IJAMAA, *Vol. 11, Nos. 2, (July-December 2016)*

CALCULATION OF THE STRESS INTENSITY FACTOR FOR TWO EQUAL CO-AXIAL CIRCULAR CRACKS

Dr. Suman Kumar Verma

ABSTRACT: The two-dimensional problem of diffraction of an arbitrary incident elastic longitudinal wave by two equal co-axial circular cracks are discussed hers. Infact these circular cracks are in an infinite, isotropic and homogeneous elastic medium. Firstly, mathematical Model developed to solved by a simple integral equation technique. Approximate expressions the stress intensity factors are derived when the wavelength is large as compared to the radius of the circular cracks. By taking appropriate limits, the corresponding results for various limiting configurations are derived for known cases.

Keywords: Stress Intensity Factor, Integral Equation, Green's Function, Hankel's Function

1. Introduction

In recent years, various authors have given attention to the problems dealing with stress and strain fields in an elastic medium containing cracks of finite dimension. These problems are of great interest in fracture mechanics, seismology and geophysics due to the nature of elastic waves is modified by the presence of cracks in the elastic medium. Specially scattering of elastic waves by cracks is a problem of considerable importance in the field of fracture mechanics, quantitative nondestructive evaluation of materials, geophysics and seismology. Recently various two-dimensional problems of diffraction of plane acoustic wave by a semi-circular soft or rigid infinite strip have been discussed by different techniques[1-5]. Shail[6] solved the problem of diffraction of low-frequency acoustic waves by an infinite circular are soft strip by integral equation techniques. These integral equation techniques give the solution of two Fredholm integral equation of the first kind with logarithmic kernels which are derived by using the well-known solutions of Carleman integral equations[7,8]. These integral equation techniques as well as their applications are quite complicated and cumbersome. Sampath and Jain[9], Jain and Jain[10-12] developed a simple independent integral equation technique to solve various two-dimensional Dirichlet as well as Neumann boundary value problems involving two equal co-axial infinite circular strips. These techniques have been further used to solve two-dimensional problems of diffraction of elastic P waves by (i) two equal co-axial circular rigid strips[13], (ii) two equal parallel and coplanar Griffith cracks[14].

It is here for the first time the solution of the two-dimensional problem of diffraction of obliquely incidence low frequency elastic P waves by two equal co-axial circular cracks are solved. The cracks are embedded, in an infinite, isotropic and homogeneous elastic medium, by these simple integral equation techniques[8-12]. With use of the usual Green's function approach, the solution of this problem is first reduced to a pair of governing simultaneous Fredholm integral equations of the first kind. When the wavelength is large as compared to the radius of circular cracks, solutions of this pair of governing integral equations is further reduced to that of a set of pairs of simultaneous Fredholm integral equations of the first kind. By making appropriate substitutions[10], the first pair of simultaneous Fredholm integral equations of the first kind of this set is solved to obtain approximate expression for the two unknown functions. Approximate expressions are derived for the far-field amplitudes, the scattering cross section by[17]

Finally, I have derived the stress intensity factors for the above problem using the technique or derivation done in detail[17]. By taking appropriate limits, we derive the corresponding known solution of the problem of diffraction of obliquely incident P waves by two parallel and coplanar Griffith cracks[14]. This serves as a check on our analysis. The other two corresponding results of the limiting configurations of a circular crack and a semi-circular crack are obtained which seem to be new.

2. Formulation of Boundary Value Problems

Consider a cylindrical polar coordinate system (r, θ, x_3) such that the two equal co-axial circular cracks are defined by the equations

$$r = a,\ 0 < \beta < |\theta| < \alpha < \pi,\ -\infty < x_3 < \infty$$

where a is the radius of the circular crack (see fig. 1),

By normalizing all the lengths with respect to 'a', the cracks are now defined by the equation

$$r = 1,\ 0 < \beta < |\theta| < \alpha < \pi,\ -\infty < x_3 < \infty.$$

Let $\boldsymbol{u}^{\mathbf{o}}\ (\mathbf{x})$ be the displacement field (the time factor e^{-iwt} is suppressed throughout the analysis) associated with the incident elastic P-waves propagating in the infinite, isotropic and homogeneous medium occupying the whole region S of the $x_1 - x_2$ plane in the direction making angle ϕ with the positive direction of x_1 axis and is defined as:

