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EXTENDED KRAUSS DEFORMATION GRADIENT DECOMPOSITION OF SINGLE DIRECTOR SHELLS MADE OF ELASO-PLASTIC MATERIALS

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ABSTRACT

The contribution of the present work is that we find the intrinsic relation between ($\mathbf{f}^e$, $\mathbf{f}^p$, $\mathbf{g}^e$, $\mathbf{g}^p$)and ($\mathbf{F}^e$, $\mathbf{F}^p$), and given a deformation decomposition, $\mathbf{f} = \mathbf{f}^e\mathbf{f}^p$ $\mathbf{g} = \mathbf{g}^e\mathbf{f}^p$, of shell by introducing extended Krauss decomposition within the framework of finite elasto-plasticity of shells. The formulations of strains and deformation rates have been given in detail for both elastic and plastic single director shell model. These formulations are very useful to complete the modelling of shells with finite elasto-plastic materials.

***Keywords**:* Krauss decomposition, deformation gradient, elasto-plasticity

1. INTRODUCTION

As we know, the shell theory is the subject, which aims to represent 3-D quantities in terms of 2-D corresponding quantities by introducing a suitable kinematics assumption. In other words, the old question of shell theory is how to express the fields given at arbitrary points in terms of one defined at mid-surface points. This means that we need a shifter, which can transform the fields out-off mid-surface onto the mid-surface. This kind of shifter plays an important role in formulating of shell theory. Krauss (1929) [1] who was the first person to introduce a shifter, which is now called as Krauss shifter tensor, and the corresponding decomposition, is called as Krauss decomposition. However, his work was not noticed until Pietraszkiewicz (1981) [2] rediscovered the decomposition from the literature. Now it is well known and used in the formulation of elastic shells, but no any work has been done on the generalization of Krauss decomposition and shifter for finite elasto-plastic deformation of shells.

In this paper, we introduced an intermediate configuration concept into the original Krauss decomposition for finite visco-elasticity-plastic deformation of shells, then we have extended Krauss decomposition. In sort of the extended decomposition, we found a remarkable result on the shell in the framework of finite deformation elasto-plasticity, $\mathbf{f}=\mathbf{f}^e\mathbf{f}^p$, $\mathbf{g}=\mathbf{g}^e\mathbf{f}^p$, $\mathbf{f}^e=\mathbf{z}^{-1}\mathbf{F}^e\overline{\mathbf{z}}$ $\mathbf{f}^p=\overline{\mathbf{z}}^{-1}\mathbf{F}^p\mathbf{Z}$, $\mathbf{g}^e=\xi^{-1}(\mathbf{F}^e\overline{\mathbf{z}}-\mathbf{f}^e)$, $\mathbf{g}^p=\xi^{-1}(\mathbf{F}^p\mathbf{Z}-\mathbf{f}^p)$, where $\mathbf{f}$ and $\mathbf{g}$ are deformation gradient of mid-surface and spatial deformation gradient of director field; ($\mathbf{f}^e$, $\mathbf{f}^p$, $\mathbf{g}^e$, $\mathbf{g}^p$) are the elastic and plastic part of $\mathbf{f}$ and $\mathbf{g}$, respectively. This can be considered as the 2-D shell form of Lee and Liu multiplicative decomposition $\mathbf{F}=\mathbf{F}^e\mathbf{F}^p$ of 3-D continuum. Using this decomposition, we formulated the kinematics of elasto-plastic shells in detail.

An outline of this paper is as follows. In section 2 the characteristics of shell deformation have been listed and single director kinematical assumption has been introduced and interpreted. In section 3, Krauss decomposition has been given, strain and rates analysis have been formulated in within elasticity. In section 4, Extended Krauss decomposition has been introduced, plastic strain and rates have been formulated in detail in sort of the extended decomposition. In section 5, we have given some conclusions.

2. SINGLE DIRECTOR KINEMATICS AND GEOMETRIC INTERPRETATION

Despite what disagreement occurs over what a shell-like body looks like. The realistic shell-like body has a unique spatial dimension associated with the through-the-thickness direction and two span-wise dimensions which are

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large, is some appropriate measure, relative to this thickness dimension. Accordingly, two surfaces can be uniquely identified as the "top" and "bottom" surfaces of the shell. If the body being analysed does not fit into broad classification, the appropriateness of using a shell theory should be brought into question.

The shell theories can be divided by Derived Approach and Direct Approach. The derived approach is that the shell theories be resulted from a systematic or approximate reduction of the three-dimensional continuum mechanics balance equations. The direct approach is that shell theories be constructed by an idealized two-dimensional continuum which is endowed with certain kinematical and kinetic ingredients that model the behaviour of the shell. However, the language of the discussion to follow and the mathematical analysis often utilizes concepts from both schools of thought, but the measure of the appropriateness of these assumptions must be the plausibility of the resulting shell behaviour relative to the three-dimensional (physical) body. Corresponding to this point of view, the basic perceptive adopted is that shell theory represents a mathematical statement of the physical assumptions imposed upon the behaviour of a three-dimensional continuum. In so far as the analysis is concerned, the three-dimensional body is replaced by a two-dimensional surface endowed with certain resultant characteristics, the definition of which stems from those physical assumptions placed upon the three-dimensional theory. The implied strategy, then, is to very clearly state the restrictions or kinematical assumptions that are introduced on the deformation of the body, to state these assumptions in mathematical language of the analysis to be performed, and to systematically reduce the general three-dimensional theory in a way that reflects the reductions imposed upon it.

It is clear that the catalogue of kinematics ingredients to be introduced requiem to capture all the important characteristics of any given problem would not necessarily be long. From physical consideration, included clearly in this list would be,

1. Membrane deformation, that is, deformation that produce reaction forces that lie entirely within the tangent plane of the shell mid-surface;
2. Bending deformation, which is the change of the curvature of the shell;
3. Transverse deformation, which rotate fibres that are initially normal to the mid-surface into fibers which are not longer normal to the mid-surface;
4. Through-the-thickness deformation, is the effect which can result as a consequence of other types of deformation, such as finite membrane strain.

Recently, someone believe that the drill rotation or deformation should be included in the list, which tend to either point-wise rotate the mid-surface of the shell or ones which tend to anti-symmetrically rotate the top surface relative to the bottom surface of the shell. Treatments relating to drill rotation appear in variational form by Reissner (1964) [3] and Naghdi (1964)[4], or can be considered to be a particular case of the constrained rotation formulation in the paper by Toupin (1964)[5]. A finite element approach to the linear theory of shells with drill rotations is given by Hughes & Brezzi (1989)[6]. In this paper, the formulation on drill rotation shall not be included.

According to the long tradition of shell research, in order to identify points off the resultant mid-surface of the shell r, the three-dimensional position-mapping **x** is expanded in a power series around the mid-surface in terms of the through-the-thickness parameter ξ and additional kinematical ingredients called directors. These directors are interpreted as vectors in R^3, the base point of which lies on the surface defined by r. This kinematical description for points x of the current configuration of the shell is written as $\mathbf{x}=\mathbf{r}+\sum_{k=0}^{K}\xi^k\mathbf{d}_k$, where $\mathbf{d}_k(\xi^1, \xi^2, t)$ (k = 1,.., K) are K directors. It is argued in Naghdi (1972) [7] that as k goes to infinite, the exact motion of the three-dimensional body is recovered. However, in this case the infinite resultant balance equations must be needed to give full information about the 3-D body. Here we only use single director assumption to capture the most important characteristics of shell deformation listed in the above consideration, the single director field in current configuration can be written as

$$\mathbf{x}=\boldsymbol{\varphi}(\xi^1,\xi^2,\xi,t)=\mathbf{r}(\xi^1,\xi^2,t)+\xi\mathbf{d}(\xi^1,\xi^2,t), \tag{1}$$

and single director field in reference or un-deformed configuration can be written as

$$\mathbf{X} = \mathbf{R}(\xi^1, \xi^2, t) + \xi\mathbf{D}(\xi^1, \xi^2, t) \tag{2}$$

Thus, the points in the body are identified the mid-surface mapping **r** or **R** plus the distance ξ along the director **d** or **D**. The kinematics assumption is illustrated in Figure 1.

