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A Mathematical Approach to Identify the Number of Distinct Mechanisms of Multiple Jointed Kinematic Chains using Concepts of MATLAB

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Abstract: The paper proposes a new method to identify the number of Distinct Mechanisms (DM) in Multiple Jointed Kinematic Chain. Using the Kinematic Graph of Multiple Jointed Kinematic Chain, a matrix namely Weighted Adjacency Matrix [WA] is developed. With the help of a MATLAB Program, the Characteristic Polynomial Coefficients of respective mechanisms of Multiple Jointed Kinematic Chain are formulated and observed to ascertain the Distinct Mechanisms. The method also enables to determine the Fixed Link (FL) in the Mechanisms. The Equivalent Links (EL), which produce the identical mechanisms, when fixed, are also enumerated.

Keywords: Distinct Mechanism, DM, WA Matrix, Fixed Link, Equivalent Link

1. INTRODUCTION

The comprehensive field of Structural Synthesis of Kinematic Structures can be visualized by the fact that, in order to develop and characterize the kinematic structures belonging to various families of Kinematic Chains, the quest for simple, efficient and economical methods has been increasing over the years. A large number of researchers have contributed in this field by proposing various techniques that are based on approaches like Generalized Matrix Notation [1], the link disposition method [2], Distance matrix [3], Adjacency matrix [4], the flow matrix method [5] and the row sum of extended distance matrix methods [6], Link path Code [7], identification code [8], minimum code [9] etc., to ascertain the kinematic chains which are distinct structurally and identification of kinematic structures. In present work the Kinematic Graph of Multiple jointed kinematic chains are pursued. A multiple Jointed Kinematic Chain is actually formed by transforming the higher order links like ternary and quaternary links into binary links by maintaining the same number of joints with the help of the graph of the multiple jointed kinematic chains. Generally, as per a kinematic graph, vertices are used for links representation while edges signify joints but since number of edges is more as compared to the number of joints in a kinematic graph of multiple jointed kinematic chains. Therefore the edges of the kinematic graphs are assigned with a joint value. It has been found that a maximum of 176 total possible assortments of Multiple jointed Kinematic Chains

can be developed from the 1 DOF, 8 Link, 10 Joint, 16 Simple Jointed Base Kinematic Chains, out of which 134 are Distinct multiple jointed kinematic Chains [10]. Using Kinematic Graph of multiple jointed Kinematic Chain, a Matrix called Modified Adjacency Matrix [MA] is developed, the diagonal elements of [MA] Matrix represents the degree of respective Links, while the other elements of [MA] shows the joint value between adjacent links, as represented by the Kinematic Graph. Further, the mutual interactive effect of Relative weights of the links connected together by a joint is also taken into account in the form of degree of the links as per its relative importance in graph. Using this interactive effect and [MA] matrix, a new matrix of the multiple jointed kinematic chains is derived and is called Weighted Adjacency Matrix [WA].Fixing n-links of kinematic chain, n-mechanisms are obtained. If the corresponding row and column elements of [WA] are changed to zero, the n-[WA] matrices will be developed for n-mechanisms. Now by using the facilities provided in the MATLAB, the Characteristic Polynomial Coefficients of these n-[WA] matrices pertaining to Mechanisms can be obtained. These Polynomial Coefficients are identical for Mechanisms which are equivalent but different for Mechanisms which are distinct. In this way distinct mechanisms can be obtained from a multiple jointed Kinematic Chain under observation.

The methodology is applied on the 134 Distinct Multiple Jointed Kinematic Chains, derived from 1DOF, 8 links, 10 joint 16 Base kinematic chains and a total of 964 distinct mechanisms are found. The results are validated and represented in understandable format.

Degree of Links d(\mathbf{I}_i): The link degree actually signify the link type i.e. binary, ternary or quaternary link etc., designated as d(\mathbf{I}_i)

d $(l_i) = 2$, for binary link d $(l_i) = 4$, for quaternary link

d $(l_i) = 3$, for ternary link d $(l_i) = n$, for n-nary link

Degree Vector (v): Degree vector represents the degree of individual link of kinematics chain and represented as:

 $v = \{ d(1_1), d(1_2), \dots, d(1n) \}$

1.1. Modified Adjacency Matrix [MA]

This matrix is used to represent the Multiple Jointed kinematic chain. We may use the graph of a multiple Jointed Kinematic Chain to represent [MA] Matrix.

 $\{d_{ii}\}$ = Joint value between adjacent ith link & jth link.

