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Dodecagonal Fuzzy Number (DDFN)

P. Selvam* A. Rajkumar and J.Sudha Easwari *****

Abstract : Nowadays fuzzy numbers has been developed and applied in various field. In this paper we define Dodecagonal Fuzzy Number with linguistic values is introduced and also includes the basic arithmetic operations like addition, subtraction, multiplication, division and symmetric image by means of alpha cut with numerical examples.

Keywords : Fuzzy numbers, Fuzzy Arithmetic, Linguistic values, Dodecagonal Fuzzy Number(DDFN), alpha cut, Function Principles.

1. INTRODUCTION

In 1965 fuzzy set theory was introduced by Lotfi.A.Zadeh and it is used to assess the graded membership function described in the interval [0,1]. But it is incomplete and imprecise. Interval arithmetic was developed by Dwyer in 1951. After that in 1978 D.Dubois and H.Prade defined any of the fuzzy numbers as a fuzzy subset of the line. A fuzzy number is a quantity whose value is precise rather than exact as in the case with ordinary single valued numbers. Based on the membership function, fuzzy numbers are classified into triangular fuzzy number, trapezoidal fuzzy number, interval value etc., fuzzy numbers are used to present real numbers in a fuzzy environment for analyzing fuzziness and fuzzy data that have various application as in linguistic, control, database system. In this paper a new operation of dodecagonal fuzzy numbers has been introduced along with uncertain linguistic term and arithmetic operations of fuzzy number is also defined. This method is useful in twelve different point.

2. PRELIMINARIES AND NOTATIONS

2.1. Definition: Fuzzy set

A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse X to the unit interval [0, 1].

A fuzzy set A in a universe of discourse X is defined as the following set of pairs:

$$A = \{(x, \mu_A(x)); x \varepsilon X\}.$$

Here $\mu_A: X \rightarrow [0, 1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \varepsilon X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

2.2. Definition : α-cut set [2][3]:

The α -cut set A_α is made up of members whose membership is not less than α.

$$A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$$

note that α is arbitrary. This α -cut set is a crisp set.

* Research scholar, Hindustan Institute of Technology and Science, Chennai -603 103, Tamil Nadu, India. & Assistant professor, Adithya Institute of Technology, Coimbatore-641 107, Tamil Nadu, India. thiruvibhaa@gmail.com

** Assistant professor, Hindustan Institute of Technology and Science, Chennai -603 103, Tamil Nadu, India. arajkumar@hindustanuniv.ac.in

*** B.T Asst. Govt. Hr. Sec. School, Thirupur- 641 606,Tamil Nadu, India. sarvapithama@gmail.com

For all x, $\alpha \in [0, 1]$ the upper α-cut of x is denoted as $x(\alpha)$ and is defined as $x^{\alpha} = \begin{cases} 1 & if \quad x \geq \alpha \\ 0 & if \quad x < \alpha \end{cases}$

and the lower α-cut of x is denoted as $x(\alpha)$ and is defined as $x_{\alpha} = \begin{cases} x & if \quad x \geq \alpha \\ 0 & if \quad x < \alpha \end{cases}$

2.3. Definition(Fuzzy number)

A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its membership function $A : \Re \longrightarrow [0,1]$ has the following definitions

- **Convex fuzzy set :** A fuzzy set A of X is called convex if ${}^{\alpha}[A]$ is a convex subset of X, $\forall\ \alpha \in [0, 1]$
- **Normal fuzzy set :** A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exists at least one $x \in$ X such that $\mu_A(x) = 1$.
- A is membership function is piecewise continuous.
- A is defined in the real number.

2.4. Definition

A linguistic variable/term is a variable whose value is not crisp number but word or sentence in a natural language.

2.5. Definition

If $S = s_0, s_1, s_2, ..., s_g$ be a finite and totally ordered set with odd linguistic terms where s_i denotes the i^{th} linguistic term, $i \in 0,1,2,...,g$ then we call set S the linguistic term set and $(g + 1)$ the cardinality of S. It is usually required that set S has the following properties:

1. **The set is ordered :** $s_i \geq s_j$ if $i > j$
2. **There is a negation operator :** neg $(s_i) = s_j$ such that $j = g - 1$
3. **Maximization operator :** max $(s_i, s_j) = s_i$ if $s_i \geq s_j$
4. **Minimization operator :** min $(s_i, sj) = s_i$ if $s_i \leq s_j$

The uncertain linguistic term is a generalization of cognitional expressions to fuzziness and uncertainty.

We introduce its definition of uncertain linguistic term below.

2.6. Definition

Let $\bar{S} = s_l, s_{l+1}, ... s_u$ where $s_l, s_{l+1}, ..., s_u \in S$, $s_l \geq s_u$, s_l and s_u are the lower and upper limits, $l, u \in 0,1,...,g$ respectively, Then we call $\bar{S}$ the uncertain linguistic term.

For simplicity, we express as . Here, the greater $(u - l)$ is, the greater the fuzziness and uncertainty degree of will be. Particularly, if $l = u$, then is reduced to a certain linguistic term.

For example, in the process of a venture decision , experts may use linguistic term set

$S = s_0$: No Influence, s_1: Very Low, s_2 : Low, s_3 : Medium, s_4 : High and s_5 : Very High to express his/her opinion on the correlation between women's education development and society development. One experts judgment may be at least High, which can be expressed by an uncertain linguistic term $[s_3, s_4]$. If his/her judgment is High, then it can be expressed by an uncertain linguistic term $[s_3, s_4]$.

3. DODECAGONAL FUZZY NUMBERS(DDFN)

Definition 3.1

A fuzzy number $\tilde{A}_{DD}$ is a dodecagonal fuzzy number denoted by $\tilde{A}_{DD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12})$ where $a_1, a_2, a_3, a_4, ..., a_9, a_{10}, a_{11}, a_{12}$ are real numbers and its membership function $\mu_{\tilde{A}_{\overline{DD}}}(x)$ is given by

$$\mu_{\tilde{A}_{\overline{DD}}} = \begin{cases} \frac{1}{5}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ \frac{1}{5}+\frac{1}{5}\left(\frac{x-a_2}{a_3-a_2}\right) & a_2 \le x \le a_3 \\ \frac{2}{5}+\frac{1}{5}\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ \frac{3}{5}+\frac{1}{5}\left(\frac{x-a_4}{a_5-a_4}\right) & a_4 \le x \le a_5 \\ \frac{4}{5}+\frac{1}{5}\left(\frac{x-a_5}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ 1 & a_6 \le x \le a_7 \\ 1-\frac{1}{5}\left(\frac{x-a_7}{a_8-a_7}\right) & a_7 \le x \le a_8 \\ \frac{4}{5}-\frac{1}{5}\left(\frac{x-a_8}{a_9-a_8}\right) & a_8 \le x \le a_9 \\ \frac{3}{5}-\frac{1}{5}\left(\frac{x-a_9}{a_{10}-a_9}\right) & a_9 \le x \le a_{10} \\ \frac{2}{5}-\frac{1}{5}\left(\frac{x-a_{10}}{a_{11}-a_{10}}\right) & a_{10} \le x \le a_{11} \\ \frac{1}{5}\left(\frac{a_{12}-x}{a_{12}-a_{11}}\right) & a_{11} \le x \le a_{12} \\ 0 & x < a_1 \,\&\, x > a_{12} \end{cases}$$

The derived dodecagonal fuzzy number from the uncertain linguistic term $[s_l, s_u]$ is shown in Fig. 2. Thus, the aggregation operations of uncertain linguistic terms can be achieved by the operations of dodecagonal fuzzy numbers.

Table. 1. Dodecagonal fuzzy linguistic scale.

Linguistic terms	*Linguistic values*
No development	(0,0,0,0,0,0,0.2,0.4,0.6,0.8,1.0,1.2)
Very Low development	(0.2,0.4,0.6,0.8,1.0,1.2,1.4,1.6,1.8,2.0,2.2,2.4)
Low development	(1.4,1.6,1.8,2.0,2.2,2.4,2.6,2.8,3.0,3.2,3.4,3.6)
Medium	(2.6,2.8,3.0,3.2,3.4,3.6,3.8,4.0,4.2,4.4,4.6,4.8)
High development	(3.8,4.0,4.2,4.4,4.6,4.8,5.0,5.2,5.4,5.6,5.8,6.0)
Very High development	(5.0,5.2,5.4,5.6,5.8,6.0,6.2,6.4,6.6,6.8,7.0,7.2)

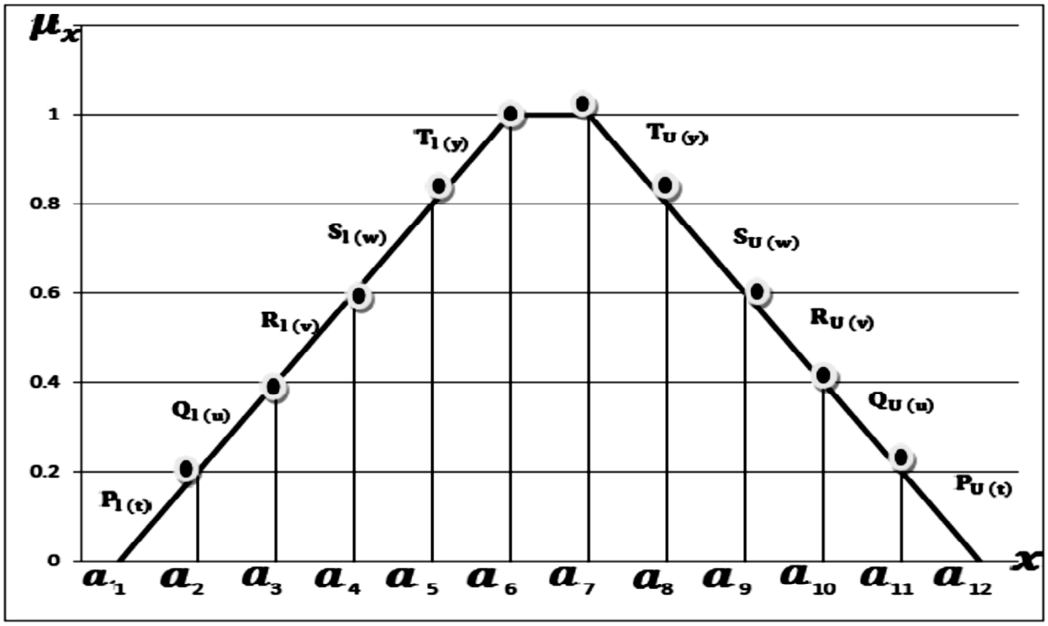

Fig. 1. Graphical representation of a normal octagonal fuzzy numbers for $x \in [0,1]$