$$u^{o}\,(\mathrm{x}) = im_1 A_0\, \hat{b} \exp\,(im_1(x.\, \hat{b}),\ \hat{b} = \hat{e}_1 \cos\phi + \hat{e}_2 \sin\phi,\ x \in \mathrm{S}, \qquad (2.1)$$

where $\boldsymbol{x} = (x_1, x_2)$, $m_1^{\,2} = (\rho_0\omega^2 a^2)/(\lambda + 2u)$, A_0 is the known constant e_1 and e_2 are the unit vectors along the x_1 and x_2 axes, λ and are the lame constants of the medium. S_0, is the density of the medium, and ω is the frequency of the incident wave. The constant stiffness tensor $C_{ijk1}(\mathrm{x})$ of the infinite host medium is defined as:

$$C_{ijkl}(\mathrm{x}) = \delta_{ij}\, \delta_{kl} + \mu(\delta_{ik}\, \delta_{jl} + \delta_{jl}\, \delta_{jk}),\ x \in \mathrm{S}, \qquad (2.2)$$

where δ' S areas Kronecker deltas and the indices are 1, 2. In the absence of body forces and the cracks, $u^{o}(x)$ satisfies the equilibrium equations

$$C_{ijkl}\, u^{o}_{k,\,lj}(x) + \mu m_2^{\,2}\, u^{o}_{1}(x)\,,\ x \in S, \tag{2.3}$$

where

$$u^{o}_{k,\,lj}(x) = \frac{\partial^2 u^{o}_{k}(x)}{\partial xi\, \partial xj};\ m_2^{\,2} = \rho_0\omega^2 a^2/\mu$$

The components $\tau^{o}r_1(x)$ and $\tau^{o}r_2(x)$ of the stress tensor $\tau^{o}(x)$ associated with the incident field are given by

$$\tau^{o}_{r1}(x) = \tau^{o}_{11}(x)\ \cos q + \tau^{o}_{12}(x)\ \sin q,\ x \in S, \tag{2.4}$$

$$\tau^{o}_{r2}(x) = \tau^{o}_{21}(x)\ \cos q + \tau^{o}_{22}(x)\ \sin q,\ x \in S, \tag{2.5}$$

where

$$\tau^{o}_{11}(x) = -A_0\,(m_1^{\,2}\mu/\tau^2)\,\{\cos^2\phi + (1 - 2\tau^2)\sin^2\phi\}\exp[im_1\,(x_1\cos\phi + x_2\sin\phi)],$$

$$\tau^{o}_{12}(x) = -A_0\, m_1^{\,2}\mu\,\{\cos^2\phi\}\exp[im_1\,(x_1\cos\phi + x_2\sin\phi)],$$

$$\tau^{o}_{22}(x) = -A_0\, m_1^{\,2}\mu\,\{(1-2\tau^2)\cos^2\phi + \sin^2\phi\}\exp[im_1\,(x_1\cos\phi + x_2\sin\phi)],$$

$$\tau = m_1/m_2 = [\mu/(\lambda + 2\mu)]^{1/2}.$$

Let the displacement vector, the stress tensor associated with the scattered field and the total field be denoted by $u^{s}(x)$ and $\tau^{s}(x)$ and $u(x)$, $\tau(x)$ respectively.

The boundary conditions are

$$\tau_{r1}(x) = \tau^{S}_{r1}(x) + \tau^{o}_{r1}(x) = 0,\ \iota = 1, 2, \text{ as } x \text{ tends to the points on C}, \tag{2.6}$$

where the arc C are defined by the equation $r = 1$, $\beta < |\theta| < \alpha$, $u_1(x)$, $\tau_{r1}(x)$, $\iota = 1, 2$ are continuous across $r = 1$, $0 \le |\theta| < \beta$, $a < |\theta| \le \pi$, (2.7)

In addition, we have to satisfy the radiation condition at infinity and the appropriate edge conditions at the tips of the cracks.

