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Image Adaptive Watermarking using JAYA Optimization Algorithm

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Abstract : The main challenge for watermarking algorithms is maintaining the tradeoff between robustness and imperceptibility. As such various optimization algorithms are popularly used to determine the optimum strength for embedding of the watermark. This paper uses JAYA optimization technique to make the scale factor adaptive. The novelty of this optimization technique is that it requires only common control parameters. Thus, there is no requirement for proper tuning of algorithm specific parameters as needed in most other optimization techniques. In this paper a comparison is performed between JAYA optimization algorithm and popularly used Hybrid Particle Swarm Optimization (HPSO) Algorithm. The performance of JAYA is also tested for an increased payload of watermark. This comparison shows a performance supremacy of JAYA over HPSO when applied in case of image watermarking.

Keywords: Watermarking, Optimization, Adaptive scale factor.

1. INTRODUCTION

The idea behind digital image watermarking is the hiding of digital data inside an image without visibly altering the image itself. This concept has applications in various domains such as broadcast monitoring, owner identification, fingerprinting, publication monitoring and copy control, image and content authentication, tamper detection, medical applications, copyright protection, content description, covert communication, digital signatures etc¹. Especially with the exponentially growing internet traffic, the protection of intellectual property rights has become a major issue. Conventionally, ownership verification is done by placing a visible signature over the image. However, such signatures can be easily removed using various image processing tools. This motivates the development of invisible watermarking techniques. For successful watermarking, the technique should be robust and imperceptible. The resistance of a watermark against several attacks such as cropping, rotation, filtering, addition of noise etc. shows its robustness. Imperceptibility is the similarity between the watermarked image and the original cover image so that the watermark is invisible to the human eye.

The watermarking techniques available today can be broadly categorized into spatial domain and frequency domain techniques. The spatial domain techniques basically deal with the image pixels directly as seen in². These techniques are relatively simpler to implement but show less robustness against most image processing attacks³. The frequency domain techniques include Discrete Cosine Transform (DCT), Discrete Fourier transform (DFT), Discrete Wavelet Transform (DWT), Singular Value Decomposition (SVD) etc. These techniques deal with modifying the transform coefficients. Although they have higher computation cost, yet they show good robustness against common image manipulations.

SVD based watermarking techniques have been popularly used throughout literature mainly due to two reasons: (1) SVD is fairly universal transform as it can be applied to both square and rectangular matrices, (2) The singular values are highly stable against small perturbations applied to the image. Several robust SVD based schemes have been proposed⁴⁻⁸. A block based watermarking scheme has been proposed by Chang et al⁸ where, SVD is applied in several blocks of the cover image and watermark is used to modify the U matrix of each block. Qingtang Su et al.⁴ have proposed a blind SVD based watermarking scheme for color images where the watermark is embedded in the first column of U matrix of the cover image. Liu and Tan⁹ have proposed a watermarking scheme in which the singular values of the original image have been modified by a pseudo Gaussian random number. Ghazy et al.¹⁰ has also designed a block based watermarking scheme in which the watermark is embedded in the singular values of each block of the cover image. SVD has also been used in combination with DWT in several watermarking schemes¹¹⁻¹⁴. O. Jane et al.³ have proposed a hybrid DWT and SVD based scheme in which the watermark is used to modify the singular values of the low frequency sub-band of the cover image. Zhang et al has shown¹⁵ that SVD based schemes show false positive problem as seen in several watermarking schemes¹⁶⁻¹⁸. Addressing this issue Jing Ming Guo et al¹⁹ have proposed a modified SVD based scheme in which the principle components of the watermark are used to modify the singular values of several blocks of the cover image.