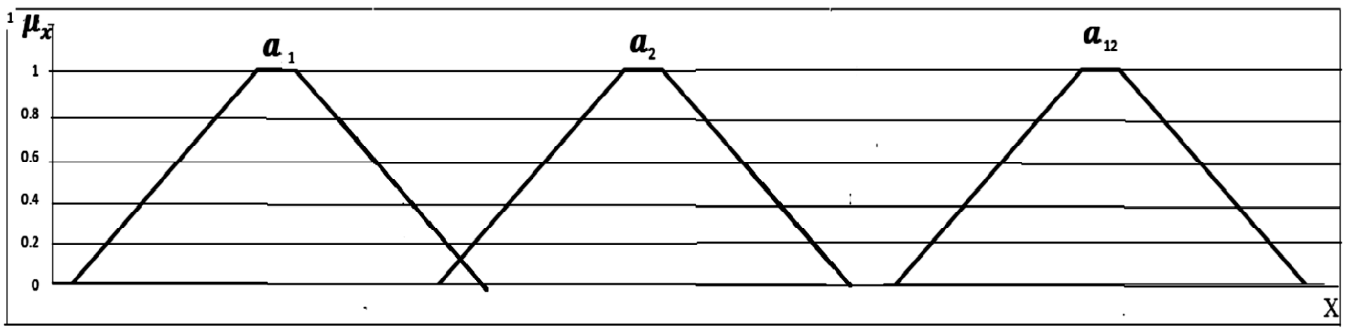

Fig. 2. The dodecagonal fuzzy numbers from uncertain linguistic term [S_l, S_u].

Notation 3.2. Arithmetic fuzzy numbers are denoted by $AA_{\overline{DD}}$ and $AB_{\overline{DD}}$. Linguistic variable transformed into dodecagonal fuzzy numbers $[s_i, s_j]$ and $[s_k, s_l]$ is denoted by $LA_{\overline{DD}}$ and $LB_{\overline{DD}}$

4. OPERATIONS OF DODECAGONAL FUZZY NUMBERS

Definition 4.1

Let $A_{\overline{DD}}$ and $B_{\overline{DD}}$ be two arbitrary uncertain linguistic terms and $A_{\overline{DD}} = (a_1, a_2, a_3, a_4, \dots, a_{10}, a_{11}, a_{12})$ and $B_{\overline{DD}} = (b_1, b_2, b_3, b_4, \dots, b_{10}, b_{11}, b_{12})$ be their corresponding dodecagonal fuzzy numbers then

1. **Addition :** $A_{\overline{DD}}(+)B_{\overline{DD}} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, \dots, a_9 + b_9, a_{10} + b_{10}, a_{11} + b_{11}, a_{12} + b_{12})$

2. **Subtraction :** $A_{\overline{DD}}(-)B_{\overline{DD}} = (a_1 - b_{12}, a_2 - b_{11}, a_3 - b_{10}, \dots, a_8 - b_5, a_9 - b_4, a_{10} - b_3, a_{11} - b_2, a_{12} - b_1)$

3. **Multiplication :** $$A_{\overline{DD}} * B_{\overline{DD}} = \begin{pmatrix} (a_1*b_1 \wedge a_1*b_{12} \wedge a_{12}*b_1 \wedge a_{12}*b_{12}),(a_2*b_2 \wedge a_2*b_{11} \wedge a_{11}*b_2 \wedge a_{11}*b_{11}),\dots \\ (a_5*b_5 \wedge a_5*b_8 \wedge a_8*b_5 \wedge a_8*b_8),(a_6*b_6 \wedge a_6*b_7 \wedge a_7*b_6 \wedge a_7*b_7), \\ (a_6*b_6 \vee a_6*b_7 \vee a_7*b_6 \vee a_7*b_7),(a_5*b_5 \vee a_5*b_8 \vee a_8*b_5 \vee a_8*b_8),\dots \\ (a_2*b_2 \vee a_2*b_{11} \vee a_{11}*b_2 \vee a_{11}*b_{11}),(a_1*b_1 \vee a_1*b_{12} \vee a_{12}*b_1 \vee a_{12}*b_{12}) \end{pmatrix}$$

4. **Division :** $$\frac{A_{\overline{DD}}}{B_{\overline{DD}}} = \begin{pmatrix} \left(\frac{a_1}{b_1} \wedge \frac{a_1}{b_{12}} \wedge \frac{a_{12}}{b_1} \wedge \frac{a_{12}}{b_{12}}\right), \left(\frac{a_2}{b_2} \wedge \frac{a_2}{b_{11}} \wedge \frac{a_{11}}{b_2} \wedge \frac{a_{11}}{b_{11}}\right) \dots \left(\frac{a_6}{b_6} \wedge \frac{a_6}{b_7} \wedge \frac{a_7}{b_6} \wedge \frac{a_7}{b_7}\right), \\ \left(\frac{a_6}{b_6} \vee \frac{a_6}{b_7} \vee \frac{a_7}{b_6} \vee \frac{a_7}{b_7}\right) \dots \left(\frac{a_2}{b_2} \vee \frac{a_2}{b_{11}} \vee \frac{a_{11}}{b_2} \vee \frac{a_{11}}{b_{11}}\right), \left(\frac{a_1}{b_1} \vee \frac{a_1}{b_{12}} \vee \frac{a_{12}}{b_1} \vee \frac{a_{12}}{b_{12}}\right) \end{pmatrix}$$

Excluding the case $b_1 = 0$ or $b_2 = 0$ or $b_3 = 0$ or $b_4 = 0$ or $b_5 = 0$ or $b_6 = 0$ or $b_7 = 0$ or $b_8 = 0$ or $b_9 = 0$ or $b_{10} = 0$ or $b_{11} = 0$ or $b_{12} = 0$

5. **Symmetric image :** $-A_{\overline{DD}} = (-a_{12}, -a_{11}, -a_{10}, -a_9 - a_8, -a_7, -a_6, -a_5, -a_4, -a_3, -a_2, -a_1)$

When previous sets $A_{\overline{DD}}$ and $B_{\overline{DD}}$ is defined in the positive real number $\Re^+$, the operations of multiplication, division, and inverse interval are written as,

(3') Multiplication : $A_{\overline{DD}} * B_{\overline{DD}} = ((a_1 * b_1), (a_2 * b_2), (a_3 * b_3), \ldots, (a_{11} * b_{11}), (a_{12} * b_{12}))$

(4') Division : $A_{\overline{DD}} (/) B_{\overline{DD}} = \left(\left(\frac{a_1}{b_{12}}\right), \left(\frac{a_2}{b_{11}}\right), \left(\frac{a_3}{b_{10}}\right), \ldots, \left(\frac{a_8}{b_5}\right), \left(\frac{a_9}{b_4}\right), \left(\frac{a_{10}}{b_3}\right), \left(\frac{a_{11}}{b_2}\right), \left(\frac{a_{12}}{b_1}\right) \right)$

Example 4.1.

Let $AA_{\overline{DD}} = (3,9,15,21,27,33,39,45,51,57,63,69)$ and $AB_{\overline{OFN}} = (2,4,6,8,10,12,14,16,18,20,22,24)$ be two dodecagonal fuzzy numbers then

- $AA_{\overline{DD}} (+) AB_{\overline{DD}} = (5,13,21,29,37,45,53,61,69,77,85,93)$
- $AA_{\overline{DD}} (-) AB_{\overline{DD}} = (-21, -13, -5, 3, 11, 19, 27, 35, 43, 51, 59, 67)$
- $AA_{\overline{DD}} (*) AB_{\overline{DD}} = (6, 36, 90, 168, 270, 396, 546, 720, 918, 1140, 1386, 1656)$
- $AA_{\overline{DD}} (/) AB_{\overline{DD}} = (0.125, 0.4091, 0.75, 1.1667, 1.6875, 2.3571, 3.25, 4.5, 6.375, 9.5, 15.75, 34.5)$
- $-AA_{\overline{DD}} = (-69, -63, -57, -51, -45, -39, -33, -27, -21, -15, -9, -3)$

Example 4.2.

Let $LA_{\overline{DD}} = (2.6, 2.8, 3.0, 3.2, 3.4, 3.6, 3.8, 4.0, 4.2, 4.4, 4.6, 4.8)$ and

$LB_{\overline{DD}} = (5.0, 5.2, 5.4, 5.6, 5.8, 6.0, 6.2, 6.4, 6.8, 7.0, 7.2)$ be two Linguistic variable transform into dodecagonal fuzzy numbers then

- $LA_{\overline{DD}} (+) LB_{\overline{DD}} = (7.6,8.0,8.4,8.8,9.2,9.6,10.0,10.4,10.8,11.2,11.6,12.0)$
- $LA_{\overline{DD}} (-) LB_{\overline{DD}} = (-4.6,-4.2,-3.8,-3.4,-3.0,-2.6,-2.2,-1.8,-1.4,-1.0,-0.6,-0.2)$
- $LA_{\overline{DD}} (*) LB_{\overline{DD}} = (13.0,14.56,16.2,17.92,19.92, 21.6,23.56,25.6,27.72,29.92,32.2,34.56)$
- $LA_{\overline{DD}} (/) LB_{\overline{DD}} = (0.3611,0.4,0.4412,0.4848,0.5313,0.5803,0.6333,0.6897,0.75,0.8148,0.8846,0.96)$
- $-LA_{\overline{DD}} = (-4.8,-4.6,-4.4,-4.2,-4.0,-3.8,-3.6,-3.4,-3.2,-3.0,-2.8,-2.6)$

Definition 4.2.