Thus, $u(x)$ satisfies the distributional formula

$$\operatorname{div}[C_{ijkl}\, u_{k,l}(x)] = \operatorname{div}[C_{ijkl}\, u_{k,l}(x)] + C_{k\alpha ilgk}\,(x_c)\,(\partial/\partial_{xl})\,\partial(x - x_c)\,\hat{n}_a(x_c)$$

$$\mu m_2^{\,2}\, u_i(x)\,,\ x \in S, \tag{2.8}$$

where the bar denotes the distributional derivative,

$$g_k(x_c) = u_k(x_c)\,|_{-} - u_k(x_c)|_{+},\ k = 1, 2$$

are the jumps in the components of the displacement vector across the arcs C and $\hat{n}(x_c)$ is the unit vector along the outward normal drown at the point x_c of the arcs C.

The above distributional formula incorporates all the continuity requirements of the components u_i, τ_{ri}, i= 1, 2, given in the above boundary conditions (2.6) and (2.7). Following the usual Green's function approach, the integral representation formulas for the components $u^S{}_m(x)$ of the displacement vector of the scattered field are given by

$$\begin{aligned} u^S{}_m(x) &= \int_{-\alpha}^{-\beta} \int_{\beta}^{\alpha} [C_{k1i1}\ g_k(x_c) \cos\theta\ \ G_{im,i}\ (x,x_c) \\ &\quad + C_{k2i\ i}\ g_k(x_c) \sin\theta\ G_{1m,\ 1}\ [(x, x_c)]\ d\theta,\ m = 1, 2, x \in D, \\ &= \mu \int_{-\alpha}^{-\beta} + \int_{\beta}^{\alpha} [g_1(x_c)\ \{\cos\theta\ [\left(\tfrac{1}{\tau^2}\right)\ G_{1m,1}\ (x, x_c) \\ &\quad + (1/\tau^2 - 2)\ G_{2m,\ 2}\ (x, x_c)] + \sin\theta\ [G_{1m,\ 2}\ (x, x_c) \\ &\quad + G_{2m,\ 1}\ (x, x_c)]\} + g_2(x_c)\ \{\cos\theta\ [G_{1m,\ 2}\ (x, x_c) \\ &\quad + (1/\tau^2)\ G_{2m,\ 2}\ (x, x_c)]\}\ d\theta, \qquad m = 1, 2, \end{aligned} \tag{2.10}$$

where $x_c = (x_1', x_2') = (\cos\theta', \sin\theta')$ and Green's function $G_{1m}(x, x_c)$ are defined as [15, 16]

$$\begin{aligned} G_{1m}(x, x_c) &= (i/4\mu m_2{}^2)\ [\delta_{1m}\ m_2 2\ H_0{}^{(1)}\ (m_2 R) \\ &\quad + (\partial^2/\partial x_1\ \partial x_m)\ [H_0{}^{(1)}\ (m_2, R) - H_0{}^{(1)}\ (m_1, R)]],\ l = 1, 2, \end{aligned} \tag{2.11}$$

where $R = |x - x_c|$ and $H_0{}^{(1)}$ is the Hankel function of the first kind of order zero.

The boundary conditions (2.6), (2.7), the formulas (2.4), (2.5), (2.10) and the relations

$$\begin{aligned} \tau^S{}_{ri}\ (x) &= \tau^S{}_{11}(x)\ \cos\theta + \tau^S{}_{12}(x)\ \sin\theta,\ i = 1, 2, \\ \tau^S{}_{11}(x) &= (\mu/\tau^2)\ \{u^S{}_{1,\ 1}(x) + (1-2\tau^2)\ u^S{}_{2,\ 2}(x)\}, \\ \tau^S{}_{12}(x) &= \tau^S{}_{21}(x) = \mu\ \{\mu^S{}_{1,\ 2}(x) + u^S{}_{2,\ 1}(x)\}, \\ \tau^S{}_{11}(x) &= (\mu/\tau^2)\ \{u^S{}_{2,\ 2}(x) + (1-2\tau^2)\ u^S{}_{1,\ 1}(x)\}, \end{aligned} \tag{2.12}$$

lead to the pair of governing integral equations of this problem and its solutions were derived first time by [17].

