by the integral sliding mode control law (34), where the constants λ_i, η_i, k_i are positive for i = 1, 2, 3.

Proof. The result is proved using Lyapunov stability theory [175].

We consider the following quadratic Lyapunov function

$$V(s_1, s_2, s_3) = \frac{1}{2}\left(s_1^2 + s_2^2 + s_3^2\right), \tag{35}$$

where S_1, S_2, S_3 are as defined in Eq. (29).

The time-derivative of V is obtained as

$$\dot{V} = s_1 \dot{s}_1 + s_2 \dot{s}_2 + s_3 \dot{s}_3 \tag{36}$$

Substituting from Eq. (31) into (36), we obtain

$$\dot{V} = s_1\left[-\eta_1 \operatorname{sgn}(s_1) - k_1 s_1\right] + s_2\left[-\eta_2 \operatorname{sgn}(s_2) - k_2 s_2\right] + s_3\left[-\eta_3 \operatorname{sgn}(s_3) - k_3 s_3\right] \tag{37}$$

Simplifying Eq. (37), we obtain

$$\dot{V} = -\eta_1 |s_1| - k_1 s_1^2 - \eta_2 |s_2| - k_2 s_2^2 - \eta_3 |s_3| - k_3 s_3^2 \tag{38}$$

Since $\eta_i > 0$ and $k_i > 0$ for i = 1, 2, 3 it is immediate that $x_i \to 0$ (i = 1, 2, 3) as $t \to \infty$ for all initial conditions $x(0) \in R^3$.

This completes the proof. ■

4.1. Numerical Results

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the system of differential equations (28) when the integral sliding mode controller (34) is implemented.

For the novel chaotic system (28), the parameter values are taken as in the chaotic case (2), i.e.

$$a = 30, \quad b = 1600, \quad c = 50 \tag{39}$$

We take the sliding constants as

$$\eta_i = \lambda_i = 0.1, \quad k_i = 25, \quad (i = 1, 2, 3) \tag{40}$$

The initial values of the chaotic system (28) are taken as

$$x_1(0) = 17.1, \; x_2(0) = 26.4, \; x_3(0) = 34.8 \tag{41}$$

Figure 6 depicts the time-history of the controlled novel chaotic system.

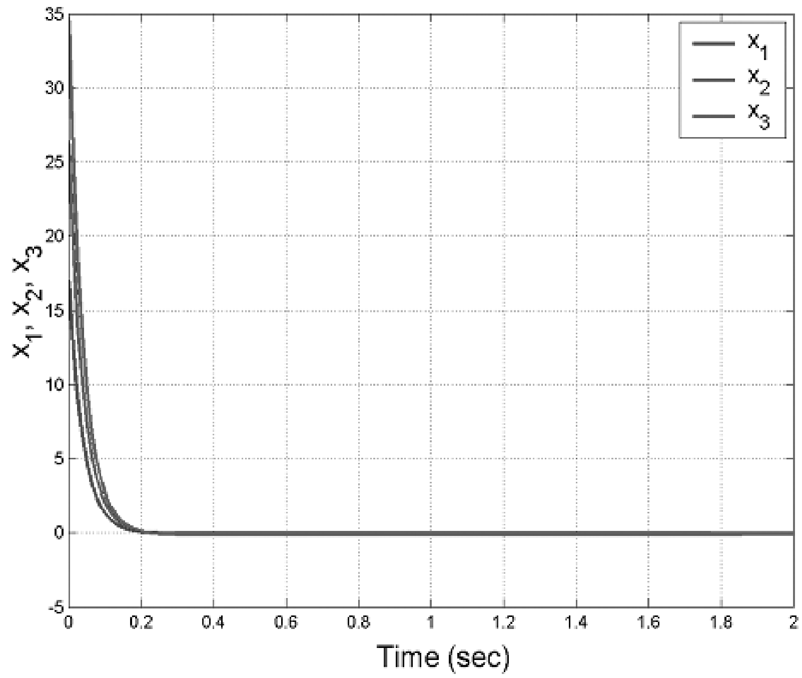

Figure 6: Time history of the controlled novel chaotic system

5. GLOBAL CHAOS SYNCHRONIZATION OF THE NOVEL HIGHLY CHAOTIC SYSTEMS VIA INTEGRAL SLIDING MODE CONTROL

In this section, we derive new results for the global chaos synchronization of the identical novel chaotic systems with unknown parameters.

