IJMS, *Vol. 19, No. 2, (June 2020), pp. 245-265*

ISSN: 0972-754X

EFFECTS OF RADIATION AND CHEMICAL REACTION ON MHD FREE CONVECTIVE FLOW ON VERTICAL PLATE IN PRESENCE OF HALL CURRENTS

Bhaskar Chandra Sarkar

Abstract: In the present paper, an analysis is carried out to investigate effects of Radiation and chemical reaction on an unsteady magnetohydrodynamic free convective flow past an exponentially accelerated infinitely long vertical plate with variable temperature and concentration in the presence of Hall Currents. The non-dimensional governing equations have been solved analytically with the help of transformed boundary conditions and the expressions are found for velocity, temperature, concentration fields, rate of heat and mass transfer and shear stress at the plate with the help of Graphs and tables. It is interesting to see that the fluid velocity distributions for externally cooled plate are in reverse order with the case of that for externally heated plate. A good agreement is found with earlier results.

Keywords: MHD, Hall current, thermal and mass Grashof number, acceleration parameter, chemical reaction parameter and radiation

1. INTRODUCTION

Frequently, transformations proceed in a moving fluid, a situation encountered in a number of technological fields. In many scientific and environmental processes, such as astrophysical flows, water evaporation from open reservoirs, heating and cooling of chambers and solar power technology the Radiative and convective flows are used. The hydrodynamic flow of an electrically conducting viscous incompressible fluid has gained considerable attention because of its numerous applications in physics and engineering. Simultaneous heat and mass transfer from different geometrics has many engineering and geophysical applications such as geothermal reservoirs, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling nuclear reactors, underground energy transport, the modeling of packed sphere beds, chemical catalytic reactors, grain storage devices fiber and granular insulation, missiles, combustion and furnace design, petroleum reservoirs, coal combustors, ground water pollution and filtration processes. In an ionized gas where the density is low and/or the magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both the electric and the magnetic fields. This current, well known in the literature, is called the Hall currents. Due to Hall currents the electrical conductivity of the fluid becomes anisotropic and this causes the secondary flow. Hall effect is important when the Hall parameter, which is the ratio between the electron-cyclotron frequency and the electron-atom-collision frequency, is high. This happens when the magnetic field is high or when the collision frequency is low. Hall currents are of great importance in many astrophysical problems, Hall accelerator and flight MHD as well as flows of plasma in a MHD power generator.

Free convective flow past an exponentially accelerated vertical plate has been described by [Singh and Kumar (1984) and Hossain and Shayo (1986)]. [Hossain and Takhar (1996)] have investigated the radiation effect on mixed convection along a vertical plate with an uniform surface temperature. [Raptis and Perdikis (1999)] have presented the radiative free convective flow past an infinitely long moving vertical plate. [Alam et al. (2006a, 2006b)] have studied the Dufour and Soret effects on MHD free convection and mass transfer flow past a vertical plate in a porous medium. They have discussed both steady and unsteady cases. Diffusion-thermo and thermal-diffusion effects on free convective heat and mass transfer flow with time dependent temperature and concentration have been investigated by [Alam et al. (2007)]. Thermal radiation effect on a transient MHD flow with mass transfer past an impulsively fixed infinitely long vertical plate has been described by [Alam and Sarmah (2009)]. [Rajesh and Varma (2009)] have presented the radiation and mass transfer effects on an MHD free convective flow past an exponentially accelerated vertical plate with variable temperature. [Muthucumaraswamy et al. (2009)] have studied an unsteady flow past an accelerated infinitely long vertical plate with variable temperature and uniform mass diffusion. [Muthucumaraswamy et al. (2009)] have also discussed the heat and mass transfer effects on a flow past an accelerated vertical plate with variable mass diffusion. Effects of transversely magnetic field on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate have been studied by [Soundalgekar et al. (1981)]. [Soundalgekar and Takhar (1992)] have studied effect of radiation on the natural convection flow of a gas past a semi-infinite plate using the CoglyVincentine-Gilles equilibrium model. The combined effects of heat and mass transfer by mixed convection MHD flow along a porous plate with chemical reaction in the presence of heat source have been depicted by [Ahamed and Zueco (2010)]. [Rajesh and Varma (2010)] have investigated the radiation effects on MHD flow with variable temperature or variable mass diffusion. [Vijaya and Verma (2011)] have described the radiation effects on an MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature. Radiation and darcy effects on an unsteady MHD heat and mass transfer flow of a chemically reacting fluid past an impulsively started vertical plate have been investigated by [Suneetha and Bhaskar (2011)]. [Manivannan et al. (2009)] have investigated the Radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion. Effects of radiation and heat transfer on flow past an exponentially accelerated vertical plate have been studied by [Mandal et al. (2011)]. [Das et al. (2011)] have presented the radiation effect on natural convection past a vertical plate with ramped wall temperature. [Pattnaik et al. (2012)] have described the radiation and mass transfer effects on an MHD free convective flow past an exponentially accelerated vertical plate with variable temperature. [Jana et al. (2012)] have discussed the radiation effects on an unsteady MHD free convective flow past an exponentially accelerated vertical plate with viscous and Joule dissipations into account. [Kishore et al. (2013)] have studied the effects of radiation and chemical reaction on an unsteady MHD free convective flow of a viscous fluid past an exponentially accelerated vertical plate. Diffusion-thermo and radiation effects on an unsteady MHD flow past an impulsively started infinitely long vertical plate with variable temperature and mass diffusion have been investigated by [Prakash et al. (2013)]. Radiation and mass transfer effects on MHD free convection flow past a vertical plate with variable temperature and concentration have been described by [Ahmmed et al. (2013)]. [Chandrakala and Bhaskar (2014)] have studied the radiation effects on an MHD flow past an impulsively started infinitely long vertical plate with mass diffusion. Hall current effects on an unsteady free convective flow past a vertical plate in the presence of radiation and thermal diffusion have been investigated by [SravanKumar et al. (2015)]. Hall current effects on unsteady MHD flow between stretching sheet and an oscillating porous upper parallel plate with constant suction have been studied by [Raju et al. (2011)]. [Sarkar et al. (2016)] have discussed the effects of diffusion-thermo and thermal radiation of an optically thick gray gas in the presence of magnetic field embedded in porous medium. Recently, effects of mass transfer with chemical reaction on MHD convective flow in a hot vertical channel with thermal radiation have been described by [Vidya et al. (2020)].