It is reduced to after detail derivation given by[17] Verma

$$I_{ij}^{(0)}(\theta') = (d/d\theta')\,[g_{ij}^{(0)}(\theta')],\ \iota, j = 1, 2 \tag{2.12}$$

and the edge conditions

$$\int_{\beta}^{\alpha} I_{\iota j}^{(0)}(\theta')\, d\theta' = 0,\ \ \iota, j = 1,2, \tag{2.14}$$

which readily follow from the edge conditions $g(\pm\alpha) = 0$, $g_1(\pm\beta) = 0$.

the following required expressions for $I_{ij}^{(0)}(\theta')$, $\iota, j = 1, 2$;

$$I_{11}^{(0)}(\theta') = \frac{[1 - A^2 - \frac{2ABJ_1}{J_0} - \frac{A^2 J_2}{J_0} - 2B\cos\theta + \cos 2\theta}{A^2[(\cos\beta - \cos\theta')(\cos\theta' - \cos\alpha)]^{1/2}}\ Ei,\ \ i=1, 2 \tag{2.15}$$

$$I_{12}^{(0)}(\theta') = \frac{(\cos\theta' - B)\sin\theta'}{A[(\cos\beta - \cos\theta')(\cos\theta' - \cos\alpha)]^{1/2}}\ Gi,\ i = 1, 2, \tag{2.16}$$

where the constants E_ι and G_ι's ($i = 1,2$) are given below.

Substituting the expressions of $I_{ij}^{(0)}(\theta')$, $i, j = 1, 2$ from eqns. (2.15) and (2.66) in the relations (2.12), we obtain the values of $g_i(\theta')$ up to the order $0(m_2^{\,2})$.

Fortunately, we do not require the values of $g_\iota(\theta')$, $i = 1, 2$, for deriving the expressions for the various physical quantities of interest in this problem. These can be readily derived by using the values of $I_{ij}^{(0)}(\theta')$, $i,j = 1, 2$, given by the relations (2.15) and (2.16).

3. STRESS INTENSITY FACTORS

The stress intensity factors K_{1i} and K_{2i} are defined as (in the physical units)

$$K_{1i} = \lim_{e->\alpha+} \sqrt{a}\,\{(\theta - \alpha)^{\frac{1}{2}}\,[\tau^{S} ri\,(1, \theta)]\}_{\theta<\alpha,}\,,\ i = 1,2, \tag{3.1}$$

$$K_{2i} = \lim_{e->\beta-} \sqrt{a}\,\{(\beta - \theta)^{\frac{1}{2}}\,[\tau^{S} ri\,(1, \theta)]\}_{\theta<\beta,}\,,\ i = 1,2, \tag{3.2}$$

From eqns. (2.12), we get

$$\tau_{r_1}^{S}(x) = -\mu \int_{-\alpha}^{-\beta} \int_{\beta}^{\alpha} \{g_1(\theta')\, M_1(\theta - \theta') + g_2(\theta')\, M_2(\theta - \theta')\} d\theta', \tag{3.3}$$

$$\tau_{r_1}^{S}(x) = \frac{\mu\ A_0 m^2 2(1-\tau^2)}{\pi}\ [\int_{\beta}^{\alpha} \{-\sin\theta\, I_{11}^{(0)}(\theta')\sin\theta' + \cos\theta\, I_{12}^{(0)}(\theta')\cos\theta'$$

$$+\mathrm{I}_{21}^{(0)}(\theta')\sin\theta'\left[\frac{1}{\cos\theta'-\cos\theta}+\cos\theta\right]+\sin\theta\ \mathrm{I}_{22}^{(0)}(\theta')\ [\frac{1}{\cos\theta'-\cos\theta}$$
$$+\cos\theta']\}\ d\theta'+0(m_2)], \tag{3.4}$$

