

# An Optimization-MCDM approach of Multi Criteria Decision Analysis

Priyanka Majumder\*, Mrinmoy Majumder\*\* and Apu Kumar Saha\*

## ABSTRACT

The accuracy of any decision making model depends mainly on their performance in estimation of the priority for each factors in respect to the decision objective. Multi Criteria Decision Making (MCDM) offers a feasible option in this regard. The priority value which is determined by the MCDM, can differentiate the factor based on their degree of importance with respect to the decision objective. But the priorities determined by the MCDM methods were identified for a common scenario. But to produce an optimal scenario, priority of the parameter must be defined for an optimal condition. So, although factors can be differentiated but the optimality of the objective function cannot be ensured. That is why; the present study is an attempt to find a way in which priorities can be estimated for an optimal scenario. In this regard, optimality of the financial feasibility of hydro power plants (HPP) was ensured by determining the priority values and using then in the representative index after finding the optimal location with the help of ensembled multiple optimization techniques (O.T.s). The advantage of using multiple O.T.s is the weakness of one can be replaced by the other. The result of the case study justifies the application of different O.T.s as well as the determination of the priority values for an optimal scenario.

**Keywords:** MCDM, Optimization

## 1. INTRODRUCTION

MCDM is a way to determine priorities of decision parameters with respect to the decision objective. This method is a relative way to estimate weights of importance of the factors which influences the decision objective. There are many MCDM methods available like Analytic Hierarchy Process (AHP)[1], Analytic Network Process (ANP)[2], Elimination Et Choix Traduisant la Realite (ELECTRE)[3-5], Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH), Multi-Attribute Utility Theory (MAUT)[6], Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE)[7], Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)[8] etc. Some tries to estimate difference of priorities by a fraction known as weights of importance and varies directly with the capacity of influences the decision objective. These MCDM methods are known as Compensatory methods [9]. The drawback of any MCDM method is it determines this weight of importance for common scenarios which includes both optimal and non-optimal cases. The difference of influence for the factors can be estimated by MCDM but it cannot present the objective function from deviating to a non-feasible region.

The present study is an approach to solve this problem of MCDM. The investigation tries to establish a new method which determines the weight of importance of the factors for maintain the decision objective within the optimal region. In this regard the new method uses different optimization techniques to estimate the weights of importance of any objective function. Application of O.T. ensures that the weights will lead the function only to the optimal region. In total three O.T. was applied in a combined manner where the disadvantage of one is solved by the advantage of other. These three O.T.s (OT) are Particle Swarm Optimization (PSO), Harmony Search (HS) Algorithm and Teaching–Learning-Based Optimization (TLBO)

\* Department of Mathematics, NIT Agartala, Agartala, India, Tripura, *Emails: majumderpriyanka94@yahoo.com, apusaha\_nita@yahoo.co.in*

\*\* Department of Civil Engineering, NIT Agartala, Agartala, India, Tripura, *Email: mmajumder15@gmail.com*

**Table 1**  
Shows a comparative represent of the strength of O.T. and weakness of MCDMs.

<i>Disadvantage of MCDM</i>	<i>Advantage of OT.</i>
Different MCDM methods yield different outcomes when applied to the same multi-criteria problem. The selection of an appropriate MCDM method from a long list of MCDM methods is often not straight forward and may possibly control the final outcome of the decisionmaking process[10].	Evolutionary Optimization (EO) with a parallel processing power achieving a computationally quick overall search[11].
Pairwise comparison can be effective because it forces the decision makers to give through consideration to all elements of decision problem [12].	O.T. to find multiple optimal solutions, thereby facilitating the solution of multi-modal and multi-objective optimization problems [11].
The pairwise comparison is seems to be insufficient and imprecise to capture the right judgments of decision-maker(s) with vagueness and uncertainty of data [13].	O.T. with the ability to normalize decision variables as well as objective and constraint functions (e.g., EO) [11].

to find the optimal weight of importance for decision factors based on an objective equation which is coherent with the decision objectives. PSO is parameter optimization technique, so variation of parameters the result may be changed, that is why in this study apply PSO. Since HS is a quality of the iteration method, then PSO, TLBO that is why in this study apply this method. Again, as the present problem is not a higher dimensional problem TLBO can be used to solve the present decision making objective for its ability to converge quickly.