As the master system, we take the novel highly chaotic system

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) - x_2 x_3 \\ \dot{x}_2 = b x_1 - x_2 + c x_1 x_3 \\ \dot{x}_3 = -x_1 x_2 - x_2^2 - x_3 \end{cases} \tag{42}$$

where x_1, x_2, x_3 are state variables and a, b, c are constant, positive, parameters of the system.

As the slave system, we take the controlled novel highly chaotic system

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) - y_2 y_3 + u_1 \\ \dot{y}_2 = b y_1 - y_2 + c y_1 y_3 + u_2 \\ \dot{y}_3 = -y_1 y_2 - y_2^2 - y_3 + u_3 \end{cases} \tag{43}$$

where y_1, y_2, y_3 are state variables and u_1, u_2, u_3 are nonlinear controllers to be designed.

The synchronization error is defined by

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{44}$$

The error dynamics is easily obtained as

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - y_2 y_3 + x_2 x_3 + u_1 \\ \dot{e}_2 = b e_1 - e_2 + c(y_1 y_3 - x_1 x_3) + u_2 \\ \dot{e}_3 = -y_1 y_2 + x_1 x_2 - y_2^2 + x_2^2 - e_3 + u_3 \end{cases} \tag{45}$$

Based on the sliding mode control theory [174], the integral sliding surface of each error variable e_i, (i = 1, 2, 3) is defined as follows:

$$s_i = \left[\frac{d}{dt} + \lambda_i \right] \left[\int_0^t e_i(\tau) d\tau \right] = e_i + \lambda_i \int_0^t e_i(\tau) d\tau, \quad (i = 1, 2, 3) \tag{46}$$

The derivative of each equation in (46) yields

$$\dot{s}_i = \dot{e}_i + \lambda_i e_i, \quad (i = 1, 2, 3) \tag{47}$$

The Hurwitz condition is satisfied if $\lambda_i > 0$ for i = 1, 2, 3.

Based on the exponential reaching law [174], we set

$$\dot{s}_i = -\eta_i \operatorname{sgn}(s_i) - k_i s_i, \quad (i = 1, 2, 3) \tag{48}$$

where sgn(·) is the sign function and η_i, k_i, (i = 1, 2, 3) are positive constants.

Comparing the equations (47) and (48), we get

$$\begin{cases} \dot{e}_1 + \lambda_1 e_1 = -\eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ \dot{e}_2 + \lambda_2 e_2 = -\eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ \dot{e}_3 + \lambda_3 e_3 = -\eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \end{cases} \tag{49}$$

Using (45), we can rewrite the equations (48) as follows. S_1, S_2, S_3

$$\begin{cases} a(e_2 - e_1) - y_2 y_3 + x_2 x_3 + u_1 + \lambda_1 e_1 = -\eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ b e_1 - e_2 + c(y_1 y_3 - x_1 x_3) + u_2 + \lambda_2 e_2 = -\eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ -y_1 y_2 + x_1 x_2 - y_2^2 + x_2^2 - e_3 + u_3 + \lambda_3 e_3 = -\eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \end{cases} \tag{50}$$

From (50), the control laws are obtained as follows.

$$\begin{cases} u_1 = -a(e_2 - e_1) + y_2 y_3 - x_2 x_3 - \lambda_1 e_1 - \eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ u_2 = -b e_1 + e_2 - c(y_1 y_3 - x_1 x_3) - \lambda_2 e_2 - \eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ u_3 = y_1 y_2 - x_1 x_2 + y_2^2 - x_2^2 + e_3 - \lambda_3 e_3 - \eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \end{cases} \tag{51}$$

Next, we state and prove the main result of this section.

Theorem 2. The novel highly chaotic system (42) and (43) with constant system parameters is globally and asymptotically synchronized for all initial conditions $x(0)$, $y(0) \in R^3$ by the integral sliding mode control law (51), where the constants λ_i, η_i, k_i are positive for $i = 1, 2, 3$.

Proof. The result is proved using Lyapunov stability theory [175].