A Dodecagonal fuzzy number $\overline{DD}$ can also be defined as $\overline{DD}$ = $P_l(t), Q_l(u), R_l(v), S_l(w), T_l(y)$, $P_u(t), Q_u(u), R_u(v), S_u(w), T_u(y)$ $t \in [0.0, 0.2], u \in [0.2, 0.4], v \in [0.4, 0.6], w \in [0.6, 0.8]$ and $y \in [0.8, 1.0]$ where $P_l(t), Q_l(u), R_l(v), S_l(w), T_l(y)$ is bounded and continuous increasing function over [0,0.2], [0.2,0.4], [0.4,0.6], [0.6,0.8] and [0.8,1.0] respectively. is bounded and continuous decreasing function over [0,0.2], [0.2,0.4], [0.4,0.6], [0.6,0.8] and [0.8,1.0] respectively.

$P_u(t), Q_u(u), R_u(v), S_u(w), T_u(y)$ is bounded and continuous decreasing function over [0,0.2], [0.2,0.4], [0.4,0.6], [0.6,0.8] and [0.8,1.0] respectively.

$P_l(t)=\frac{1}{5}\left(\frac{x-a_1}{a_2-a_1}\right)$	$P_u(t)=\frac{1}{5}\left(\frac{a_{12}-x}{a_{12}-a_{11}}\right)$
$Q_l(u)=\frac{1}{5}+\frac{1}{5}\left(\frac{x-a_2}{a_3-a_2}\right)$	$Q_u(u)=\frac{2}{5}-\frac{1}{5}\left(\frac{x-a_{10}}{a_{11}-a_{10}}\right)$
$R_l(v)=\frac{2}{5}+\frac{1}{5}\left(\frac{x-a_3}{a_4-a_3}\right)$	$R_u(v)=\frac{3}{5}-\frac{1}{5}\left(\frac{x-a_9}{a_{10}-a_9}\right)$
$S_l(w)=\frac{3}{5}+\frac{1}{5}\left(\frac{x-a_4}{a_5-a_4}\right)$	$S_u(w)=\frac{4}{5}-\frac{1}{5}\left(\frac{x-a_8}{a_9-a_8}\right)$
$T_l(y)=\frac{4}{5}+\frac{1}{5}\left(\frac{x-a_5}{a_6-a_5}\right)$	$T_u(y)=1-\frac{1}{5}\left(\frac{x-a_7}{a_8-a_7}\right)$

Definition 4.3.

The α- cut of the fuzzy set of the universe of discourse X is defined as $\overline{DD}=\left\{x\in X/\mu_{\overline{A}}(x)\geq\alpha\right\}$ where $\alpha\in[0,1]$

$$\overline{DD}_\alpha = \begin{cases} [P_l(\alpha),P_u(\alpha)] & \text{for} \quad \alpha\in[0.0,0.2) \\ [Q_l(\alpha),Q_u(\alpha)] & \text{for} \quad \alpha\in[0.2,0.4) \\ [R_l(\alpha),R_u(\alpha)] & \text{for} \quad \alpha\in[0.4,0.6) \\ [S_l(\alpha),S_u(\alpha)] & \text{for} \quad \alpha\in[0.6,0.8) \\ [T_l(\alpha),T_u(\alpha)] & \text{for} \quad \alpha\in[0.8,1.0] \end{cases}$$

Definition 4.4.

If $P_l(x)=\alpha$ and $P_u(x)=\alpha$, then α-cut operations interval $\overline{DD}_\alpha$ is obtained as

- $[P_l(\alpha),P_u(\alpha)]=[5\alpha(a_2-a_1)+a_1, 5\alpha(a_{11}-a_{12})+a_{12}]$

  Similarly we can obtain α-cut operation interval $\overline{DD}_\alpha$ for $[Q_l(\alpha),Q_u(\alpha)]$, $[R_l(\alpha),R_u(\alpha)]$, $[S_l(\alpha),S_u(\alpha)]$ and $[T_l(\alpha),T_u(\alpha)]$ as follows:
- $[Q_l(\alpha),Q_u(\alpha)]=[5\alpha(a_3-a_2)+2a_2-a_3,\ 5\alpha(a_{10}-a_{11})+2a_{11}-a_{10}]$
- $[R_l(\alpha),R_u(\alpha)]=[5\alpha(a_4-a_3)+3a_3-2a_4,\ 5\alpha(a_9-a_{10})+3a_{10}-2a_9]$
- $[S_l(\alpha),S_u(\alpha)]=[5\alpha(a_5-a_4)+4a_4-3a_5,\ 5\alpha(a_8-a_9)+4a_9-3a_8]$
- $[T_l(\alpha),T_u(\alpha)]=[5\alpha(a_6-a_5)+5a_5-4a_6, 5\alpha(a_7-a_8)+5a_8-4a_7]$

  Hence α-cut of dodecagonal fuzzy number

$$\overline{DD}_\alpha = \begin{cases} [5\alpha(a_2-a_1)+a_1, 5\alpha(a_{11}-a_{12})+a_{12}] & \textit{for} \quad \alpha\in[0.0,0.2) \\ [5\alpha(a_3-a_2)+2a_2-a_3, 5\alpha(a_{10}-a_{11})+2a_{11}-a_{10}] & \textit{for} \quad \alpha\in[0.2,0.4) \\ [5\alpha(a_4-a_3)+3a_3-2a_4, 5\alpha(a_9-a_{10})+3a_{10}-2a_9] & \textit{for} \quad \alpha\in[0.4,0.6) \\ [5\alpha(a_5-a_4)+4a_4-3a_5, 5\alpha(a_8-a_9)+4a_9-3a_8] & \textit{for} \quad \alpha\in[0.6,0.8) \\ [5\alpha(a_6-a_5)+5a_5-4a_6, 5\alpha(a_7-a_8)+5a_8-4a_7] & \textit{for} \quad \alpha\in[0.8,1.0] \end{cases}$$

5. A NEW OPERATION FOR ADDITION, SUBTRACTION , MULTIPLICATION ,DIVISION AND SYMMETRIC IMAGE ON DODECAGONAL FUZZY NUMBER

5.1. Addition of Two Dodecagonal Fuzzy Numbers

Definition 5.1.

Let $A_{\overline{DD}} = (a_1, a_2, a_3, a_4, ..., a_{10}, a_{11}, a_{12})$ and $B_{\overline{DD}} = (a_1, a_2, a_3, a_4, ..., a_{10}, a_{11}, a_{12})$ be their corresponding dodecagonal fuzzy numbers for all $\alpha \in [0,1]$. Let us add α- cut $^{\alpha}A$ and $^{\alpha}B$ of $A_{\overline{DD}}$ and $B_{\overline{DD}}$ using interval arithmetic.

$$A_{\overline{DD}} + B_{\overline{DD}} = \begin{cases} \begin{pmatrix} [5\alpha((a_2+b_2)-(a_1+b_1))+(a_1+b_1), \\ 5\alpha((a_{11}+b_{11})-(a_{12}+b_{12}))+(a_{12}+b_{12})] \end{pmatrix} & for \quad \alpha \in [0.0, 0.2) \\ \begin{pmatrix} [5\alpha((a_3+b_3)-(a_2+b_2))+2(a_2+b_2)-(a_3+b_3), \\ 5\alpha((a_{10}+b_{10})-(a_{11}+b_{11}))+2(a_{11}+b_{11})-(a_{10}+b_{10})] \end{pmatrix} & for \quad \alpha \in [0.2, 0.4) \\ \begin{pmatrix} [5\alpha((a_4+b_4)-(a_3+b_3))+3(a_3+b_3)-2(a_4+b_4), \\ 5\alpha((a_9+b_9)-(a_{10}+b_{10}))+3(a_{10}+b_{10})-2(a_9+b_9)] \end{pmatrix} & for \quad \alpha \in [0.4, 0.6) \\ \begin{pmatrix} [5\alpha((a_5+b_5)-(a_4+b_4))+4(a_4+b_4)-3(a_5+b_5)], \\ 5\alpha((a_8+b_8)-(a_9+b_9))+4(a_9+b_9)-3(a_8+b_8)] \end{pmatrix} & for \quad \alpha \in [0.6, 0.8) \\ \begin{pmatrix} [5\alpha((a_6+b_6)-(a_5+b_5))+5(a_5+b_5)-4(a_6+b_6), \\ 5\alpha((a_7+b_7)-(a_8+b_8))+5(a_8+b_8)-4(a_7+b_7)] \end{pmatrix} & for \quad \alpha \in [0.8, 1.0] \end{cases} \quad ...(1)$$

To verify this new operation with addition operation , we take the arithmetic example 4.1

$AA_{\overline{DD}} = (3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69)$ and $AB_{\overline{OFN}} = (2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24)$

For $\alpha \in [0.0, 0.2)$ For $\alpha \in [0.2, 0.4)$ For $\alpha \in [0.4, 0.6)$ For $\alpha \in [0.6, 0.8)$ For $\alpha \in [0.8, 1.0]$	$AA_{\alpha} = (30\alpha + 3, -30\alpha + 69)$ $AB_{\alpha} = (10\alpha + 2, -10\alpha + 24)$	$AA_{\alpha}(+)AB_{\alpha} = (40\alpha + 5, -40\alpha + 93)$

