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PERIODIC VIBRATION OF A MAGNETOSTRICTIVE ACTUATOR —
A Numerical Solution based on the Transfer Matrix Method with Broken-line Reduction

Xinchun Shang[1], Liping Qin[2] and Ernie Pan[3]

ABSTRACT

A new numerical approach is proposed to solve the periodic vibration problem of a magnetostrictive actuator. The periodic vibration of the magnetostrictive rod is excited by an alternating current and the system is modeled as a boundary-value problem of a partial differential equation with variable coefficients and with time periodicity condition. The computation approach is based on the numerical integration in space domain and transfer matrix method combining with broken-line reduction in time domain. Numerical results show that the present solutions are in good agreement with both the experiment data and results from other numerical methods. They also indicate that the output displacement is synchronized with the input current when a high bias magnetic field is applied, and the double frequency effect appears only in the absence of the bias magnetic field. For potential applications of the magnetostrictive actuator, the displacement responses vs. the excitation frequency and the peak current are also given.

***Keywords*:** magnetostrictive Terfenol-D actuator, periodic vibration, transfer matrix method

1. INTRODUCTION

With the discovery of Terfenol-D [1], research in magnetostrictive actuators has been greatly developed in the last twenty years [2]. As a giant magnetostrictive material (GMM), Terfenol-D is of large strain, high energy density, small hysteretic loss and less performance degradation in room temperature. These fine characteristics make Terfenol-D rods as the principal driving element in designing electromechanical sensors and actuators [3~6]. A simple one-dimensional constitutive relation of the magnetostrictive materials, called standard square law, was investigated both theoretically and experimentally [7]. It revealed that nonlinearity exists under a high driving magnetic field. Some new nonlinear constitutive relations have been also proposed in recent research works [8, 9]. The mechanical field generated by a magnetostrictive actuator in response to an applied input current can be modeled as the vibration of a magnetostrictive rod in the actuator. On the basis of the standard square law, a mathematical model of and various numerical approaches to the vibration problem of magnetostrictive actuators were developed by first author and coworkers [10, 11]. Based on this model, the problem is reduced to an initial and boundary value problem of a partial differential equation with time variable coefficient. The numerical approaches are developed from the transfer matrix method, which were implemented in some recent works [12, 13]. There are also other vibration analyses to magnetostrictive actuators in references, such as [14~16].

In this paper, we propose a new numerical method for the analysis of periodic vibration of a magnetostrictive actuator. The problem is mathematically modeled as a boundary value problem of a partial differential equation with time periodicity condition. The difficulty in solving the problem comes from the fact that there is a time variable coefficient in the governing equation and boundary condition. The main idea of the new numerical approach

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1. Department of Mathematics and Mechanics

2. Department of Civil Engineering, University of Science and Technology Beijing, Beijing 100083, China

3. Department of Civil Engineering, University of Akron, Akron, Ohio 44325, USA

is based on the numerical integration in space domain and transfer matrix method with *broken-line reduction* in time domain. In some previous works [11~13], the variable coefficient was approximated as constant in each small time segment, an idea similar to the well-known step reduction method suggested by Prof. K. Y. Yeh in non-homogenous mechanics. To obtain an approximate solution with high accuracy, here we make use of the *broken-line reduction* approach proposed by Yeh and Shang [17]. That is, the variable coefficient is replaced approximately by a linear function of time t in each time segment. Unlike step reduction method in the previous works where the transfer matrix in each segment from the broken-line reduction is not expressed by a matrix exponential, here we calculate the matrix by constructing a recursive relation [18]. As a numerical example, displacement responses to the given current are investigated and the results demonstrate the accuracy and efficiency of the present method.

2. MATHEMATIC FORMULATION

The structure of a typical Terfenol-D actuator to be considered is shown in Figure 1. The Terfenol-D rod is a dominant element of the actuator. The mechanical action of the actuator or the rod vibration is excited by the periodic alternating current running through a coil. The input current produces an alternating magnetic field in the rod, which would induce the rod vibration due to the magnetostriction of the Terfenol-D material.