Most of these above mentioned schemes however use a scalar scale factor. The scale factor determines the strength with which the watermark is embedded inside the cover image. If the strength of embedding is very high, robustness increases, however, the imperceptibility becomes very poor. On the other hand, if the strength of embedding is low, the robustness of the scheme becomes poor. The scale factor that gives best trade-off between robustness and imperceptibility varies with images. Thus, to make a watermarking scheme universal, we need to make the scale factor adaptive with respect to the image content. Several optimization algorithms that are being used in contemporary watermarking techniques include Particle Swarm Optimization (PSO), Genetic Algorithm(GA), Ant colony optimization (ACA), Differential Evolution (DE), Bee algorithm (BA), Cuckoo Search algorithm (CS), Firefly Algorithm (FA) etc. Hybrid Particle Swarm Optimization (HPSO) Algorithm is a hybrid optimization technique between PSO and GA. GA is based on the concept of natural selection and survival of fittest. A population of individuals representing possible solutions of a given problem are coded with a finite length vector just like human chromosomes. A fitness value is calculated for each of these individuals and the individuals showing good fitness value are considered for recombination while others are discarded. In image watermarking GA can be used to determine the locations suitable for insertion of watermark²⁰⁻²². In several other work²³⁻²⁶ GA has been used to determine the optimum strength factors. PSO developed by Kennedy and Elberhart²⁷, is a popular technique for multi-dimensional optimization problems. In this technique, a random population of individuals is generated and the best individual is found iteratively based on velocity vector, position vector and fitness value of each individual. Just like GA, PSO is also used to optimize the scale factors²⁸⁻³⁰ and also to determine the best positions of embedding³¹⁻³⁴.

Hai Tao et al.³⁵ have combined GA and PSO to make their algorithm image adaptive. In this scheme, the cover image is divided into several blocks and SVD is applied to each block. A preprocessed watermark is embedded in the largest singular values of each block using a scale factor. The scale factor is optimized using the hybrid PSO algorithm. This scheme shows good robustness against image processing and geometric attacks, as well as good imperceptibility.

In this paper JAYA algorithm is used in determining the optimum scale factor. The scheme of Hai Tao et al is used with JAYA algorithm and the results are compared with the results of HPSO algorithm.

The rest of the paper is organized as follows: Section II gives the preliminaries, the embedding and extraction algorithms are given in Section III, Section IV gives the experimental results and the conclusion is given in section V.

2. PRELIMINARIES

2.1. JAYA Optimization Algorithm

Most optimization algorithms as discussed in the previous section require algorithm specific parameters in addition to common control parameters. The common control parameters include population size, number of iterations etc. Examples of algorithm specific parameters may include mutation rate, crossover rate, selection rate as found in GA, or inertia weight, cognitive parameter and social parameter as found in PSO etc. These algorithms thus require proper tuning of the algorithm specific parameters, necessary to obtain the optimum results. Rao³⁶ developed a new simple optimization technique called the JAYA algorithm which doesn't require any algorithm specific parameter. The algorithm is based on the idea that the solution of any problem should move away from the worst solution and towards the best solution.

Let $f(x)$ be the fitness function to be optimized. We initialize n number of candidate solutions such that ($k = 1, 2 \dots n$) and set the total number of iterations to be carried out. For each candidate solution the $f(x)$ is evaluated and the best and worst solutions of $f(x)$ are determined. The candidates are then updated by the following equation.

$$X'_{k,i} = X_{k,i} + r_{1,i}(X_{best,i} - |X_{k,i}|) - r_{2,i}(X_{worst,i} - |X_{k,i}|) \quad (1)$$