Since $\alpha \in [0.0, 0.2)$, $\alpha \in [0.2, 0.4)$, $\alpha \in [0.4, 0.6)$, $\alpha \in [0.6, 0.8)$ and $\alpha \in [0.8, 1.0]$,arithmetic intervals are same $AA_{\alpha}(+)AB_{\alpha} = (40\alpha + 5, -40\alpha + 93)$ for all $\alpha \in [0,1]$

when

$$\begin{aligned} \alpha = 0.0 &\Rightarrow AA_{\alpha}(+)AB_{\alpha} = [5, 93] \\ \alpha = 0.2 &\Rightarrow AA_{\alpha}(+)AB_{\alpha} = [13, 85] \\ \alpha = 0.4 &\Rightarrow AA_{\alpha}(+)AB_{\alpha} = [21, 77] \\ \alpha = 0.6 &\Rightarrow AA_{\alpha}(+)AB_{\alpha} = [29, 69] \\ \alpha = 0.8 &\Rightarrow AA_{\alpha}(+)AB_{\alpha} = [37, 61] \\ \alpha = 1.0 &\Rightarrow AA_{\alpha}(+)AB_{\alpha} = [45, 53] \end{aligned}$$

Hence $AA_{\alpha}(+)AB_{\alpha} = (5,13,21,29,37,45,53,61,69,77,85,93)$. Hence all the points coincides with the addition of the two dodecagonal fuzzy number.

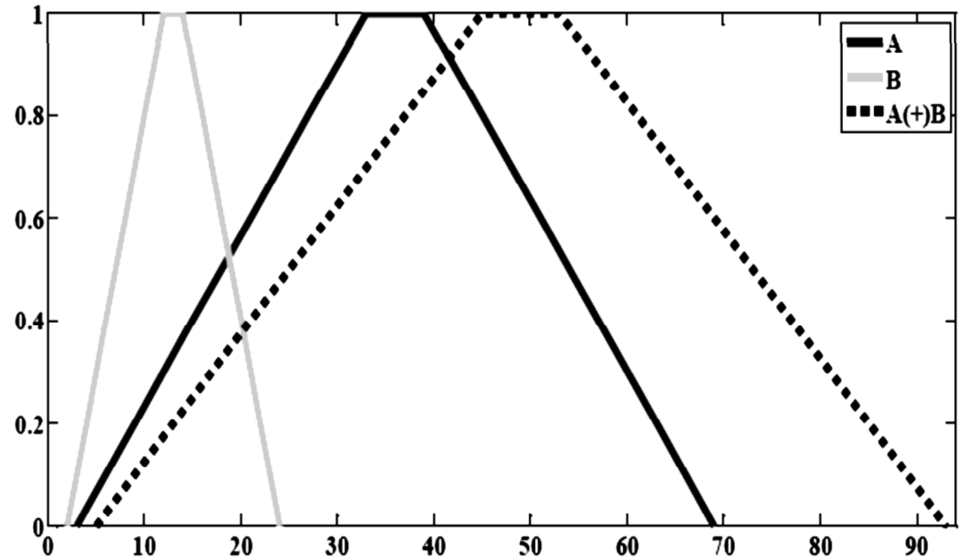

Fig. 3. Summation of two dodecagonal fuzzy numbers.

Definition 5.2.

Let LA and LB be two arbitrary uncertain linguistic terms $\mathrm{LA}_{\overline{\mathrm{DD}}} = (c_1, c_2, c_3, c_4, c_5 ...., c_{10}, c_{11}, c_{12})$ and $\mathrm{LB}_{\overline{\mathrm{DD}}} = (e_1, e_2, e_3, e_4, e_5 ...., e_{10}, e_{11}, e_{12})$ be their corresponding dodecagonal fuzzy numbers, the addition operation of α cuts $\mathrm{LC}_{\overline{\mathrm{DD}}}$ and $\mathrm{LE}_{\overline{\mathrm{DD}}}$ using interval arithmetic defined as $\mathrm{LA}_{\overline{\mathrm{DD}}} + \mathrm{LB}_{\overline{\mathrm{DD}}}$ same as by (1) replacing a by *c* and *b* by *e*.

To verify this a new operation with ordinary addition operation , we take example 4.2. Let $\mathrm{LA}_{\overline{\mathrm{DD}}} = (2.6, 2.8, 3.0, 3.2, 3.4, 3.6, 3.8, 4.0, 4.2, 4.4, 4.6, 4.8)$ and $\mathrm{LB}_{\overline{\mathrm{DD}}} = (5.0, 5.2, 5.4, 5.6, 5.8, 6.0, 6.2, 6.4, 6.8, 7.0, 7.2)$

For $\alpha \in [0.0, 0.2)$ For $\alpha \in [0.2, 0.4)$ For $\alpha \in [0.4, 0.6)$ For $\alpha \in [0.6, 0.8)$ For $\alpha \in [0.8, 1.0]$	$\mathrm{LA}_\alpha = (\alpha + 2.6, -\alpha + 4.8)$ $\mathrm{LB}_\alpha = (\alpha + 5.0, -\alpha + 7.2)$	$\mathrm{LA}_\alpha (+) \mathrm{LB}_\alpha = (2\alpha + 7.6, -2\alpha + 12.0)$

Since $\alpha \in [0.0, 0.2)$, $\alpha \in [0.2, 0.4)$, $\alpha \in [0.4, 0.6)$, $\alpha \in [0.6, 0.8)$ and $\alpha \in [0.8, 1.0]$, arithmetic intervals are same $\mathrm{LA}_\alpha (+) \mathrm{LB}_\alpha = (2\alpha + 7.6, -2\alpha + 12.0)$ for all forall$\alpha \in [0,1]$

when

$$
\begin{aligned}
\alpha = 0.0 \quad &\Rightarrow \quad \mathrm{LA}_\alpha (+) \mathrm{LB}_\alpha = [7.6, 12.0] \\
\alpha = 0.2 \quad &\Rightarrow \quad \mathrm{LA}_\alpha (+) \mathrm{LB}_\alpha = [8.0, 11.6] \\
\alpha = 0.4 \quad &\Rightarrow \quad \mathrm{LA}_\alpha (+) \mathrm{LB}_\alpha = [8.4, 11.2] \\
\alpha = 0.6 \quad &\Rightarrow \quad \mathrm{LA}_\alpha (+) \mathrm{LB}_\alpha = [8.8, 10.8] \\
\alpha = 0.8 \quad &\Rightarrow \quad \mathrm{LA}_\alpha (+) \mathrm{LB}_\alpha = [9.2, 10.4] \\
\alpha = 1.0 \quad &\Rightarrow \quad \mathrm{LA}_\alpha (+) \mathrm{LB}_\alpha = [9.6, 10.0]
\end{aligned}
$$

Hence $\mathrm{LA}_\alpha (+) \mathrm{LB}_\alpha = (7.6, 8.0, 8.4, 8.8, 9.2, 9.6, 10.0, 10.4, 10.8, 11.2, 11.6, 12.0)$. Hence all the points coincides with the addition of the two dodecagonal fuzzy number.

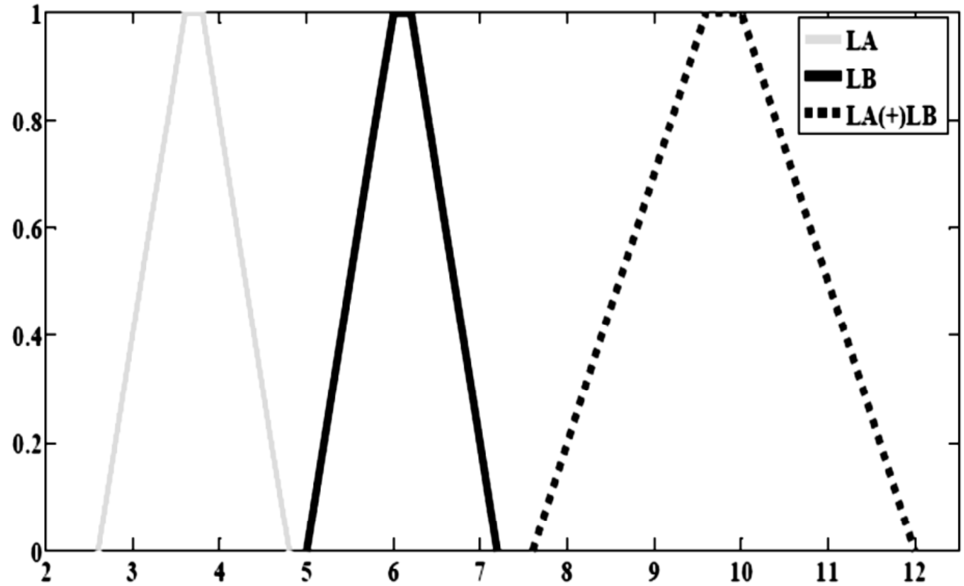

Fig. 4. Additional two arbitary uncertain linguistic terms of DDFNs A and B.

Therefore addition of two α- cuts lies within the interval. Hence it is verified.

5.2. Subtraction of Two Dodecagonal Fuzzy Numbers

Definition 5.3.