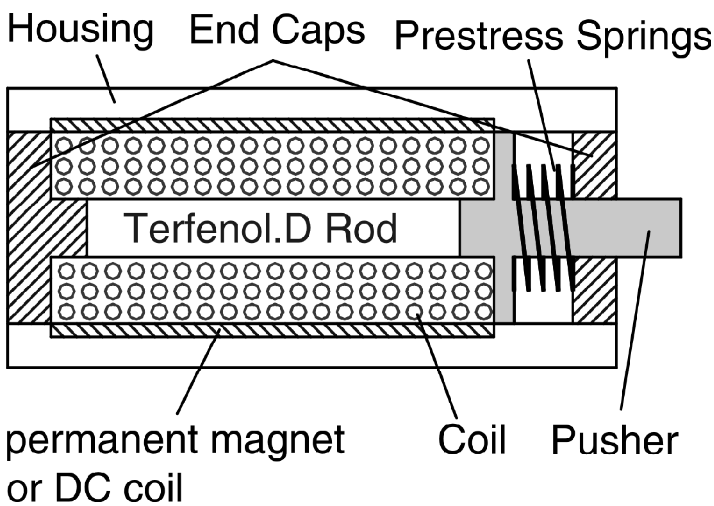

Figure 1: Schematic Section of a Typical Terfenol Actuator

Under the bias magnetic field H_b, applied by the permanent magnet and/or direct current (DC) coil, the induced magnetic field H in the rod is assumed to be uniform. Using the Ampere's law [3], we have the following formula:

$$H(t) = H_b + \frac{n}{l} i(t) \tag{1}$$

where n is the number of turns in the coil and l is the length of the rod, $i(t)$ is input alternating current with period T:

$$i(t + T) = i(t) \tag{2}$$

Based on the standard square law [1, 8], a simple one-dimensional constitutive relationship for the magnetostrictive material can be expressed as

$$\varepsilon = (1/E + rH^2)\sigma + mH^2, \; B = (\mu + m\sigma + r\sigma^2)H \tag{3}$$

where ε ($= \partial u / \partial x$) is the strain, σ the stress, and B the magnetic induction. The material parameters are the Young's modulus E, the magnetic permeability μ, the magnetostrictive modulus m, and the magnetoelastic coefficient r.

The mathematic model for the longitudinal vibration of the rod can be derived using the Hamilton's principle, as given below [10, 13]

$$\frac{\partial^2 u}{\partial t^2}=\frac{a^2}{c(t)}\frac{\partial^2 u}{\partial x^2}\ (0\le x\le l,\ t\ge 0) \tag{4}$$

$$u(x,\ 0)=u(x,T),\frac{\partial u}{\partial t}(x,0)=\frac{\partial u}{\partial t}(x,T)\ (0\le x\le l) \tag{5}$$

$$u(0,\ t)=0,\left[\frac{\partial u}{\partial x}+bc(t)u\right]_{x=l}=mH^2(t)-mH_b^2\ (t\ge 0) \tag{6}$$

where the coefficients: $a=\sqrt{E/\rho}$, $b = K_0 / (EA)$ and $c(t) = [1 + rEH^2 (t)]^2$ with r the density of rod, K_0 the pre-stress spring stiffness, A the cross-section area of the rod. It should be pointed out that the term $-mH_b^2$ in boundary condition (6) is to remove the static displacement produced by the bias magnetic field H_b.

Thus, mathematically the rod vibration problem is described by a boundary value problem (4)-(6) for the partial differential equation (4) with time periodicity condition (5). The difficulty to solve the problem comes from the variable coefficients in equation (4) and in boundary condition (6). Therefore, a new numerical method is proposed in the next section to solve the problem (4)-(6).

3. NUMERICAL METHOD

3.1. Numerical Integration

We introduce the velocity and the strain as new unknowns:

$$p=\frac{\partial u}{\partial t},q=\frac{\partial u}{\partial x} \tag{7}$$

Based on these new variables, the governing equation (4) can be equivalently written as the following system of first-order partial differential equations:

$$\begin{cases}\dfrac{\partial p}{\partial t}=\dfrac{a^2}{c(t)}\dfrac{\partial q}{\partial x}\\[2mm] \dfrac{\partial q}{\partial t}=\dfrac{\partial p}{\partial x}\end{cases} \tag{8}$$

To numerically solve the problem, we divide the space interval $x\in[0, l]$ into N subintervals with uniform length $h = l/N$. The space nodes are $0 = x_0 < x_1 < \cdots < x_{N-1} < x_N = l$. We then denote the unknowns at the node x_j for any time $t \ge 0$ as

$$p_j = p_j(t) \cong p(x_j, t),\ q_j = q_j(t) \cong q(x_j, t)\ (j = 0,1, \ldots, N) \tag{9}$$