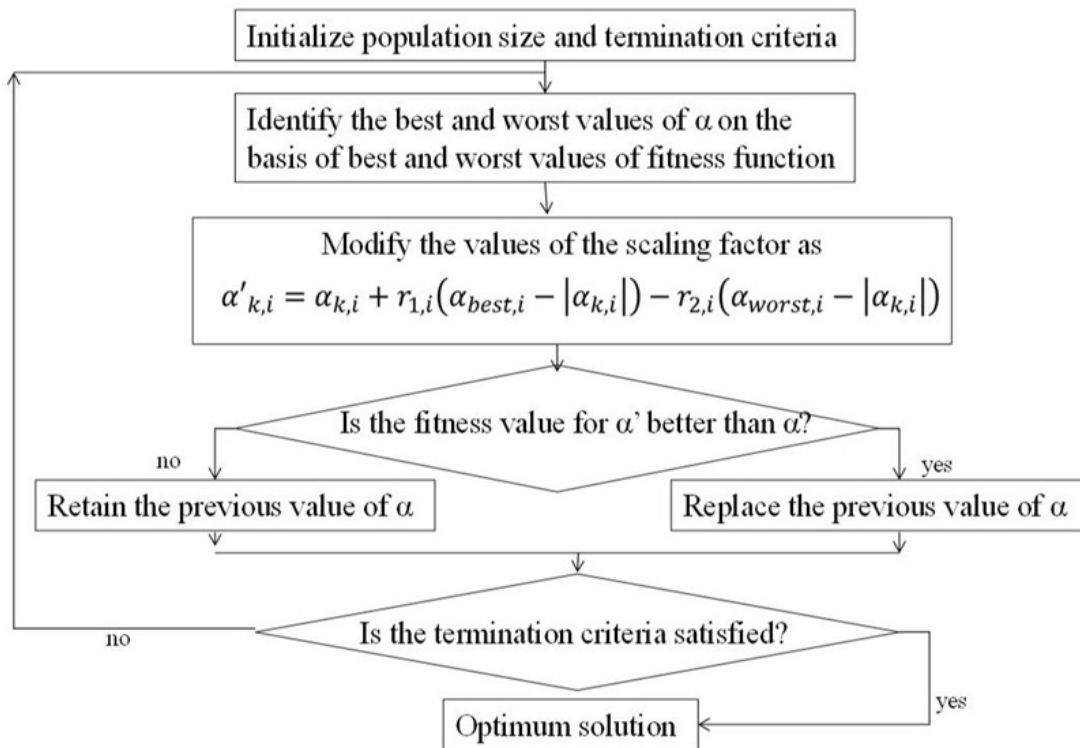


Figure 1: JAYA algorithm to optimize the scale factor. (Here α depicts the scale factor, k is the candidate in the population, i is the number of iteration)

$X_{\text{best},i}$ is the value of the best candidate for the i^{th} iteration and $X_{\text{worst},i}$ is the value of the worst candidate for the i^{th} iteration. $X'_{k,i}$ is the updated value of the candidate k . If the updated value gives better solution to the fitness function than the current value for the k^{th} candidate, then the updated value is sent for the next iteration. Else the current value is retained. $r_{1,i}$ and $r_{2,i}$ are two random values chosen for i^{th} iteration.

In this paper JAYA is used to determine the best possible scaling factor to maintain both robustness and imperceptibility as shown in Fig. 1.

2.2. Singular Value Decomposition (SVD)

SVD is a mathematical tool for decomposition of a matrix into three components U, S and V as given by the following equation.

$$A = U \times S \times V^T \quad (2)$$

If A is a matrix of size $M \times N$ then, the columns of matrix U (size $M \times M$) are called the left singular vectors, the columns of matrix V (size $N \times N$) are called the right singular vectors and the diagonal elements of matrix S (size $M \times N$) are called the singular values (SVs). If A is an image, then matrix U and V will give the horizontal and vertical details of the image respectively while matrix S will give the gray scale values of the image layers formed by U and V. In image watermarking techniques SVD finds huge applications because even after an image is rotated, translated or transposed, the SVs of the image doesnot change.