- If ith link and jth link are adjacent, then joint value will be equal to the value as calculated by using the graph of the kinematic chain.
- If ith link and jth link are not adjacent to each other, then,

 $d_{ii} = 0$

• The Diagonal elements of the [MA] Matrix, i.e. when $i = j (d_{ij} \text{ or } d_{ij})$, represent the Degree of Links.

$$[\mathbf{MA}] = \{\mathbf{d}_{ii}\}_{nxn}$$

1.2. Mutual Interactive Effect of Relative Weights (W_{ii})

The mutual interactive effect of Relative weights of the links connected together by a joint is also taken into account in the form of degree of the links as per its relative importance in graph and is given as,

$$W_{ij} = (W_{ij} + W_{ji}) / 2$$
$$W_{ij} = \frac{1}{2} [d(l_i) / d(l_j) + d(l_j) / d(l_j)]$$

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1.3. Weighted Adjacency Matrix [WA]

Weighted Adjacency Matrix incorporates both, the influence of Modified Adjacency Matrix [MA] and Mutual effect of weights (W_{ii}), thus we obtained a square matrix of size nxn, represented below,

$$[WA] = \{g_{ii}\} n \times r$$

Where;

$$g_{ij} = \{d_{ij}\}^* (W_{ij})$$

1.4. Characteristic Polynomial of [WA] Matrix

The weighted adjacency matrix is further analyzed to derive the characteristic polynomial. It is pertinent to mention here that the roots of nth order characteristic polynomial are nothing but a set of n-Eigen values, also known by the name Eigen spectrum. Since the effect of mutual interactive weight of links has already been taken into account in the form of degree of link therefore the proposed [WA] matrix is well equipped to identify the different link type. Hence it can be established that the characteristic polynomial thus obtained and its roots are unique and corroborate in identification of distinct kinematic chains and Mechanisms.

Using facilities provided by the software MATLAB, a predefined function called *poly* carried out, which in return provide the Characteristic Polynomial of the Matrix.

Brief Facts about function poly

Poly: Polynomial with specified roots

Syntax:

p = poly(A)

Description:

p = poly(A) where A is an n-by-n matrix and returns an n+1 element row vector, the elements of which forms the coefficients of the characteristic polynomial.

The characteristic Polynomial equation of [WA] matrix is given as:

$$(WA - \lambda_j) = 0$$
$$= \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \dots + a_{n-1} \lambda + a_n$$

Poly (WA) provides for the characteristic polynomial of [WA], as *poly* use eigen values, which is based on similarity transformations.

1.5. Characteristic Polynomial Equation (Coefficients) for Mechanisms

For finding the polynomial representation of mechanisms of multiple jointed kinematic chains, the elements of a particular row and column of [WA] matrix are converted into zero, while remaining elements remains same as in the base [WA] matrix. e.g, For calculating the Characteristic Polynomial equation for Ist mechanism (say, P1) of multiple jointed kinematic chain, then the base [WA] matrix of that chain is used and the 1st row and 1st column of [WA] are converted into Zero and the using function *poly* the desired polynomial equation is calculated.

Similarly, for calculating the Characteristic Polynomial equation for 2^{nd} mechanism (say, P2) of multiple jointed kinematic chain, then the base [WA] matrix of that chain is used and the 2^{nd} row and 2^{nd} column of [WA] are changed into Zero, while all the elements remains same as in base [WA] matrix and then using function *poly* the desired polynomial equation is calculated. Similarly, P(3) for 3^{rd} Mechanism , P(4), P(5).....P(n) are

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calculated. This procedure is used for finding the Characteristic polynomial equation of nth Mechanism of the multiple jointed kinematic chains.

2. ILLUSTRATIVE EXAMPLE

Considering the configuration of Multiple jointed Kinematic chain and its Kinematic Graph shown in figure-1(a) and 1(b) respectively.

2.1. Degree Vector (V)

The degree vector for the above chain can be written as,



Figure 1:



2.2. Modified Adjacency Matrix [MA]

Writing the Modified Adjacency Matrix of the multiple jointed kinematic chain figure-1(a) using its graphic structure figure-1(b)

	3	0	1	0	1	1	0	0
	0	3	0	1	1	0	0	1
	1	0	2	0	0	0	0.67	0.67
	0	1	0	2	0	0.67	0.67	0
	1	1	0	0	2	0	0	0
MA =	1	0	0	0.67	0	2	0.67	0
	0	0	0.67	0.67	0	0.67	2	0.67
	0	1	0.67	0	0	0	0.67	2

Weighted Adjacency matrix [WA]

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	3	0	1.0833	0	1.0833	1.0833	0	0
	0	3	0	1.0833	1.0833	0	0	1.0833
	1.0833	0	2	0	0	0	0.67	0.67
	0	1.0833	0	2	0	0.67	0.67	0
***	1.0833	1.0833	0	0	2	0	0	0
WA =	1.0833	0	0	0.67	0	2	0.67	0
	0	0	0.67	0.67	0	0.67	2	0.67
	0	1.0833	0.67	0	0	0	0.67	2