Integrating the two sides of equation (8) with expect to x from x_{j-1} to x_{j+1}, we have

$$\begin{cases}\dfrac{\partial}{\partial t}\left[\displaystyle\int_{x_{j-1}}^{x_{j+1}}p(x,t)dx\right]=\dfrac{a^2}{c(t)}(q_{j+1}-q_{j-1})\\[2mm] \dfrac{\partial}{\partial t}\left[\displaystyle\int_{x_{j-1}}^{x_{j+1}}q(x,t)dx\right]=(p_{j+1}-p_{j-1})\end{cases}\quad (j=1,\cdots,N-1) \tag{10}$$

By applying Simpson's rule for integration to the above equation (10), we obtain a numerical approximation to equation (8) as follows:

$$\begin{cases}\dfrac{d}{dt}(p_{j-1}+4p_j+p_{j+1})=\dfrac{3a^2}{c(t)h}(q_{j+1}-q_{j-1})\\[2mm] \dfrac{d}{dt}(q_{j-1}+4q_j+q_{j+1})=\dfrac{3}{h}(p_{j+1}-p_{j-1})\end{cases}\quad (j=1,\cdots,N-1) \tag{11}$$

Similarly, using the trapezoidal rule of integrations to equation (8) from x_{N-1} to x_N and from x_0 to x_1, we get

$$\frac{d}{dt}(q_1 - q_0) = \frac{2}{h}(p_1 - p_0) \tag{12}$$

$$\frac{d}{dt}(p_N - p_{N-1}) = \frac{2a^2}{c(t)h}(q_N - q_{N-1}) \tag{13}$$

From the boundary conditions (6), we derive:

$$\frac{d}{dt} p_0 = 0 \tag{14}$$

$$\frac{d}{dt} q_N = -c(t) b\, p_N + \frac{\dot{c}(t)}{c(t)} q_N + g(t) \tag{15}$$

where the known function $g(t) = mH(t)[2\dot{H}(t) - H(t)\dot{c}(t)/c(t)]$.

The periodicity condition (5) becomes

$$p_j(0) = p_j(T),\ q_j(0) = q_j(T) \quad (j = 0, 1, \cdots, \mathrm{N}) \tag{16}$$

3.2. A System of First-order Ordinary Differential Equations

Introduce the following $2N + 2$ dimensions state vector function:

$$\mathbf{y}(t) = \left[y_1(t), y_2(t), \cdots, y_{2N+2}(t)\right]^{\mathrm{T}} = \left[p_0(t), p_1(t), \cdots, p_N(t), q_0(t), q_1(t), \cdots, q_N(t)\right]^{\mathrm{T}} \tag{17}$$

Thus, equations (11)-(15) and the periodicity condition (16) can be expressed as

$$\begin{cases} \dfrac{d\mathbf{y}}{dt} = \mathbf{A}(t)\mathbf{y} + \mathbf{f}(t) \quad (t > 0) \\ \mathbf{y}(0) = \mathbf{y}(T) \end{cases} \tag{18}$$

where the coefficient matrix is

$$\mathbf{A}(t) = \begin{bmatrix} \mathbf{O} & \mathbf{G}_1^{-1}\mathbf{B}_1(t) \\ \mathbf{G}_2^{-1}\mathbf{B}_2(t) & \mathbf{G}_2^{-1}\mathbf{B}_3(t) \end{bmatrix} \tag{19}$$

and the inhomogeneous term is

$$\mathbf{f}(t) = \begin{bmatrix} \mathbf{0}_{N+1} \\ \boldsymbol{\varphi}(t) \end{bmatrix} \quad \Big(\mathbf{0}_{N+1} = \underbrace{\left[0, 0 \cdots, 0\right]}_{N+1}^{\mathrm{T}}, \quad \boldsymbol{\varphi}(t) = \mathbf{G}_2^{-1} \underbrace{\left[0, 0 \cdots, g(t)\right]}_{N+1}^{\mathrm{T}}\Big) \tag{20}$$