2.3. Normalized Correlation Coefficient (NC)

$$\rho(W, W^*) = \frac{\sum_{i=1}^N \sum_{j=1}^N (w_{ij} - \mu_w)(w^*_{ij} - \mu_w^*)}{\sqrt{\sum_{i=1}^N \sum_{j=1}^N (w_{ij} - \mu_w)^2} \sqrt{\sum_{i=1}^N \sum_{j=1}^N (w^*_{ij} - \mu_w^*)^2}} \quad (3)$$

Normalized correlation coefficient measures the similarity between the embedded and the extracted watermark. Here W is the embedded watermark, W^* is the extracted watermark, μ_w is the mean value of the embedded watermark and μ_w^* is the mean value of the extracted watermark; N is the total number of pixels in the watermark. A correlation coefficient value close to 1 is desired. It is used in this paper as a measure of robustness.

3. WATERMARK EMBEDDING AND EXTRACTION ALGORITHM

3.1. Preprocessing of the Watermark

The binary watermark (BW) of size $M \times M$ is scrambled using Arnold scrambling to remove any correlation present between the image pixels. This enhances the robustness of the technique. The watermark is then modulated by a binary pseudo random sequence $p \in \{-1, 1\}$ to obtain the preprocessed watermark BW'.

3.2. Watermark Embedding

The cover image (A) of size $N \times N$ is decomposed into blocks of size $m \times m$ such that $m = N/M$.

SVD is performed on each block and the highest singular value of each block is extracted. The highest singular value is the first element of the S matrix.

The highest singular value of each block is modified by the preprocessed binary watermark pixels as:

$$s'(i, j) = s(i, j) + \alpha * BW'(i, j) \quad (4)$$

Inverse SVD is performed to obtain the modified blocks which are then recombined to obtain the watermarked image (IW).

The entire process of embedding is shown in Fig. 2.

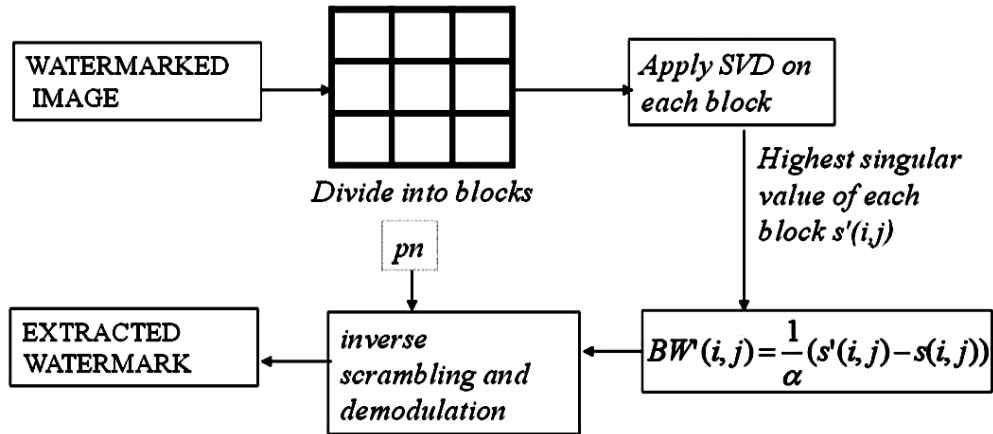


Figure 2: Watermark embedding

3.3. Watermark Extraction:

1. The possibly distorted watermarked image (IW') is decomposed into blocks of size $m \times m$.
2. SVD is performed on each block and the highest singular value of each block is extracted.
3. The watermark bits are then extracted as

$$BW'(i, j) = \frac{1}{\alpha}(s'(i, j) - s(i, j)) \quad (5)$$

4. BW' is then demodulated using the same binary pseudo random sequence used to modulate the watermark and then inverse scrambling is carried out. A thresholding operation is performed to obtain the extracted watermark.

The entire process of extraction of watermark is shown in Fig. 3.