The aim of the present paper is to study the effect of Hall currents and radiation on MHD natural convective flow of a viscous incompressible electrically conducting fluid past a moving vertical plate with variable temperature and concentration. The governing equations are normalized and reduced to a set of ordinary differential equation and then solved using the help of transformed boundary conditions. The results are discussed with the help of graphs and tables.

FORMULATION OF THE PROBLEM AND ITS SOLUTION:

Consider an unsteady magnetohydrodynamic free convective flow of a viscous incompressible electrically conducting fluid past an infinitely long vertical flat plate with variable temperature and mass diffusion. The convection current is induced due to both the temperature and concentration differences. Choose a cartesian co-ordinates system such that the x and z-axes are taken at the plane of the plate with x-axis is in the vertically upward direction and the y-axis is normal to the plane of the plate in the fluid [See Fig.1]. Initially, at time $t \le 0$, both the plate and the fluid are assumed to be at the same temperature T_∞ with concentration level C_∞. At time $t > 0$, the plate at $y = 0$ starts to move in its own plane with a velocity $u_0 e^{a't}$, the temperature of the plate and the concentration level are raised to $T_\infty + (T_w - T_\infty)e^{a't}$ and $C_\infty + (C_w - C_\infty)e^{a't}$ respectively, T_w being the temperature of the plate and C_w the concentration of the fluid near the plate. A uniform transverse magnetic field of strength B_0 is applied upright to plane of plate.

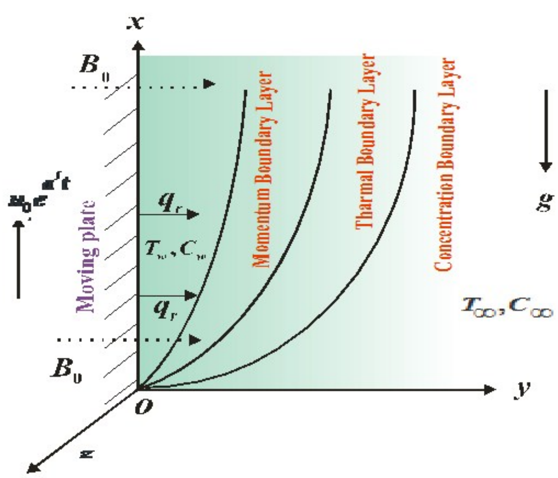

Fig.1: Geometry of the problem

Generalized Ohm's law on taking Hall current into account is [see [Cowling (1957)]]

$$\vec{j} + \frac{\omega_e \tau_e}{B_0}\left(\vec{j} \times \vec{B}\right) = \sigma\left(\vec{E} + \vec{q} \times \vec{B}\right), \tag{1}$$

where $\vec{q}$, $\vec{B}$, $\vec{E}$, $\vec{j}$, σ, ω_e and τ_e are respectively the velocity vector, the magnetic field vector, the electric field vector, the current density vector, electric conductivity, cyclotron frequency and electron collision time. In writing the equation (1), the ion-slip and the thermoelectric effects as well as the electron pressure gradient are neglected.

The equation of continuity $\nabla \cdot \vec{q} = 0$ with no-slip condition at the plate gives $v = 0$ everywhere in the flow where $\vec{q} \equiv (u, v, w)$, u, v and w are respectively velocity components along the coordinate axes. The solenoidal relation $\nabla \cdot \vec{B} = 0$ gives

$B_y = \text{constant} = B_0$ everywhere in the flow where $\vec{B} \equiv (0, B_y, 0)$. The conservation of electric current $\nabla \cdot \vec{j} = 0$ yields $j_y =$ constant where $\vec{j} \equiv \left(j_x, j_y, j_z\right)$. This constant is zero since $j_y = 0$ at the plate which is electrically non-conducting. Hence, $j_y = 0$ everywhere in the flow. As the induced magnetic field is neglected, Maxwell's equation $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$ becomes $\nabla \times \vec{E} = 0$ which gives $\dfrac{\partial E_x}{\partial y} = 0$ and $\dfrac{\partial E_z}{\partial y} = 0$. This implies that $E_x =$ constant and $E_z =$ constant everywhere in the flow.

In view of the above assumption, equation (1) yields

$$j_x - mj_z = \sigma(E_x - wB_0), \tag{2}$$

$$j_z + mj_x = \sigma(E_z + uB_0), \tag{3}$$

where $m = \omega_e \tau_e$ is the Hall parameter.

As $y \to \infty$, $j_x \to 0$, $j_z \to 0$, since the magnetic field is uniform at infinity. Also, $u \to 0$, $w \to 0$ as $y \to \infty$. Using these conditions, equations (2) and (3) give $E_x = 0$ and $E_y = 0$ everywhere in the flow.

Solving for j_x and j_z from (2) and (3), on taking $E_x = E_y = 0$, we have

$$j_x = \frac{\sigma B_0}{1+m^2}\left(mu - w\right), \tag{4}$$

$$j_z = \frac{\sigma B_0}{1+m^2}\left(mw + u\right). \tag{5}$$

On the use of (4) and (5) and under the usual Boussinesq approximation, the governing boundary layer equations of the fluid are

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0{}^2}{\rho(1+m^2)}\left(mw + u\right), \tag{6}$$

$$\frac{\partial w}{\partial t} = \nu \frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_0{}^2}{\rho(1+m^2)}\left(w - mu\right), \tag{7}$$

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \tag{8}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} - k_1(C - C_\infty), \tag{9}$$

where T is the temperature of the fluid, C the concentration in the fluid, D the mass diffusivity, t the time, g the acceleration due to gravity, ν the kinematic viscosity, ρ the fluid density, β the thermal expansion coefficient, β^* the concentration expansion coefficient, c_p the specific heat at constant pressure, k the thermal conductivity, k_1 the reaction rate constant and q_r the radiative heat flux. For small velocities the heating due to viscous dissipation is neglected in the energy equation (8).