Comparing the position maps Eq. (1), Eq. (2) in current and reference configuration, then we can find that the deformation of shell is such that points initially defined along straight fibers, i.e., identified as the distance ξ above or below the mid-surface along the line defined by the initial director D, remain along strain fibres. From physical point of view, the solution to the shell problem, under the single director kinematical assumption, can be considered as the solution for the shell mid-surface plus the solution for the director, which orients points along thickness fibres.

Let us now check, from geometric understanding, whether the single director kinematical assumption could capture the main characteristics of shell deformation, for instance, deformation of membrane, curvature, shear, through-the-thickness, and drill, before we go to deep. Indeed, from Eq.(1) and Eq.(2), finite membrane stretch is captured by the deformation of the mid-surface map **r**, finite bending strains are modelled by the spatial gradient of the director **d**, transverse shear strain is accounted for by measuring the relative rotation of the director **d** with respect to the normal to the mid-surface. The thickness change can be represented by the difference of magnitude of initial and current director. However, since there is no an additional rotation of the director **d** about itself, so the single director assumption here cannot be serviced to the problem which the drill rotation should be considered. This is the limitation of this kind kinematics assumption. It can be modified to take the drill rotation into the account by introducing extended kinematics.

3. KINEMATICS ANALYSIS OF SINGLE DIRECTOR MODEL AND KRAUSS DECOMPOSITION WITHIN ELASTICITY

The purpose of this section is to calculate most of the three-dimensional, as well as the surface, kinematics quantities which are required in the formulation of shells.

3.1. Tangent Basis and Deformation Gradient

From the definition in section 2, the convected tangent basis vectors in current shell configuration S is defined by

$$\mathbf{g}_\alpha = \mathbf{x},_\alpha = \mathbf{r},_\alpha + \xi\mathbf{d}_{,\alpha,}\ \ \mathbf{g}_3 = \mathbf{x},_3 = \mathbf{d}, \tag{3}$$

and ones in reference configuration B is defined by

$$\mathbf{G}_\alpha=\mathbf{X},_\alpha+\xi\mathbf{D},_\alpha,\ \ \mathbf{G}_3=\mathbf{D}., \tag{4}$$

The dual basis vectors are also defined by the relationships $\mathbf{g}^i \bullet \mathbf{g}_j=\delta^i_j$, and $\mathbf{G}^I \bullet \mathbf{G}_J=\delta^I_J$. If we put $\xi = 0$ in the Eq. (3) and Eq.(4), then the basis vectors of the mid-surface in the current and reference configuration can be obtained, $\mathbf{a}_i = \mathbf{g}_i|_{\xi=0}$, and $\mathbf{A}_I=\mathbf{G}_I|_{\xi=0}$, respectively. That is the

$$\mathbf{a}_\alpha=\mathbf{r},_\alpha,\ \ \mathbf{a}_3=\mathbf{d};\ \ \mathbf{A}_\alpha=\mathbf{R},_\alpha,\ \ \mathbf{A}_3=\mathbf{D}, \tag{5}$$

corresponding dual basis vectors in the mid-surface are defined by $\mathbf{a}^i \bullet \mathbf{a}_j=\delta^i_j$, $\mathbf{A}^I \bullet \mathbf{A}_J=\delta^I_J$. Then, tangent of deformation or deformation gradient **F** can be defined by

$$\mathbf{F} = \mathbf{g}_i \otimes \mathbf{G}^I = \mathbf{r},_\alpha \otimes \mathbf{G}^\alpha + \mathbf{d} \otimes \mathbf{G}^3 + \xi\mathbf{d},_\alpha \otimes \mathbf{G}^\alpha., \tag{6}$$

The associated Jacobean determinant is $J = \det(\mathbf{F})=j/j^0$, $j=\mathbf{g}_1 \times \mathbf{g}_2 \bullet \mathbf{g}_3$, $j^0=\mathbf{G}_1 \times \mathbf{G}_2 \bullet \mathbf{G}_3$. And from Eq.(4) and Eq. (5) and if we denote the director field gradient as

$$\mathbf{d},_\alpha=\kappa^\beta_\alpha\mathbf{a}_\beta+\kappa^3_\alpha\mathbf{d},\ \ \mathbf{D},_\alpha=K^\beta_\alpha\mathbf{A}_\alpha+K^3_\alpha\mathbf{D}., \tag{7}$$

we have $j = \bar{j}(1 + 2\xi h + \xi^2\kappa)$, $\bar{j} = j|_{\xi=0} =\mathbf{r},_1\times\mathbf{r},_2 \bullet \mathbf{d}$, $j^0=\bar{j}^0(1+2\xi H+\xi^2 K)$, $\bar{j}^0=j^0|_{\xi=0}=\mathbf{R},_1\times\mathbf{R},_2 \bullet\mathbf{D}$, where $h = \frac{1}{2}(\kappa^1_1 + \kappa^2_2)$, $\kappa = \kappa^1_1\kappa^2_2 - \kappa^1_2\kappa^2_1$ can be thought of as the mean and Gaussian curvature in the current configuration, and $H = \frac{1}{2}(K_1^1 + K_2^2), K = K_1^1 K_2^2 - K_1^2 K_2^1$ can be thought of as mean and Gaussian curvature in the reference configuration.

3.2. Krauss Decomposition of Deformation Gradient and Shifter

According to the aim of shell theories, which the information of points off the mid-surface is to be represented in terms of mid-surface quantities. The deformation gradient **F** of shell can be decomposed into

$$\mathbf{F} = \mathbf{g}_i \otimes \mathbf{G}^I = (\mathbf{g}_i \otimes \mathbf{a}^i)(\mathbf{a}_i \otimes \mathbf{A}^I)(\mathbf{A}_I \otimes \mathbf{G}^i) = \mathbf{z}\mathbf{F}_0\mathbf{Z}^{-1}, \quad (8)$$

where $\mathbf{F}_0 = \mathbf{F}|_{\xi=0} = \mathbf{r},_i \otimes \mathbf{A}^i = \mathbf{r},_\alpha \otimes \mathbf{A}^\alpha + \mathbf{d} \otimes \mathbf{A}^3$ is the mid-surface/director field deformation gradient, and which map the basis vectors on the mid-surface in reference configuration into the ones in the current configuration, that is, $\mathbf{a}_i = \mathbf{F}_0\, \mathbf{A}_1$. **Z**. and **Z** are called Krauss shifter tensors, which map the basis vectors of mid-surface into the points off the mid-surface, they are defined by $\mathbf{z} = \mathbf{g}_i \otimes \mathbf{a}^i$, and $\mathbf{Z} = \mathbf{G}_I \otimes \mathbf{A}^I$, with $\mathbf{Z}^{-1}$, $\mathbf{A}_I \otimes \mathbf{G}^I$. Krauss (1929)[1] firstly gave the decomposition in Eq.(5), its geometric interpretation was illustrated in Figure 1.

