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Anti-Synchronization of Identical Chaotic Systems via Novel Sliding Control Method with Application to Vaidyanathan-Madhavan Chaotic System

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Abstract: First, this paper proposes a general procedure for the anti-synchronization of identical chaotic systems using novel sliding mode control method. The general result derived using novel sliding mode control method is established via Lyapunov stability theory. As an application of the general result, the problem of anti-synchronization of identical Vaidyanathan-Madhavan chaotic systems (2013) is studied and a new sliding mode controller is derived. Numerical simulations with MATLAB have been shown to illustrate the phase portraits of Vaidyanathan-Madhavan chaotic system and the sliding mode controller design for the ant-synchronization of identical Vaidyanathan-Madhavan chaotic systems.

Keywords: Chaos, chaotic systems, chaos synchronization, sliding mode control.

1. INTRODUCTION

A *chaotic system* is commonly defined as a nonlinear dissipative dynamical system that is highly sensitive to even small perturbations in its initial conditions [1]. In other words, a chaotic system is a nonlinear dynamical system with at least one positive Lyapunov exponent. Some paradigms of chaotic systems can be listed as Lorenz's system [2], Rössler system [3], Shimizu-Morioka system [4], Shaw system [5], Chen system [6], Lü system [7], Chen-Lee system [8], Cai system [9], Tigan system [10], Li system [11], etc. Many new 3-D chaotic systems have been discovered in the recent years such as Sundarapandian systems [12-13], Vaidyanathan systems [14-20], Vaidyanathan-Madhavan system [21], Vaidyanathan-Azar system [22], Vaidyanathan-Volos system [23-24], Pehlivan-Moroz system [25], Pham system [26], etc.

Hyperchaotic systems are the chaotic systems with more than one positive Lyapunov exponent. They have important applications in control and communication engineering. Some recently discovered 4-D hyperchaotic systems are hyperchaotic Vaidyanathan systems [27-28], hyperchaotic Vaidyanathan-Azar system [29], etc. A 5-D hyperchaotic system with three positive Lyapunov exponents was also recently found [30].

Chaos theory has several applications in a variety of fields such as lasers [31], oscillators [32-33], chemical reactors [34-35], biology [36-38], ecology [39-40], neural networks [41-43], robotics [44-45], memristors [46-48], fuzzy systems [49-50], etc.

The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [51-52]. Some popular methods for chaos control are active control [53-57], adaptive control [58-59], sliding mode control [60-62], etc.

Chaos synchronization problem can be stated as follows. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of

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the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

The synchronization of chaotic systems has applications in secure communications [63-65], cryptosystems [66-67], encryption [68-70], etc.

The chaos synchronization problem has been paid great attention in the literature and a variety of impressive approaches have been proposed. Since the pioneering work by Pecora and Carroll [71-72] for the chaos synchronization problem, many different methods have been proposed in the control literature such as active control method [73-80], adaptive control method [81-107], sampled-data feedback control method [108-109], time-delay feedback approach [110], backstepping method [111-122], sliding mode control method [123-131], etc.

In this paper, new results have been derived for the anti-synchronization of identical chaotic systems using novel sliding control method. The sliding mode control method has advantages of low sensitivity to parameter variations in the plant and disturbances affecting the plant.

In Section 2, we describe the anti-synchronization of identical chaotic systems. In Section 3, we derive a general result for the anti-synchronization of identical chaotic systems using novel sliding control method. In Section 4, we describe the Vaidyanathan-Madhavan chaotic system ([21], 2013) and its qualitative properties. The phase portraits of the Vaidyanathan-Madhavan system are described using MATLAB. In Section 5, we describe the sliding mode controller design for the anti-synchronization of the identical Vaidyanathan-Madhavan chaotic systems using novel sliding control method and its numerical simulations using MATLAB. Section 6 contains a summary of the main results derived in this paper.

2. ANTI-SYNCHRONIZATION OF IDENTICAL CHAOTIC SYSTEMS

In this section, we provide a problem statement for the anti-synchronization of identical chaotic systems.

As the master system, we consider the chaotic system given by

$$\dot{x} = Ax + \varphi(x) \tag{1}$$

In Eq. (1), $x \in R^n$ denotes the state of the system, A denotes the matrix of system parameters and φ contains the nonlinear parts of the system.

As the slave or response system, we take the controlled chaotic system given by

$$\dot{y} = Ay + \varphi(y) + u \tag{2}$$

In Eq. (2), $y \in R^n$ denotes the state of the system and $u \in R^n$ is the control.

The anti-synchronization error between the systems (1) and (2) is defined as

$$e = y + x \tag{3}$$

A simple calculation yields the error dynamics as

$$\dot{e} = Ae + \eta(x, y) + u, \tag{4}$$

where

$$n(x, y) = \varphi(y) + \varphi(x). \tag{5}$$

Thus, the anti-synchronization problem for the chaotic systems (1) and (2) can be defined as follows: Find a controller u so as to render the anti-synchronization error $e(t)$ to be globally asymptotically stable for all values of $e(0) \in R^n$, i.e.

$$\lim_{t\to\infty}\|e(t)\| = 0 \text{ for all } e(0)\in R^n. \tag{6}$$

3. SLIDING CONTROLLER DESIGN

First, we set the design by setting the control as

$$u(t) = -\eta(x, y) + Bv(t) \tag{7}$$

In Eq. (7), $B \in R^n$ is chosen such that (A, B) is completely controllable.