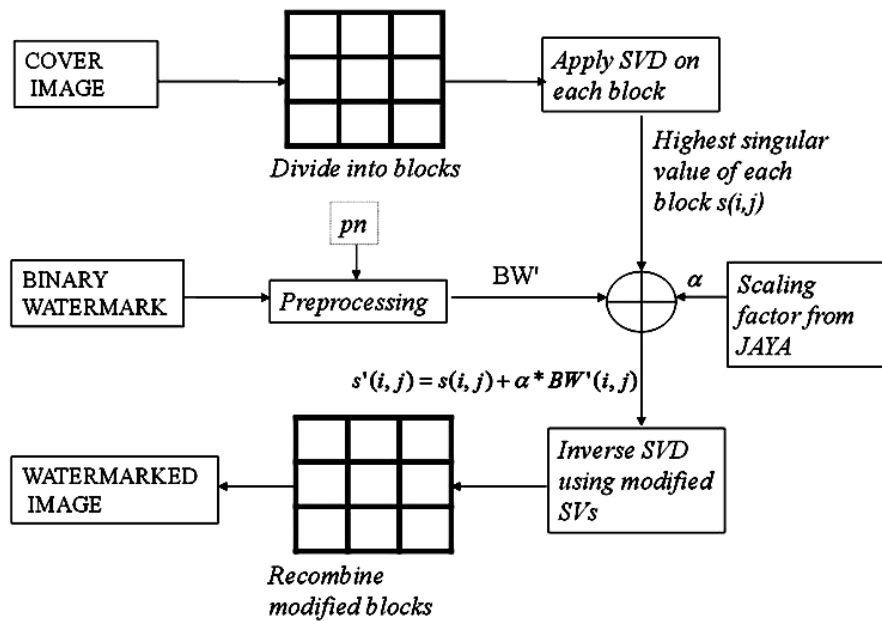


Figure 3: Watermark extraction

3.4. JAYA Optimization

The fitness function is chosen as

$$f(x) = NC_1 + \frac{1}{n} \sum_{i=1}^n NC_i \quad (6)$$

Here, NC_1 is the correlation coefficient value between the watermarked image and the original cover image. The second term of the equation gives the average of the correlation coefficient values between the embedded and the extracted watermarks after applying n attacks. The flowchart of JAYA algorithm for choosing the optimum scaling factor is shown in Fig. 1.

4. EXPERIMENTAL RESULTS

The experiment was carried out for three cover images “Lena”, “Barbara” and “Mandrill” of size 256×256 each. The cover images are shown in Fig. 4. The binary watermark of size 32×32 is shown in Fig. 4(d). The termination criteria for JAYA algorithm is set for 30 iterations and population size is chosen to be 20. For the fitness function three different attacks were used salt and pepper noise (0.01), JPEG compression ($Q = 40$) and median filtering (3×3 window). The fitness values for all three cover images after 30 iterations remains above 0.98. Imperceptibility is measured using normalized correlation coefficient (NC_1) values between watermarked image and cover image. The watermarked images with their NC_1 values are seen in Fig. 5.



Figure 4: Cover images (a) Lena (b) Barbara (c) Mandrill (d) Binary watermark



Figure 5: Watermarked images (a) Lena ($NC_1 = 0.9953$) (b) Barbara ($NC_1 = 0.9967$) (c) Mandrill ($NC_1 = 0.9877$)



Figure 6: Watermarked images after attacks (a) Resize (50%) (b) JPEG compression (Q = 40) (c) LPF (window 3x3) (d) s & p (0.01) (e) Gaussian noise (0.01) (f) Cropping (10%) (g) Tampering