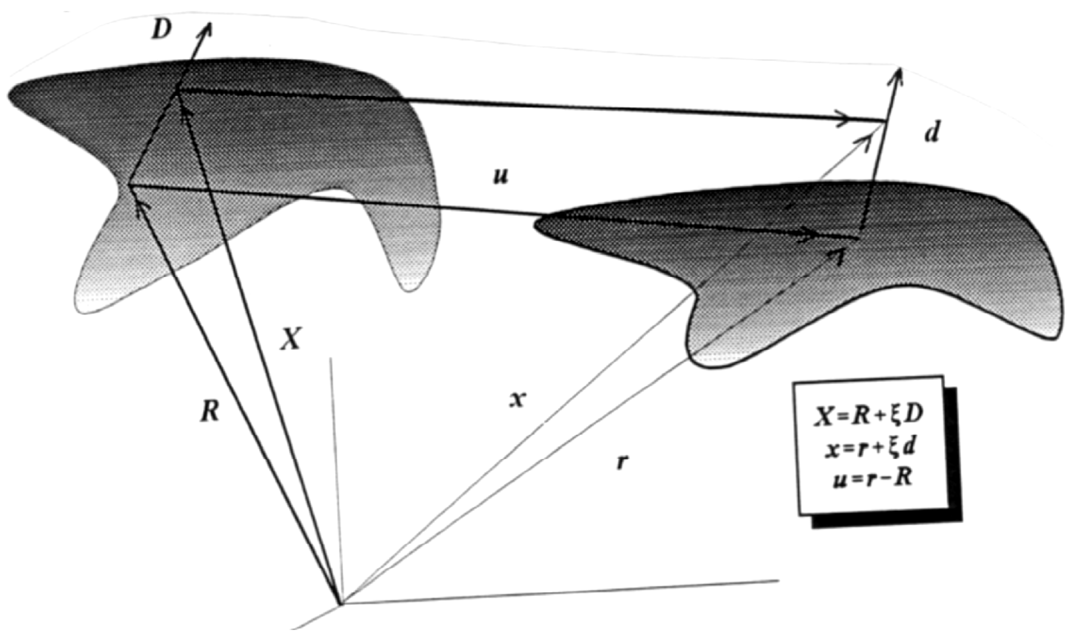

Figure 1: Single Director Model and Geometric Interpretation

If we introduce the metric tensor **a** and director metric tensor **b** in the current configuration as $\mathbf{a} = \mathbf{a}_i \otimes \mathbf{a}^i = \mathbf{r},_\alpha \otimes \mathbf{a}^\alpha + \mathbf{d} \otimes \mathbf{a}^3$, and $\mathbf{b} = \mathbf{d},_\alpha \otimes \mathbf{a}^\alpha$, and the metric tensor **A** and director metric tensor **B** in the reference configuration as $\mathbf{A} = \mathbf{A}_i \otimes \mathbf{A}^i$, and $\mathbf{B} = \mathbf{D},_\alpha \otimes \mathbf{A}^\alpha$, then the shifter tensors can be rewritten as

$$\begin{aligned} \mathbf{z} &= \mathbf{g}_i \otimes \mathbf{a}^i = \mathbf{a}_\alpha \otimes \mathbf{a}^\alpha + \mathbf{d} \otimes \mathbf{a}^3 + \xi \mathbf{d},_\alpha \otimes \mathbf{a}^\alpha = \mathbf{a} + \xi \mathbf{b}, \\ \mathbf{Z} &= \mathbf{G}_i \otimes \mathbf{A}^i = \mathbf{A}_\alpha \otimes \mathbf{A}^\alpha + \mathbf{D} \otimes \mathbf{A}^3 + \xi \mathbf{D},_\alpha \otimes \mathbf{A}^\alpha = \mathbf{A} + \xi \mathbf{B}.' \end{aligned} \quad (9)$$

For the given shape of shell, **Z** is known after we define the mid-surface. For convenient, let us write $\mathbf{zF}_0 = \mathbf{f} + \xi\mathbf{g}$, and since we have

$$\mathbf{zF}_0 = (\mathbf{g}_i \otimes \mathbf{a}^i)(\mathbf{a}_i \otimes \mathbf{A}^I) = \mathbf{g}_i \otimes \mathbf{A}^I = \mathbf{a},_\alpha \otimes \mathbf{A}^\alpha + \mathbf{d} \otimes \mathbf{A}^3 + \xi \mathbf{d},_\alpha \otimes \mathbf{A}^\alpha \equiv \mathbf{f} + \xi \mathbf{g}, \quad (10)$$

we have $\mathbf{f} = \mathbf{a}_\alpha \otimes \mathbf{A}^\alpha + \mathbf{d} \otimes \mathbf{A}^3 \equiv \mathbf{F}_0$, and $\mathbf{g} = \mathbf{d},_\alpha \otimes \mathbf{A}^\alpha$ which is called director field gradient.

Well the deformation in Eq.(10) can be rewritten as

$$\mathbf{F} = (\mathbf{f} + \xi\mathbf{g})\mathbf{Z}^{-1}, \quad (11)$$

If the thickness of shell is very small then we have approximation, $\mathbf{Z} \approx \mathbf{Z}^{-1} \approx 1$, and $\mathbf{F} \approx \mathbf{f} + \xi\mathbf{g}$.

From this representation of the deformation gradient **F**, we can find that the **F** can be replaced by (**f, g**). This can be considered as one of the mathematical properties of shell, which is different from three-dimensional body. More precisely, for the three-dimensional theory, the configuration manifold consists of simply one quantity: the three-dimensional position map. Corresponding, one configuration gradient field is required namely, the deformation gradient **F**. The gradient **F**, then, is the mapping which transforms tangent vectors in the reference configuration into tangent vectors in the current configuration. For the present shell theory, the configuration manifold consists of pairs: (**r, d**), in which **r** is the mid-surface position map and **d** is the director field. Corresponding, two configuration gradient fields are required. One, i.e., **f**, analogous to the three-dimensional case, maps the tangent vectors in the reference configuration into tangent vectors in the current configuration. The second, i.e., **g**, maps tangent vectors in the reference configuration into the director gradient field in the current configuration.

3.3. Strain Measures and Velocity Gradient

After having the deformation gradient **F**, then the proper strain measures can be defined according to the theorem of polar decomposition of the deformation gradient.

The Green left stretching tensor C, which is the metric tensor in the current configuration, can be defined with the polar decomposition as

$$\mathbf{C}=\mathbf{U}^2=\mathbf{F}^{\mathrm{T}}\mathbf{F}=\mathbf{Z}^{-\mathrm{T}}\mathbf{C}_0\mathbf{Z}^{-1}=\mathbf{Z}^{-\mathrm{T}}\mathbf{U}_0^2\mathbf{Z}^{-1}, \tag{12}$$

where $\mathbf{C}_0=\mathbf{U}_0^2=\mathbf{f}^{\mathrm{T}}\mathbf{f}+\xi(\mathbf{f}^{\mathrm{T}}\mathbf{g}+\mathbf{g}^{\mathrm{T}}\mathbf{f})+\xi^2\mathbf{g}^{\mathrm{T}}\mathbf{g}$ is defined on the mid-surface in the reference configuration, this can be easily identified by the definition of f and g, that is, $\mathbf{C}_0=\mathrm{C}_{0IJ}\mathbf{A}^I\otimes\mathbf{A}^J=\mathrm{U}_{0IJ}^2\mathbf{A}^I\otimes\mathbf{A}^J$. Then the definition Eq. (12) can be understand as that the C or U^2 can be obtained by the push-forward operation of $\mathbf{C}_0$ or $\mathbf{U}_0^2$ through the shifter **Z**, i.e., $\mathbf{C}=\mathbf{U}^2=\phi_{*\mathbf{Z}}(\mathbf{C}_0)=\phi_{*\mathbf{Z}}(\mathbf{U}_0^2)$. Clearly, the **C** and $\mathbf{U}^2$ are defined in the points off the mid-surface.