$$\tau_{r_2}^{S}(\mathrm{x}) = -\mu\int_{-\alpha}^{-\beta}\int_{\beta}^{\alpha}\{g_1(\theta')\,\mathrm{L}_1(\theta-\theta')+g_2(\theta')\,\mathrm{M}_1(\theta-\theta')\}d\theta', \tag{3.5}$$

$$\tau_{r_2}^{S}(\mathrm{x}) = \frac{\mu\ \mathrm{A}_0 m^2 2(1-\tau^2)}{\pi}\ [\int_{\beta}^{\alpha}\{\mathrm{I}_{11}^{(0)}(\theta')\ \left[\frac{1}{\cos\theta'-\cos\theta}-\cos\theta\right]\sin\theta'$$
$$+\sin\theta\,\mathrm{I}_{12}^{(0)}(\theta')\left[\frac{1}{\cos\theta'-\cos\theta}-\cos\theta'\right]-\sin\theta\ \mathrm{I}_{21}^{(0)}(\theta')\sin\theta'$$
$$+\cos\theta\,\mathrm{I}_{22}^{(0)}(\theta')\cos\theta'\}\ d\theta'+0(m_2)], \tag{3.6}$$

where we have used the expressions for $L_1(\theta, \theta)$, $M_1(\theta, \theta)$, $M_2(\theta, \theta)$ as given below and its more details are given by[17] .

$$\mathrm{M}_1(\theta, \theta') = \frac{(1-\tau^2)}{4\pi\sin^2 ½(\theta-\theta')}[\sin(\theta+\theta')(1-\cos(\theta-\theta')]+0(m_2),$$
$$\mathrm{M}_2(\theta, \theta') = \frac{(1-\tau^2)}{4\pi\sin^2 ½(\theta-\theta')}[1+\cos(\theta+\theta')-\cos(\theta+\theta')\cos(\theta-\theta')]+0(m_2), \tag{3.7}$$
$$\mathrm{L}_1(\theta, \theta') = \frac{(1-\tau^2)}{4\pi\sin^2 ½(\theta-\theta')}[1-\cos(\theta-\theta')+\cos(\theta-\theta')\cos(\theta+\theta')]+0(m_2).$$

The expansions of the density functions $g_i(\theta)$, $i = 1, 2$ derived by Verma [ ]

The values of the stress components $\tau^S r_i(\mathrm{x})$, $i = 1, 2$ can be easily evaluated when $\theta<\beta$ or $\theta>\alpha$ from eqns. (3.4) and (3.6) after substituting the values of functions $I_{ij}^{(0)}(\theta)$, i, $j = 1,2$

Where $I_{ij}^{(0)}(\theta)$, are

$$\mathrm{I}_{11}^{(0)}(\theta') = \frac{[1-\mathrm{A}^2-\frac{2\mathrm{ABJ}_1}{\mathrm{J}_0}-\frac{\mathrm{A}^2\mathrm{J}_2}{\mathrm{J}_0}-2\mathrm{B}\cos\theta+\cos 2\theta}{\mathrm{A}^2[(\cos\beta-\cos\theta')(\cos\theta'-\cos\alpha)]^{1/2}}\ \mathrm{Ei}, \quad \mathrm{i}=1, 2 \tag{3.8}$$

$$\mathrm{I}_{12}^{(0)}(\theta') = \frac{(\cos\theta'-B)\sin\theta'}{A[(\cos\beta-\cos\theta')(\cos\theta'-\cos\alpha)]^{1/2}}\ \mathrm{Gi}, \quad =1, 2, \tag{3.9}$$

$$\mathrm{A} = ½(\cos\beta-\cos\alpha),\ \mathrm{B} = ½(\cos\beta+\cos\alpha),\ 0 < x, y < \pi, \text{ when } \beta < \theta, \theta' < \alpha. \tag{3.10}$$

$$J_n = \int_0^{\pi} \frac{\cos nt}{[1-(A\cos t+B)^2]^{1/2}}\,dt, \; n = 0, 1, 2, \ldots.$$

$$E_1 = \frac{(\tau^2 \sin 2\phi) A^2}{(1-\tau^2)\left(2+A^2+\frac{2ABJ_1}{J_0}+\frac{A^2J_2}{J_0}\right)},$$