By substituting (7) into (4), we get the closed-loop error dynamics

$$\dot{e} = Ae + Bv \tag{8}$$

The system (8) is a linear time-variant control system with single input v.

Hence, the anti-synchronization of the identical chaotic systems (1) and (2) has been converted to an equivalent control problem of globally stabilizing the error system (8) by a suitable choice of the feedback control (7).

We start the sliding controller design by defining the sliding variable as

$$s(e) = Ce = c_1e_1 + c_2e_2 + \cdots + c_ne_n, \tag{9}$$

where $C \in R^{1\times n}$ is a constant vector to be determined.

The *sliding manifold* S is defined as the hyperplane

$$S = \left\{e\in R^n : s(e) = Ce = 0\right\}. \tag{10}$$

We shall assume that a sliding motion occurs on the hyperplane S.

In sliding mode, the following equations must be satisfied:

$$s \equiv 0 \text{ and } \dot{s} \equiv CAe + CBv = 0 \tag{11}$$

We assume that

$$CB \neq 0 \tag{12}$$

The sliding motion is influenced by the equivalent control derived from (11) as

$$v_{eq}(t) = -(CB)^{-1}CAe(t) \tag{13}$$

By substituting (13) into (8), we obtain the equivalent error dynamics in the sliding phase as

$$\dot{e} = Ae - (CB)^{-1}CAe = Ee, \tag{14}$$

where

$$E = [I - B(CB)^{-1}C]A \tag{15}$$

We note that E is independent of the control and has at most $(n–1)$ nonzero eigenvalues, depending on the chosen switching surface, while the associated eigenvectors belong to ker (C).

Since (A, B) is controllable, we can use sliding control theory to choose B and C so that E has any desired $(n–1)$ stable eigenvalues.

This shows that the dynamics in the sliding mode is globally asymptotically stable.

Finally, for the sliding controller design, we apply a *novel sliding control law*, viz.

$$\dot{s} = -ks - qs^2 \text{sgn}(s) \tag{16}$$

In Eq. (16), $\text{sgn}(\cdot)$ denotes the sign function and the sliding mode control constants $k > 0,\ q > 0$ are found in such a way that the sliding condition is satisfied and that the sliding motion will occur.

By combining equations (11), (13) and (16), we finally obtain the sliding mode control (SMC) $v(t)$ as

$$v(t) = -(CB)^{-1}[C(kI + A)e + qs^2 \text{ sgn}(s)] \tag{17}$$

Next, we establish the main result of this section.

Theorem 1: The sliding mode controller law defined by (7) achieves global and asymptotic anti-synchronization of the identical chaotic systems (1) and (2) for all initial conditions $x(0), y(0) \in R^n$, where v is defined by the novel sliding control law (16), $B \in R^{n\times 1}$ is such that (A, B) is controllable, $C \in R^{1\times n}$ is such that $CB \neq 0$ and that the matrix E defined by (15) has $(n-1)$ stable eigenvalues.

Proof. Upon substitution of the control laws (7) and (17) into the error dynamics (4), we get the closed-loop error dynamics as

$$\dot{e} = Ae - B(CB)^{-1}\left[C(kI + A)e + qs^2 \text{ sgn}(s)\right] \tag{18}$$

We shall show that the error system (18) is globally asymptotically stable by considering the quadratic Lyapunov function

$$V(e) = \frac{1}{2}s^2(e) \tag{19}$$

The sliding mode motion is characterized by the equations

$$s(e) = 0 \text{ and } \dot{s}(e) = 0 \tag{20}$$

By the choice of E, the dynamics in the sliding mode is globally asymptotically stable.

When $s(e) \neq 0, V(e) > 0$.

Also, when $s(e) \neq 0$, differentiating V along the error dynamics (18) or the equivalent dynamics (16), we get

$$\dot{V}(e) = s\dot{s} = -ks^2 - qs^3 \text{ sgn}(s) < 0. \tag{21}$$

Hence, by Lyapunov stability theory [132], the error dynamics (18) is globally asymptotically stable for all $e(0) \in R^n$. This completes the proof.

4. VAIDYANATHAN-MADHAVAN CHAOTIC SYSTEM

The Vaidyanathan-Madhavan system ([21], 2013) is described by the 3-D dynamics

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\
\dot{x}_2 &= bx_1 + cx_1 x_3 \\
\dot{x}_3 &= -dx_3 - x_1 x_2 - x_1^2
\end{aligned} \tag{22}$$

where x_1, x_2, x_3 are state variables and a, b, c, d are constant, positive, parameters of the system.