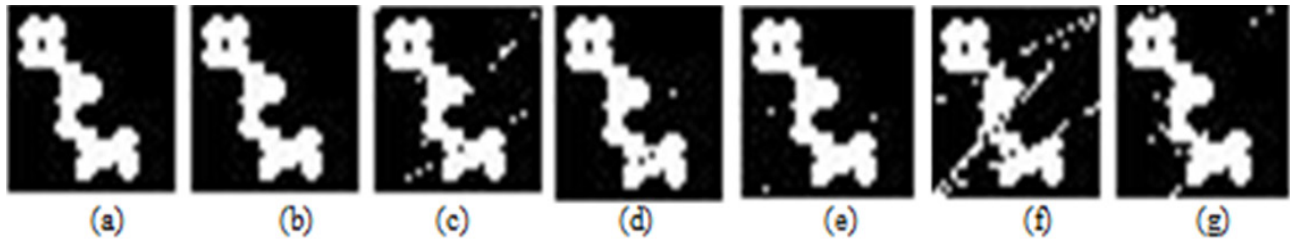


Figure 7: Extracted watermark images after attacks (a) Resize (50%) (b) JPEG compression (Q = 40) (c) LPF (window 3x3) (d) s & p (0.01) (e) Gaussian noise (0.01) (f) Cropping (10%) (g) Tampering

The optimized value of scale factor was then used to determine the robustness against attacks. Seven different types of attacks were applied on the watermarked image to check the robustness of the watermarking scheme namely resizing (50%), JPEG compression (Q = 40), low pass filtering (LPF) (window size 3×3), salt and pepper noise (s&p) (0.01), Gaussian noise (0.01), cropping (10%) and tampering. The watermarked images after attacks along with the intensity of each attack is given in Fig. 6. The extracted watermarks after attacks is given in Fig. 7. The correlation coefficient values after applying attacks is listed in Table 1. A correlation coefficient value of 1 shows that the extracted watermarks after JPEG compression (Q = 40) and resizing (50%) remains exactly same as the embedded watermark. Also for most other types of attacks the scheme optimized with JAYA algorithm gives much better results showing performance supremacy over HPSO algorithm. The comparison of Normalized correlation coefficient values after different attacks between HPSO (as documented in³⁵) and JAYA is shown in Fig. 8.

Table 1
Correlation Coefficient values for different cover images (HPSO results obtained from ³⁵)

| Attacks | Lena | | Barbara | | Mandrill | |
|------------------------|--------|--------|---------|--------|----------|--------|
| | JAYA | HPSO | JAYA | HPSO | JAYA | HPSO |
| Resizing (50%) | 1 | 0.8127 | 1 | 0.7122 | 1 | 0.7866 |
| JPEG (Q=40) | 1 | 0.9524 | 1 | 0.9035 | 1 | 0.8711 |
| Low pass filter (3x3) | 0.9426 | 0.7849 | 0.9531 | 0.7721 | 0.9886 | 0.7687 |
| Salt and pepper (0.01) | 0.9828 | 0.8916 | 0.9666 | 0.8264 | 0.9914 | 0.8969 |
| Gaussian noise (0.001) | 0.9153 | 0.7716 | 0.8954 | 0.7690 | 0.9915 | 0.7823 |
| Cropping (10%) | 0.8189 | 0.8029 | 0.8317 | 0.7949 | 0.8343 | 0.7855 |
| Tampering | 0.9605 | 0.8429 | 0.9576 | 0.7792 | 0.9747 | 0.7826 |