The above sub-matrices have the following expressions

$$\mathbf{G}_1 = \begin{bmatrix} 1 & 0 & & & \\ 1 & 4 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 4 & 1 \\ & & & -1 & 1 \end{bmatrix}_{(N+1)\times(N+1)}, \quad \mathbf{G}_2 = \begin{bmatrix} -1 & 1 & & & \\ 1 & 4 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 4 & 1 \\ & & & 0 & 1 \end{bmatrix}_{(N+1)\times(N+1)} \tag{21}$$

$$\mathbf{B}_1(t) = \frac{a^2}{c(t)h}\begin{bmatrix} 0 & 0 & & & \\ -3 & 0 & 3 & & \\ & \ddots & \ddots & \ddots & \\ & & -3 & 0 & 3 \\ & & & -2 & 2 \end{bmatrix}_{(N+1)\times(N+1)}, \quad \mathbf{B}_2 = \frac{1}{h}\begin{bmatrix} -2 & 2 & & & 0 \\ -3 & 0 & 3 & & \\ & \ddots & \ddots & \ddots & \\ & & -3 & 0 & 3 \\ 0 & & & 0 & -c(t)bh \end{bmatrix}_{(N+1)\times(N+1)} \tag{22}$$

$$\mathbf{B}_3 = diag[0,\cdots,0,\dot{c}(t)/c(t)] \ (N+1)\times(N+1). \tag{23}$$

Therefore, the system of first-order ordinary differential equations (ODE) with the periodicity condition (18) is the numerical discretization in the space domain to the original problem (4)-(6).

3.3. Transfer Matrix Method and Broken-line Reduction

To find the numerical solution for problem (15), the transfer matrix method [18] and the *broken-line reduction* approach [17] are employed. First, the time domain for one period [0, T] is divided into M equal segments with temporal nodes at $t_{k-1} = (k-1)\,\Delta t$ $(k = 1, 2, \ldots, M)$ $\Delta t = T / M$. On the basis of ODE theory [19], the solution to equation (18) can be expressed as

$$\mathbf{y}(t) = \mathbf{X}_k(t)\left[\mathbf{y}(t_{k-1}) + \int_{t_{k-1}}^{t}\mathbf{X}_k^{-1}(\tau)\mathbf{f}(\tau)d\tau\right] \quad (t_{k-1} \le t < t_k) \tag{24}$$

where $\mathbf{X}_k(t)$ is the fundamental matrix of the following initial value problem:

$$\begin{cases} \dfrac{d}{dt}\mathbf{X}_k(t) = \mathbf{A}(t)\mathbf{X}_k(t) \quad (t_{k-1} \le t < t_k) \\ \mathbf{X}_k(t_{k-1}) = \mathbf{I}_{2N+2} \end{cases} \tag{25}$$

with I_{2N+2} being the identity matrix of dimension $2N$+2. Hence, the problem is reduced to find the fundamental matrix $\mathbf{X}_k(t)$ which satisfies (25).

Now, in each segment $[t_{k-1}, t_k)$ (k = 1, 2,..., M), using the *broken-line reduction* for the coefficient matrix $\mathbf{A}(t)$ and the inhomogeneous term $\mathbf{f}(t)$ in (25), we have approximations:

$$\begin{aligned} &\mathbf{A}(t) \cong \mathbf{A}_k^{(0)} + (t-t_{k-1})\mathbf{A}_k^{(1)} \quad (t_{k-1} \le t \le t_k), \quad \mathbf{A}_k^{(0)} = \mathbf{A}(t_{k-1}), \quad \mathbf{A}_k^{(1)} = [\mathbf{A}(t_k) - \mathbf{A}(t_{k-1})]/h \\ &\mathbf{f}(t) \cong \mathbf{f}_k^{(0)} + (t-t_{k-1})\mathbf{f}_k^{(1)} \quad (t_{k-1} \le t \le t_k), \quad \mathbf{f}_k^{(0)} = \mathbf{f}(t_{k-1}), \quad \mathbf{f}_k^{(1)} = [\mathbf{f}(t_k) - \mathbf{f}(t_{k-1})]/h \end{aligned} \tag{26}$$

Thus, in segment $[t_{k-1}, t_k)$ the solution to the initial value problem (25) is approximated as an L-order truncated power series:

$$\mathbf{X}_k(t) \cong \sum_{i=0}^{L}(t-t_{k-1})^i\,\mathbf{C}_k^{(i)} \quad (t_{k-1} \le t < t_k) \tag{27}$$