The system (22) exhibits a chaotic attractor for the values

$$a = 22, \quad b = 400, \quad c = 50, \quad d = 0.5 \tag{23}$$

The Lyapunov exponents of the system (22) are numerically obtained with MATLAB as

$$L_1 = 3.3226, \quad L_2 = 0, \quad L_3 = -30.3406 \tag{24}$$

The Lyapunov dimension of the chaotic system (1) is determined as

$$D_L = \frac{L_1 + L_2}{|L_3|} = 2.1095 \tag{25}$$

For numerical simulations, the initial values of the Vaidyanathan-Madhavan system (22) are taken as $x_1(0) = 0.6$, $x_2(0) = 1.8$ and $x_3(0) = 1.2$.

Figure 1 shows the strange chaotic attractor of the Vaidyanathan-Madhavan system (22). Figures 2-4 show the 2-D view of the chaotic attractor of the system (1) in (x_1, x_2), (x_2, x_3) and (x_1, x_3) planes respectively.

5. ANTI-SYNCHRONIZATION OF VAIDYANATHAN-MADHAVAN CHAOTIC SYSTEMS VIA NOVEL SLIDING CONTROLLER

In this section, we describe novel sliding controller design for the anti-synchronization of Vaidyanathan-Madhavan chaotic systems.

As the master system, we consider the Vaidyanathan-Madhavan chaotic system given by