Polynomial Coefficients (Say, P) of Base [WA] Matrix

P = Poly(WA)

P = 1, -18, 131.26, -499.95, 1068, -1276.6, 801.89, - 226.22, 20.756

Expression of the above polynomial will be (say in terms of variable x):

 $P = x^8 - 18x^7 + 131.26x^6 - 499.95x^5 + 1068x^4 - 1276.6x^3 + 801.89x^2 - 226.22x + 20.756$

2.3. Polynomial Coefficients of Mechanisms of Multiple Jointed Kinematic Chain(figure-1a)

i) Fixing Link 1st, the Matrix obtained, (say M1),

	0	0	0	0	0	0	0	0]
	0	3	0	1.0833	1.0833	0	0	1.0833
	0	0	2	0	0	0	0.67	0.67
	0	1.0833	0	2	0	0.67	0.67	0
	0	1.0833	0	0	2	0	0	0
MI =	0	0	0	0.67	0	2	0.67	0
	0	0	0.67	0.67	0	0.67	2	0.67
	0	1.0833	0.67	0	0	0	0.67	2

Polynomial Coefficients of M1(say P1):

P1 = Poly(M1)

P1 = 1, -15, 89.786, -276.37, 468.04, -429.34, 191.12, -29.079,0

ii) Fixing Link 2nd, the Matrix obtained, (say M2),

	3	0	1.0833	0	1.0833	1.0833	0	0]
	0	3	0	0	0	0	0	0
	1.0833	0	2	0	0	0	0.67	0.67
	0	0	0	2	0	0.67	0.67	0
	1.0833	0	0	0	2	0	0	0
M2 =	1.0833	0	0	0.67	0	2	0.67	0
	0	0	0.67	0.67	0	0.67	2	0.67
	0	0	0.67	0	0	0	0.67	2

Polynomial Coefficients of M2(say P2):

P2 = Poly(M2)

P2 = 1, -15, 89.786, -276.37, 468.04, -429.34, 191.12, -29.079,0

iii) Fixing Link 3rd, the Matrix obtained, (say M3),

	3	0	0	0	1.0833	1.0833	0	0
	0	3	0	1.0833	1.0833	0	0	1.0833
	0	0	0	0	0	0	0	0
M2_	0	1.0833	0	2	0	0.67	0.67	0
M 3 –	1.0833	1.0833	0	0	2	0	0	0
	1.0833	0	0	0.67	0	2	0.67	0
	0	0	0.67	0.67	0	0.67	2	0.67
	0	1.0833	0	0	0	0	0.67	2

Polynomial Coefficients of M3, (sayP3):

P3=Poly(M3)

P3 = 1, -16, 101.34, -324.51, 555.94, -494.68, 201.5, -25.976, -1.1536e-014

iv) Fixing Link 4th, the Matrix obtained, (say M4),

	3	0	1.0833	0	1.0833	1.0833	0	0]
	0	3	0	0	1.0833	0	0	1.0833
	1.0833	0	2	0	0	0	0.67	0.67
M 4 _	0	0	0	0	0	0	0	0
<i>M</i> 4 –	1.0833	1.0833	0	0	2	0	0	0
	1.0833	0	0	0	0	2	0.67	0
	0	0	0.67	0	0	0.67	2	0.67
	0	1.0833	0.67	0	0	0	0.67	2

Polynomial Coefficients of M4, (say P4):

P4 = poly(M4)

P4 = 1, -16, 101.34, -324.51, 555.94, -494.68, 201.5, -25.976, 1.1536e-014

v) Fixing Link 5th, the Matrix obtained, (say M5),

	3	0	1.0833	0	0	1.0833	0	0
	0	3	0	1.0833	0	0	0	1.0833
	1.0833	0	2	0	0	0	0.67	0.67
M5 -	0	1.0833	0	2	0	0.67	0.67	0
M J =	0	0	0	0	0	0	0	0
	1.0833	0	0	0.67	0	2	0.67	0
	0	0	0.67	0.67	0	0.67	2	0.67
	0	1.0833	0.67	0	0	0	0.67	2

Polynomial Coefficients of M5(say P5);

$$P5 = Poly(M5)$$

P5 = 1, -16, 101.61, -327.24, 566.01, -512.21, 219.05, -34.645, -1.1113e-027

vi) Fixing Link 6th, the Matrix obtained, (say M6)