Substituting expressions (26) and (27) into the initial value problem (25), we therefore obtain the following recursive relations for the coefficient matrices in (27)

$$\begin{aligned} &\mathbf{C}_k^{(0)} = \mathbf{I}_{2N+2}, \quad \mathbf{C}_k^{(1)} = \mathbf{A}_k^{(0)} \\ &\mathbf{C}_k^{(i)} = [\mathbf{A}_k^{(0)}\mathbf{C}_k^{(i-1)} + \mathbf{A}_k^{(1)}\mathbf{C}_k^{(i-2)}]/i \quad (i = 2,\cdots,L), \quad (k = 1,2,\cdots,M) \end{aligned} \tag{28}$$

In order to calculate the inverse matrix $\mathbf{X}_k^{-1}(t)$ in expression (24), we make use of the identical relation $\mathbf{X}_k(t)\mathbf{X}_k^{-1}(t) = \mathbf{I}_{2N+2}$ and equations (25) to derive

$$\begin{cases} \dfrac{d}{dt}\mathbf{X}_k^{-1}(t) = -\mathbf{X}_k^{-1}(t)\mathbf{A}(t) \\ \mathbf{X}_k^{-1}(t_{k-1}) = \mathbf{I}_{2N+2} \end{cases} \tag{29}$$

Since this expression is similar to (25), the approximate solution of problem (29) is hence assumed as

$$\mathbf{X}_k^{-1}(t) \cong \sum_{i=0}^{L}(t-t_{k-1})^i\ \mathbf{D}_k^{(i)} \quad (t_{k-1} \le t < t_k) \tag{30}$$

with the recursive relations similar to (28) as follows:

$$\begin{aligned} &\mathbf{D}_k^{(0)} = \mathbf{I}_{2N+2}, \quad \mathbf{D}_k^{(1)} = -\mathbf{A}_k^{(0)} \\ &\mathbf{D}_k^{(i)} = -[\mathbf{D}_k^{(i-1)}\mathbf{A}_k^{(0)} + \mathbf{D}_k^{(i-2)}\mathbf{A}_k^{(1)}]/i \quad (i = 2,\cdots,L) \end{aligned} \tag{31}$$

Letting $t = t_k$ in solution (24), it follows that the transfer relation for the solution $\mathbf{y}(t)$ between the left and right nodes of segment $[t_k\text{-}1, t_k)$ is given by

$$\mathbf{y}(t_k) = \mathbf{T}_k[\mathbf{y}(t_{k-1}) + \mathbf{g}_k] \quad (k = 1,2, ..., M) \tag{32}$$

where the expressions of the segment transfer matrix $\mathbf{T}_k$ and the vector $\mathbf{g}_k$ ($k = 1,2,..., M$) are

$$\mathbf{T}_k = \mathbf{X}_k(t_k) = \sum_{i=0}^{L}(\Delta t)^i\ \mathbf{C}_k^{(i)}, \mathbf{g}_k = \int_{t_{k-1}}^{t_k}\mathbf{X}_k^{-1}(\tau)\,\mathbf{f}(\tau)d\tau = \sum_{i=0}^{L}\frac{(\Delta t)^{i+1}}{i+2}\mathbf{D}_k^{(i)}[\mathbf{f}(t_k) + \frac{1}{i+1}\mathbf{f}(t_{k-1})] \tag{33}$$

By making use of the transfer relation (24) iteratively, we find that

$$\mathbf{y}(T) = \mathbf{Z}_N\mathbf{y}(0) + \sum_{\alpha=1}^{N}\mathbf{Z}_\alpha\mathbf{g}_{N+\alpha-1},\ \mathbf{Z}_\alpha = \mathbf{T}_N\,\mathbf{T}_{N-1}\cdots\mathbf{T}_{N-\alpha+1},\ \mathbf{Z}_N = \mathbf{T}_N\,\mathbf{T}_{N-1}\cdots\mathbf{T}_1 \tag{34}$$

Moreover, introducing the periodicity condition in (18) into relation (34), we obtain the initial value of the state vector function:

$$\mathbf{y}(0) = [\mathbf{I}_{2N+2} - \mathbf{Z}_N]^{-1}\sum_{\alpha=1}^{N}\mathbf{Z}_\alpha\mathbf{g}_{N+\alpha-1} \tag{35}$$