Let $A_{\overline{DD}} = (a_1, a_2, a_3, a_4, ..., a_{10}, a_{11}, a_{12})$ and $B_{\overline{DD}} = (a_1, a_2, a_3, a_4, ..., a_{10}, a_{11}, a_{12})$ be their corresponding dodecagonal fuzzy numbers for all $\alpha \in [0,1]$. Let us subtract add $\alpha-$ cut ${}^{\alpha}A$ and ${}^{\alpha}B$ of $AA_{\overline{DD}}$ and $AB_{\overline{DD}}$ using interval arithmetic.

$$A_{\overline{DD}}(-)B_{\overline{DD}} = \begin{cases} \begin{pmatrix} [5\alpha((a_2+b_{12})-(a_1+b_{11}))+(a_1-b_{12}), \\ 5\alpha((a_{11}+b_1)-(a_{12}+b_2))+(a_{12}-b_1)] \end{pmatrix} & for \quad \alpha \in [0.0, 0.2) \\ \begin{pmatrix} [5\alpha((a_3+b_{11})-(a_2+b_{10}))+2(a_2-b_{11})-(a_3-b_{10}), \\ 5\alpha((a_{10}+b_2)-(a_{11}+b_3))+2(a_{11}-b_2)-(a_{10}-b_3)] \end{pmatrix} & for \quad \alpha \in [0.2, 0.4) \\ \begin{pmatrix} [5\alpha((a_4+b_{10})-(a_3+b_9))+3(a_3-b_{10})-2(a_4-b_9), \\ 5\alpha((a_9+b_3)-(a_{10}+b_4))+3(a_{10}-b_3)-2(a_9-b_4)] \end{pmatrix} & for \quad \alpha \in [0.4, 0.6) \\ \begin{pmatrix} [5\alpha((a_5+b_9)-(a_4+b_8))+4(a_4-b_9)-3(a_5-b_8)], \\ 5\alpha((a_8+b_4)-(a_9+b_5))+4(a_9-b_4)-3(a_8-b_5)] \end{pmatrix} & for \quad \alpha \in [0.6, 0.8) \\ \begin{pmatrix} [5\alpha((a_6+b_8)-(a_5+b_7))+5(a_5-b_8)-4(a_6-b_7), \\ 5\alpha((a_7+b_5)-(a_8+b_6))+5(a_8-b_5)-4(a_7-b_6)] \end{pmatrix} & for \quad \alpha \in [0.8, 1.0] \end{cases} \quad ...(2)$$

To verify this new operation with subtraction operation , we take the arithmetic example 4.1 $AA_{\overline{DD}} = (3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69)$ and $AB_{\overline{OFN}} = (2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24)$

For $\alpha \in [0.0, 0.2)$ For $\alpha \in [0.2, 0.4)$	$AA_{\alpha} = (30\alpha + 3, -30\alpha + 69)$	$AA_{\alpha}(-)AB_{\alpha} = (40\alpha - 21, -40\alpha + 67)$
For $\alpha \in [0.4, 0.6)$		
For $\alpha \in [0.6, 0.8)$ For $\alpha \in [0.8, 1.0]$	$AB_{\alpha} = (10\alpha + 2, -10\alpha + 24)$	

Since $\alpha \in [0.0, 0.2)$, $\alpha \in [0.2, 0.4)$, $\alpha \in [0.4, 0.6)$, $\alpha \in [0.6, 0.8)$ and $\alpha \in [0.8, 1.0]$, arithmetic intervals are same $AA_\alpha(-)AB_\alpha = (40\alpha - 21, -40\alpha + 67)$ for all $\alpha \in [0,1]$

when

$$\begin{aligned}
\alpha = 0.0 &\Rightarrow AA_\alpha(-)AB_\alpha = [-21, 67] \\
\alpha = 0.2 &\Rightarrow AA_\alpha(-)AB_\alpha = [-13, 59] \\
\alpha = 0.4 &\Rightarrow AA_\alpha(-)AB_\alpha = [-5, 51] \\
\alpha = 0.6 &\Rightarrow AA_\alpha(-)AB_\alpha = [3, 43] \\
\alpha = 0.8 &\Rightarrow AA_\alpha(-)AB_\alpha = [11, 35] \\
\alpha = 1.0 &\Rightarrow AA_\alpha(-)AB_\alpha = [19, 27]
\end{aligned}$$

Hence $AA_\alpha(-)AB_\alpha =$ (-21,-13,-5,3,11,19,27,35,43,51,59,67). Hence all the points coincides with the subtraction of the two dodecagonal fuzzy number.

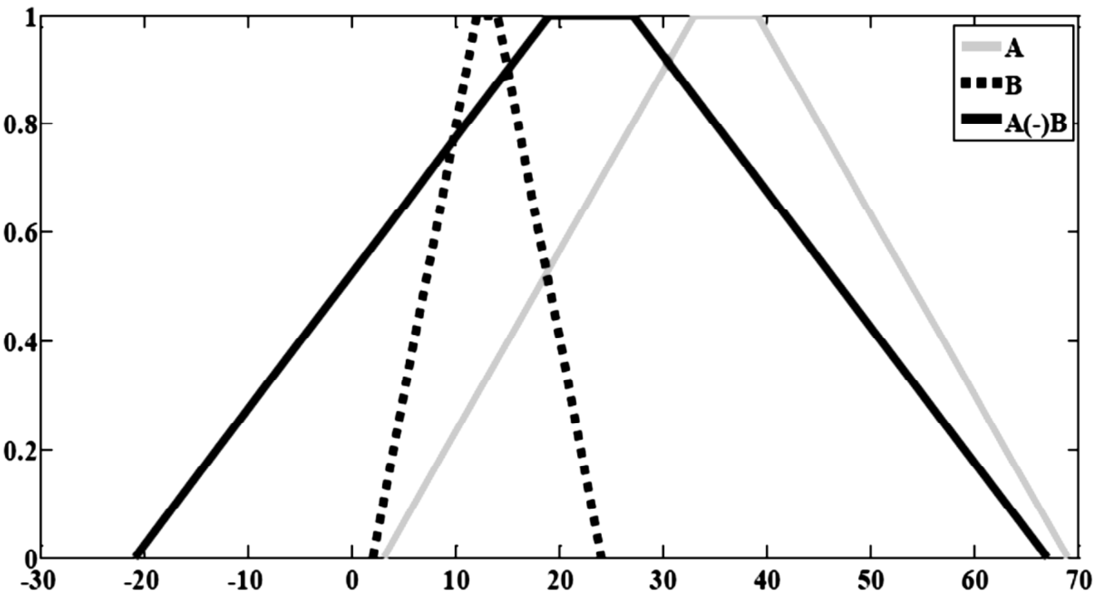

Fig. 5. Subtraction of two DDFNs A and B.

Definition 5.4.

Let LA and LB be two arbitrary uncertain linguistic terms $LA_{\overline{DD}} = (c_1, c_2, c_3, c_4, c_5 ...., c_{10}, c_{11}, c_{12})$ and $LB_{\overline{DD}} = (e_1, e_2, e_3, e_4, e_5 ...., e_{10}, e_{11}, e_{12})$ be their corresponding dodecagonal fuzzy numbers, the subtraction operation of α cuts $LC_{\overline{DD}}$ and $LE_{\overline{DD}}$ using interval arithmetic defined as $LA_{\overline{DD}} + LB_{\overline{DD}}$ same as by (2) replacing a by *c* and *b* by *e*.

To verify this a new operation with ordinary subtraction operation , we take example 4.2 Let $LA_{\overline{DD}} = (2.6, 2.8, 3.0, 3.2, 3.4, 3.6, 3.8, 4.0, 4.2, 4.4, 4.6, 4.8)$ and $LB_{\overline{DD}} = (5.0, 5.2, 5.4, 5.6, 5.8, 6.0, 6.2, 6.4, 6.8, 7.0, 7.2)$

For $\alpha \in [0.0, 0.2)$ For $\alpha \in [0.2, 0.4)$ For $\alpha \in [0.4, 0.6)$ For $\alpha \in [0.6, 0.8)$ For $\alpha \in [0.8, 1.0]$	$LA_\alpha = (\alpha + 2.6, -\alpha + 4.8)$ $LB_\alpha = (\alpha + 5.0, -\alpha + 7.2)$	$LA_\alpha(-)LB_\alpha = (2\alpha - 4.6, -2\alpha - 0.2)$

Since $\alpha \in [0.0, 0.2)$, $\alpha \in [0.2, 0.4)$, $\alpha \in [0.4, 0.6)$, $\alpha \in [0.6, 0.8)$ and $\alpha \in [0.8, 1.0]$, arithmetic intervals are same $LA_\alpha(-)LB_\alpha = (2\alpha - 4.6, -2\alpha - 0.2)$ for all $\alpha \in [0,1]$

when

$$\alpha = 0.0 \quad \Rightarrow \quad LA_\alpha(-)LB_\alpha = [-4.6, -0.2]$$
$$\alpha = 0.2 \quad \Rightarrow \quad LA_\alpha(-)LB_\alpha = [-4.2, 0.6]$$
$$\alpha = 0.4 \quad \Rightarrow \quad LA_\alpha(-)LB_\alpha = [-3.8, -1.0]$$
$$\alpha = 0.6 \quad \Rightarrow \quad LA_\alpha(-)LB_\alpha = [-3.4, -1.4]$$
$$\alpha = 0.8 \quad \Rightarrow \quad LA_\alpha(-)LB_\alpha = [-3.0, -1.8]$$
$$\alpha = 1.0 \quad \Rightarrow \quad LA_\alpha(-)LB_\alpha = [-2.6, -2.2]$$

Hence $LA_\alpha(-)LB_\alpha = (\text{-4.6,-4.2,-3.8,-3.4,-3.0,-2.6,-2.2,-1.8,-1.4,-1.0,-0.6,-0.2})$. Hence all the points coincides with the subtraction of the two dodecagonal fuzzy number.