The boundary conditions are

$$t > 0 : u = u_0 e^{a' t},\ w = 0,\ T = T_\infty + (T_w - T_\infty)e^{a' t},\ C = C_\infty + (C_w - C_\infty)e^{a' t} \text{ at } y = 0$$
$$t > 0 : u \to 0,\ w \to 0,\ T \to T_\infty,\ C \to C_\infty \text{ as } y \to \infty \tag{10}$$

The radiative heat flux can be found from Rosseland approximation [see [Siegel and Howell(2002)]] and its formula is derived from the diffusion concept of radiative heat transfer in the following way

$$q_r = -\frac{4\sigma^*}{3k_R}\frac{\partial T^4}{\partial y}, \tag{11}$$

where σ^* is the Stefan-Boltzman constant and k_R the Rosseland mean absorption coefficient of the medium. It should be noted that by using the Rosseland approximation the present analysis is limited to optically thick fluids. If the temperature differences within the flow are sufficiently small, then the equation (11) can be linearized by expanding T^4 into the Taylor series about T_∞ as follows:

$$T^4 = T_\infty^4 + 3T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 + \cdots \tag{12}$$

from which neglecting higher order terms to give

$$T^4 = 4T_\infty^3 T - 3T_\infty^4. \tag{13}$$

In view of (11) and (13), equation (8) becomes

$$\rho c_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k_R}\frac{\partial^2 T}{\partial y^2}. \tag{14}$$

Introducing non-dimensionless variables

$$\eta = \frac{yu_0}{\nu},\ \tau = \frac{tu_0^2}{\nu},\ (u_1, w_1) = \frac{(u, w)}{u_0},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ \varphi = \frac{C - C_\infty}{C_w - C_\infty}, \tag{15}$$

equations (6), (7), (14) and (9) become

$$\frac{\partial u_1}{\partial \tau} = \frac{\partial^2 u_1}{\partial \eta^2} + Gr\theta + Gc\varphi - \frac{M^2}{1 + m^2}\left(mw_1 + u_1\right), \tag{16}$$

$$\frac{\partial w_1}{\partial \tau} = \frac{\partial^2 w_1}{\partial \eta^2} - \frac{M^2}{1 + m^2}\left(w_1 - mu_1\right), \tag{17}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{Pr}\left(1 + \frac{4}{3R}\right)\frac{\partial^2 \theta}{\partial \eta^2}, \tag{18}$$

$$Sc\frac{\partial \varphi}{\partial \tau} = \frac{\partial^2 \varphi}{\partial \eta^2} - K\varphi, \tag{19}$$

where $M^2 = \frac{\sigma B_0^2 \nu}{\rho u_0^2}$ is the magnetic parameter, $R = \frac{k k_R}{4\sigma^* T_\infty^3}$ the radiation parameter, $Pr = \frac{\rho \nu c_p}{k}$ the Prandtl number, $Gr = \frac{g\beta(T_w - T_\infty)\nu}{u_0^3}$ the thermal Grashof number, $Gc = \frac{g\beta^*(C_w - C_\infty)\nu}{u_0^3}$ the mass Grashof number, $Sc = \frac{\nu}{D}$ Schmidt number, $a = \frac{a'\nu}{u_0^2}$ the accelerating parameter and $K = \frac{k_1 \nu^2}{Du_0^2}$ the chemical reaction parameter.

The boundary conditions for u_1, w_1, φ and θ are

$$t > 0 : u_1 = e^{a\tau},\ w_1 = 0,\ \varphi = e^{a\tau},\ \theta = e^{a\tau}\ \text{ at }\ \eta = 0 \tag{20}$$

$$t > 0 : u_1 \to 0,\ w_1 \to 0,\ \varphi \to 0,\ \theta \to 0\ \text{ as }\ \eta \to \infty$$

Combining equations (16) and (17), we get

$$\frac{\partial f}{\partial \tau} = \frac{\partial^2 f}{\partial \eta^2} + Gr\theta + Gc\varphi - \left[\frac{M^2}{1 + m^2}(1 - im) + \frac{1}{k_1}\right]f, \tag{21}$$

where

$$f = u_1 + iw_1 \ \text{ and } \ i = \sqrt{-1}. \tag{22}$$

The boundary conditions become

$$t > 0 : f = e^{a\tau}, \ \varphi = e^{a\tau}, \ \theta = e^{a\tau} \ \text{ at } \ \eta = \mathbf{0} \tag{23}$$

$$t > 0 : f \to 0, \ \varphi \to 0, \ \theta \to 0 \ \text{ as } \ \eta \to \infty$$

Using transformed boundary conditions, we take

$$f = f_0 e^{a\tau}, \ \theta = \theta_0 e^{a\tau} \ \text{ and } \ \varphi = \varphi_0 e^{a\tau}. \tag{24}$$

On the use of (24), equations (21), (18) and (19) become

$$f_0'' - \left[\frac{M^2}{1+m^2}(1 - im) + a\right] f_0 + Gr\theta + Gc\varphi = 0, \tag{25}$$

$$\theta_0'' - a\alpha\theta_0 = 0, \tag{26}$$

$$\varphi_0'' - (K + aSc)\varphi_0 = 0, \tag{27}$$

where

$$\alpha = \frac{3RPr}{3R+4}. \tag{28}$$

Corresponding boundary conditions become

$$t > 0 : f_0 = 1, \ \varphi_0 = 1, \ \theta_0 = 1 \ \text{ at } \ \eta = 0 \tag{29}$$

$$t > 0 : f_0 \to 0, \ \varphi_0 \to 0, \ \theta_0 \to 0 \ \text{ as } \ \eta \to \infty$$

Solutions of equations (25)-(27) subject to the conditions (29) are easily obtained and are given by

$$f_0 = \left(1 + \frac{Gr}{\lambda_2 - \lambda_1^2} + \frac{Gc}{\lambda_3 - \lambda_1^2}\right) e^{-\lambda_1 \eta} - \frac{Gr}{\lambda_2 - \lambda_1^2} e^{-\sqrt{\lambda_2}\,\eta} - \frac{Gc}{\lambda_3 - \lambda_1^2} e^{-\sqrt{\lambda_3}\,\eta}, \tag{30}$$

$$\theta_0 = e^{-\sqrt{\lambda_2}\,\eta}, \tag{31}$$

$$\varphi_0 = e^{-\sqrt{\lambda_3}\,\eta} \tag{32}$$

where

$$\lambda_1 = \alpha_1 - i\beta_1,\ \lambda_2 = a\alpha,\ \lambda_3 = K + aSc,$$

$$\alpha_1, \beta_1 = \frac{1}{\sqrt{2}}\left[\left\{\left(\frac{M^2}{1+m^2}+a\right)^2 + \frac{m^2M^4}{(1+m^2)^2}\right\}^{\frac{1}{2}} \pm \left(\frac{M^2}{1+m^2}+a\right)\right]^{\frac{1}{2}}. \tag{33}$$

Hence the solutions for the velocity field, temperature and concentration distributions are as follows

$$f = \left(1 + \frac{Gr}{\lambda_2-\lambda_1^2} + \frac{Gc}{\lambda_3-\lambda_1^2}\right)e^{a\tau-\lambda_1\eta} - \frac{Gr}{\lambda_2-\lambda_1^2}e^{a\tau-\sqrt{\lambda_2}\,\eta} - \frac{Gc}{\lambda_3-\lambda_1^2}e^{a\tau-\sqrt{\lambda_3}\,\eta}, \tag{34}$$

$$\theta = e^{a\tau-\sqrt{\lambda_2}\,\eta}, \tag{35}$$

$$\varphi = e^{a\tau-\sqrt{\lambda_3}\,\eta} \tag{36}$$

where λ_1, λ_2 and λ_3 are given by (33).