We consider the following quadratic Lyapunov function

$$V(s_1, s_2, s_3) = \frac{1}{2}\left(s_1^2 + s_2^2 + s_3^2\right), \tag{52}$$

where S_1, S_2, S_3 are as defined in Eq. (46).

The time-derivative of V is obtained as

$$\dot{V} = s_1\dot{s}_1 + s_2\dot{s}_2 + s_3\dot{s}_3 \tag{53}$$

Substituting from Eq. (48) into (53), we obtain

$$\dot{V} = s_1\left[-\eta_1 \operatorname{sgn}(s_1) - k_1 s_1\right] + s_2\left[-\eta_2 \operatorname{sgn}(s_2) - k_2 s_2\right] + s_3\left[-\eta_3 \operatorname{sgn}(s_3) - k_3 s_3\right] \tag{54}$$

Simplifying Eq. (54), we obtain

$$\dot{V} = -\eta_1 |s_1| - k_1 s_1^2 - \eta_2 |s_2| - k_2 s_2^2 - \eta_3 |s_3| - k_3 s_3^2 \tag{55}$$

Since $\eta_i > 0$ and $k_i > 0$ for $i = 1, 2, 3$, it is immediate that $e_i \to 0$ ($i = 1, 2, 3$) as $t \to \infty$ for all initial conditions $e(0) \in R^3$.

This completes the proof. ■

4.2. Numerical Results

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the system of differential equations (42) and (43) when the integral sliding mode controller (51) is implemented.

For the novel highly chaotic systems (42) and (43), the parameter values are taken as in the chaotic case (2), i.e.

$$a = 30, \quad b = 1600, \quad c = 50 \tag{56}$$

We take the sliding constants as

$$\eta_i = \lambda_i = 0.1, \quad k_i = 25, \quad (i = 1, 2, 3) \tag{57}$$

The initial values of the chaotic system (42) are taken as

$$x_1(0) = 12.1, \; x_2(0) = 6.9, \; x_3(0) = 20.5 \tag{58}$$

The initial values of the chaotic system (43) are taken as

$$y_1(0) = 9.5, \; y_2(0) = 15.3, \; y_3(0) = 7.2 \tag{59}$$

Figures 7-9 depict the complete synchronization of the highly chaotic systems (42) and (43).

Figure 10 depicts the time-history of the synchronization errors e_1, e_2, e_3.