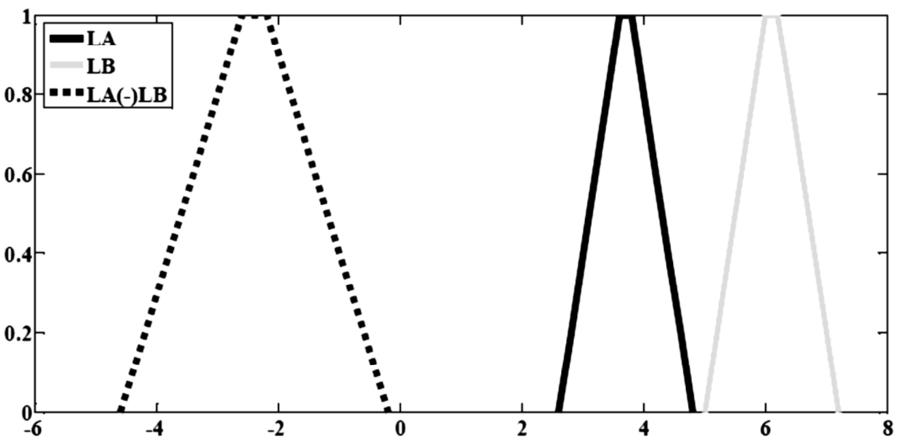

Fig. 6. Subtraction of two arbitary uncertain linguistic tem of DDFNs A and B.

Therefore subtraction of two $\alpha-$ cuts lies within the interval. Hence it is verified.

5.3. Multiplication of Two Dodecagonal Fuzzy Numbers

Definition 5.5.

Let $A_{\overline{DD}} = (a_1, a_2, a_3, a_4, ..., a_{10}, a_{11}, a_{12})$ and $B_{\overline{DD}} = (a_1, a_2, a_3, a_4, ..., a_{10}, a_{11}, a_{12})$ be their corresponding dodecagonal fuzzy numbers for all $\alpha \in [0,1]$. Let us multiply add $\alpha-$cut ${}^{\alpha}A$ and ${}^{\alpha}B$ of $AA_{\overline{DD}}$ and $AB_{\overline{DD}}$ using interval arithmetic.

$$A_{\overline{DD}}(*)B_{\overline{DD}} = \begin{cases} \begin{pmatrix} [5\alpha(a_2 - a_1) + a_1, 5\alpha(a_{11} - a_{12}) + a_{12}]* \\ [5\alpha(b_2 - b_1) + b_1, 5\alpha(b_{11} - b_{12}) + b_{12}] \end{pmatrix} & for \quad \alpha \in [0.0, 0.2) \\ \begin{pmatrix} [5\alpha(a_3 - a_2) + 2a_2 - a_3, 5\alpha(a_{10} - a_{11}) + 2a_{11} - a_{10}]* \\ [5\alpha(b_3 - b_2) + 2b_2 - b_3, 5\alpha(b_{10} - b_{11}) + 2b_{11} - b_{10}] \end{pmatrix} & for \quad \alpha \in [0.2, 0.4) \\ \begin{pmatrix} [[5\alpha(a_4 - a_3) + 3a_3 - 2a_4, 5\alpha(a_9 - a_{10}) + 3a_{10} - 2a_9]* \\ [5\alpha(b_4 - b_3) + 3b_3 - 2b_4, 5\alpha(b_9 - b_{10}) + 3b_{10} - 2b_9] \end{pmatrix} & for \quad \alpha \in [0.4, 0.6) \\ \begin{pmatrix} [5\alpha(a_5 - a_4) + 4a_4 - 3a_5, 5\alpha(a_8 - a_9) + 4a_9 - 3a_8]* \\ [5\alpha(b_5 - a_4) + 4b_4 - 3b_5, 5\alpha(b_8 - b_9) + 4b_9 - 3b_8] \end{pmatrix} & for \quad \alpha \in [0.6, 0.8) \\ \begin{pmatrix} [5\alpha(a_6 - a_5) + 5a_5 - 4a_6, 5\alpha(a_7 - a_8) + 5a_8 - 4a_7]* \\ [5\alpha(b_6 - b_5) + 5b_5 - 4b_6, 5\alpha(b_7 - b_8) + 5b_8 - 4b_7] \end{pmatrix} & for \quad \alpha \in [0.8, 1.0] \end{cases} \quad ...(3)$$

To verify this new operation with multiply operation , we take the arithmetic example 4.1

$AA_{\overline{DD}} = (3,9,15,21,27,33,39,45,51,57,63,69)$ and $AB_{\overline{OFN}} = (2,4,6,8,10,12,14,16,18,20,22,24)$

For $\alpha \in [0.0,0.2)$ For $\alpha \in [0.2,0.4)$ For $\alpha \in [0.4,0.6)$ For $\alpha \in [0.6,0.8)$ For $\alpha \in [0.8,1.0]$	$AA_\alpha = (30\alpha+3, -30\alpha+69)$ $AB_\alpha = (10\alpha+2, -10\alpha+24)$	$AA_\alpha(*)AB_\alpha = \begin{pmatrix} 300\alpha^2+90\alpha+6, \\ 300\alpha^2-1410\alpha+1656 \end{pmatrix}$

Since $\alpha \in [0.0,0.2)$, $\alpha \in [0.2,0.4)$, $\alpha \in [0.4,0.6)$, $\alpha \in [0.6,0.8)$ and $\alpha \in [0.8,1.0]$,arithmetic intervals are same $AA_\alpha(*)AB_\alpha = (300\alpha^2+90\alpha+6, 300\alpha^2-1410\alpha+1656)$ for all $\alpha \in [0,1]$

when

$$\alpha = 0.0 \Rightarrow AA_\alpha(*)AB_\alpha = [6,1656]$$
$$\alpha = 0.2 \Rightarrow AA_\alpha(*)AB_\alpha = [36,1386]$$
$$\alpha = 0.4 \Rightarrow AA_\alpha(*)AB_\alpha = [90,1140]$$
$$\alpha = 0.6 \Rightarrow AA_\alpha(*)AB_\alpha = [168,918]$$
$$\alpha = 0.8 \Rightarrow AA_\alpha(*)AB_\alpha = [270,720]$$
$$\alpha = 1.0 \Rightarrow AA_\alpha(*)AB_\alpha = [396,546]$$

Hence $AA_\alpha(*)AB_\alpha = (6,36,90,168,270,396,546,720,918,1140,1386,1656)$. Hence all the points coincides with the multiplication of the two dodecagonal fuzzy number.

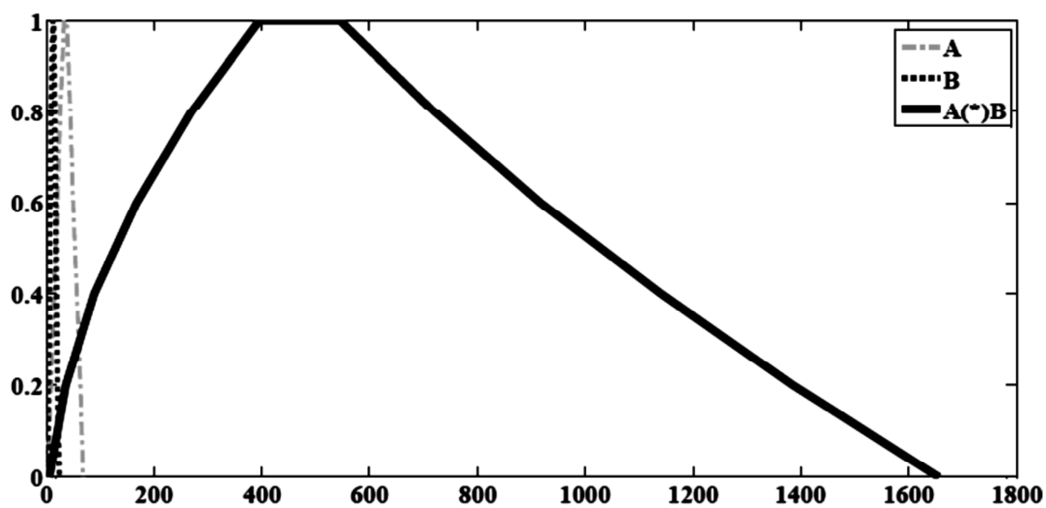

Fig. 7. Multiplication of two DDFNs A and B.

Definition 5.6.

Let LA and LB be two arbitrary uncertain linguistic terms $LA_{\overline{DD}} = (c_1, c_2, c_3, c_4, c_5 ...., c_{10}, c_{11}, c_{12})$ and $LB_{\overline{DD}} = (e_1, e_2, e_3, e_4, e_5 ...., e_{10}, e_{11}, e_{12})$ be their corresponding dodecagonal fuzzy numbers, the multiply operation of α cuts $LC_{\overline{DD}}$ and $LE_{\overline{DD}}$ using interval arithmetic defined as $LA_{\overline{DD}} + LB_{\overline{DD}}$ same as by (3) replacing a by *c* and *b* by *e*.