If $m = 0$ and $K = 0$ then the equations (34)-(36) are similar type with the equations (16)-(18) of [Ahmmed et al. (2013)].

RESULTS AND DISCUSSION:

The uneven temperature of the plate shifts in time & space, which is hydrodynamic convective flow is analyzed. An accurate solution of the problem is attained in the presence of transverse magnetic field. In order to get a physical insight into the flow problem, analytic solutions are conducted for various values of the parameters such as magnetic parameter M^2, Hall parameter m, accelerating parameter a, thermal Grashof number Gr, mass Grashof number Gc, chemical reaction parameter K, Prandtl number Pr, radiation parameter R, Schmidt number Sc and time τ that describe the flow characteristics and the results are illustrated either graphically or in tabular form. Here we restrict our discussion to the aiding of favourable case only. To be realistic, the values of Schmidt number Sc are chosen for hydrogen $(Sc = 0.22)$, water vapour $(Sc = 0.6)$, ammonia $(Sc = 0.78)$ and Ethyl benzene $(Sc = 2.01)$ at temperature $25^0\,C$

and one atmospheric pressure. The value of the Prandtl number is chosen to be $Pr = 0.71$, which represents air at $25^0 C$ and one atmospheric pressure. The velocity profiles are presented in Figs.2-11 for the cases of cooling $(Gr > 0, Gc > 0)$ and heating $(Gr < 0, Gc < 0)$ of the plate. The cooling and heating take place by setting up free convection current due to temperature and concentration gradient. $Gr = 0$ and $Gc = 0$ corresponds the absence of free convection current. The cooling problem is often encountered in engineering applications. For example in the cooling of electronic components and nuclear reactors. We have made use of the following parameter values $M^2 = 5$, $m = 0.5$, $Sc = 0.22$, $a = 0.4$, $K = 10$, $R = 10$ and $\tau = 0.5$. These will be the default values of such parameters in this work.

EFFECTS OF PARAMETERS ON THE VELOCITY PROFILES:

It is seen from Fig.2 that both the primary and secondary fluid velocity components u_1 and w_1 decrease with an increase in magnetic parameter M^2 in case of cooling of the plate. It is due to the presence of magnetic field normal to the flow in an electrically conducting fluid introduces a Lorentz force which acts against the flow and hence tends to reduce the fluid flow velocities. For an externally heated plate (see Fig.3) the results are observed in reverse order. It is observed from Fig.4 that for an externally cooled plate the fluid velocity components u_1 and w_1 increase with an increase in Hall parameter m. It means that Hall current has an accelerating influence on the velocity components. In the momentum equation (7), the term $-\frac{M^2}{1+m^2}\left(w_1 - mu_1\right)$ contributes effectively to the primary velocity u_1, according to the expression $\frac{mM^2}{1+m^2}u_1$ indicating that a rise in m causes a direct rise in the hydromagnetic adding body force term affecting the primary velocity u_1. On the other hand, the Hall parameter m has a marked effect on the secondary velocity profiles. This is because the effective conductivity $\frac{\sigma}{1+m^2}$ decreases as m increases. Thus, an increasing in Hall parameter reduecs the resistive Lorentz force and the motion of the fluid particles is reinforced and hence the secondary velocity component is enhanced. This is a new phenomenon, which appears as a result of including the Hall term. The case $m = 0$ corresponds to the neglect of the Hall effects. The mechanism by which Hall currents influence hydromagnetic channel flow (whether translational or rotational) is therefore via secondary effects and coupling in the momentum equations. This has important applications in practical MHD energy systems where better performance or control can be achieved of flows using Hall currents. For an externally heated plate (see Fig 5) the results are observed in reverse order. It is seen from Fig.6 that for an externally cooled plate the fluid velocity components u_1 and w_1 increase with an increase in mass Grashof number Gc. The mass Grashof number Gc defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, it is observed that there is a rise in the fluid velocity due to the enhancement of the species buoyancy force and this is clearly reflected in the progressive increase in the velocity of the flow. For an externally heated plate (see Fig.7) the results are observed in reverse order. The thermal Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. There is a rise in the fluid velocity components u_1 and w_1 due to the enhancement of the thermal buoyancy force which is elucidated from Fig.8. For an externally heated plate (see Fig.9) the results are observed in reverse order. It is interesting to note that near the plate in the vicinity of the boundary layer, the magnitude of the flow velocity is considerably high and gradually and uniformly decrease thereafter and then decays to zero asymptotically as $\eta \to \infty$ for either an externally cooled plate or an externally heated plate. It is also interesting to see that the value of the fluid velocity components become negative in most of the region for an externally heated plate which indicates that there occurs a reverse flow at that region due to the occurrence of heating of the plate in the presence of thermal and concentration buoyancy effects. The occurrence of primary flow has a good agreement with the figures of [Ahmmed et al. (2013)]. A good agreement is found with earlier results.