With the definition of Green strain tensors, we have

$$\mathbf{E}=\mathbf{Z}^{-\mathrm{T}}\mathbf{E}_0\mathbf{Z}^{-1}=\phi_{*\mathbf{Z}}(\mathbf{E}_0),\ \ \mathbf{E}_0=\frac{1}{2}(\mathbf{U}^2-\mathbf{G}_0)=\mathbf{E}_f+\xi\mathbf{E}_g+\xi^2\mathbf{E}_2, \tag{13}$$

Where

$$\mathbf{G}_0=\mathbf{1}_0=\mathbf{Z}^{\mathrm{T}}\mathbf{Z}=\mathrm{G}_{0IJ}\mathbf{A}^I\otimes\mathbf{A}^J,\ \mathbf{E}_f=\frac{1}{2}(\mathbf{f}^{\mathrm{T}}\mathbf{f}-\mathbf{A}^{\mathrm{T}}\mathbf{A}),\ \ \mathbf{E}_g=\frac{1}{2}(\mathbf{f}^{\mathrm{T}}\mathbf{g}+\mathbf{g}^{\mathrm{T}}\mathbf{f}-\mathbf{A}^{\mathrm{T}}\mathbf{B}-\mathbf{B}^{\mathrm{T}}\mathbf{A}),\ \ \mathbf{E}_2=\frac{1}{2}(\mathbf{g}^{\mathrm{T}}\mathbf{g}-\mathbf{B}^{\mathrm{T}}\mathbf{B}).,\tag{14}$$

Here $\mathbf{E}_f$, $\mathbf{E}_g$ are measures of membrane strain and curvature of mid-surface. Let us introduce the following notations

$$\begin{aligned}\mathbf{C}_f &\overset{def}{=} \mathbf{f}^{\mathrm{T}}\mathbf{f}=a_{\alpha\beta}\mathbf{A}^\alpha\otimes\mathbf{A}^\beta+\gamma_\alpha(\mathbf{A}^\alpha\otimes\mathbf{A}^3+\mathbf{A}^3\otimes\mathbf{A}^\alpha)+\lambda^2\mathbf{A}^3\otimes\mathbf{A}^3,\\ \mathbf{C}_g &\overset{def}{=} \mathbf{f}^{\mathrm{T}}\mathbf{g}=\kappa_{\alpha\beta}\mathbf{A}^\alpha\otimes\mathbf{A}^\beta+\kappa_{\alpha3}\mathbf{A}^\alpha\otimes\mathbf{A}^3,\quad \mathbf{C}_d \overset{def}{=} \mathbf{g}^{\mathrm{T}}\mathbf{g}=\eta_{\alpha\beta}\mathbf{A}^\alpha\otimes\mathbf{A}^\beta,\end{aligned}, \tag{15}$$

and $a_{\alpha\beta}=\mathbf{a}_\alpha\cdot\mathbf{a}_\beta$, $\gamma_\alpha=\mathbf{a}_\alpha\cdot\mathbf{d}$, $\kappa_{\alpha\beta}=\mathbf{a}_\alpha\cdot\mathbf{d}_{,\beta}$, $\kappa_{\alpha3}=\mathbf{d}_{,\alpha}\cdot\mathbf{d}$, $\lambda^2=\mathbf{d}\cdot\mathbf{d}$, $\eta_{\alpha\beta}=\mathbf{d}_{,\alpha}\cdot\mathbf{d}_{,\beta}$, and $A_{\alpha\beta}=\mathbf{A}_\alpha\cdot\mathbf{A}_{,\beta}$, $\Gamma_\alpha=\mathbf{A}_\alpha\cdot\mathbf{D}$, , $\mathrm{K}_{\alpha\beta}=\mathbf{A}_\alpha\cdot\mathbf{D}_{,\beta}$, $\mathrm{K}_{\alpha3}=\mathbf{D}\cdot\mathbf{D}_{,\alpha}$, $\Lambda^2=\mathbf{D}\cdot\mathbf{D}$, $\mathrm{H}_{\alpha\beta}=\mathbf{D}_{,\alpha}\cdot\mathbf{D}_{,\beta}$. Then we have the components form of Green strain tensor as follows

$$\begin{aligned}\mathbf{E}_f &=\frac{1}{2}(a_{\alpha\beta}-A_{\alpha\beta})\mathbf{A}^\alpha\otimes\mathbf{A}^\beta+(\gamma_{\alpha3}-\Gamma_{\alpha3})(\mathbf{A}^\alpha\otimes\mathbf{A}^3+\mathbf{A}^3\otimes\mathbf{A}^\alpha)+\frac{1}{2}(\lambda^2-\Lambda^2)\mathbf{A}^3\otimes\mathbf{A}^3\\ \mathbf{E}_g &=(\kappa_{\alpha\beta}-\mathrm{K}_{\alpha\beta})\mathbf{A}^\alpha\otimes\mathbf{A}^\beta+(\kappa_{\alpha3}-\mathrm{K}_{\alpha3})(\mathbf{A}^\alpha\otimes\mathbf{A}^3+\mathbf{A}^3\otimes\mathbf{A}^\alpha),\ \mathbf{E}_2=\frac{1}{2}(\eta_{\alpha\beta}-\mathrm{H}_{\alpha\beta})\mathbf{A}^\alpha\otimes\mathbf{A}^\beta.\end{aligned}, \tag{16}$$

In (15) and (16), are classical surface metric measures in the current/reference configuration, respectively; $a_{\alpha\beta}$ and $A_{\alpha\beta}$ are transverse shear deformation measures in current/reference configuration, and which measure how much **d** or **D** are rotated away from the normal to the surface, then $\frac{1}{2}(\gamma_\alpha - \Gamma_\alpha)$ can measure the shear deformation; similarly, $\frac{1}{2}(\lambda^2 - \Lambda^2)$ can measure the thickness change of shell; $(\kappa_{\alpha\beta} - K_{\alpha\beta}), (\kappa_{\alpha 3} - K_{\alpha 3})$ *and* $(\eta_{\alpha\beta} - H_{\alpha\beta})$ can measure the bending or curvature of shell configuration space. This means that the strain measure can capture all deformation behaviours in the single director kinematics.

Since the time differential of deformation gradient is $\dot{\mathbf{F}}=\dot{\mathbf{g}}_i \otimes \mathbf{G}^I=\mathbf{r},_\alpha \otimes \mathbf{G}^\alpha+\dot{\mathbf{d}} \otimes \mathbf{G}^3+\xi\mathbf{d},_\alpha \otimes \mathbf{G}^\alpha$, then, using the relation $\mathbf{G}^I\mathbf{F}^{-1}=\mathbf{g}^i$, the velocity gradient can be got

$$\mathbf{l}=\dot{\mathbf{F}}\mathbf{F}^{-1}=\dot{\mathbf{g}}_i \otimes \mathbf{g}^i=\mathrm{r},_\alpha \otimes \mathbf{g}^\alpha+\dot{\mathbf{d}} \otimes \mathbf{g}^3+\xi\dot{\mathbf{d}},_\alpha \otimes \mathbf{g}^\alpha, \tag{17}$$

with relation $\mathbf{l}^T=\mathbf{g}^i \otimes \dot{\mathbf{g}}_i$.

