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ANALYSIS OF A BULK SERVICE QUEUE WITH UNRELIABLE SERVER, MULTIPLE VACATION, STAND-BY SERVER, SETUP AND N-POLICY

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Abstract: We studied the behavior of a bulk service queueing system with unreliable server, stand-by server, setup with N-policy and multiple vacation, in this paper. The stand-by server is utilized only during main server's repair period. The Probability Generating Function(PGF) of queue size and some important performance measures are derived. An extensive numerical result is illustrated.

Keywords: Multiple vacation, Setup time with N-policy, Unreliable server and Stand-by server.

INTRODUCTION

The General bulk service rule was first introduced by Neuts (1967). The book by Chaudhry and Templeton explains the bulk service queueing systems in-depth. The General bulk service rule states that the server will start to provide services only when at least $'a'$ units are present in the queue, and maximum service capacity is $'b'$.

Setup time is preliminary work of the server before starting the service. In N-policy queueing system, the server begins service from vacation period only when the queue length reaches at least N. Artalejo et.al (2005) analyzed an multiserver queue with setup times. Jeyakumar et. al (2005), Krishna Reddy et.al (1998), Tadj et. al (2006, 2012) studied setup times with N-policy. Haridass et.al (2012), Senthilnathan et. al (2014) and Arumuganathan et.al (2006), studied setup time without N-policy.

Sudden failure or a breakdown of a system or the service channel is common in many queueing situations. As a result of a sudden breakdown, the service of a customer or a unit undergoing service has to be suspended and the customers have to wait till the server returns to the system or the system becomes operable again. Queueing systems which are subject to breakdown in bulk queueing models have been studied previously by many authors. Jeyakumar and Senthilnathan (2012) examined $M^{[X]}/G(a,b)/1$ queueing system with server breakdown without interruption, multiple

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vacations and closedown times. Jeyakumar and Senthilnathan (2014) analysed server breakdown without interruption in an queueing system with multiple vacations and setup times.

The present paper considers a single server queueing system, standby server, multiple vacation, setup time and N-policy. In order to improve efficiency of service, stand-by service is utilized just throughout breakdown periods of the regular server.

MODEL DESCRIPTION

In this paper, we consider that the arrival follows a compound Poisson process with arrival rate λ. Both server's service time, multiple vacation and setup time of main server are follow general distributions. The main server's breakdown and repair times follows exponential distributions with rate $'\alpha'$ and $'\eta'$ respectively. The main server gets breakdown at any instant during busy in service. In such case, he immediately sent for repair and current batch service is transferred to stand-by server who starts service to that batch afresh. The stand-by server retain in the system until the main server's repair completion. Suppose at the moment of main repair completion the stand-by service is busy the current batch of customers is transferred to the main server who starts service to that batch afresh. At the instant of main server's busy completion or repair completion, if the number of customers in the queue length is less than 'a', the main server avails multiple vacation repeatedly until finds minimum 'N' in the queue. After a vacation, if he finds minimum '' customers in the system, he requires a setup before start a service. After setup time he starts service to a batch of '' customers.

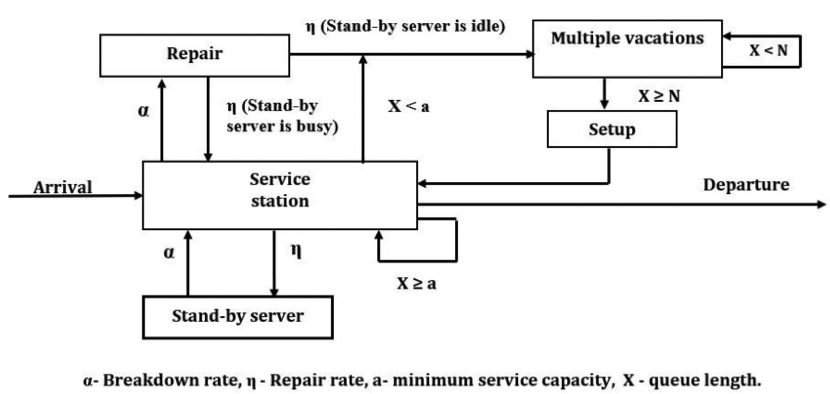

Figure 1: Pictorial representation of the proposed model

NOTATIONS

Let X be the group size random variable of arrival, g_k be the probability of 'k' customers arrive in a batch and $X(z)$ be its PGF.