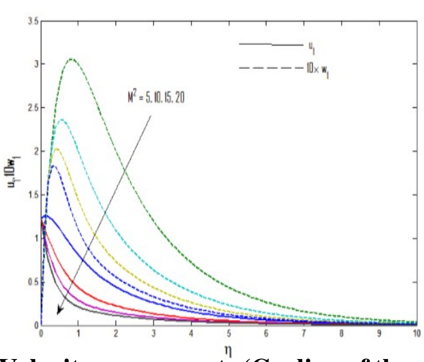

Fig.2: Velocity components (Cooling of the plate) for M^2 when $Gr = 5$ and $Gc = 5$

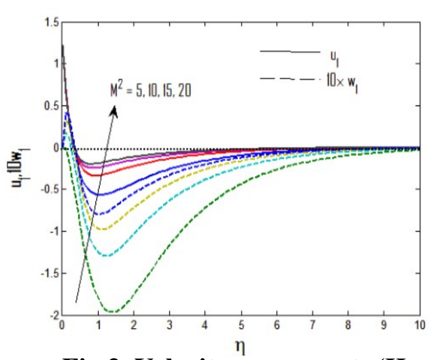

Fig.3: Velocity components (Heating of the plate) for M^2 when $Gr = -5$ and $Gc = -5$

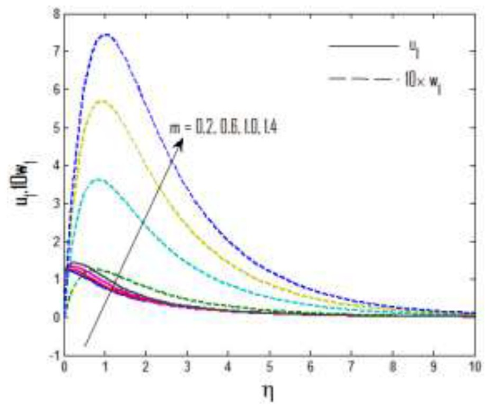

Fig.4: Velocity components (Cooling of the plate) for m **when** Gr = 5 **and** Gc = 5

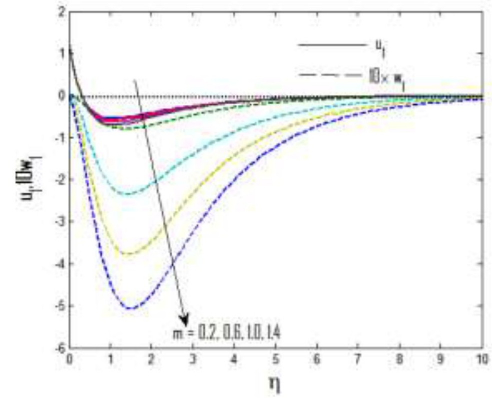

Fig.5: Velocity components (Heating of the plate) for m **when** Gr = –5 **and** Gc = –5

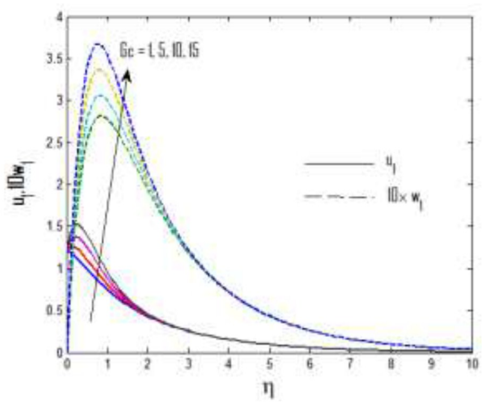

Fig.6: Velocity components (Cooling of the plate) for Gc **when** Gr = 5

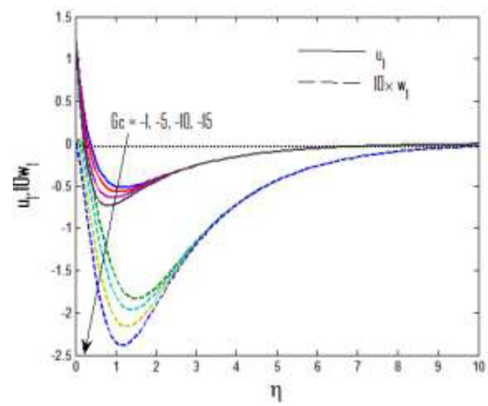

Fig.7: Velocity components (Heating of the plate) for Gc **when** Gr = –5

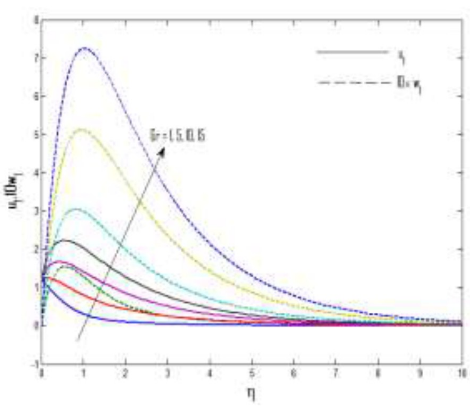

Fig.8: Velocity components (Cooling of the plate) for Gr **when** Gc = 5

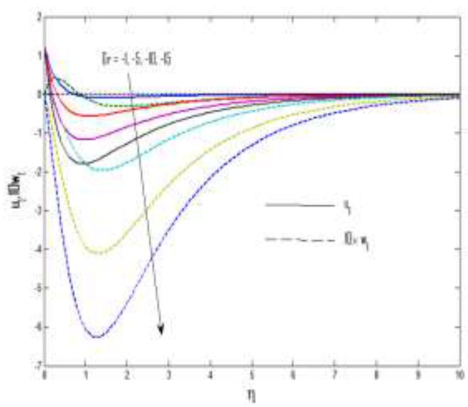

Fig.9: Velocity components (Heating of the plate) for Gr **when** Gc = -5

EFFECTS OF PARAMETERS ON THE TEMPERATURE DISTRIBUTION:

It is observed from Fig.10 that the fluid temperature θ sketches drop with rise in radiation parameter R. This result qualitatively agrees with expectations, since the radiation causes a faster dissipation of heat and consequently lowers the temperature. Initially for lower values of the radiation parameter R, the heat transfer is dominated by conduction, as the values of R increases the radiation absorption in boundary layer increases. i.e., an increase in the radiation parameter R results, decrease in the temperature. This can be mathematically explained as a decrease in radiation parameter for given and means a decrease in Rosseland radiation absorption coefficient . Since divergence of the radiative heat flux increases, decreases which in turn causes to increase the rate of radiative heat transfer at the plate and hence the fluid temperature decreases. This means the thermal boundary layer decreases and more uniform temperature distribution across the boundary layer. This phenomenon is of interest in very high temperature (e.g. glass) flows in the mechanical and chemical process industries and is currently under investigation. Further, it is seen from Fig.10 that the temperature profile approaches its classical values when the Prandtl number becomes large. The thermal conductivity of fluid decreases with an increase in , resulting a decrease in thermal boundary layer thickness and the heat is able to diffuse away from the heated surface. Therefore, thermal diffusion has a tendency to reduce the fluid temperature. Fig.11 shows that an increase in time leads to rise in both the fluid temperature distribution and concentration distribution . The chemical reaction reduces the local concentration, thus increasing its concentration gradient and its flux. As seen from the Fig.12, an increase in chemical reaction parameter causes a decrease in the concentration of the chemical species in the boundary layer. Further, Fig.12 shows that the concentration sketches drop with rise in Schmidt number . Physically, it is true since increase of means decrease of molecular diffusivity which results in decreasing of concentration boundary layer. Hence, the concentration of species is higher for small values of . The profiles have the common feature that the maximum values of temperature and concentration distributions are found at the surface and then decrease in a monotone fashion from the surface to a zero value far away in the free stream.