Then the rate of deformation $\underset{\sim}{\mathbf{d}}$ and spin **w** can be defined as

$$\underset{\sim}{\mathbf{d}}=\frac{1}{2}(\mathbf{l}+\mathbf{l}^T)=\mathrm{d}_{ij}\mathbf{g}^i \otimes \mathbf{g}^j=\frac{1}{2}\dot{\mathrm{g}}_{ij}\mathbf{g}^i \otimes \mathbf{g}^j, \quad \mathbf{w}=\frac{1}{2}(\mathbf{l}-\mathbf{l}^T), \tag{18}$$

According to the Krauss decomposition, the velocity gradient can also be rewritten as

$$\mathbf{l}=[(\dot{\mathbf{f}}+\xi\dot{\mathbf{g}})\mathbf{Z}^{-1}](\mathbf{z}\mathbf{f}\mathbf{Z}^{-1})^{-1}=(\dot{\varphi}+\xi\dot{\mathbf{g}})\mathbf{f}^{-1}\mathbf{z}^{-1}=[\dot{\mathbf{f}}\mathbf{f}^{-1}+\xi\dot{\mathbf{g}}\mathbf{f}^{-1}]\mathbf{z}^{-1}. \tag{19}$$

Set $\xi = 0$ in the above, then we have velocity gradient of the mid-surface as

$$\mathbf{l}_f=\mathbf{l}|_{\xi=0}=\dot{\mathbf{f}}\mathbf{f}^{-1}=\dot{\mathbf{a}}_i \otimes \mathbf{a}^i=\dot{\mathbf{d}}_\alpha \otimes \mathbf{a}^\alpha+\dot{\mathbf{d}} \otimes \mathbf{a}^3., \tag{20}$$

After understanding of Krauss decomposition, let us go to find the extended Krauss decomposition for elasto-plastic shells.

4. EXTENDED KRAUSS DECOMPOSITION, STRAINS AND RATES WITHIN FINITE ELASTO-PLASTICITY

In this section, we will generalise Krauss decomposition into the frame of finite deformation of shell by introducing intermediate configuration.

4.1. Extended Krauss Decomposition

The proper definition of plastic strain and plastic strain rate are very important to develop plastic constitutive equations of shell in terms of resultant quantities. In this section, we will define proper plastic strain and plastic rate by extended Krauss decomposition, which is first introduced in present paper. In order to give a clear picture of these definitions, let us firstly introduce extended Krauss decomposition and it has been illustrated in Figure 3.

we have following decompositions of deformation gradient

$$\begin{aligned}
&\mathbf{F}^e=\mathbf{z}\mathbf{f}^e\overline{\mathbf{z}}^{-1}=(\mathbf{f}^e+\xi\mathbf{g}^e)\overline{\mathbf{z}}^{-1}, \quad \mathbf{F}^p=\overline{\mathbf{z}}\mathbf{f}^p\mathbf{Z}^{-1}=(\mathbf{f}^p+\xi\mathbf{g}^p)\mathbf{Z}^{-1},\\
&\mathbf{F}=\mathbf{F}^e\mathbf{F}^p=\mathbf{z}\mathbf{f}^e\mathbf{f}^p\mathbf{Z}^{-1}=(\mathbf{f}^e+\xi\mathbf{g}^e)\mathbf{f}^p\mathbf{Z}^{-1}=\mathbf{z}\mathbf{f}^e\overline{\mathbf{z}}^{-1}(\mathbf{f}^p+\xi\mathbf{g}^p)\mathbf{Z}^{-1},\\
&\quad=(\mathbf{f}^e+\xi\mathbf{g}^e)\overline{\mathbf{z}}^{-1}(\mathbf{f}^p+\xi\mathbf{g}^p)\mathbf{Z}^{-1}\equiv\mathbf{z}\mathbf{f}\mathbf{Z}^{-1}\equiv(\mathbf{f}+\xi\mathbf{g})\mathbf{Z}^{-1}.
\end{aligned} \tag{21}$$

where the shifter tensor is defined by $\overline{\mathbf{z}}=\overline{\mathbf{g}}_i \otimes \overline{\mathbf{a}}^i$, the elastic and plastic parts of deformation gradient on the surface are defined by $\mathbf{f}^e=\mathbf{a}_i \otimes \overline{\mathbf{a}}^i$, and $\mathbf{f}^p=\overline{\mathbf{a}}_i \otimes \mathbf{A}^I$, and elastic and plastic parts of director field gradient $\mathbf{g}^e=\mathbf{d},_\alpha \otimes \overline{\mathbf{a}}^\alpha$, $\mathbf{g}^p=\overline{\mathbf{d}},_\alpha \otimes \mathbf{A}^\alpha$, and $\overline{\mathbf{a}}_i=\overline{\mathbf{g}}_i|_{\xi=0}$.