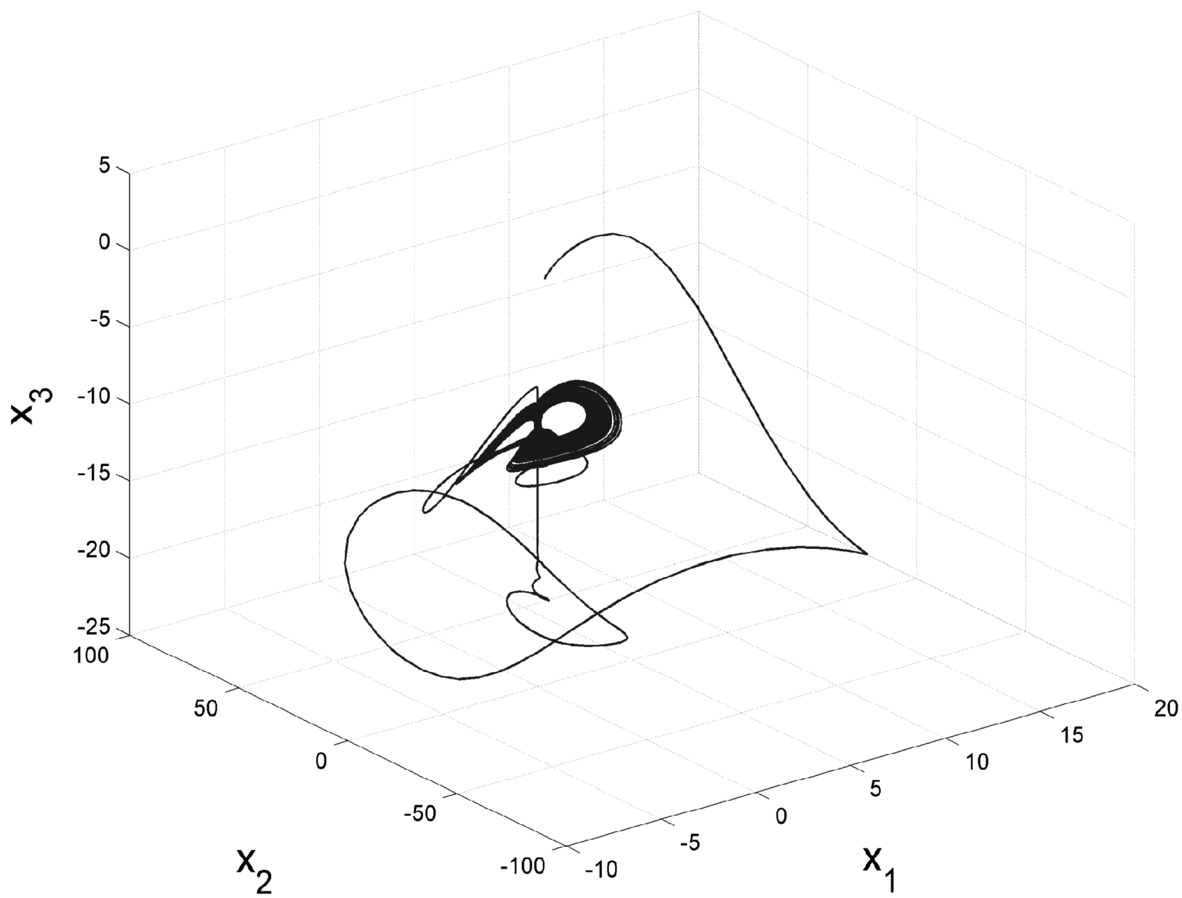

Figure 1: Strange attractor of the novel chaotic system

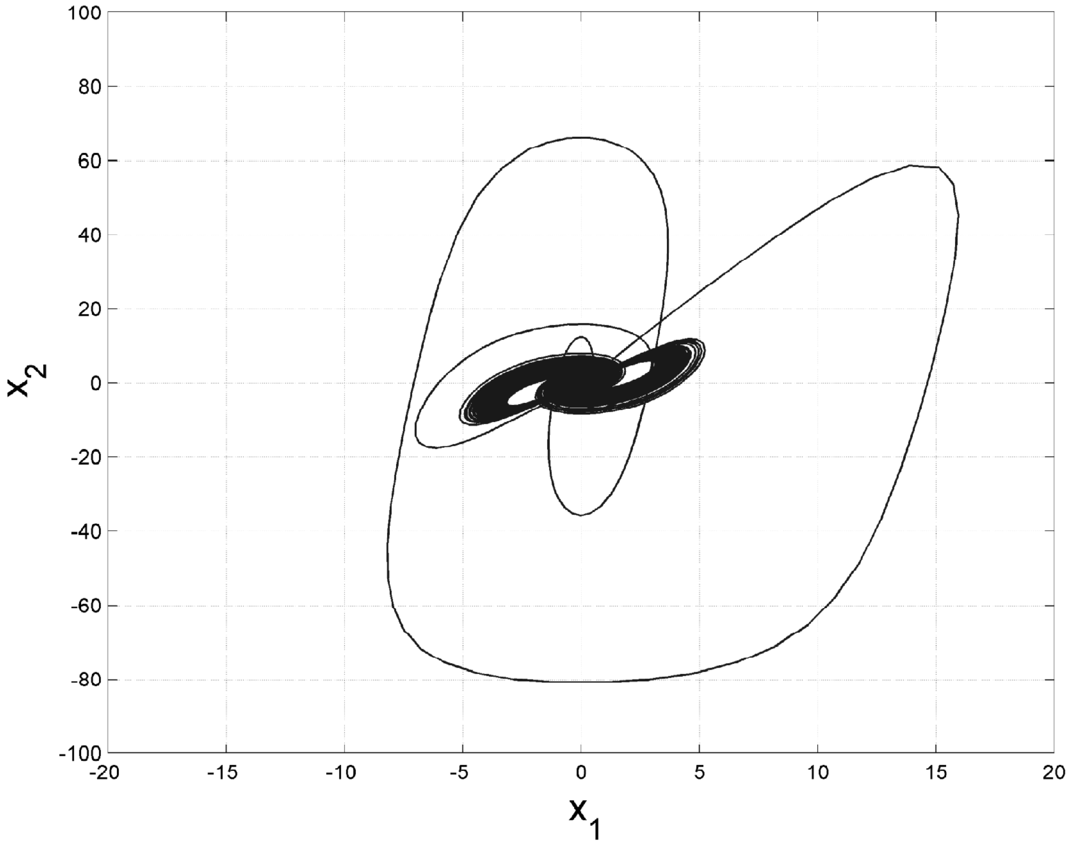

Figure 2: 2-D view of the novel chaotic system in (x_1, x_2) plane

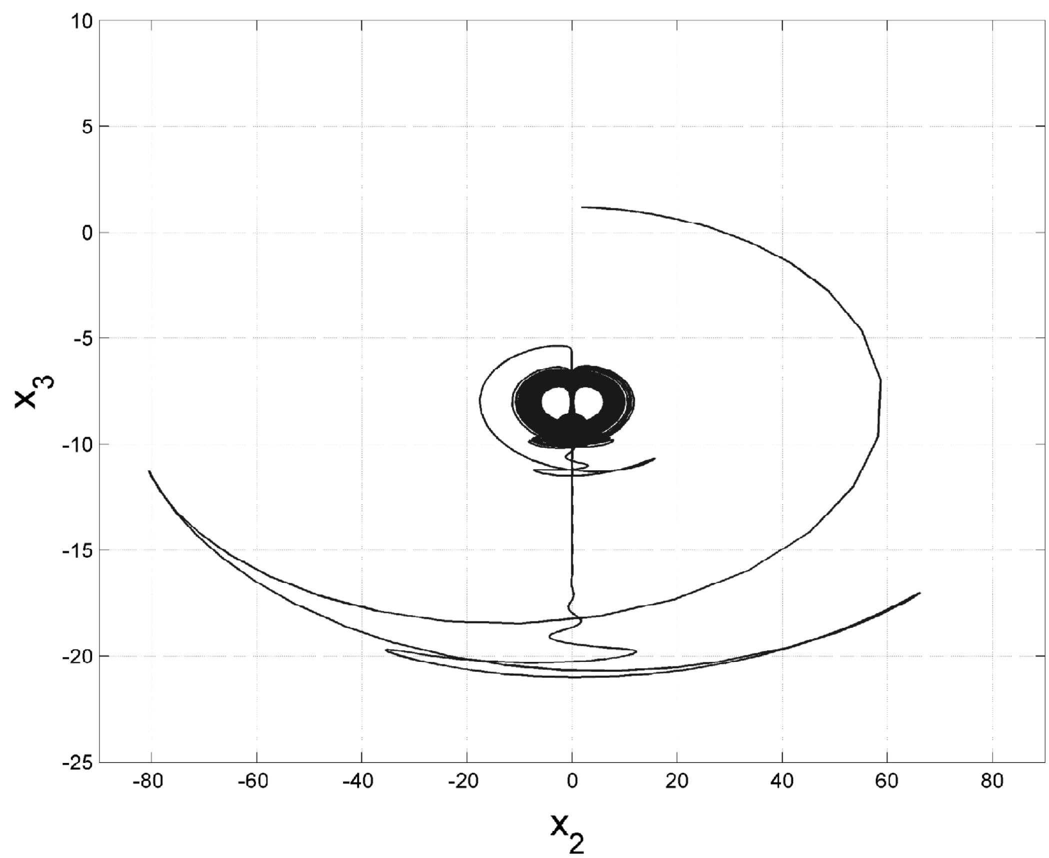

Figure 3: 2-D view of the novel chaotic system in (x_2, x_3) plane

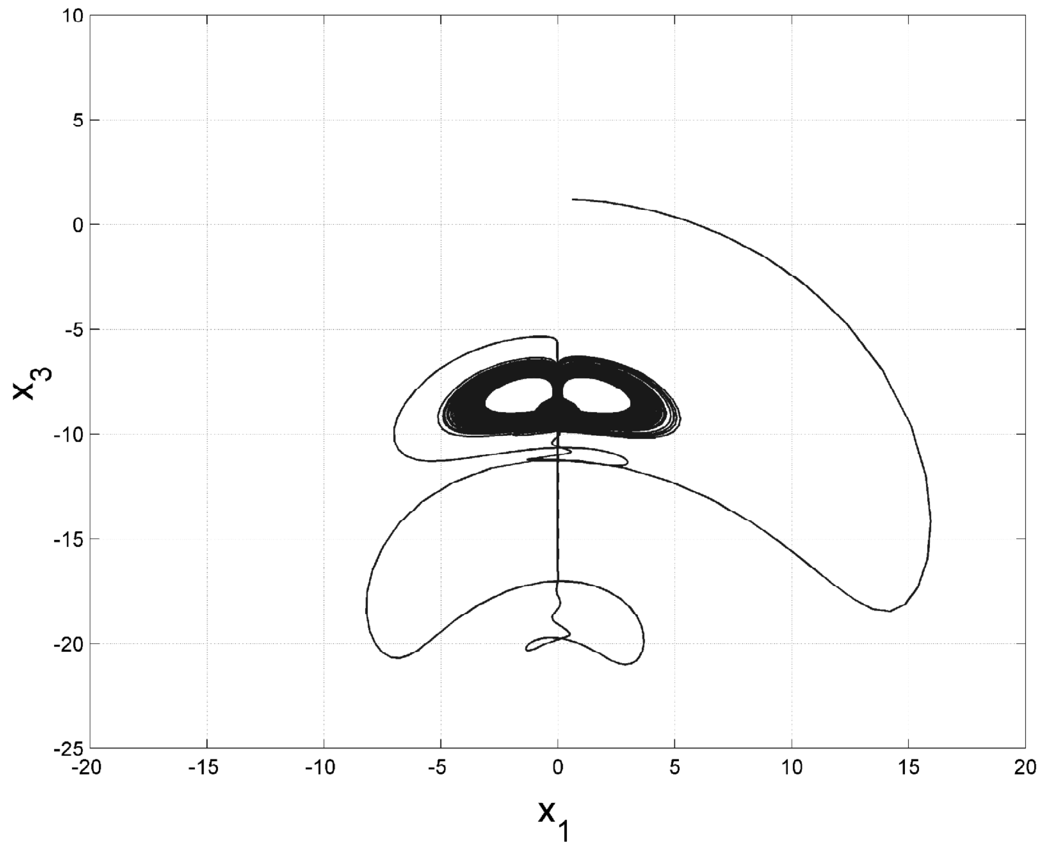

Figure 4: 2-D view of the novel chaotic system in (x_1, x_3) plane

$$
\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\
\dot{x}_2 &= b x_1 + c x_1 x_3 \\
\dot{x}_3 &= -d x_3 - x_1 x_2 - x_1^2
\end{aligned} \tag{26}
$$

where x_1, x_2, x_3 are the state variables and a, b, c, d are positive parameters.

As the slave system, we consider the controlled Zhu chaotic system given by

$$
\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + y_2 y_3 + u_1 \\
\dot{y}_2 &= b y_1 + c y_1 y_3 + u_2 \\
\dot{y}_3 &= -d y_3 - y_1 y_2 - y_1^2 + u_3
\end{aligned} \tag{27}
$$