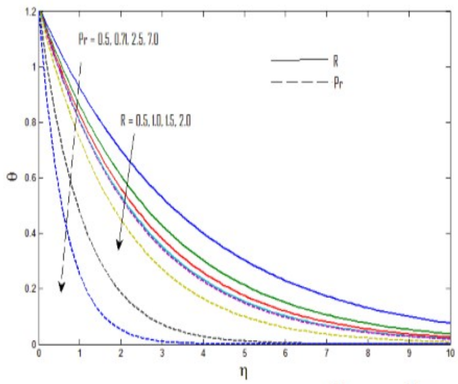

Fig.10: Temperature profiles for R and Pr

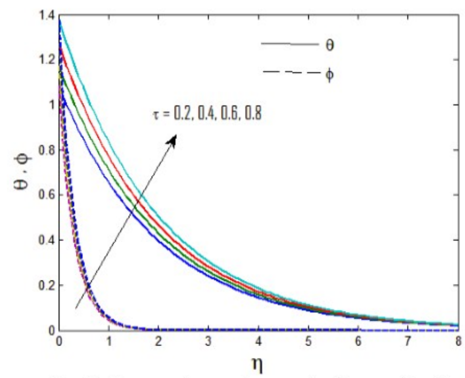

Fig.11: Temperature and concentration profiles for τ

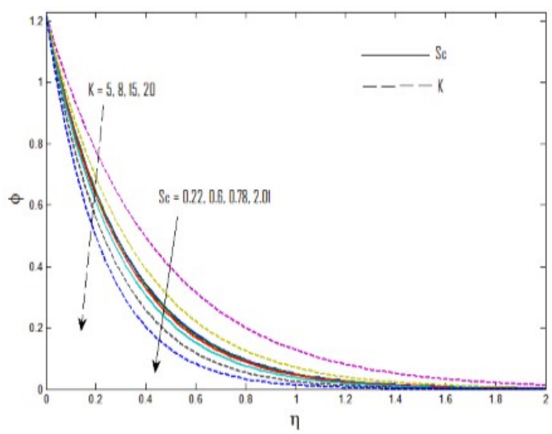

Fig 12. Concentration profiles for Sc and K

EFFECTS OF PARAMETERS ON THE RATE OF MASS TRANSFER

The rate of mass transfer (Sherwood number) at the moving plate $\eta = 0$ is given by

$$\mathrm{Sh} = -\left(\frac{\partial \varphi}{\partial \eta}\right)_{\eta=0}$$

$$= \sqrt{\lambda_3}\, e^{a\tau}, \tag{37}$$

where λ_3 is given by (33).

The negative sign indicates that the mass transfers from the plate to fluid.

Results of the rate of mass transfer Sh at the plate $\eta = 0$ are presented in Table 1 for several values of chemical reaction parameter K, Schmidt number Sc and time τ. It is seen from Table 1 that the rate of mass transfer Sh rises with rise in either Schmidt number Sc or chemical reaction parameter K. The variations of Schmidt number Sc as well as chemical reaction parameter K show that lesser the molecular diffusivity enhance the rate of mass transfer at the plate. With respect to time τ it is noticed that the rate of mass transfer increases in progressing of time. The effect of increase in time span is to enhance the rate of mass transfer at the plate.

Table 1. Rate of mass transfer Sh at the moving plate $\eta = 0$.

	K			Sc		
τ	2	5	10	0.22	0.6	2.01
0.2	1.565341	2.443527	3.440695	3.440695	3.466519	3.560704
0.4	1.695713	2.647041	3.727260	3.727260	3.755235	3.857265
0.6	1.836944	2.867505	4.037693	4.037693	4.067997	4.178525
0.8	1.989938	3.106331	4.373980	4.373980	4.406809	4.526542

EFFECTS OF PARAMETERS ON THE RATE OF HEAT TRANSFER:

The rate of heat transfer (Nusselt number) at the moving plate $\eta = 0$ is given by

$$Nu = -\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0}$$

$$= \sqrt{\lambda_2}\, e^{a\tau}, \tag{38}$$

where λ_2 is given by (33).

Numerical results of the rate of heat transfer Nu at the plate $\eta = 0$ are presented in Table 2 for several values of accelerating parameter a, Prandtl number PR, radiation parameter R and time τ. It is seen from Table 2 that the rate of heat transfer Nu at the plate increases with an increase in radiation parameter R. In conclusion, it is very obvious from our results that thermal radiation intensifies the convective flow. It is also observed that Nusselt number for water (PR=7) is higher than that of air (PR=0.71). The reason is that smaller values of PR are equivalent to increasing the thermal conductivities and therefore heat is able to diffuse away from the plate more rapidly than higher values of Pr, hence the rate of heat transfer is reduced. It is also observed from Table 2 that the rate of heat transfer Nu at the plate rises with an increase in either accelerating parameter a or time τ.

Table 2. Rate of heat transfer Nu at the moving plate $\eta = 0$.

τ	a			Pr			R		
	0.2	0.4	0.6	0.71	2.5	7.0	2	5	10
0.2	0.368415	0.542280	0.691260	0.542280	1.017571	1.702722	0.447176	0.512946	0.542280
0.4	0.383450	0.587445	0.779393	0.587445	1.102322	1.844537	0.484420	0.555668	0.587445
0.6	0.399099	0.636372	0.878763	0.636372	1.194131	1.998163	0.524766	0.601948	0.636372
0.8	0.415387	0.689373	0.990803	0.689373	1.293586	2.164584	0.568472	0.652082	0.689373

EFFECTS OF PARAMETERS ON THE SHEAR STRESS:

From the engineering point of view, the most important characteristic of the flow is the resultant non-dimensional shear stress at the moving plate $\eta = 0$ which is given by

$$\tau_x + i\tau_z = \left(\frac{\partial f}{\partial \eta}\right)_{\eta=0}$$

$$= e^{a\tau}\left[-\lambda_1\left(1+\frac{Gr}{\lambda_2-\lambda_1^2}+\frac{Gc}{\lambda_3-\lambda_1^2}\right)+\frac{Gr\sqrt{\lambda_2}}{\lambda_2-\lambda_1^2}+\frac{Gc\sqrt{\lambda_3}}{\lambda_3-\lambda_1^2}\right] \tag{39}$$

where λ_1, λ_2 and λ_3 are given by (33).