To verify this a new operation with ordinary multiply operation , we take example 4.2 Let $LA_{\overline{DD}} = (2.6,2.8,3.0,3.2,3.4,3.6,3.8,4.0,4.2,4.4,4.6,4.8)$ and $LB_{\overline{DD}} = (5.0,5.2,5.4,5.6,5.8,6.0,6.2,6.4,6.8,7.0,7.2)$

For $\alpha \in [0.0, 0.2)$ For $\alpha \in [0.2, 0.4)$ For $\alpha \in [0.4, 0.6)$ For $\alpha \in [0.6, 0.8)$ For $\alpha \in [0.8, 1.0]$	$LA_\alpha = (\alpha + 2.6, -\alpha + 4.8)$ $LB_\alpha = (\alpha + 5.0, -\alpha + 7.2)$	$LA_\alpha(*)LB_\alpha = \begin{pmatrix} \alpha^2 + 7.6\alpha + 13, \\ \alpha^2 - 12\alpha + 34.56 \end{pmatrix}$

Since $\alpha \in [0.0, 0.2)$, $\alpha \in [0.2, 0.4)$, $\alpha \in [0.4, 0.6)$, $\alpha \in [0.6, 0.8)$ and $\alpha \in [0.8, 1.0]$, arithmetic intervals are same $LA_\alpha(-)LB_\alpha = (2\alpha - 4.6, -2\alpha - 0.2)$ for all $\alpha \in [0,1]$

when

$$\begin{aligned}
\alpha = 0.0 &\Rightarrow LA_\alpha(*)LB_\alpha = [13.00, 34.56] \\
\alpha = 0.2 &\Rightarrow LA_\alpha(*)LB_\alpha = [14.56, 32.20] \\
\alpha = 0.4 &\Rightarrow LA_\alpha(*)LB_\alpha = [16.20, 29.92] \\
\alpha = 0.6 &\Rightarrow LA_\alpha(*)LB_\alpha = [17.92, 27.72] \\
\alpha = 0.8 &\Rightarrow LA_\alpha(*)LB_\alpha = [19.72, 25.60] \\
\alpha = 1.0 &\Rightarrow LA_\alpha(*)LB_\alpha = [21.60, 23.56]
\end{aligned}$$

Hence $LA_\alpha(-)LB_\alpha = (13.00, 14.56, 16.20, 17.92, 19.72, 21.60, 23.56, 25.60, 27.72, 29.92, 32.20, 34.56)$. Hence all the points coincides with the multiply of the two dodecagonal fuzzy number.

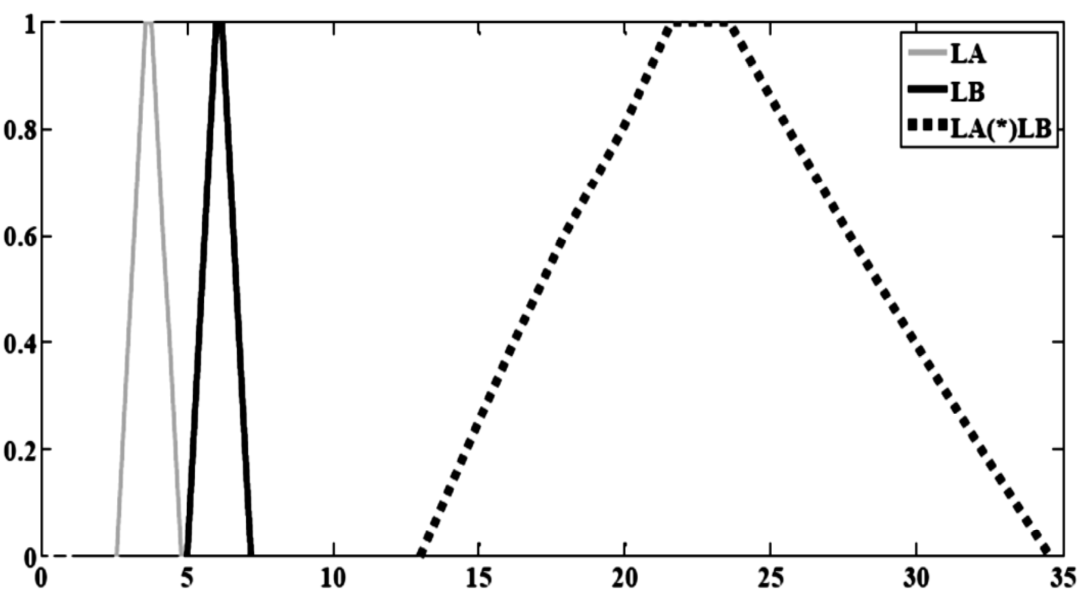

Fig. 8. Multiplication of two arbitary uncertain linguistic term of DDFNs A and B.

Therefore multiplication of two α- cuts lies within the interval. Hence it is verified.

5.4. Division of Two Dodecagonal Fuzzy Numbers

Definition 5.7.

Let $A_{\overline{DD}} = (a_1, a_2, a_3, a_4, ..., a_{10}, a_{11}, a_{12})$ and $B_{\overline{DD}} = (a_1, a_2, a_3, a_4, ..., a_{10}, a_{11}, a_{12})$ be their corresponding dodecagonal fuzzy numbers for all $\alpha \in [0,1]$. Let us division add α– cut ${}^{\alpha}A$ and ${}^{\alpha}B$ of $AA_{\overline{DD}}$ and $AB_{\overline{DD}}$ using interval arithmetic.

$$A_{\overline{DD}}(/)B_{\overline{DD}} = \begin{cases} \left[\dfrac{5\alpha(a_2-a_1)+a_1}{5\alpha(b_{11}-b_{12})+b_{12}}, \dfrac{5\alpha(a_{11}-a_{12})+a_{12}}{5\alpha(b_2-b_1)+b_1}\right] & for \quad \alpha \in [0.0, 0.2) \\ \left[\dfrac{5\alpha(a_3-a_2)+2a_2-a_3}{5\alpha(b_{10}-b_{11})+2b_{11}-b_{10}}, \dfrac{5\alpha(a_{10}-a_{11})+2a_{11}-a_{10}}{5\alpha(b_3-b_2)+2b_2-b_3}\right] & for \quad \alpha \in [0.2, 0.4) \\ \left[\dfrac{5\alpha(a_4-a_3)+3a_3-2a_4}{5\alpha(b_9-b_{10})+3b_{10}-2b_9}, \dfrac{5\alpha(a_9-a_{10})+3a_{10}-2a_9}{5\alpha(b_4-b_3)+3b_3-2b_4}\right] & for \quad \alpha \in [0.4, 0.6) \\ \left[\dfrac{5\alpha(a_5-a_4)+4a_4-3a_5}{5\alpha(b_8-b_9)+4b_9-3b_8}, \dfrac{5\alpha(a_8-a_9)+4a_9-3a_8}{5\alpha(b_5-a_4)+4b_4-3b_5}\right] & for \quad \alpha \in [0.6, 0.8) \\ \left[\dfrac{5\alpha(a_6-a_5)+5a_5-4a_6}{5\alpha(b_7-b_8)+5b_8-4b_7}, \dfrac{5\alpha(a_7-a_8)+5a_8-4a_7}{5\alpha(b_6-b_5)+5b_5-4b_6}\right] & for \quad \alpha \in [0.8, 1.0] \end{cases} \quad ...(4)$$

To verify this new operation with division operation, we take the arithmetic example 4.1

$AA_{\overline{DD}} = (3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69)$ and $AB_{\overline{OFN}} = (2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24)$

For $\alpha \in [0.0, 0.2)$ For $\alpha \in [0.2, 0.4)$ For $\alpha \in [0.4, 0.6)$ For $\alpha \in [0.6, 0.8)$ For $\alpha \in [0.8, 1.0]$	$AA_\alpha = (30\alpha+3, -30\alpha+69)$ $AB_\alpha = (10\alpha+2, -10\alpha+24)$	$AA_\alpha(/)AB_\alpha = \left(\dfrac{30\alpha+3}{-10\alpha+24}, \dfrac{-30\alpha+69}{10\alpha+2}\right)$

Since $\alpha \in [0.0, 0.2)$, $\alpha \in [0.2, 0.4)$, $\alpha \in [0.4, 0.6)$, $\alpha \in [0.6, 0.8)$ and $\alpha \in [0.8, 1.0]$, arithmetic intervals are same $AA_\alpha(/)AB_\alpha = \left(\dfrac{30\alpha+3}{-10\alpha+24}, \dfrac{-30\alpha+69}{10\alpha+2}\right)$ for all $\alpha \in [0,1]$

when

$$\begin{aligned} \alpha = 0.0 &\Rightarrow AA_\alpha(/)AB_\alpha = [0.125, 34.50] \\ \alpha = 0.2 &\Rightarrow AA_\alpha(/)AB_\alpha = [0.4091, 15.75] \\ \alpha = 0.4 &\Rightarrow AA_\alpha(/)AB_\alpha = [0.750, 9.500] \\ \alpha = 0.6 &\Rightarrow AA_\alpha(/)AB_\alpha = [1.667, 6.375] \\ \alpha = 0.8 &\Rightarrow AA_\alpha(/)AB_\alpha = [1.6875, 4.50] \\ \alpha = 1.0 &\Rightarrow AA_\alpha(/)AB_\alpha = [2.3571, 3.25] \end{aligned}$$

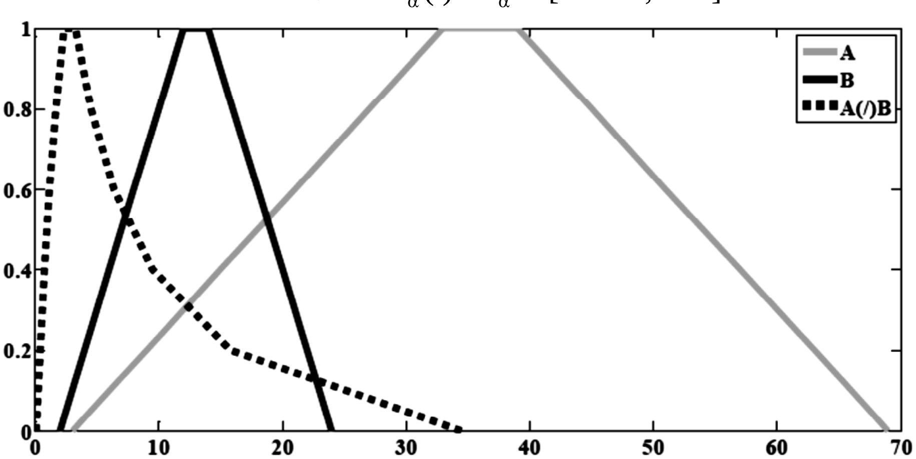

Fig. 9. Division of two DDFNs A and B.

Hence $AA_\alpha(/)AB_\alpha = (0.152, 0.4091, 0.75, 1.1667, 1.6875, 2.3571, 3.25, 4.5, 6.375, 9.5, 15.75, 34.5)$. Hence all the points coincides with the division of the two dodecagonal fuzzy number.