where y_1, y_2, y_3 are the state variables and u_1, u_2, u_3 are the controls.

The anti-synchronization error is defined by

$$
e_i = y_i + x_i, \quad (i = 1, 2, 3) \tag{28}
$$

Then the error dynamics is obtained as

$$
\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + y_2 y_3 + x_2 x_3 + u_1 \\
\dot{e}_2 &= b e_1 + c(y_1 y_3 + x_1 x_3) + u_2 \\
\dot{e}_3 &= -d e_3 - y_1 y_2 - x_1 x_2 - y_1^2 - x_1^2 + u_3
\end{aligned} \tag{29}
$$

In matrix form, we can write the error dynamics (28) as

$$\dot{e} = Ae + \varphi(x, y) + u, \tag{30}$$

where

$$A = \begin{bmatrix} -a & a & 0 \\ b & 0 & 0 \\ 0 & 0 & -d \end{bmatrix}, \quad \varphi(x, y) = \begin{bmatrix} y_2 y_3 + x_2 x_3 \\ c(y_1 y_3 + x_1 x_3) \\ -y_1 y_2 - y_1^2 - x_1 x_2 - x_1^2 \end{bmatrix}, \text{ and } u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{31}$$

First, we set u as

$$u(t) = -\varphi(x, y) + Bv(t), \tag{32}$$

where B is selected such that (A, B) is completely controllable.