From Eq.(9), it is easy to find the following decomposition

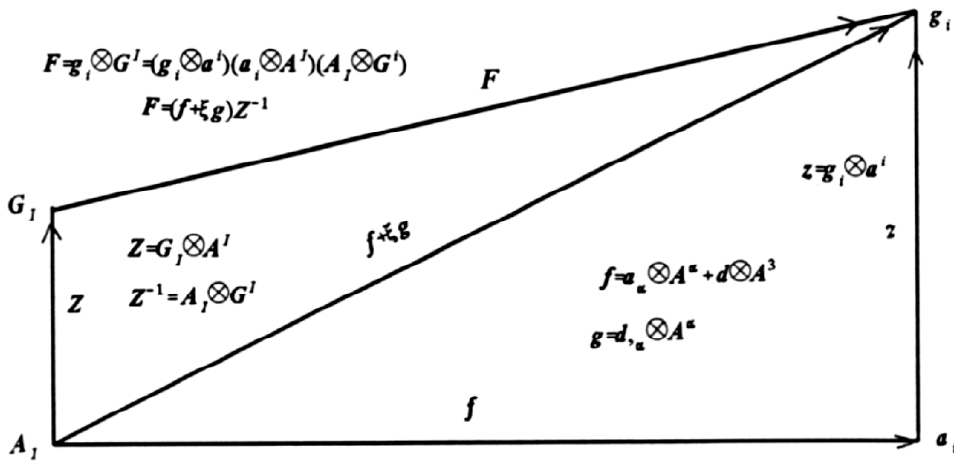

Figure 2: Krauss Decomposition

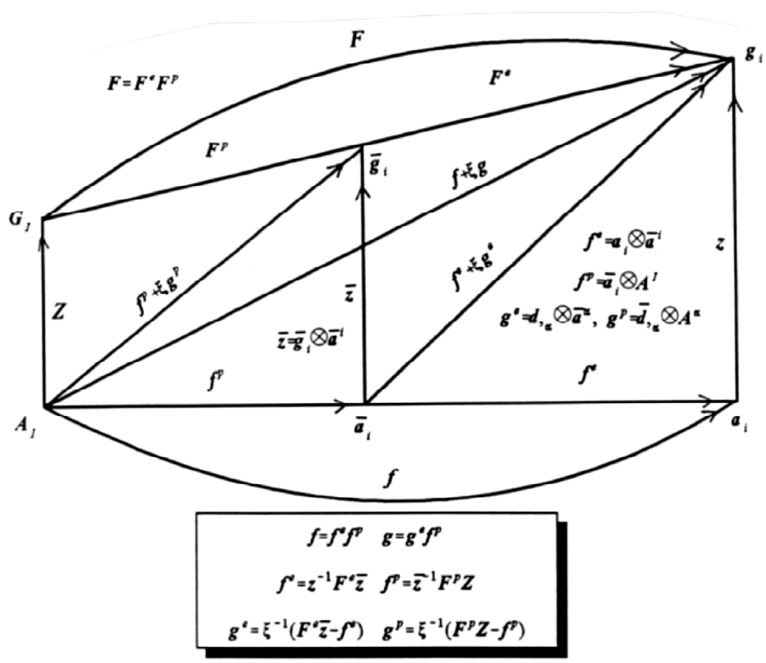

Figure 3: Extended Krauss Decomposition within Elasto-plasticity

$$
\begin{aligned}
&\mathbf{f}=\mathbf{f}^e\mathbf{f}^p, \quad \mathbf{g}=\mathbf{g}^e\mathbf{f}^p \\
&\mathbf{f}^e=\mathbf{z}^{-1}\mathbf{F}^e\bar{\mathbf{z}}, \quad \mathbf{f}^p=\bar{\mathbf{z}}^{-1}\mathbf{F}^p\mathbf{Z} \\
&\mathbf{g}^e=\xi^{-1}(\mathbf{F}^e\bar{\mathbf{z}}-\mathbf{f}^e), \quad \mathbf{g}^p=\xi^{-1}(\mathbf{F}^p\mathbf{Z}-\mathbf{f}^p)
\end{aligned} \tag{22}
$$

This means that the deformation gradient on the surface and director field gradient can also be multiplicatively decomposed into elastic and plastic parts. These remarkable results are due to the extended Krauss decomposition. These intrinsic relations are the keys to formulate the kinematics of shells with finite elasto-plasticity.

It means that the elastic strain is defined with respect to the intermediate configuration. These formulations can be directly used to complete modelling of shells.

4.2. Plastic Strain Measure and Plastic Strain Rate

Using the above decompositions and the definitions of elastic/plastic strains, we have

$$2\overline{\mathbf{E}}^{e}=\mathbf{F}^{eT}\mathbf{F}^{e}-\mathbf{1}=\overline{\mathbf{Z}}^{-T}[\mathbf{f}^{eT}\mathbf{f}^{e}+\xi(\mathbf{f}^{eT}\mathbf{g}^{e}+\mathbf{g}^{eT}\mathbf{f}^{e})+\xi^{2}\mathbf{g}^{eT}\mathbf{g}^{e}]\overline{\mathbf{Z}}^{-1}-\overline{\mathbf{Z}}^{T}(\overline{\mathbf{Z}}^{T}\overline{\mathbf{Z}})\overline{\mathbf{Z}}^{-1}\equiv\overline{\mathbf{Z}}^{-T}\overline{\mathbf{E}}_{0}^{e}\overline{\mathbf{Z}}^{-1}\equiv\overline{\mathbf{Z}}^{-T}[\overline{\mathbf{E}}_{f}^{e}+\xi\overline{\mathbf{E}}_{g}^{e}+\xi^{2}\overline{\mathbf{E}}_{2}^{e}]\overline{\mathbf{Z}}^{-1},$$
$$2\mathbf{E}^{p}=\mathbf{F}^{pT}\mathbf{F}^{p}-\mathbf{1}=\mathbf{Z}^{-T}[\mathbf{f}^{pT}\mathbf{f}^{p}+\xi(\mathbf{f}^{p}\mathbf{g}^{p}+\mathbf{g}^{pT}\mathbf{f}^{p}+\xi^{2}\mathbf{g}^{pT}\mathbf{g}^{p}]\mathbf{Z}^{-1}-\mathbf{Z}^{-T}(\mathbf{Z}^{T}\mathbf{Z})\mathbf{Z}^{-1}\equiv\mathbf{Z}^{-T}\mathbf{E}_{0}^{p}\mathbf{Z}^{-1}\equiv\mathbf{Z}^{-T}[\mathbf{E}_{f}^{p}+\xi\mathbf{E}_{g}^{p}+\xi^{2}\mathbf{E}_{2}^{p}]\mathbf{Z}^{-1}. \quad (23)$$

where

$$\overline{\mathbf{E}}_{f}^{e}=\frac{1}{2}(\mathbf{f}^{eT}\mathbf{f}^{e}\overline{\mathbf{a}}^{T}\overline{\mathbf{a}}),\ \overline{\mathbf{E}}_{g}^{e}=\frac{1}{2}(\mathbf{f}^{eT}\mathbf{g}^{e}+\mathbf{g}^{eT}\mathbf{f}^{e}\overline{\mathbf{a}}^{T}\overline{\mathbf{b}}\overline{\mathbf{b}}^{T}\overline{\mathbf{a}}),\ \overline{\mathbf{E}}_{2}^{e}=\frac{1}{2}(\mathbf{g}^{eT}\mathbf{g}^{e}\overline{\mathbf{b}}^{T}\overline{\mathbf{b}}).,\quad (24)$$

and

$$\overline{\mathbf{E}}_{f}^{p}=\frac{1}{2}(\mathbf{f}^{pT}\mathbf{f}^{p}\overline{\mathbf{a}}^{T}\overline{\mathbf{a}}),\ \overline{\mathbf{E}}_{g}^{p}=\frac{1}{2}(\mathbf{f}^{pT}\mathbf{g}^{p}+\mathbf{g}^{pT}\mathbf{f}^{p}\overline{\mathbf{a}}^{T}\overline{\mathbf{b}}\overline{\mathbf{b}}^{T}\overline{\mathbf{a}}),\ \overline{\mathbf{E}}_{2}^{p}=\frac{1}{2}(\mathbf{g}^{pT}\mathbf{g}^{p}\overline{\mathbf{b}}^{T}\overline{\mathbf{b}}).,\quad (25)$$

In which the metric tensor $\overline{\mathbf{a}}$ and director metric $\overline{\mathbf{b}}$ in the intermediate configuration as $\overline{\mathbf{a}}=\overline{\mathbf{a}}_{i}\otimes\overline{\mathbf{a}}^{i}=\overline{\mathbf{d}},_{\alpha}\otimes\overline{\mathbf{a}}^{\alpha}+\overline{\mathbf{d}}\otimes\overline{\mathbf{a}}^{3}$, and $\overline{\mathbf{b}}=\overline{\mathbf{d}},_{\alpha}\otimes\overline{\mathbf{a}}^{\alpha}$. Similarly, we have elastic strain referred to reference configuration as follows

$$\mathbf{E}_{*}^{e}=\mathbf{F}^{pT}\overline{\mathbf{E}}_{0}^{e}\mathbf{F}^{p}=\mathbf{F}^{pT}\overline{\mathbf{Z}}^{-T}\overline{\mathbf{E}}_{0}^{e}\overline{\mathbf{Z}}^{-1}\mathbf{F}^{p}=\mathbf{Z}^{-T}\mathbf{f}^{pT}\overline{\mathbf{E}}_{0}^{e}\mathbf{f}^{p}\mathbf{Z}^{-1}.,\quad (26)$$