Numerical values of the non-dimensional shear stress components τ_x and τ_z due to the primary and secondary flows at the externally cooled plate $\eta = 0$ are presented in Figs.13-15 for several values of magnetic parameter M^2, thermal Grashof number Gr and mass Grashof number Gc. It is seen from Fig.13 that the shear stress component due to the primary flow drops down whereas the shear stress component due to the secondary flow rises with an increase in magnetic parameter . It is due to fact that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force, which tends to resist the fluid flow as well as the shear stress due to the primary flow. On the other hand, greater Lorentz gives an additional momentum in the boundary layer on the shear stress due to the secondary flow. The shear stress component is negative for the high values of magnetic parameter which retard the flow in the boundary layer to such an extent that reversal of the flow is caused. The effects of buoyancy parameters decelerate the frictional drag at the moving plate. Therefore, the shear stress components and due to the primary and secondary flows at the plate increase with an increase in either thermal Grashof number or mass Grashof number which are elucidated from Figs.14 and 15. We note that values become negative for very low values of either or since the magnetic impedance force will dominate and have a greater inhibiting influence with low buoyancy that is, flow reversal accompanies lower thermal buoyancy forces for higher permeability regimes.

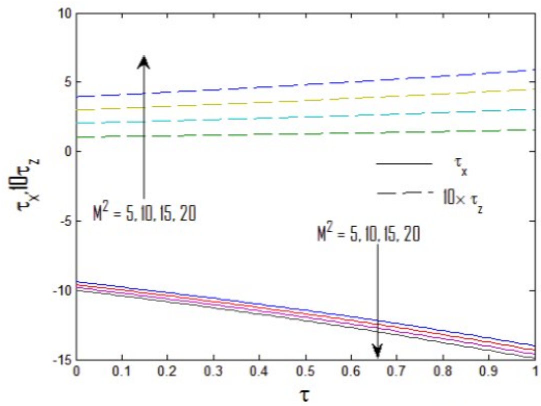

Fig.13: Shear stress components τ_x and τ_z for M^2 when $Gr = 5$ and $Gc = 5$

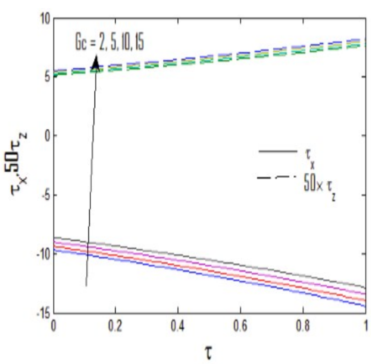

Fig.14: Shear stress components τ_x and τ_z for Gc when $Gr = 5$

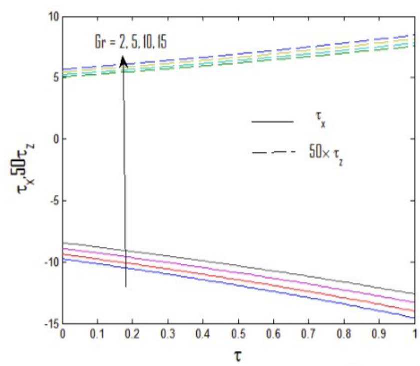

Fig.15: Shear stress components τ_x and τ_z for Gr when $Gc = 5$

CONCLUSIONS

The effects of Hall currents and radiation on an unsteady magnetohydrodynamic free convective flow of a viscous incompressible electrically conducting fluid past an exponentially accelerated vertical flat plate with variable plate temperature and concentration in the presence of transverse applied magnetic field have been studied. Magnetic field has a retarding influence whereas Hall currents has an accelerating influence on the fluid velocity components in case of cooling of the plate. For an externally cooled plate either mass Grashof number or thermal Grashof number or time has an accelerating influence on the fluid velocity distributions as well as on the shear stresses at the plate. The maximum values of temperature and concentration distributions are found at the plate surface. The fluid temperature distribution drops down with rise in Prandtl number and radiation whereas they tend to enhance the Nusselt number at the plate. Chemical reaction parameter has a retarding influence on the concentration distribution whereas it tends to enhance the Sherwood number at the plate. The shear stress component τ_x is negative for strong magnetic field and for low buoyancy force. Otherwise it is always positive.

REFERENCES

[1] Singh, A. K. and Kumar, N. (1984) :Free convection flow past an exponentially accelerated vertical plate; *Astrophys. Space Sci.,* vol.98, pp.245-258.

[2] Hossain, M. A. and Shayo, L. K. (1986): The Skin friction in the unsteady free convection flow past an accelerated plate; *Astrophys. Space Sci.* vol.125, pp.315-324.

[3] Hossain, M. A. and Takhar, H. S. (1996): Radiation effect on mixed convection along a vertical plate with uniform surface temperature; *Heat and Mass Trans.,* vol.31, pp.243-248.

[4] Raptis, A. and Perdikis, C. (1999): Radiation and free convection flow past a moving plate; *Int. J. Appl. Mech. and Eng.,* vol.4, pp.817-821.

[5] Alam, M. S., Rahman, M. M. and Smad, M. A. (2006a): Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium; *Nonlinear Anal. Model. Control,* vol.11, no.3, pp.217-226.

[6] Alam, M. S., Ferdows, M., Ota, M. and Maleque, M. A. (2006b): Dufour and Soret effects on steady free convection and mass transfer flow past a semi-infinite vertical porous plate in a porous medium; *Int. J. Appl. Mech. Eng.,* vol.11, no.3, pp.535-545.

[7] Alam, M. S., Rahman, M. M., Ferdows, M., Kaino, K., Mureithi, E. and Postelnicu, A.(2007): Diffusion-thermo and thermal-diffusion effects on free convective heat and mass transfer flow on a porous medium with time dependent temperature and concentration*; Int. J. Appl. Engg. Res.,* vol.2, no.1, pp.81-96.