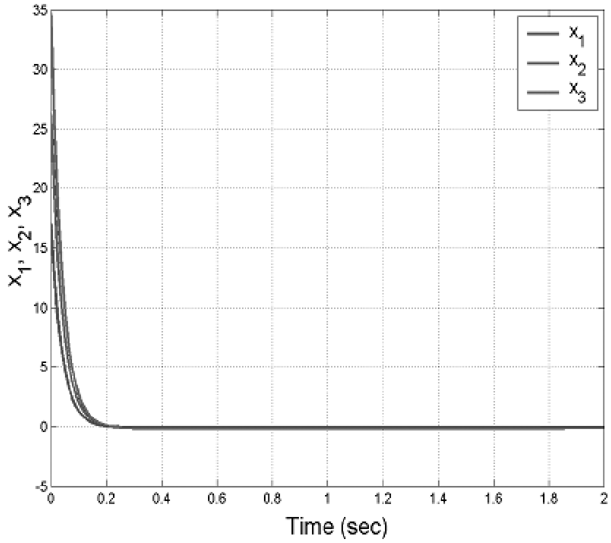

Figure 7: Complete synchronization of the states x_1 and y_1

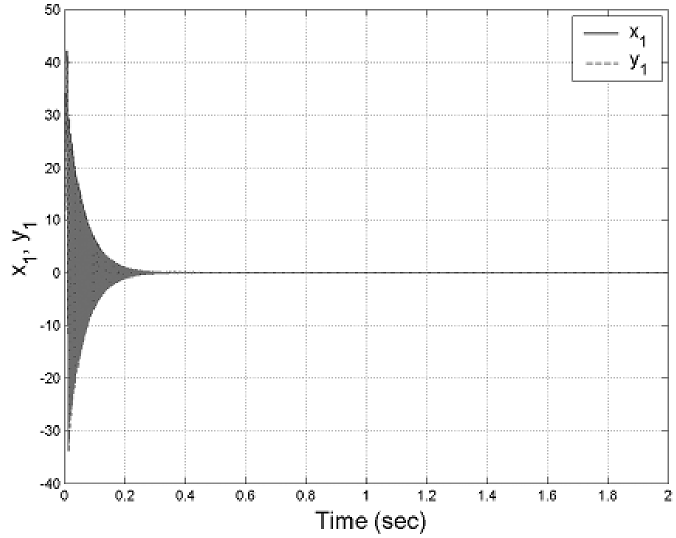

Figure 8: Complete synchronization of the states x_2 and y_2

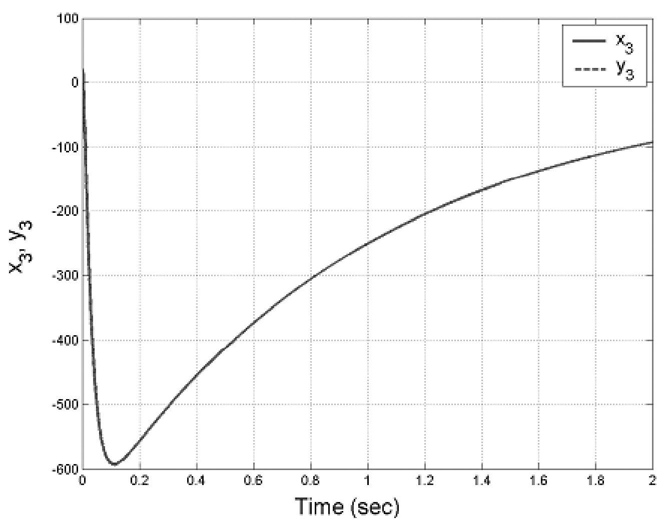

Figure 9: Complete synchronization of the states x_3 and y_3

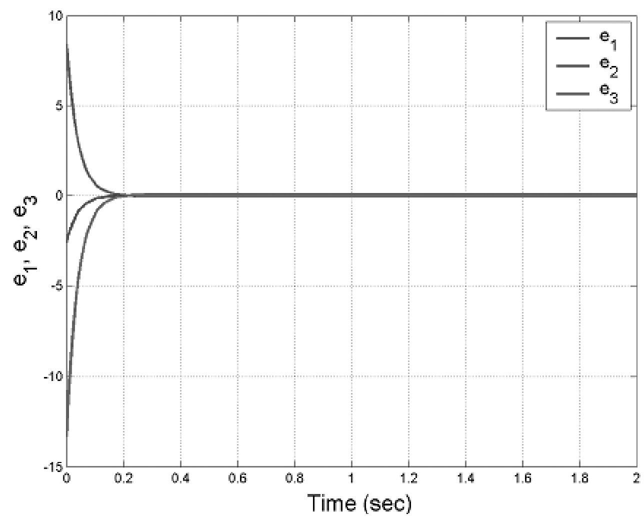

Figure 10: Time history of the chaos synchronization errors e_1, e_2, e_3, e_4

6. CONCLUSIONS

In this paper, we have described a nine-term novel 3-D chaotic system with four quadratic nonlinearities. The phase portraits of the novel chaotic system were displayed and the qualitative properties were described in detail. The Lyapunov exponents of the novel 3-D chaotic system have been obtained as and The maximal Lyapunov exponent (MLE) for the novel chaotic system is obtained as The large value of shows that the novel chaotic system is highly chaotic and exhibits highly complex behaviour. Also, the Lyapunov dimension of the novel chaotic system is derived as Next, we derived new results for the global chaos control of the novel highly chaotic system via integral sliding mode control (ISMC). We also derived new results for the global chaos synchronization design of the identical novel highly chaotic systems via integral sliding mode control (ISMC). The global chaos control and synchronization results for the novel highly chaotic system have been established using Lyapunov stability theory. In contrast with conventional sliding mode control (SMC), the system motion under integral sliding mode has a dimension equal to that of the state space. In ISMC, the system trajectory always starts from the sliding surface. Accordingly, the reaching phase is eliminated and robustness in the whole state space is promised. Numerical simulations with MATLAB were shown to validate and demonstrate all the new results derived in this paper for the novel highly chaotic system using integral sliding mode control.

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