Then we have total strain on the mid-surface

$$\mathbf{E}_{0}=\mathbf{f}^{pT}\overline{\mathbf{E}}_{o}^{e}\mathbf{f}^{p}+\mathbf{E}_{0}^{p},\quad \mathbf{E}_{f}=\mathbf{f}^{pT}\overline{\mathbf{E}}_{f}^{e}\mathbf{f}^{p}+\mathbf{E}_{f}^{p},\quad \mathbf{E}_{g}=\mathbf{f}^{pT}\overline{\mathbf{E}}_{f}^{e}\mathbf{f}^{p}+\mathbf{E}_{g}^{p},\quad \mathbf{E}_{2}=\mathbf{f}^{pT}\overline{\mathbf{E}}_{2}^{e}\mathbf{f}^{p}+\mathbf{E}_{2}^{p},\quad (27)$$

and the strain rates

$$\dot{\mathbf{E}}_{f}=\mathbf{f}^{pT}\overset{\square}{\mathbf{E}}{}_{f}^{e}\ \mathbf{f}^{p}+\dot{\mathbf{E}}_{f}^{p},\ \dot{\mathbf{E}}_{g}=\mathbf{f}^{pT}\overset{\square}{\mathbf{E}}{}_{g}^{e}\ \mathbf{f}^{p}+\dot{\mathbf{E}}_{g}^{p}.,\quad (28)$$

where the objective derivatives are defined by $\overset{\square}{\mathbf{E}}{}_{f}^{e}=\dot{\overline{\mathbf{E}}}{}_{f}^{e}+\overline{\mathbf{l}}_{f}^{pT}\overline{\mathbf{E}}_{f}^{e}+\overline{\mathbf{E}}_{f}^{e}\overline{\mathbf{l}}_{f}^{p}$, and $\overset{\square}{\mathbf{E}}{}_{f}^{e}=\dot{\overline{\mathbf{E}}}{}_{g}^{e}+\overline{\mathbf{l}}_{g}^{pT}\overline{\mathbf{E}}_{g}^{e}+\overline{\mathbf{E}}_{g}^{e}\overline{\mathbf{l}}_{g}^{p}$, $\overline{\mathbf{l}}_{f}^{p}=\dot{\mathbf{f}}^{p}\mathbf{f}^{p-1}$.

For easily understanding, let us write out the component form of the above strains and strains rates as follows. Introduce the following notations

$$\mathbf{C}_{f}^{e}\overset{\text{def}}{=}\mathbf{f}^{eT}\mathbf{f}^{e}=a_{\alpha\beta}\overline{\mathbf{a}}^{\alpha}\otimes\overline{\mathbf{a}}^{\beta}+\gamma_{\alpha}(\overline{\mathbf{a}}^{\alpha}\otimes\overline{\mathbf{a}}^{3}+\overline{\mathbf{a}}^{3}\otimes\overline{\mathbf{a}}^{\alpha})+\lambda^{2}\overline{\mathbf{a}}^{3}\otimes\overline{\mathbf{a}}^{3},$$
$$\mathbf{C}_{g}^{e}\overset{\text{def}}{=}\mathbf{f}^{eT}\mathbf{g}^{e}=\kappa_{\alpha\beta}\overline{\mathbf{a}}^{\alpha}\otimes\overline{\mathbf{a}}^{\beta}+\kappa_{\alpha3}\overline{\mathbf{a}}^{\alpha}\otimes\overline{\mathbf{a}}^{3},\quad \mathbf{C}_{d}^{e}\overset{\text{def}}{=}\mathbf{g}^{eT}\mathbf{g}^{e}=\eta_{\alpha\beta}\overline{\mathbf{a}}^{\alpha}\otimes\overline{\mathbf{a}}^{\beta},\quad (29)$$

where, $a_{\alpha\beta}=\mathbf{a}_{\alpha}\bullet\mathbf{a}_{\beta}$, $\gamma_{\alpha}=\mathbf{a}_{\alpha}\bullet\mathbf{d}$, $\kappa_{\alpha\beta}=\mathbf{a}_{\alpha}\bullet\mathbf{d},_{\beta}$, $\kappa_{\alpha3}=\mathbf{d},_{\alpha}\bullet\mathbf{d}$, $\lambda^{2}=\mathbf{d}\bullet\mathbf{d}$, $\eta_{\alpha\beta}=\mathbf{d},_{\alpha}\bullet\mathbf{d},_{\beta}$; $A_{\alpha\beta}=\mathbf{A}_{\alpha}\bullet\mathbf{A},_{\beta}$, $\Gamma_{\alpha}=\mathbf{A}_{\alpha}\bullet\mathbf{D}$, $\mathrm{K}_{\alpha\beta}=\mathbf{A}_{\alpha}\bullet\mathbf{D},_{\beta}$, $\mathrm{K}_{\alpha3}=\mathbf{D}\bullet\mathbf{D},_{\alpha}$, $\Lambda^{2}=\mathbf{D}\bullet\mathbf{D}$, $\mathrm{H}_{\alpha\beta}=\mathbf{D},_{\alpha}\bullet\mathbf{D},_{\beta}$; $\overline{a}_{\alpha\beta}=\overline{\mathbf{a}}_{\alpha}\bullet\overline{\mathbf{a}}_{\beta}$, $\gamma_{\alpha}=\overline{\mathbf{a}}_{\alpha}\bullet\overline{\mathbf{d}}$, $\overline{\kappa}_{\alpha\beta}=\overline{\mathbf{a}}_{\alpha}\bullet\overline{\mathbf{d}},_{\beta}$, $\overline{\kappa}_{\alpha3}=\overline{\mathbf{d}},_{\alpha}\bullet\overline{\mathbf{d}}$, $\overline{\lambda}^{2}=\overline{\mathbf{d}}\bullet\overline{\mathbf{d}}$, $\overline{\eta}_{\alpha\beta}=\overline{\mathbf{d}},_{\alpha}\bullet\overline{\mathbf{d}},_{\beta}$.

Then we have the components form of Green strain tensor as follows

$$\mathbf{E}_{f}=\frac{1}{2}(a_{\alpha\beta}A_{\alpha\beta})\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}+(\gamma_{\alpha3}\Gamma_{\alpha3})(\mathbf{A}^{\alpha}\otimes\mathbf{A}^{3}+\mathbf{A}^{3}\otimes\mathbf{A}^{\alpha})+\frac{1}{2}(\lambda^{2}\Lambda^{2})\mathbf{A}^{3}\otimes\mathbf{A}^{3}$$
$$\mathbf{E}_{g}=(\kappa_{\alpha\beta}\mathrm{K}_{\alpha\beta})\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}+(\kappa_{\alpha3}\mathrm{K}_{\alpha3})(\mathbf{A}^{\alpha}\otimes\mathbf{A}^{3}+\mathbf{A}^{3}\otimes\mathbf{A}^{\alpha}),\ \mathbf{E}_{2}=\frac{1}{2}(\eta_{\alpha\beta}\mathrm{H}_{\alpha\beta})\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta},\quad (30)$$

and elastic strain with respect to the intermediate configuration

$$\overline{\mathbf{E}}_{f}^{e}=\frac{1}{2}(a_{\alpha\beta}-\overline{a}_{\alpha\beta})\overline{\mathbf{a}}^{\alpha}\otimes\overline{\mathbf{a}}^{\beta}+(\gamma_{\alpha3}-\overline{\gamma}_{\alpha3})(\overline{\mathbf{a}}^{\alpha}\otimes\overline{\mathbf{a}}^{3}+\overline{\mathbf{a}}^{3}\otimes\overline{\mathbf{a}}^{\alpha})+\frac{1}{2}(\lambda^{2}-\overline{\lambda}^{2})\overline{\mathbf{a}}^{3}\otimes\overline{\mathbf{a}}^{3}$$
$$\overline{\mathbf{E}}_{g}^{e}=(\kappa_{\alpha\beta}-\overline{\kappa}_{\alpha\beta})\overline{\mathbf{a}}^{\alpha}\otimes\overline{\mathbf{a}}^{\beta}+(\kappa_{\alpha3}-\overline{\kappa}_{\alpha3})(\overline{\mathbf{a}}^{\alpha}\otimes\overline{\mathbf{a}}^{3}+\overline{\mathbf{a}}^{3}\otimes\overline{\mathbf{a}}^{\alpha}),\quad \overline{\mathbf{E}}_{2}^{e}=\frac{1}{2}(\eta_{\alpha\beta}-\overline{\eta}_{\alpha\beta})\overline{\mathbf{a}}^{\alpha}\otimes\overline{\mathbf{a}}^{\beta},\quad (31)$$