We choose B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{33}$$

We choose the parameters of the Vaidyanathan-Madhavan systems as in the chaotic case, viz.

$$a = 22, \quad b = 400, \quad c = 50, \quad d = 0.5 \tag{34}$$

The sliding mode variable is selected as

$$s = Ce = [1 \quad -4 \quad 1]e = e_1 - 4e_2 + e_3 \tag{35}$$

which renders the sliding motion globally asymptotically stable.

Next, we take the sliding mode gains as

$$k = 6 \text{ and } q = 0.2. \tag{36}$$

From Eq. (17) of Section 3, we obtain the novel sliding control v as

$$v = -808e_1 - e_2 + 2.75e_3 + 0.1s^2 \,\text{sgn}(s) \tag{37}$$

As an application of Theorem 1 to the identical Zhu chaotic systems, we obtain the following main result of this section.

Theorem 2. The identical Vaidyanathan-Madhavan chaotic systems (26) and (27) are globally and asymptotically anti- $x(0), y(0) \in R^3$ synchronized for all initial conditions with the sliding controller u defined by (32), where $\varphi(x, y)$ and B are defined by (31) and v is defined by (37).

As an initial condition for the master system (25), we take $x_1(0) = 3.7,\ x_2(0) = 1.3$ and $x_3(0) = 5.8.$ As an initial condition for the slave system (26), we take $y_1(0) = -7.6,\ y_2(0) = 6.8,$ and $y_3(0) = -4.5.$

Figures 5-7 depicts the anti-synchronization of the identical Vaidyanathan-Madhavan chaotic systems. Figure 8 depicts the time-history of the anti-synchronization errors.