$$E_2 = \frac{[(1-2\tau^2\sin 2\phi) - A^2(1-\tau^2)]A^2}{(1-\tau^2)\;(2+A^2+\frac{2ABJ_1}{J_0}+A^2J_2/J_0)}, \tag{3.11}$$

$$C_1 = (2BJ_1/J_0 + AJ_2/J_0)E_1/A,\; D_1 = (2B/A)E_1,\; i = 1, 2,$$

$$G_1 = 2A - 2E_2/A,\; G_2 = 2E_1/A,\; H_1 = F_1 = 0,\; i = 1, 2. \tag{3.12}$$

After evaluating the values of $\tau^S r_i(x)$, $i = 1, 2$ and putting these in the relations (3.01) and (3.02) we readily obtain.

$$K_{11} = A_0\mu m^2_2(1-\tau^2)\sqrt{2}\,[(3 - Q_1 - 2B\cos\alpha + \cos 2\alpha)\,P_1/Q_1$$
$$+ [1 - (P_2 - A^2)/Q_2]\,(\sin 2\alpha - 2B\sin\alpha)\}/[\sin\alpha\,(\cos\beta - \cos\alpha)]^{1/2}$$
$$+ 0(m_2)], \tag{3.13}$$

$$K_{12} = A_0\mu m^2_2\,(1-\tau^2)\sqrt{a}\,[\{Q_2 - 1 - 2B\cos\alpha + \cos 2\alpha)\,(P_2 - A^2)/Q_2$$
$$+ (\sin^2\alpha - 2B\sin\alpha)\,P_1/Q_1)\,/\,[\sin\alpha\,(\cos\beta - \cos\alpha]^{1/2} + 0(m_2)] \tag{3.14}$$

$$K_{21} = -A_0\mu^2_2\,(1-\tau^2)\sqrt{a}\,[(3 - Q_1 - 2B\cos\beta + \cos 2\beta)\,P_1/Q_1$$
$$+ [1 - (P_2 - A^2)\,/\,Q_2]\,(\sin 2\beta - 2B\sin\beta)\}\,/\,[\sin\beta\,(\cos\beta - \cos\alpha)]^{1/2}$$
$$+ 0(m_2)], \tag{3.15}$$

$$K_{22} = -A_0\mu^2_2\,(1-\tau^2)\sqrt{a}\,[Q_2 - 1 - 2B\cos\beta + \cos 2\beta)\,(P_2 - A^2)/Q_2$$
$$+ (\sin 2\beta - 2B\sin\beta)\,P_1/Q_1\}/[\sin\beta(\cos\beta - \cos\alpha)\,]^{1/2} + 0(m_2)], \tag{3.16}$$

where the values of constants P_1, Q_1's , $i = 1, 2$ are given below

$$P_1 = \tau^2\sin^2\phi/(1-\tau^2),\; P_2 = (1-2\tau^2\sin^2\phi)/(1-\tau^2), \tag{3.17}$$

$$Q_1 = 2 + A^2 + 2AB\ J_1/J_0 + A_2J_2/J_1, \tag{3.18}$$

$$Q_2 = 2 - A^2 - 2AB\ J_1/J_0 - A_2J_2/J_1, \tag{3.19}$$

As far as the author knows, the above results seem to be new.

4. LIMITING RESULTS

A. (i) An Infinite Circular Crack

When, $\beta \to 0$, all defined stress intensity factor are derived from eqns. (3. 13) & (3.14) for the following results for the corresponding problem of an infinite circular crack; $r = 1, -\alpha < \theta < \alpha$, $-\infty < x_3 < \infty$.