From the above table 1 it is clear that MCDM techniques give priority value (p.v.) at normal scenarios but these MCDM techniques unable to give p.v. at optimal scenario as well as pairwise comparison is more difficult for qualitative and quantitative information of factors, for overcoming these disadvantages of MCDM O.T.s apply as MCDM. For using O.T. at least one objective must be required. When applying O.T.s as MCDM then construct an objective function by this factors which are decided alternatives for MCDM. This objective function must be equivalent to objective of the MCDM. This new objective function is constructed by ratio of weights of importance of beneficiary and non-beneficiary factors, so this objective function is non-linear. Using O.T.s it is always found p.v. of each factor in optimal scenario. Also it is clear that using new way of MCDM there is not required any pairwise comparison. Since in MCDM all p.v. values of each factors calculate in normalized form, that is why study more preferred T.O.s as MCDM because it has ability to normalize decision variables as well as objective and constraint functions. The new method was applied to estimate the priority of factors with respect to the estimation of financial liability of hydro power projects. The index was applied for evaluation of financial feasibility of HPP located in Tripura. The plant was selected as the financial liability of the power house is unsuitable and hard to predict.

## 2. METHODOLOGY

The new method has two steps. First step is determination of the index and second step is application of O.T. of identifying p.v. at an optimal stage. Figure depicts the methodology of the proposed method.

### 2.1. Model Input

In the present study, input parameters are selected on the basis of the scores received for the literature survey(Eqn.3), Stakeholder survey(Eqn.4) and local survey(Eqn.5).

$$\text{Parameter selection} = f(\text{literature survey, expert survey, Stakeholder survey}) \quad (1)$$

$$\text{Percentage of literature survey} = \frac{\text{No of literature which prefer the criteria}}{\text{Total number literature studied}} \times 100 \quad (2)$$

$$\text{Percentage of expert survey} = \frac{\text{No of expert survey which prefer the criteria}}{\text{Total number expert survey}} \times 100 \tag{3}$$

$$\text{Percentage of Stakeholder survey} = \frac{\text{No of local survey which prefer the criteria}}{\text{Total number local survey}} \times 100 \tag{4}$$

**2.2. Model Output:**

If *m* (p.v. of the beneficiary parameter) and *n* (p.v. of the non-beneficiary parameter) are the priorities of the input parameters represented by a priority value, the objective equation used in the optimization procedure is given as:

$$I = \frac{\sum mB_m}{\sum nNB_n} \tag{5}$$

Subject to:  $m_{min} < m < m_{max}$  and  $n_{min} < n < n_{max}$

The present study utilizes three O.T.s to find the point at which the objective function, which is proportional to the objective of the given problem, will be optimum. The priority value was used as the design variables and the magnitude of the factors are included as a control variable. The domain of the priority value is set at 0 to 1 whereas the same for the control variable is the maximum and minimum value of the factors for the selected location. The governing equation, strength, weakness, previous applications and the justification for using the method in the present study was shown in table 2.

**Table 2**  
**Describes about the O.T.s.**

Name of method	Proposed by	Governing Equation	Strengthens	weakness	Appli-cation	Why used
PSO	Kennedy and Eberhart [14]	Eqn.6	Capability	Unsure	[16,	For its ability
		Eqn.7	to escape local minima	about the location of global optima	17, 18]	to escape local minima
		$v_i(k+1) = v_i(k) + P(i) + G(i)$				
		$x_i(k+1) = v_i(k+1) + x_i(k)$	[15]			
HSA	Geem (2001) [19]	Eqn.8	Insensitive	Large	[21,	The
		$x_i^j = x_i^j \in Y, rand \leq HMCR$	to initial	convergence	22,	iteration
		$x_i^j \in X_i, otherwise$	values [20]	time	23]	quality
		Eqn.9				ignores
		$x_i^j = x_i^j \pm c_e, rand \leq HMCR$				initial value
		$x_i^j, otherwise$				
TLBO	Rao (2011) [24]	Eqn.10	Parameter-free	Large	[26,	Due to its
		Eqn.11	technique [25]	convergence time	27, 28]	parameter independence
		$new x_i = \begin{cases} x_i + Y(i) \text{ iff } (x_i) < f(x_j) \\ x_i + Y(i) \text{ otherwise} \end{cases}$				