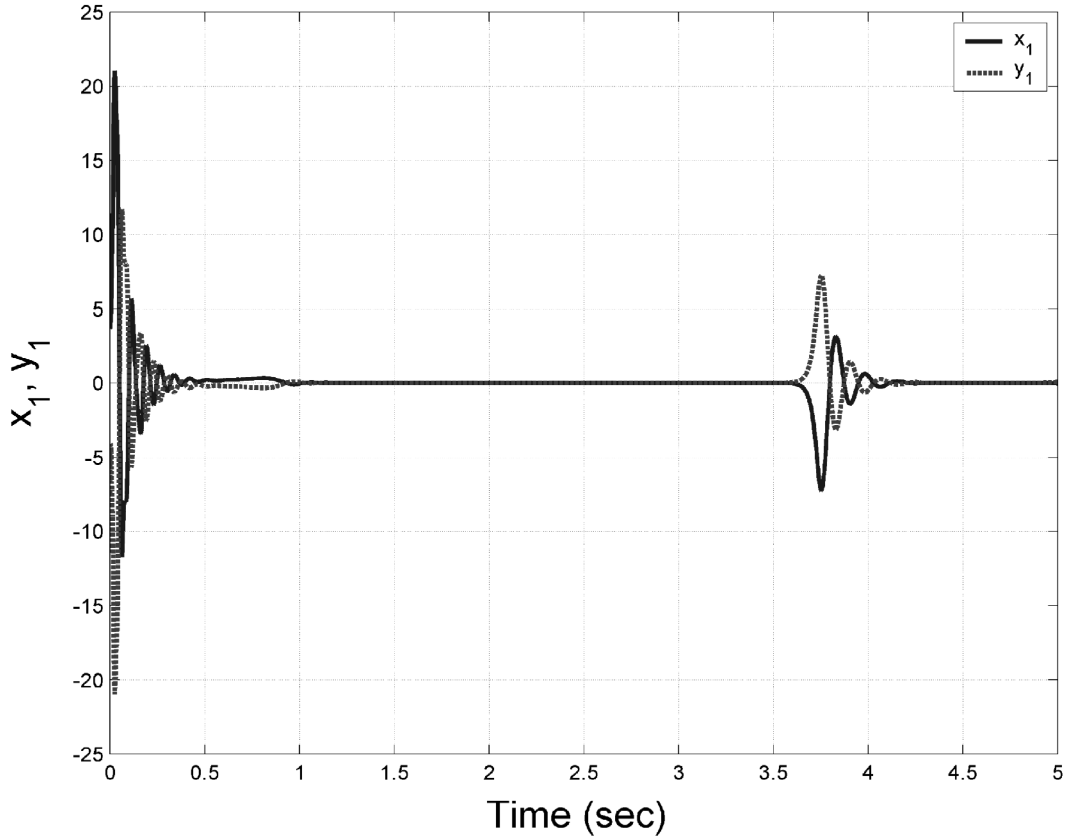

Figure 5: Anti-synchronization of the states x_1 and y_1

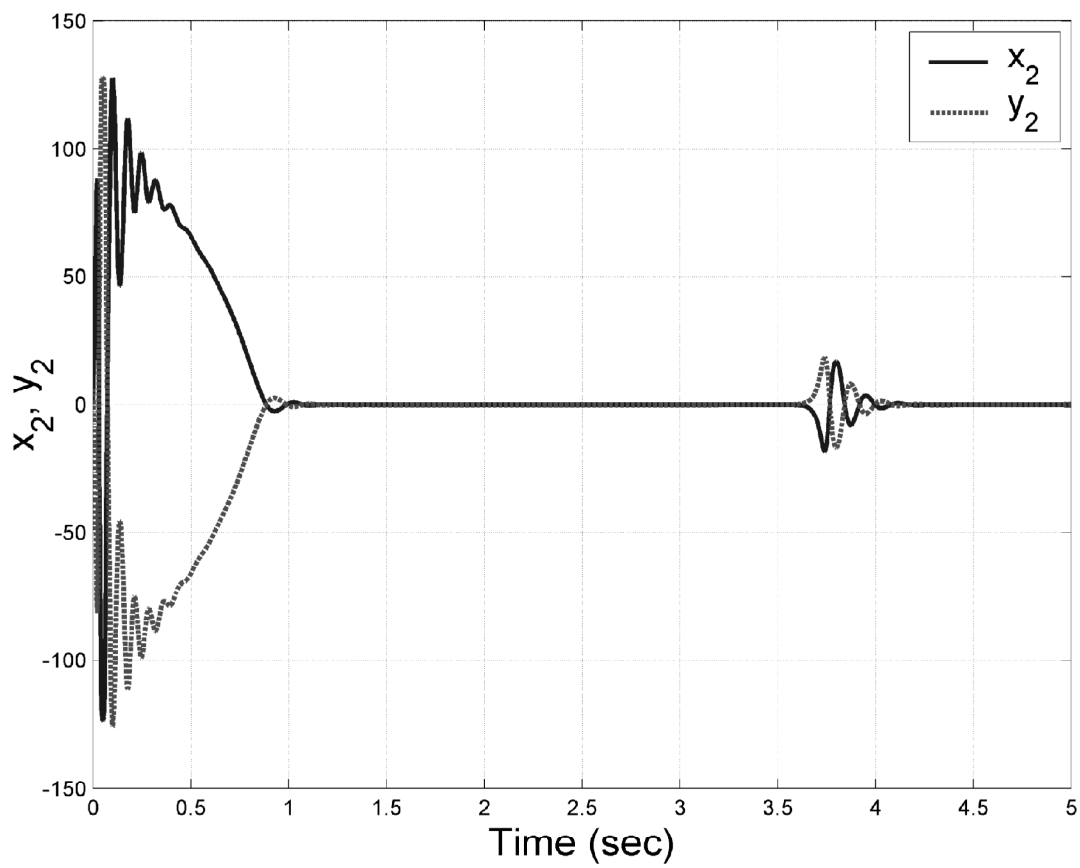

Figure 6: Anti-synchronization of the states x_2 and y_2

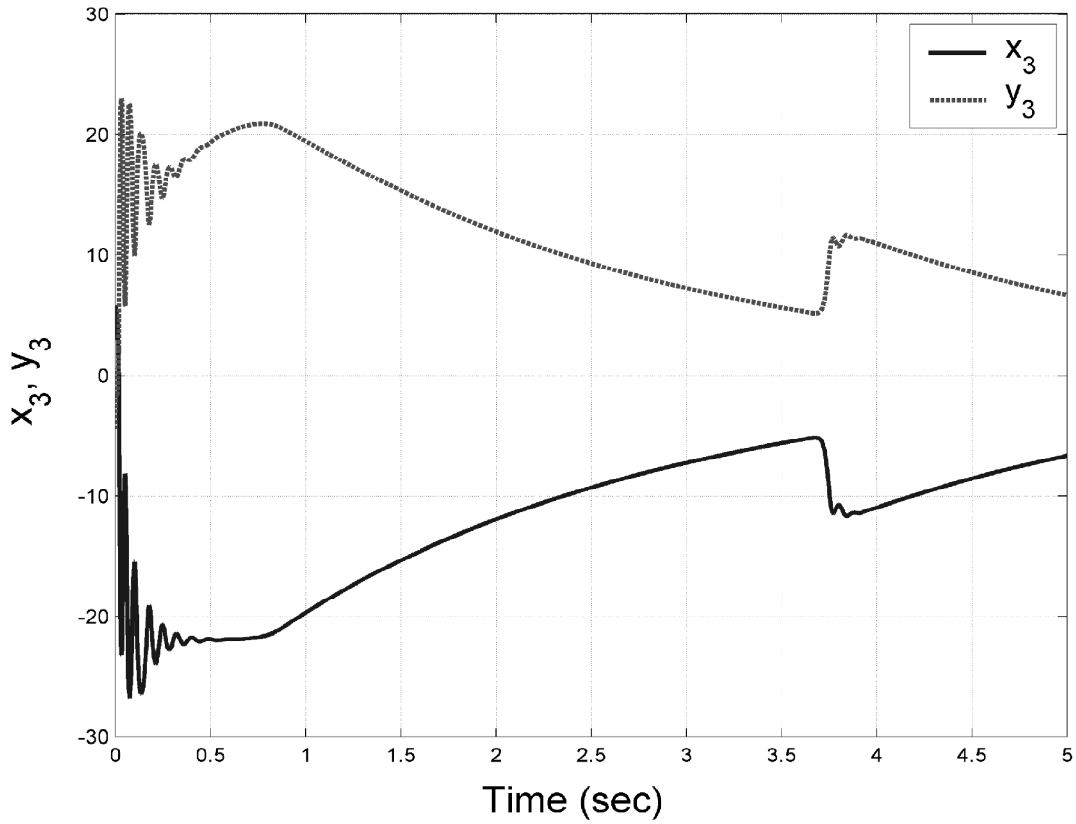

Figure 7: Anti-synchronization of the states x_3 and y_3

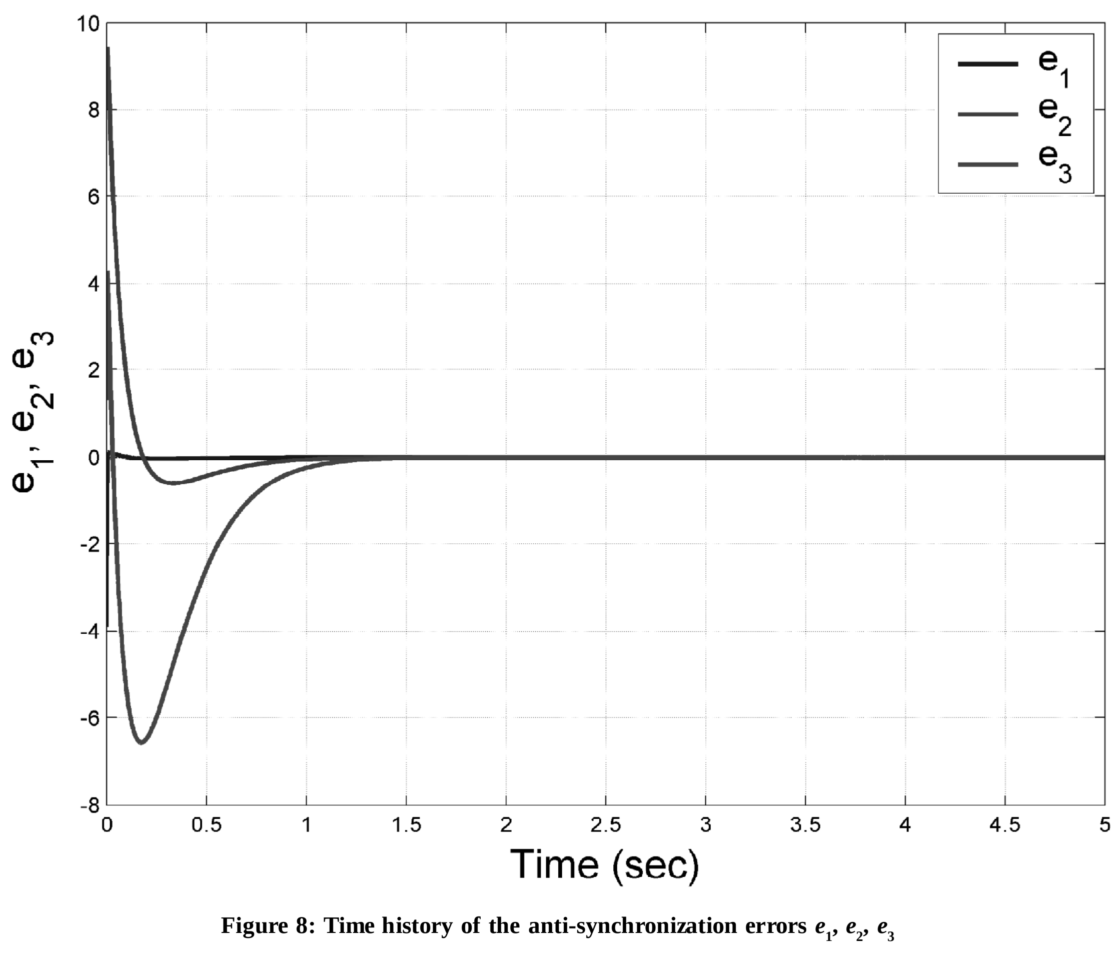

Figure 8: Time history of the anti-synchronization errors e_1, e_2, e_3

6. CONCLUSIONS

In this paper, a novel sliding mode controller has been designed for the anti-synchronization of identical chaotic systems. Lyapunov stability theory has been used to prove this main result of the work. Next, as an application of the main result, a sliding controller has been designed for achieving anti-synchronization of identical Vaidyanathan-Madhavan chaotic systems (2013). Numerical simulations have been provided to illustrate phase portraits of the Vaidyanathan-Madhavan chaotic system and the novel sliding mode controller for the anti-synchronization of identical Vaidyanathan-Madhavan chaotic systems.

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