$$K_{11} = \frac{A_0\mu m^2 2(1-\tau^2)\sqrt{a}}{\sin\left(\frac{\alpha}{2}\right)\sqrt{(sin a)}} \left[\{3\text{-}Q_1{}' - 2\cos^2\left(\frac{\alpha}{2}\right)\cos\alpha + \cos 2\alpha)\frac{P1}{Q1} + [1-(P_2-\sin^4\left(\frac{\alpha}{2}\right) / Q_2{}'\ ] \right. \\ \left. (\sin\alpha 2\alpha - 2\cos^2\left(\frac{\alpha}{2}\right)\sin\alpha)\} + 0(m_2)\ \right], \tag{4.1}$$

$$K_{12} = \frac{A_0\mu m^2 2(1-\tau^2)\sqrt{\alpha}}{\sin\left(\frac{\alpha}{2}\right)\sqrt{(sin a)}} \left[\{Q_2{}' - 1 - 2\cos^2\left(\frac{\alpha}{2}\right)\cos\alpha \right. \\ + \cos 2\alpha\left(P_2 - \sin^4\left(\frac{\alpha}{2}\right)\right) / Q_2{}' + (\sin 2\alpha - 2\cos^2\left(\frac{\alpha}{2}\right)\sin\alpha)\ P_2 / Q_1{}'\} \\ \left. + 0\ (m_2)\ \right], \tag{4.2}$$

K_{21} and K_{22} are not defined,

where,

$$J_n{}' = \int_0^\pi \frac{\cos(nt)}{\left[1-\left(\sin^2\left(\frac{\alpha}{2}\right)\cos\tau + \cos^2\left(\frac{\alpha}{2}\right)\right)2\right]\frac{1}{2}}\ dt, \tag{4.3}$$

$$Q_1{}' = 2 + \sin^4\left(\frac{\alpha}{2}\right) + \frac{1}{2}\sin^2\alpha J_1{}' / J_0{}' + \sin^4\left(\frac{\alpha}{2}\right)J_2{}' / J_0{}', \tag{4.4}$$

$$Q_2{}' = 4 - Q_1{}', \tag{4.5}$$

(ii) An Infinite Semi-Circular Crack

When $\alpha \to \frac{\pi}{2}$ in the above results (4.1) – (4.2), we get the corresponding problem of an infinite semi-circular crack occupying the region[17] : $r = 1, -\frac{\pi}{2} < |\theta| < \frac{\pi}{2}, -\infty < x < \infty$.

As far as the authors know, even the above results for the two limiting configurations also seem to be new.

B. Two equal Griffith cracks

We let β, α, and $a \to \infty$ such that $a\beta \to a_2$, $a\alpha \to a_1$, $A_0 a^2/a^2_1 \to B_0$, ($c = \frac{a2}{a1} < 1$, when we assume that $0 < \alpha + \beta < \pi$), in the eqns. (3.13) – (3.16) and obtain the following corresponding results for two equal and parallel Griffith cracks; $a_2 < |x_2| < a_1$, $x_1 = \infty$, $- < x_3 < \infty$:

$$K_{11} = -\frac{B_0\, M^2 2(1-\tau^2)\sqrt{a_1}\, P_i}{[2(1-c^2)]^{\frac{1}{2}}} \left[\left(1 - \frac{E}{F} + 0(M_2)\right)\right], \; i = 1, 2, \tag{4.6}$$

$$K_{21} = -\frac{B_0\, M^2 2(1-\tau^2)\sqrt{a_1}\, P_i}{[2c(1-c^2)]^{\frac{1}{2}}} \left[\left(\frac{E}{F} - c^2\right) + 0(M_2)\right], \; i = 1, 2, \tag{4.7}$$

Where

$$M_2 = \rho\omega^2 a^2_1/\mu,$$

Bo is known constant and

$$F = F\left(\frac{\pi}{2}, \sqrt{(1-c^2)}\right),$$

$$E = E\left(\frac{\pi}{2}, \sqrt{(1-c^2)}\right)$$

are the complete elliptical integrals of first and second kind respectively. The above results agree with the known results[14, 17], for two equal parallel coplanar Griffith cracks $b < |x_1| < a$, $x_2 = 0$, $-\infty < x_3 < \infty$, when we interchange the values for P_1 and P_2, and change the values of a_1 to a, a_2 to b and ϕ to $\left(\frac{\pi}{2} - \gamma\right)$. This serves as a check on our analysis presented here.