$S_v(.)$, $S_u(.)$, $V(.)$ and $R(.)$ represent the Cumulative Distribution Functions (CDF) of service time of stand-by server, service time of main server, vacation time and setup time of main server with corresponding probability density functions are $s_u(w)$, $s_v(w)$, $v(w)$ and $r(w)$ respectively.

$S_u^0(t)$, $S_v^0(t)$, $V^{(0)}(t)$ and $R^{(0)}(t)$ represent the remaining service time of main server, remaining service time of stand-by server and remaining vacation time and setup time of main server at time 't' respectively.

$\tilde{S}_u(\tau)$, $\tilde{S}_v(\tau)$, $\tilde{V}(\tau)$ and $\tilde{R}(\tau)$ represent the Laplace Stieltjes Transform (LST) of S_u, S_v, V and R respectively.

$N_1(t)$ and $N_2(t)$ represents No. of. customers in service station and queue at time "" respectively. , if the server is on the vacation.

DEFINE THE PROBABILITIES

$$M_{i,e}(w,t)\Delta t = \Pr\{N_1(t) = i, N_2(t) = e, w \le S_u^0(t) \le w + \Delta t, \varepsilon(t) = 3\}, \quad a \le i \le b, e \ge 0,$$
$$L_{i,e}(w,t)\Delta t = \Pr\{N_1(t) = i, N_2(t) = e, w \le S_v^0(t) \le w + \Delta t, \varepsilon(t) = 1\}, \ a \le i \le b, e \ge 0$$
$$S_e(t)\Delta t = \Pr\{N_2(t) = e, \varepsilon(t) = 2\}, \ 0 \le e \le a - 1,$$
$$V_{l,e}(w,t)\Delta t = \Pr\{\chi(t) = l, N_2(t) = e, w \le V^0(t) \le w + \Delta t, \varepsilon(t) = 4\}, \quad l \ge 1, e \ge 0,$$
$$R_e(w,t)\Delta t = \Pr\{N_2(t) = e, w \le R^0(t) \le w + \Delta t, \varepsilon(t) = 5\}, e \ge N,$$

$\varepsilon(t) = 1, 2, 3, 4$ and 5 denotes stand-by server is busy, stand-by server idle, main server's busy, on vacation and setup, respectively,

The steady state equation for the proposed model is:

$$(\lambda + \eta)S_0 = \sum_{r=a}^{b} L_{r,0}(0), \tag{1}$$
$$(\lambda + \eta)S_e = \sum_{r=a}^{b} L_{r,e}(0) + \sum_{k=1}^{e} S_{e-k}\lambda g_k, 1 \le e \le a - 1, \tag{2}$$
$$-M_{i,0}'(w) = -(\lambda + \alpha)M_{i,0}(w) + \sum_{r=a}^{b} M_{r,i}(0)s_u(w) + \eta \int_0^\infty L_{i,0}(y)dys_u(w), \quad a \le i \le b. \tag{3}$$
$$-M_{i,e}'(w) = -(\lambda + \alpha)M_{i,e}(w) + \eta \int_0^\infty L_{i,e}(y)dys_u(w) + \sum_{k=1}^{e} M_{i,e-k}(w)\lambda g_k, \quad e \ge 1, a \le i \le b - 1, \tag{4}$$
$$-M_{b,e}'(w) = -(\lambda + \alpha)M_{b,e}(w) + \eta \int_0^\infty L_{b,e}(y)dys_u(w) + \sum_{r=a}^{b} M_{r,b+e}(0)s_u(w)$$