	3	0	1.0833	0	1.0833	0	0	0]
	0	3	0	1.0833	1.0833	0	0	1.0833
	1.0833	0	2	0	0	0	0.67	0.67
M6 -	0	1.0833	0	2	0	0	0.67	0
M 0 =	1.0833	1.0833	0	0	2	0	0	0
	0	0	0	0	0	0	0	0
	0	0	0.67	0.67	0	0	2	0.67
	0	1.0833	0.67	0	0	0	0.67	2

Polynomial Coefficients of M6 (say P6);

P6 = poly (M6)

P6 = 1, -16, 101.34, -324.51, 555.94, -494.68, 201.5, -25.976, -2.1269e-014

vii) Fixing Link 7th, the Matrix obtained, (say M7)

	3	0	1.0833	0	1.0833	1.0833	0	0]
	0	3	0	1.0833	1.0833	0	0	1.0833
	1.0833	0	2	0	0	0	0	0.67
M7 _	0	1.0833	0	2	0	0.67	0	0
MI =	1.0833	1.0833	0	0	2	0	0	0
	1.0833	0	0	0.67	0	2	0	0
	0	0	0	0	0	0	0	0
	0	1.0833	0.67	0	0	0	0	2

Polynomial Coefficients of M7 (say P7);

P7 = poly(M7)

P7 = 1, -16, 101.06, -321.77, 546.26, -480.28, 196.52, -29.509, -1.6381e-014

viii) Fixing Link 8th, the Matrix obtained, (say M8)

	3	0	1.0833	0	1.0833	1.0833	0	0
	0	3	0	1.0833	1.0833	0	0	0
	1.0833	0	2	0	0	0	0.67	0
MO	0	1.0833	0	2	0	0.67	0.67	0
<i>M</i> 8 =	1.0833	1.0833	0	0	2	0	0	0
	1.0833	0	0	0.67	0	2	0.67	0
	0	0	0.67	0.67	0	0.67	2	0
	0	0	0	0	0	0	0	0

P8 = poly(M8) P8 = 1, -16, 101.34, -324.51, 555.94, -494.68, 201.5, -25.976, 0

2.4. Identification of the Number of Distinct Mechanisms

Summarize the results obtained above and observed them to ascertain the Distinct mechnisms in the given Multiple Jointed Kinematic Chain (figure-1a), where we find 5 Distinct Mechanisms.

3. RESULT

The Number of Distinct Mechanisms of the Multiple jointed kinematic chain shown in figure-1(a), developed from 1-F, 8- links, 10- Joints, 16 simple jointed base kinematic chains, including the Fixed Links (FL), Equivalent Links (EL) and the Characteristic Polynomial Coefficients are shown in Table 1.

Multiple Jointed	F	E	Polynomial Coefficients	D
Kinematic Chain	L	L		Μ
	1	2	1, -15, 89.786, -276.37, 468.04, -429.34,	
			191.12, -29.079, 0	
	-		1, -15, 89.786, -276.37, 468.04, -429.34,	
$ \rangle \wedge \rangle $	-		191.12, -29.079, 0	
	3	4	1, -16, 101.34, -324.51, 555.94, -494.68,	5
			201.5, -25.976, -1.1536e-014	
	-		1, -16, 101.34, -324.51, 555.94, -494.68,	
	-		201.5, -25.976, 1.1536e-014	
	5		1, -16, 101.61, -327.24, 566.01, -512.21,	
			219.05, -34.645, -1.1113e-027	
118. 131.26	6	8	1, -16, 101.34, -324.51, 555.94, -494.68,	
499.95, 1068, -			201.5, -25.976, -2.1269e-014	
1276.6. 801.89	7		1, -16, 101.06, -321.77, 546.26, -480.28,	
226.22, 20.756			196.52, -29.509, -1.6381e-014	
,	-		1, -16, 101.34, -324.51, 555.94, -494.68,	
	-		201.5, -25.976, 0	

Table 1

4. CONCLUSION

The proposed method has the potential to develop mechanisms pertaining to multiple jointed kinematic chains and also provides a way to differentiate between similar and distinct mechanisms. Being computationally efficient, the proposed study also capable of producing the results in a much more understandable format and as well as makes it less time consuming for the user to obtain the results. With these characteristics a much more improvised and innovative method has been developed for detection and identification of the distinct mechanisms. These insights in structural analysis of kinematic chains and mechanisms have a profound impact on further research in the area and will bolster the efforts of establishing a more robust and effective mechanical conceptual design. The further scope of this study lies in the fact that it can be used for developing the Mechanisms of multiple jointed kinematic chains with much higher number of links, joints and more degrees of freedom. Thus it may prove to be a very convenient, much more efficient and less time consuming for the designers engaged in the development of robust and economical kinematic structures.

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