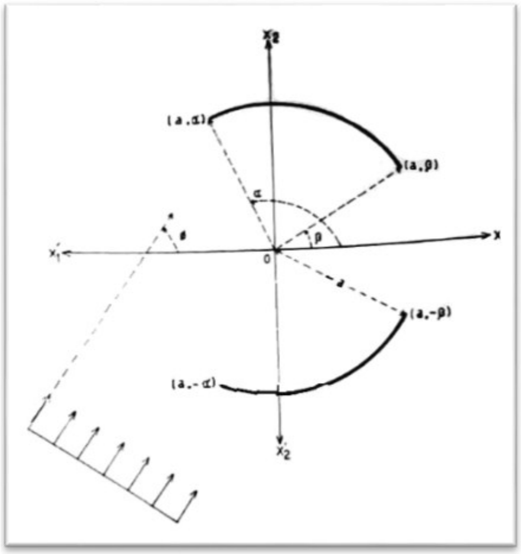

Fig. 1. Section of the two equal circular cracks in the $x_1 - x_2$ plane.

REFERENCES

1. P. Wolfe 1975 Q. J. Mech. Appl. Math. 28, 355-369. Diffraction of plane waves by a semi-circular strip.
2. B. Lal and D.L. Jain 1977 J. Acoust. Soc. Am. 62, 250-254. Acoustic diffraction of a plane wave by a sem-circular infinite soft strip.
3. B. Lal and D.L. Jain 1987 J. Acoust. Soc. Am., 81, 190-191. Erratum: Acoustic diffraction of a plane wave by a semi-circular infinite soft strip [J. Acoust. Soc. Am., 62 (1977), 250-254].
4. D.L. Jain and B. Lal 1980 Radio Sci. 15, 711-719. Diffraction of an H-polarized electromagnetic wave by a semi-circular strip.
5. D.L. Jain and B. Lal 1987 Radio Sci.22, 168. Correction to "Diffraction of an H-polarized electromagnetic wave by a semi-circular strip." [Radio Sci., 15 (1980), 711-719].
6. R. Shail 1984 Int. J. Math. Educ. Sci. Technol. 15, 359-374. A class of singular integral equation with some applications.
7. J.C. Cooke 1970 Glasgow Math J. 11, 9-20. The solution of some integral equations and their connection with dual integral equations and series.
8. W.E. Williams 1972 Glasgow Math. J. 13, 119-121. A note on integral equation.
9. C. Sampath and D.L. Jain 1988 Internal. J. Math. & Math. Sci. 11, 751-762. On solution of the integral equations for the potential problems of two circular strips.
10. S. Jain and D.L. Jain 1987 J. Acoust. Soc. Am. 81, 647-653. Diffraction of a plane acoustic wave by two equal circular infinite soft strips.
11. S. Jain and D.L. Jain 1989 J. Acoust. Soc. Am. 85 1785. Erratum: Diffraction of a plane acoustic wave by two equal circular infinite soft strips. [J. Acoust. Soc. Am. 82 (1987), 647-

653].

12. S. Jain and D.L. Jain 1989 Radio Sci. 24(4), 443-454. Diffraction of an H-polarized electromagnetic wave by two circular infinite strips.
13. S.K. Verma and D.L. Jain 1991 Europ. J. Mech. A/Solid MS # A121. Diffraction of elastic P waves by two equal co-axial circular rigid strips.
14. S.K. Verma and D.L. Jain 1990 J. Appl. Phy. 67(11), 6787-6793. Diffraction of obliquely incident P waves by two parallel and coplanar Griffith cracks.
15. D.L. Jain and R.P. Kanwal 1982 J. Appl. Phys. 53(4), 4208-4217. The Born Approximation for the scattering theory of elastic waves by two dimensional flaws.
16. A.C. Eringen and E.S. Suhubi 1975 Academic Press, New York, Vol.II. Elastodynamics.
17. S.K. Verma 1991 Ph.D. Submitted to the University of Delhi, Delhi, India . Solution of some mixed boundary value problems by Integral Equation Techniques.

Dr. Suman Kumar Verma
Department of Mathematics,
School of Open Learning
University of Delhi,
Delhi-110007, India