and plastic strains with respect to the un-deformed configuration

$$\mathbf{E}_f^{\,p}=\frac{1}{2}(\bar{a}_{\alpha\beta}A_{\alpha\beta})\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}+(\bar{\gamma}_{\alpha3}\Gamma_{\alpha3})(\mathbf{A}^{\alpha}\otimes\mathbf{A}^{3}+\mathbf{A}^{3}\otimes\mathbf{A}^{\alpha})+\frac{1}{2}(\bar{\lambda}^{2}\Lambda^{2})\mathbf{A}^{3}\otimes\mathbf{A}^{3}$$

$$\mathbf{E}_g^{\,p}=(\bar{\kappa}_{\alpha\beta}K_{\alpha\beta})\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}+(\bar{\kappa}_{\alpha3}K_{\alpha3})(\mathbf{A}^{\alpha}\otimes\mathbf{A}^{3}+\mathbf{A}^{3}\otimes\mathbf{A}^{\alpha}),\quad E_2^{p}=\frac{1}{2}(\bar{\eta}_{\alpha\beta}H_{\alpha\beta})\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}, \tag{32}$$

And the total strain rates

$$\dot{\mathbf{E}}_f=\frac{1}{2}\dot{a}_{\alpha\beta}\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}+\dot{\gamma}_{\alpha3}(\mathbf{A}^{\alpha}\otimes\mathbf{A}^{3}+\mathbf{A}^{3}\otimes\mathbf{A}^{\alpha})+\frac{1}{2}\dot{\lambda}^{2}\mathbf{A}^{3}\otimes\mathbf{A}^{3}$$

$$\dot{\mathbf{E}}_g=\dot{\kappa}_{\alpha\beta}\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}+\dot{\kappa}_{\alpha3}(\mathbf{A}^{\alpha}\otimes\mathbf{A}^{3}+\mathbf{A}^{3}\otimes\mathbf{A}^{\alpha}),\quad \dot{\mathbf{E}}_2=\frac{1}{2}\dot{\eta}_{\alpha\beta}\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}, \tag{33}$$

And

$$\dot{\mathbf{E}}_f^{\,p}=\frac{1}{2}\dot{\bar{a}}_{\alpha\beta}\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}+\dot{\bar{\gamma}}_{\alpha3}(\mathbf{A}^{\alpha}\otimes\mathbf{A}^{3}+\mathbf{A}^{3}\otimes\mathbf{A}^{\alpha})+\frac{1}{2}\dot{\bar{\lambda}}^{2}\mathbf{A}^{3}\otimes\mathbf{A}^{3}$$

$$\dot{\mathbf{E}}_g^{\,p}=\dot{\bar{\kappa}}_{\alpha\beta}\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}+\dot{\bar{\kappa}}_{\alpha3}(\mathbf{A}^{\alpha}\otimes\mathbf{A}^{3}+\mathbf{A}^{3}\otimes\mathbf{A}^{\alpha}),\quad \dot{\mathbf{E}}_2^{p}=\frac{1}{2}\dot{\bar{\eta}}_{\alpha\beta}\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}, \tag{34}$$

and objective elastic strain rates

$$\overset{\square}{\mathbf{E}}{}_f^{e}=\mathbf{f}^{-pT}(\dot{\bar{\mathbf{E}}}_f-\dot{\bar{\mathbf{E}}}_f^{\,p})\mathbf{f}^{-p}=\frac{1}{2}(\dot{a}_{\alpha\beta}-\dot{\bar{a}}_{\alpha\beta})\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}+(\dot{\gamma}_{\alpha3}-\dot{\bar{\gamma}}_{\alpha3})(\mathbf{A}^{\alpha}\otimes\mathbf{A}^{3}+\mathbf{A}^{3}\otimes\mathbf{A}^{\alpha})+\frac{1}{2}(\dot{\lambda}^{2}-\dot{\bar{\lambda}}^{2})\mathbf{A}^{3}\otimes\mathbf{A}^{3}$$

$$\overset{\square}{\mathbf{E}}{}_g^{e}=\mathbf{f}^{-pT}(\dot{\bar{\mathbf{E}}}_g-\dot{\bar{\mathbf{E}}}_g^{\,p})\mathbf{f}^{-p}=(\dot{\kappa}_{\alpha\beta}-\dot{\bar{\kappa}}_{\alpha\beta})\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}+(\dot{\kappa}_{\alpha3}-\dot{\bar{\kappa}}_{\alpha3})(\mathbf{A}^{\alpha}\otimes\mathbf{A}^{3}+\mathbf{A}^{3}\otimes\mathbf{A}^{\alpha}),\quad \overset{\square}{\mathbf{E}}{}_2^{e}=\frac{1}{2}(\dot{\eta}_{\alpha\beta}-\dot{\bar{\eta}}_{\alpha\beta})\mathbf{A}^{\alpha}\otimes\mathbf{A}^{\beta}. \tag{35}$$

It means that the elastic strain is defined with respect to the intermediate configuration. These formulations can be directly used to complete modelling of shells.

5. CONCLUSIONS

We believe that there are following features in our formulations:

1. Extended Krauss decomposition, $\mathbf{F} = (\mathbf{f}^e\mathbf{f}^p + \mathbf{g}^e\mathbf{f}^p)\mathbf{Z}^{-1}$, is important for the formulation of elasto-plastic shell with finite deformation;
2. The intrinsic relations we found between the deformation gradients $\mathbf{f}^e$, $\mathbf{f}^p$, $\mathbf{g}^e$, $\mathbf{g}^p$ of shell with the deformation gradients $\mathbf{F}^e$, $\mathbf{F}^p$ of 3-D continuum are key relations for the formulation of shells;
3. The form of multiplicative decomposition for elasto-plastic shell takes $\mathbf{f}=\mathbf{f}^e\mathbf{f}^p$, $\mathbf{g}=\mathbf{g}^e\mathbf{f}^p$, which can be considered as the extension of Lee and Liu's work on the shells;
4. The formulation in this paper can be directly used to formulate the elasto-plastic constitutive equations of shells.

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References

[1] Krauss, F., (1929), Über die Grundgleidingen der Elastizitätetheorie Schwach Deformierter Schalen. *Math.Annalen*, 101, 61-92.

[2] Pietraszkiewicz, W., (1981), On Consistent Approximations in the Geometrically Non-linear theory of Shells, *Mitt. f Mech.* Nr.26. Ruhr-Universität Bochum.

[3] Reissner, E., (1964), On the form of Variationally Derived Shell Equations, *J. of Appl. Mech.*, **31**, 233-238.

[4] Naghdi, P. M., (1964), On a Variational Theorem in Elasticity and its Application to Shell Theory, *J. Appl. Mech.* **31**, 647-653.

[5] Toupin, R. A., (1964), Theories of Elasticity with Couple-stress, *Archive for Rational Mechanics and Analysis*, **17**, 85-112.

[6] Hughes, T. J. R. & Brezzi, F., On Drill Degrees of Freedom, *Computer Methods in Appl. Mech. & Engng.*, **17**, 105-121, 1989.

[7] Naghdi, P. M., (1972), The Theory of Shells, Handbuch der Physik, Vol Via/2, Mechanics of Solids II, C. Truesdell, ed., Springer-Verlag, Berlin.