Definition 5.8.

Let LA and LB be two arbitrary uncertain linguistic terms $LA_{\overline{DD}} = (c_1, c_2, c_3, c_4, c_5...., c_{10}, c_{11}, c_{12})$ and $LB_{\overline{DD}} = (e_1, e_2, e_3, e_4, e_5...., e_{10}, e_{11}, e_{12})$ be their corresponding dodecagonal fuzzy numbers, the division operation of $\alpha-$ cuts $LC_{\overline{DD}}$ and $LE_{\overline{DD}}$ using interval arithmetic defined as $LA_{\overline{DD}} + LB_{\overline{DD}}$ same as by (4) replacing a by *c* and *b* by *e*.

To verify this a new operation with ordinary division operation , we take example 4.2 . Let $LA_{\overline{DD}} = (2.6, 2.8, 3.0, 3.2, 3.4, 3.6, 3.8, 4.0, 4.2, 4.4, 4.6, 4.8)$ and $LB_{\overline{DD}} = (5.0, 5.2, 5.4, 5.6, 5.8, 6.0, 6.2, 6.4, 6.8, 7.0, 7.2)$

For $\alpha \in [0.0, 0.2)$ For $\alpha \in [0.2, 0.4)$ For $\alpha \in [0.4, 0.6)$ For $\alpha \in [0.6, 0.8)$ For $\alpha \in [0.8, 1.0]$	$LA_\alpha = (\alpha + 2.6, -\alpha + 4.8)$ $LB_\alpha = (\alpha + 5.0, -\alpha + 7.2)$	$LA_\alpha(/)LB_\alpha = \left(\frac{\alpha+2.6}{-\alpha+7.2}, \frac{-\alpha+4.8}{\alpha+5}\right)$

Since $\alpha \in [0.0, 0.2)$, $\alpha \in [0.2, 0.4)$, $\alpha \in [0.4, 0.6)$, $\alpha \in [0.6, 0.8)$ and $\alpha \in [0.8, 1.0]$, arithmetic intervals are same $LA_\alpha(/)LB_\alpha = \left(\frac{\alpha+2.6}{-\alpha+7.2}, \frac{-\alpha+4.8}{\alpha+5}\right)$ for all $\alpha \in [0,1]$

when

$$\alpha = 0.0 \Rightarrow LA_\alpha(/)LB_\alpha = [0.3611, 0.96]$$
$$\alpha = 0.2 \Rightarrow LA_\alpha(/)LB_\alpha = [0.4, 0.8846]$$
$$\alpha = 0.4 \Rightarrow LA_\alpha(/)LB_\alpha = [0.4412, 0.8148]$$
$$\alpha = 0.6 \Rightarrow LA_\alpha(/)LB_\alpha = [0.4848, 0.75]$$
$$\alpha = 0.8 \Rightarrow LA_\alpha(/)LB_\alpha = [0.5313, 0.6897]$$
$$\alpha = 1.0 \Rightarrow LA_\alpha(/)LB_\alpha = [0.5806, 0.6333]$$

Hence $LA_\alpha(/)LB_\alpha = (0.3611, 0.4, 0.4412, 0.4848, 0.5313, 0.5806, 0.6333, 0.6897, 0.75, 0.8148, 0.8846, 0.96)$. Hence all the points coincides with the division of the two dodecagonal fuzzy number.

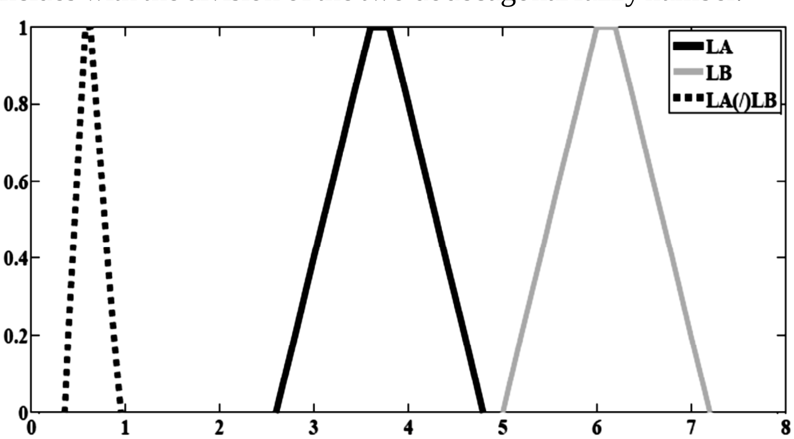

Fig. 10. Division of two arbitary uncertain linguistic term of DDFNs A and B.

Therefore division of two α- cuts lies within the interval. Hence it is verified.

5.5. Symmetric image of a Dodecagonal Fuzzy Numbers

Definition 5.9.

Let $A_{\overline{DD}} = (a_1, a_2, a_3, a_4, ..., a_{10}, a_{11}, a_{12})$ be the corresponding dodecagonal fuzzy numbers for all $\alpha \in [0,1]$. Let us symmetric image α- cut ${}^{\alpha}A$ of $AA_{\overline{DD}}$ using interval arithmetic.

$$AA_{\alpha} = \begin{cases} [5\alpha(a_2 - a_1) + a_1, 5\alpha(a_{11} - a_{12}) + a_{12}] & for \quad \alpha \in [0.0, 0.2) \\ [5\alpha(a_3 - a_2) + 2a_2 - a_3, 5\alpha(a_{10} - a_{11}) + 2a_{11} - a_{10}] & for \quad \alpha \in [0.2, 0.4) \\ [5\alpha(a_4 - a_3) + 3a_3 - 2a_4, 5\alpha(a_9 - a_{10}) + 3a_{10} - 2a_9] & for \quad \alpha \in [0.4, 0.6) \\ [5\alpha(a_5 - a_4) + 4a_4 - 3a_5, 5\alpha(a_8 - a_9) + 4a_9 - 3a_8] & for \quad \alpha \in [0.6, 0.8) \\ [5\alpha(a_6 - a_5) + 5a_5 - 4a_6, 5\alpha(a_7 - a_8) + 5a_8 - 4a_7] & for \quad \alpha \in [0.8, 1.0] \end{cases} \quad ...(5)$$

To verify this new operation with symmetric image operation , we take the arithmetic example 4.1 $AA_{\overline{DD}} = (3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69)$

For $\alpha \in [0.0, 0.2)$ For $\alpha \in [0.2, 0.4)$ For $\alpha \in [0.4, 0.6)$ For $\alpha \in [0.6, 0.8)$ For $\alpha \in [0.8, 1.0]$	$AA_{\alpha} = (30\alpha + 3, -30\alpha + 69)$	$-AA_{\alpha} = (30\alpha - 69, -30\alpha - 3)$

Since $\alpha \in [0.0, 0.2)$, $\alpha \in [0.2, 0.4)$, $\alpha \in [0.4, 0.6)$, $\alpha \in [0.6, 0.8)$ and $\alpha \in [0.8, 1.0]$, arithmetic intervals are same $-AA_{\alpha} = (30\alpha - 69, -30\alpha - 3)$ for all for all $\alpha \in [0,1]$

when

$$\begin{aligned} \alpha = 0.0 &\Rightarrow -AA_{\alpha} = [-69, -3] \\ \alpha = 0.2 &\Rightarrow -AA_{\alpha} = [-63, -9] \\ \alpha = 0.4 &\Rightarrow -AA_{\alpha} = [-57, -15] \\ \alpha = 0.6 &\Rightarrow -AA_{\alpha} = [-51, -21] \\ \alpha = 0.8 &\Rightarrow -AA_{\alpha} = [-45, -27] \\ \alpha = 1.0 &\Rightarrow -AA_{\alpha} = [-39, -33] \end{aligned}$$

Hence $-AA_{\alpha} =$ (-69,-63,-57,-51,-45,-39,-33,-27,-21,-15,-9,-3) . Hence all the points coincides with the symmetric image of the two dodecagonal fuzzy number.

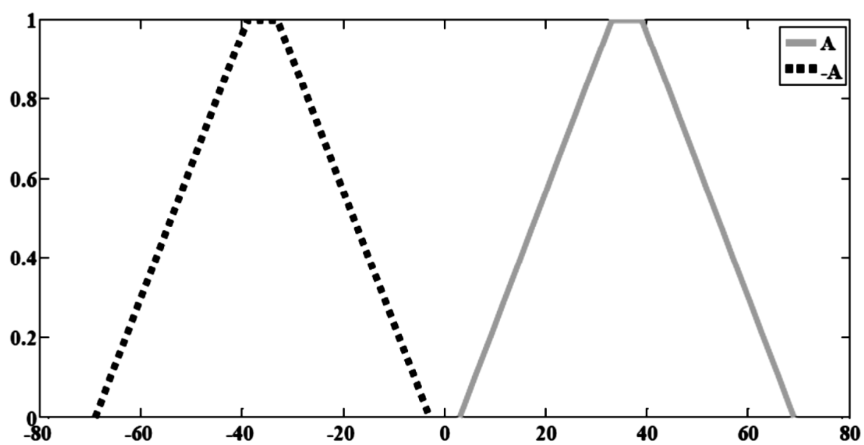

Fig. 11. Symmetric image of DDFN A.

6. CONCLUSION

In this paper dodecagonal fuzzy number has been newly introduced and the alpha cut operation of both arithmetic function and arbitrary linguistic variable principles using addition, subtraction multiplication division and symmetric image has been fully modified with some conditions and has been explained with numerical examples. Dodecagonal Fuzzy Number can be applied to that problem which has twelve points in representation. We observe that fuzzy number concepts could be applied to many real life problems. In future, it may be applied in many operations/ research problems, which has twelve arithmetic and linguistic variables.

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