Then, from the recursive relation (32) with the initial value (35), we can calculate the state vector function $\mathbf{y}(t_k)$ ($k = 0, 1, ..., M$) at any time node. Hence, by using the trapezoidal rule in numerical integration, we finally obtain the approximate values of the displacement at different space and time nodes as:

$$u(x_j, t_k) = \sum_{i=1}^{j}\int_{x_{i-1}}^{x_i} q(s,\, t_k)ds = \frac{h}{2}\sum_{i=1}^{j}\left[y_{N+2+i}(t_k) + y_{N+1+i}(t_k)\right]\,(j = 0,1,\cdots,N;\, k = 0,1,\,\cdots,M) \tag{36}$$

4. NUMERICAL RESULTS AND CONCLUSIONS

As a numerical example, periodic vibration of a magnetostrictive rod in Terfenol-D actuator is analyzed. The geometric parameters of the rod are: Length $l = 0.1$m and cross-section area $A = 4.91\times10^{-4}$ m^2 (the radius $R = 0.0125$m). The material parameters of Terfenol-D are: Young's modulus $E = 26.5$GPa and mass density $\rho = 9250$kg/m^3. The turn number of the coil $n = 700$ and the stiffness of the pre-stress spring $K_0 = 3.24\times10^5$ N/m. The applied bias magnetic field is $H_b = 20$ kA m^{-1} [21]. The piezomagnetic coefficient of the material $d = 1.37\times10^{-8}$ mA^{-1}, which is calculated from the test data in [21]. Thus, using the formula in [8], $d = 2H_bm$, we found that the magnetostrictive modulus $m = 0.34\times10^{-12}$m^{2}A^{-2}. The magnetoelastic coefficient is taken from reference [8]: $r = -2.77\times10^{-20}$m^{2}A^{-2}Pa^{-1}, and the input alternating current is $i(t) = i_{\max}\sin\omega t$.

In the numerical computation, excellent convergence is observed when the space subinterval number $N \ge 15$ and time segment number $M \ge 40$ with a truncated order $L \ge 6$. Figure 2 shows the response curves of the pusher end displacement in the range of three periods for different values of the bias magnetic fields $H_b = 0$, 10 kA m^{-1}, 20 kA m^{-1} with fixed alternating current frequency $f = 10$Hz and peak current $i_{max} = 0.5$A. It is noted that the frequency of displacement response obtained is twice that of the exciting current, which is so-called the *double frequency effect*. Such an effect was actually observed in the experiment for Terfenol-D actuator [20], and was numerically demonstrated previously [11~13]. It is clear from Figure 2 that the double frequency effect appears in the absence of the bias magnetic field; however the output displacement is synchronized with the input current when the bias magnetic field is applied.

Figure 3 displays the peak displacements versus current frequency for various peak current values. The curves in figure 3 are flat within a large range of the frequency. In other words, the peak displacements remain constant, which implies that the actuator may be provided with higher work frequency. Also in Figure 3 the present numerical solutions are compared with both experimental data available in [21] and numerical results in [13]. It is obvious that our solutions are in good agreement with both the experimental and other numerical results.

The relations between the peak displacement output at the pusher end and the peak current input are shown in Figure 4. These curves indicate that the relation between these two physical quantities is nonlinear at low bias magnetic field, but becomes linear with increasing bias magnetic field.

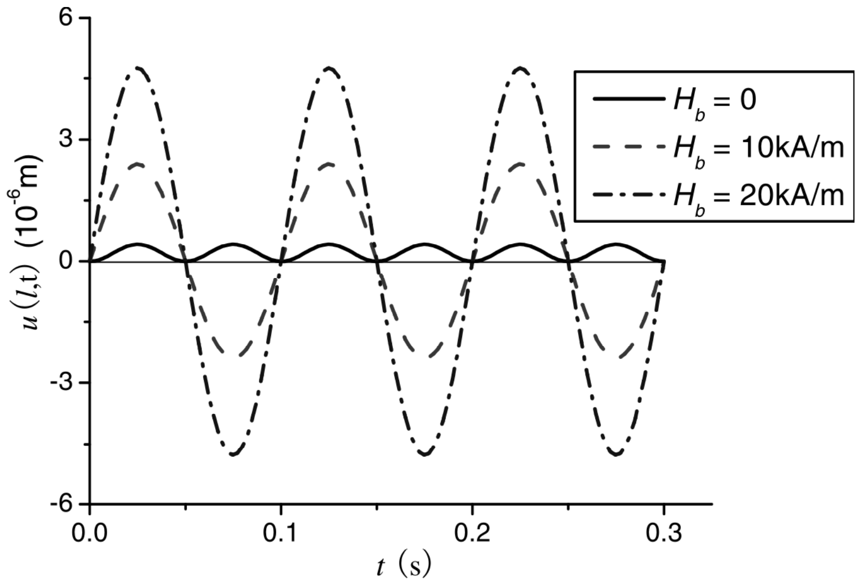

Figure 2: Displacement Responses at the Pusher End verses Time for Different Values of the Bias Magnetic Fields. The Current Frequency is Fixed at f = 10Hz and the Peak Current is at i_{max}= 0.5A.