Eqn. 6 and Eqn. 7 represent the equations by which the particles in the PSO Technique updates their position and velocity respectively. For chosen component of new generated harmony vector Eqn. 9 and

Eqn. 8 are used which represent a pitch adjustment decision for design variables. For finding best mean results using Eqn.10 and updating the learner’s knowledge by Eqn.11. The present study considers PSO, HSA and TLBO algorithms for optimization of the objective function. The PSO is a swarm based optimization technique which has an advantage of escaping local minima [15] but it does not ensure the identification of global optima. HSA in the other hand is known for its insensitivity towards initial values [20] but it has the drawback of a long convergence time. Although the quality of iteration in HSA is most reliable [29]. TLBO is popular for its quick convergence time in case of lower dimensional problems and its parametric free technique [25] but this method takes a long time to converge in case of higher dimensional problem[30].

**2.3. Detail Methodology**

Nearly 100 literatures were analysed to find out the significant factors controlling expenditure in HPP. The Eqn.2 is utilized to find a score of importance of the identified parameters with represent to literature survey. A survey was carried out within the experts of related fields where participants were asked to suggest and rank about the cost parameters which can induce effect on the plant income. According to the response received from the experts a score was calculated for each to the factors according to Eqn. 3. A survey was carried out within local people of the plant area where participants were asked to suggest about the cost parameters which can induce effect on the plant income. According to response received from the stakeholder survey a percentage was given to the factors according to Eqn.4. From the above surveys it was found that Rate of Electricity changed from the industrial and agricultural consumers ( $I$ ), domestic consumer ( $D$ ), civil cost ( $C$ ), energy equipment cost ( $E$ ) as well as labour and engineering cost ( $N$ ) are the five most parameters which influences the overall financial performance of any HPP. The objective function for optimization is given in Eqn.12. The bounds of the priority vector are taken as 0 and 1. The bound of the control variable is taken from the maximum and minimum value of the parameter observed in Gomati HPP

(table 3) within the last year. In the objective function (Eqn. 13)  $\frac{\partial F_j}{\partial w_i}$  represent the partial differentiation  $F_j$  with respect to  $w_i$ , where the dependent variable  $F_j$  is a function of more than two independent variables and independent variable  $w_i$  represent as design variable. Also  $1 < i \in \mathbb{N}$  (Set of natural numbers).

$$\begin{aligned}
 \text{Max}P = & \begin{vmatrix} \frac{1}{2} \frac{\partial}{\partial w_1} & \frac{1}{2} \frac{\partial}{\partial w_2} \\ Aw_1^2 - Dw_2^2 & Aw_1^2 + Dw_2^2 \\ \frac{\partial}{\partial w_3} & \frac{\partial}{\partial w_4} & \frac{\partial}{\partial w_5} \\ \frac{1}{2} w_5^2 & \frac{1}{2} w_3^2 & \frac{1}{2} w_4^2 \\ C & E & N \end{vmatrix} \tag{12}
 \end{aligned}$$

*Subject to*  $0 < w \leq U$  Where  $0 = [0 \ 0 \ 0 \ 0 \ 0]^T$ ,  $U = [1 \ 1 \ 1 \ 1 \ 1]^T$ ,  $W = [w_1 \ w_2 \ w_3 \ w_4 \ w_5]^T$ ,  $[IENAD]^T = [I_L \ E_L \ N_L \ A_L \ D_L]^T + r$  and  $(0, 1) \times S$  and  $S = [(I_H - I_L) \ (E_H - E_L) \ (N_H - N_L) \ (A_H - A_L) \ (D_H - D_L)]^T$ .