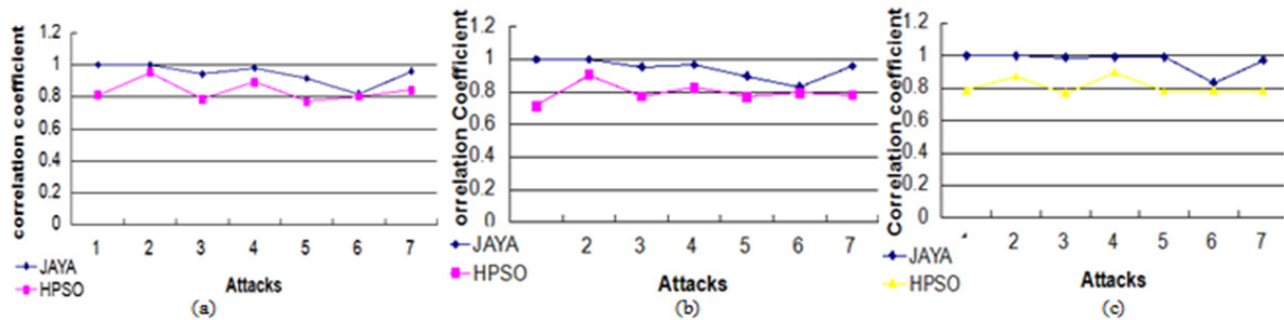


Figure 8: Comparison of robustness after using JAYA optimization algorithm and HPSO algorithm for cover images (a) Lena (b) Barbara (c) Mandrill. (Attacks 1. Resizing (50%) 2. JPEG compression (Q = 40) 3. LPF (window 3x3) 4. S&p (0.01) 5. Gaussian noise (0.01) 6. Cropping (10%) 7. Tampering)

Another experiment was carried out by increasing the payload i.e. the size of the watermarked used was 64x64. The correlation coefficient values for increased payload is listed in Table 2. The extracted watermarks for the same is seen in Fig. 9.

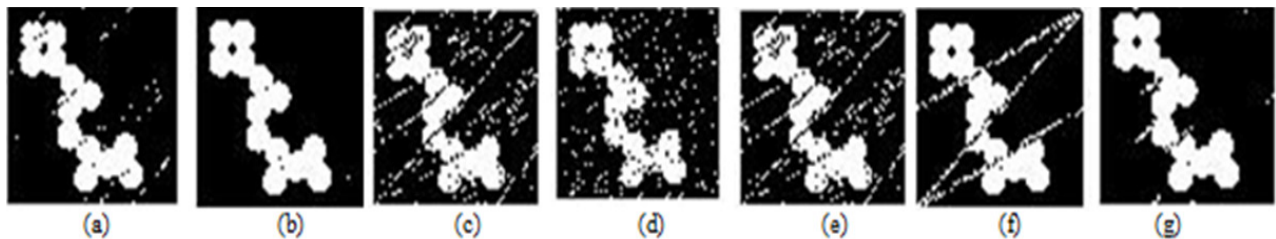


Figure 9: Extracted watermark images after attacks (a) Resize (50%) (b) JPEG compression (Q = 40) (c) LPF (window 3x3) (d) s&p (0.01) (e) Gaussian noise (0.01) (f) Cropping (10%) (g) Tampering taken for watermark of size 64x64 and cover image “Lena”

Table 2
Correlation Coefficient values for different cover images taken for watermark of size 64x64

| Cover Image | NCI | JPEG (Q=40) | S&p (.01) | Resize (50%) | Gaussian noise(.01) | Cropping (10%) | LPF (3x3) | Tampering |
|-------------|--------|-------------|-----------|--------------|---------------------|----------------|-----------|-----------|
| Lena | 0.9953 | 1 | 0.8503 | 0.9600 | 0.7095 | 0.8669 | 0.7934 | 0.9630 |
| Barbara | 0.9898 | 1 | 0.8651 | 0.9949 | 0.7181 | 0.8833 | 0.8739 | 0.9700 |
| Mandril | 0.9829 | 1 | 0.8865 | 0.9877 | 0.6814 | 0.8781 | 0.8317 | 0.9757 |

5. CONCLUSION

In this paper JAYA algorithm has been applied to image watermarking to find suitable scale factor. The algorithm proves successful in maintaining a trade-off between imperceptibility and robustness. When compared to HPSO algorithm, JAYA algorithm shows better results for most attacks thus showing good robustness. In case of imperceptibility, the correlation coefficient values between watermarked and original images always remains above 0.98. When applied for higher payload, the watermarks could be successfully extracted even after attacks. Comparable performance is obtained with lesser number of iterations and smaller population sizes. Thus, it requires lesser number of computations as compared to HPSO. Also due to the absence of algorithm specific parameters the algorithm is easier to implement. The new JAYA optimization technique can thus be successfully used for optimization problems related to image watermarking.

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