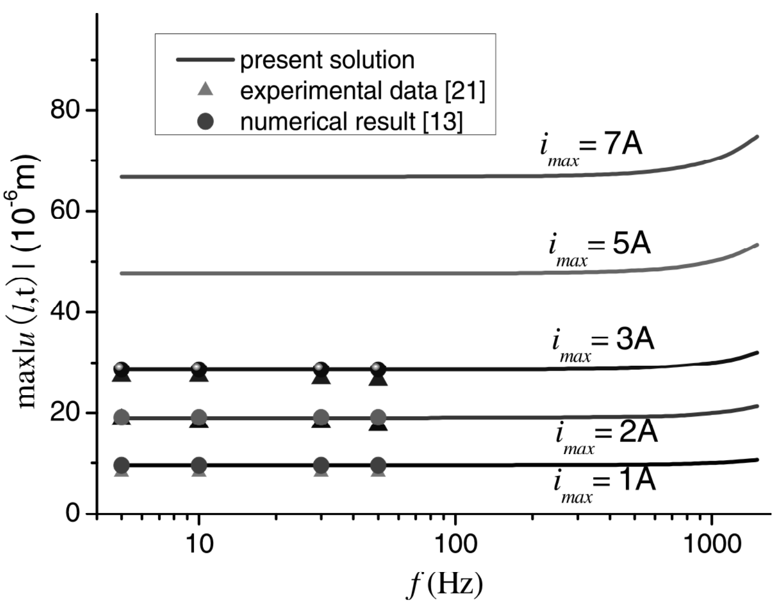

Figure 3: Peak Displacement Output versus Current Frequency for Various Peak Current Values, as Compared with Experimental Data and other Numerical Results

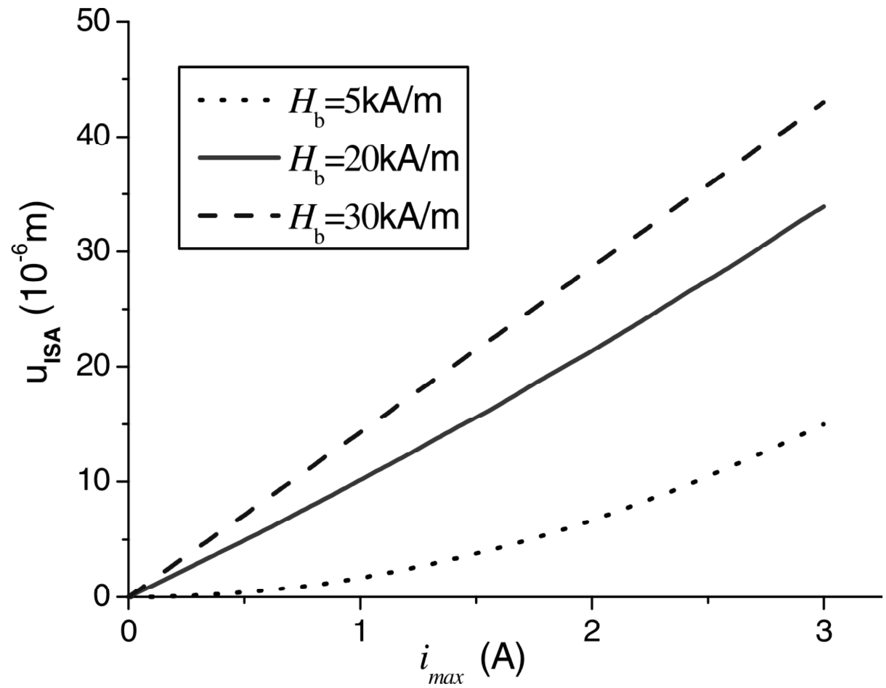

Figure 4: Peak Displacement Output at the Pusher End versus Peak Current Input

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