$$+\sum_{k=1}^{e} M_{b,e-k}(w)\lambda g_k, 1 \le e \le N-b-1, \quad (5)$$

$$-M'_{b,e}(w) = -(\lambda+\alpha)M_{b,e}(w) + \eta\int_0^\infty L_{b,e}(y)dys_u(w) + \sum_{k=1}^{e} M_{b,e-k}(w)\lambda g_k$$
$$+\sum_{r=a}^{b} M_{r,b+e}(0)s_u(w) + R_{b+e}(0)s_u(w), e \ge N-b, \quad (6)$$

$$-L'_{i,0}(w) = -(\lambda+\eta)L_{i,0}(w) + \sum_{r=a}^{b} L_{r,i}(0)s_v(w) + \alpha\int_0^\infty M_{i,0}(y)dys_v(w)$$
$$+\sum_{k=0}^{a-1} S_k\lambda g_{i-k}s_v(w), a \le i \le b, \quad (7)$$

$$-L'_{i,e}(w) = -(\lambda+\eta)L_{i,e}(w) + \alpha\int_0^\infty M_{i,e}(y)dys_v(w) + \sum_{k=1}^{e} L_{i,e-k}(w)\lambda g_k,$$
$$e \ge 1, a \le i \le b-1, \quad (8)$$

$$-L'_{b,e}(w) = -(\lambda+\eta)L_{b,e}(w) + \sum_{r=a}^{b} L_{r,b+e}(0)s_v(w) + \sum_{k=1}^{e} L_{b,e-k}(w)\lambda g_k$$
$$+\alpha\int_0^\infty M_{b,e}(y)dys_v(w) + \sum_{k=0}^{a-1} S_k\lambda g_{b+e-k}s_v(w), e \ge 1, \quad (9)$$

$$-R'_N(w) = -\lambda R_N(w) + \sum_{l=1}^{\infty} V_{l,N}(0)r(w), \quad (10)$$

$$-R'_e(w) = -\lambda R_e(w) + \sum_{l=1}^{\infty} V_{l,e}(0)r(w) + \sum_{k=1}^{e-N} R_{e-k}(w)\lambda g_k,$$
$$e \ge N+1, \quad (11)$$

$$-V'_{1,0}(w) = -\lambda V_{1,0}(w) + \sum_{r=a}^{b} M_{r,0}(0)v(w) + \eta S_0 v(w), \quad (12)$$

$$-V'_{1,e}(w) = -\lambda V_{1,e}(w) + \sum_{r=a}^{b} M_{r,e}(0)v(w) + \eta S_e v(w) + \sum_{k=1}^{e} V_{1,e-k}(w)\lambda g_k,$$
$$1 \le e \le a-1, \quad (13)$$

$$-V'_{1,e}(w) = -\lambda V_{1,e}(w) + \sum_{k=1}^{e} V_{1,e-k}(w)\lambda g_k, e \ge a, \quad (14)$$

$$-V'_{l,0}(w) = -\lambda V_{l,0}(w) + V_{l-1,0}(0)v(w), l \ge 2, \quad (15)$$

$$-V'_{l,e}(w) = -\lambda V_{l,e}(w) + V_{l-1,e}(0)v(w) + \sum_{k=1}^{e} V_{l,e-k}(w)\lambda g_k, l \ge 2,$$
$$1 \le e \le N-1, \quad (16)$$

$$-V'_{l,e}(w) = -\lambda V_{l,e}(w) + \sum_{k=1}^{e} V_{l,e-k}(w)\lambda g_k, l \ge 2, e \ge N. \quad (17)$$

After some mathematical manipulation, we get,

$$\widetilde{M}_i(z,\tau) = \frac{(\tilde{S}_u(f(z)) - \tilde{S}_u(\tau))}{(\tau - f(z))}\left[\sum_{r=a}^{b} M_{r,i}(0) + \eta\tilde{L}_i(z,0)\right], a \le i \le b-1, \quad (18)$$

$$\widetilde{M}_b(z,\tau) = \frac{(\tilde{S}_u(f(z)) - \tilde{S}_u(\tau))}{(\tau - f(z))(z^b - \tilde{S}_u(f(z)))}\left[z^b\eta\tilde{L}_b(z,0) + R(z,0)\right.$$
$$\left.+\sum_{r=a}^{b-1} M_r(z,0) - \sum_{e=0}^{b-1}\sum_{r=a}^{b-1} M_{r,e}(0)z^e\right], \quad (19)$$