**2.4. Case study**

The new method is applied to identify the most important parameter which can control the profitability of Gomati HPP in Tripura. The actual energy shortage in Tripura is 4.3 % which were lower than the forecasted shortages of 13.5%. The present peak power demand and deficit of the State is 810 MW and 12 MW. Total available generating capacity of the state was 298 MW [31]. The generation capacity of 95

**Table 3**  
**Table showing the upper and lower bound of each parameter**  
**(Gomati peak and Off Peak of the parameter in last one year).**

<i>Weights Assigned</i>	<i>Design variables</i>	<i>Lower Bound (L)</i>	<i>Upper Bound (H)</i>
$W_1$	Rate of Electricity collected from industrial and agricultural consumers ( $A$ )	540000 Rs $A_L$	600000 Rs $A_H$
$W_2$	Rate of Electricity collected from domestic consumers ( $D$ )	3500 Rs $D_L$	4000 Rs $D_H$
$W_3$	Cost due to infrastructure development ( $C$ )	100000 Rs $I_L$	110000 Rs $I_H$
$W_4$	Cost due to energy equipment ( $E$ )	750000 Rs $E_L$	800000 Rs $E_H$
$W_5$	Cost due to the labour and engineers ( $N$ )	250000 Rs $N_L$	300000 Rs $N_H$

MW shares was available from North-Eastern Power grid. For reduction of power shortfall, construction of one unit of 21 MW GT set at Rokhia was taken up and started generation from August, 2013. After commissioning of unit-1 of Palatana power project, Tripura HSA become a surplus power state with 50 MW in peak hour and 100 MW in off peak hour. The surplus power could not be despatched to other power deficit region of the country due to transmission corridor constraints. Overall unit cost of supply increased by 21%. Maximum increase was in the interest payments (65%), followed by increase in establishment and administration expenses (24%), power purchase (21%) and depreciation (21%). The Operation and Maintenance (O&M) expenses and other miscellaneous expenses was decreased by 36% each during the same period.

**Table-4: Table showing the priority value of each cost factors.**

<b>Algorithm</b>	<b>PSO</b>	<b>PSO</b>	<b>PSO</b>	<b>HS</b>	<b>HS</b>	<b>HS</b>	<b>TLBO</b>	<b>TLBO</b>	<b>TLBO</b>
<b>Change in Parameter</b>	c1,c2-1,3; population size-20; iteration no.-100	c1,c2-2,2; population size-30; iteration no.-100	c1,c2-3,1; population size-50; iteration no.-100	HMCR-0.3117; PAR-0.8481; population size-20; iteration no.-100	HMCR-0.4425; PAR-0.1208; population size-30; iteration no.-100	HMCR-0.9320; PAR-0.8329; population size-50; iteration no.-100	population size-20; iteration no.-100	population size-30; iteration no.-100	population size-50; iteration no.-100
<b>Normal</b>									
<b>W<sub>1</sub></b>	0.9998	0.9985	0.9934	0.9854	0.9799	0.9716	0.9970	0.9848	0.9981
<b>W<sub>2</sub></b>	0.5677	0.4188	0.9340	0.5938	0.7977	0.2475	0.8144	0.3000	0.6912
<b>W<sub>3</sub></b>	0.1002	0.1024	0.1001	0.1356	0.1328	0.1228	0.1000	0.1000	0.1032
<b>W<sub>4</sub></b>	0.1002	0.1002	0.1000	0.1050	0.1029	0.1002	0.1009	0.1002	0.1002
<b>W<sub>5</sub></b>	0.1003	0.1011	0.1003	0.1030	0.1023	0.1035	0.1003	0.1000	0.1013
<b>Optimal Value</b>	5.2245	5.1788	4.5843	4.3361	4.3969	4.7176	5.1511	4.9808	4.8299

### 3. RESULTAND DISCUSSION

The study results show that Rate of Electricity from Industry and Agriculture Consumer is the most important parameter (showing in table 4) in influencing the financial performance of HPP to an optimal region. Lee [32], Takashi [33] and Bo [34] in their research work has highlighted the importance of the selected parameter in great details.

#### 4. CONCLUSION

The present investigation is an attempt to find p.v. of decision variables at an optimal scenario with the help of the bagging of different O.T.s. The aim was to replace the weakness of one MCDM technique with the strength of the other. Accordingly an index was made to estimate the degree of financial feasibility of new HPP at an optimal scenario. The result selected Rate of Electricity charged from Industry and Agriculture Consumer to be the most influential parameter in maximizing financial performance of HPP. The decision variable as well as the magnitude of the priority values of the parameters may change if different methods of optimization is selected but still the present procedure was able to identify the priorities of the related parameter successfully and if uniform methods were applied while comparing different HPP projects a reliable decision making can be executed.

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