[8] Ahmed, N.and Sarmah, H. K. (2009): Thermal radiation effect on a transient MHD flow with mass transfer past an impulsively fixed infinite vertical plate; *Int. J. of Appl. Math and Mech*., vol.5, no.5, pp.87-98.

[9] Rajesh, V. and Varma, S. V. K. (2009): Radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature; *ARPN J. Engg. and Appl. Sci*., vol.4, no.6, pp.20-26.

[10] Muthucumaraswamy, R., Sundar Raj M. and Subramanian, V. S. A. (2009): Unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion; *Int. J. Appl. Math and Mech.,* vol.5, no.6, pp.51-56.

[11] Muthucumaraswamy, R. , Sundar Raj, M. and Subramanian, V. S. A. (2009): Heat and mass transfer effects on flow past an accelerated vertical plate with variable mass diffusion; *Int. J. Appl. Math. and Engg Sci*., vol.3, no.1, pp.55-60.

[12] Soundalgekhar, V. M., Gupta, S. K. and Birajdar, N. S. (1981): Effects of mass transfer and free convection flow past currents on MHD stokes problem for a vertical plate; *Nuclear Eng. Des.,* Vol. 53, pp. 339-346.

[13] Soundalgekar, V. M. and Takhar, H. S. (1992): Radiative convective flow past a semiinfinite vertical plate; *Modelling Measure and Cont.,* Vol. 51, pp. 31-40.

[14] Ahamed, S. and Zueco, J. (2010): Combined heat and mass transfer by mixed convection MHD flow along a porous plate with chemical reaction in presence of heat source; Appl Math Mech., vol.31, no.10, pp.1217-1230.

[15] Rajesh, V. and Varma, S. V. K. (2010): Radiation effects on MHD flow through a porous medium with variable temperature or variable mass diffusion; *Int. J. of Appl. Math and Mech.,* vol.6, no.1, pp.39-57.

[16] Vijaya, A. G. K. and Verma, S. V. K. (2011): Radiation effects on MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature in the presence of heat generation; *Int. J. of Engg. Sci. and Tech.,* vol.3, no.4, pp.2898-2909.

[17] Suneetha, S. and Bhaskar Reddy, N. (2011): Radiation and darcy effects on unsteady MHD heat and mass transfer flow of a chemically reacting fluid past an impulsively started vertical plate with heat generation; Int. J. Appl. Math. and Mech., vol.7, no.7, pp.1-19.

[18] Manivannan , K., Muthucumarswamy, R. and Thangaraj, V. (2009): Radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion; *Thermal Science,* Vol. 13, No. 2, pp. 155-162.

[19] Mandal, C., Maji, S. L., Das, S. and Jana, R. N. (2011): Effects of radiation and heat transfer on flow past an exponentially accelerated vertical plate with constant heat flux; *Advances in Theo. and Appl. Math*., vol.6, no.5, pp.579-590.

[20] Das, S., Jana, M. and Jana, R. N. (2011) : Radiation effect on natural convection near a vertical plate embedded in porous medium with ramped wall temperature; *Open J. Fl. Dynamics*, vol.1, pp.1-11.

[21] Pattnaik, J. R., Dash, G. C. and Singh, S. (2012) : Radiation and mass transfer effects on MHD free convection flow through porous medium past an exponentially accelerated vertical plate with variable temperature; *Int. J. Engg., V*om X*,* pp.175-182.

[22] Jana, M., Das, S. and Jana, R. N. (2012): Radiation effects on unsteady MHD free convective flow past an exponentially accelerated vertical plate with viscous and Joule dissipations; *Int. J. Engg. Res. and Appl.,* vol.2, no.5, pp.270-278.

[23] Kishore, P. M. , Prasada Rao, N. V. R. V., Varma, S. V. K. and Venkataramana, S. (2013): The effects of radiation and chemical reaction on unsteady MHD free convection flow of viscous fluid past an exponentially accelerated vertical plate; *Int. J. Phys. and Math. Sci.,* vol.4, no.1, pp.300-317.

[24] Prakash, J., Bhanumathi, D., Vijaya, A. G. K. and Varma, S. V. K. (2013): Diffusion-thermo and radiation effects on unsteady MHD flow through porous medium past an impulsively started infinite vertical plate with variable temperature and mass diffusion; *Transp. Porous Med.,* vol.96, pp.135-151.

[25] Ahmmed, S. F., Parvin, S. and Morshed, M. (2013) : Radiation and mass transfer effects on MHD free convection flow past a vertical plate with variable temperature and concentration;

Int. J. Phy. and Math. Sc., vol.3, no.4, pp.54-61.

[26] Chandrakala, P. and Bhaskar, N. (2014): Radiation effects on MHD flow past an impulsively started infinite vertical plate with mass diffusion; *Int. J. of Appl. Mech. and Engg.,* vol.19, no.1, pp.17-26.

[27] SravanKumar, T. , Rushi Kumar, B. and Vijaya Kumar, A. G. (2015) : Hall current effects on unsteady free convective flow past a vertical plate in the presence of radiation and thermal diffusion; *Int. J. Appl. Eng. Res.,* vol.10, no.55, pp.3773-3778.

[28] Raju, M. C. , Reddy, N. A. and Varma, S. V. K. (2011): Hall current effects on unsteady MHD flow between stretching sheet and an oscillating porous upper parallel plate with constant suction; *Thermal science*, Vol. 15, No. 2 , pp. 527-536.

[29] Sarkar, B. C., Jana, R. N. and Das, S. (2016): Diffusion-thermo and thermal radiation of an optically thick gray gas in presence of magnetic field and porous medium; *J. Appl. Fl. Mech.,* vol.9, no.4, pp.2037-2051.

[30] Vidhya, M., Lawanya, T., Govindaraja, A. and Bianchini, S. (2020): Effect of mass transfer with chemical reaction on MHD convective flow through a porous medium in a hot vertical channel with thermal radiation; *AIP Conference Proceedings,* vol. 2277, no. 1, pp.(030005)1-9.

[31] Cowling, T. G. (1957) : Magnetohydrodynamics; *New York, Interscience*, pp.101.

[32] Siegel, R. and Howell, J. R. (2002): Thermal radiation heat transfer; *Taylor and Francis, New York*.

Bhaskar Chandra Sarkar
Department of Mathematics, Ramananda College,
Bishnupur 722 122,W.B, India
Mail: bhaskar.sarkar2045@gmail.com