$$\tilde{L}_i(z,\tau) = \frac{(\tilde{S}_v(g(z)) - \tilde{S}_v(\tau))}{(\tau - g(z))}\left[\alpha\widetilde{M}_i(z,0) + \sum_{r=a}^{b} L_{r,i}(0) + \sum_{k=0}^{a-1} S_k\lambda g_{i-k}\right],$$
$$a \le i \le b-1, \quad (20)$$

$$\tilde{L}_b(z,\tau) = \frac{(\tilde{S}_v(g(z)) - \tilde{S}_v(\tau))}{(\tau - g(z))(z^b - \tilde{S}_v(g(z)))}\left[z^b\alpha\widetilde{M}_b(z,0) - \sum_{e=0}^{b-1}\sum_{r=a}^{b} L_{r,e}(0)z^e\right.$$
$$\left.+\sum_{r=a}^{b-1} L_r(z,0) + \lambda\sum_{k=0}^{a-1} S_k z^k\sum_{e=b}^{\infty} g_{e-k}z^{e-k}\right], \quad (21)$$

$$\tilde{R}(z,\tau) = \frac{(\tilde{R}(h(z))-\tilde{R}(\tau))}{(\tau-h(z))}\sum_{l=1}^{\infty}[V_l(z,0) - \sum_{e=0}^{N-1} V_{l,e}(0)z^e], \tag{22}$$

$$\tilde{V}_1(z,\tau) = \frac{(\tilde{V}(h(z))-\tilde{V}(\tau))}{(\tau-h(z))}\sum_{e=0}^{a-1}[\sum_{r=a}^{b} M_{r,e}(0)z^e + \eta S_e z^e], \tag{23}$$

$$\tilde{V}_l(z,\tau) = \frac{(\tilde{V}(h(z))-\tilde{V}(\tau))}{(\tau-h(z))}\sum_{e=0}^{N-1} V_{l-1,e}(0)z^e, l \geq 2. \tag{24}$$

PROBABILITY GENERATING FUNCTION FOR QUEUE SIZE

3.1 The PGF of the queue size at an arbitrary time epoch

Let $P(z)$ be the PGF of the queue size at an arbitrary time epoch. Then,

$$P(z) = \sum_{r=a}^{b} \widetilde{M}_r(z,0) + \sum_{r=a}^{b} \tilde{L}_r(z,0) + \tilde{R}(z,0) + \sum_{l=1}^{\infty} \tilde{V}_l(z,0) + S(z). \tag{25}$$

By substituting $\tau = 0$ in equations (18) to (24) then equation (25) becomes

$$P(z) = \frac{\begin{aligned}&\{h(z)A_1(z)(1-\tilde{S}_u(f(z)))\textstyle\sum_{i=a}^{b-1}(z^b-z^i)m_i\\ &+h(z)A_2(z)(1-\tilde{S}_v(g(z)))\textstyle\sum_{i=a}^{b-1}(z^b-z^i)(q_i+\sum_{k=0}^{a-1}S_k\lambda g_{i-k})\\ &+\textstyle\sum_{k=0}^{a-1}S_k z^k[\eta h(z)\tilde{R}(h(z))\tilde{V}(h(z))A_1(z)(1-\tilde{S}_u(f(z)))\\ &+Y_1(z)[h(z)+\eta(1-\tilde{R}(h(z))\tilde{V}(h(z)))]-g(z)h(z)A_2(z)(1-\tilde{S}_v(g(z)))]\\ &+[Y_1(z)+h(z)A_1(z)(\tilde{S}_u(f(z))-1)]\times\\ &[(1-\tilde{R}(h(z))\tilde{V}(h(z)))\textstyle\sum_{n=0}^{a-1}m_n z^n+\tilde{R}(h(z))(1-\tilde{V}(h(z)))\sum_{n=0}^{N-1}v_n z^n]\}\end{aligned}}{h(z)Y_1(z)}, \tag{26}$$

$$\text{where} m_i = \sum_{r=a}^{b} M_{r,i}(0), v_i = \sum_{l=1}^{\infty} V_{l,i}(0), q_i = \sum_{r=a}^{b} L_{r,i}(0)$$

and the expressions for A_1 (z),A_2 (z) and Y_1 (z) are defined in Appendix-(I).

3.2 Steady state condition

The probability generating function has to satisfy $P(1) = 1$. In order to satisfy this condition applying L' Hopital's rule and evaluating $\lim_{z\to 1} P(z)$, then equating the expression to 1, we have $H = T_{10}$, where the expressions H and T_{10} are defined in Appendix-(II).

Since m_i, v_i, q_i and S_i are probabilities of 'i' customers being in the queue during main server's busy and vacation completion epoch, stand-by server's busy completion epoch and idle respectively, it follows that must be positive. Thus is satisfied iff $T_{10}>0$. If

$$\rho = \frac{\lambda X_1(\alpha+\eta)(1-\tilde{S}_u(\alpha))(1-\tilde{S}_v(\eta))}{b\alpha\eta[\tilde{S}_u(\alpha)(1-\tilde{S}_v(\eta))+\tilde{S}_v(\eta)(1-\tilde{S}_u(\alpha))]} < 1$$

is the condition for the existence of steady state for the model under consideration.

3.3 Computational aspects

Equation (26) has '$2b+N$' unknowns $m_0, m_1, \ldots, m_{b-1}, q_a, \ldots, q_{b-1}, S_0, S_1, \ldots, S_{a-1}$ and $v_0, v_1, \ldots, v_{N-1}$. We can express v_i in terms of m_i and S_i in such a way that numerator has only $2b$ constants. Now equation gives the PGF of the number of customers involving only '$2b$' unknowns. By Rouche's theorem, it can be proved that "Y_1 (Z) has $2b$-1 zeros inside and one on the unit circle $|z|=1$. Since $P(Z)$ is analytic within and on the unit circle, the numerator must vanish at these points, which gives $2b$ equations in $2b$ unknowns". We can solve these equations by any suitable numerical technique.

3.4 Result

Probability that at the main server's vacation completion epoch, there are 'i' $(0 \leq i \leq N-1)$ customers in the queue v_i can be expressed as sum of the probabilities of 'i' customers in the queue during main server's busy and stand-by server's idle period as,

Case: 1

$$v_n = \sum_{i=0}^{n} K_i(\eta S_{n-i} + m_{n-i}), n = 0,1,2,\ldots,a-1, \text{where} K_0 = \frac{\beta_0}{1-\beta_0},$$

$$K_n = \frac{\beta_n + \sum_{i=1}^{n} \beta_i K_{n-i}}{1-\beta_0}, n = 1,2,\ldots,a-1,$$

$\beta_i' s$ s are the probabilities of the 'i' customers arrive during vacation time.

$Case2$: For $\mathrm{n} = \mathrm{a}, \mathrm{a}+1, \ldots, \mathrm{N}-1$

$$\theta_n = \sum_{i=0}^{a-1} (\beta_{n-i} + \sum_{j=0}^{a-1-i} K_j \beta_{n-j-i})(m_i + \eta S_i), n = a, a+1, \ldots, N-1, \text{ then}$$

$$v_n = \frac{[\theta_n + \sum_{i=1}^{n-a} \beta_i v_{n-i}]}{1-\beta_0}, n = a+1, \ldots, N-1,$$

Where $v_a = \frac{\theta_a}{1-\beta_0}$

3.5 Particular cases

Case 1: When there are no breakdown, set-up and N-policy then equation (26) becomes

$$P(z) = \frac{(\tilde{S}_u(h(z))-1)\sum_{i=a}^{b-1}(z^b-z^i)m_i + (z^b-1)(\tilde{V}(h(z))-1)(\sum_{n=0}^{a-1}(m_n z^n+v_n z^n))}{(h(z))(\tilde{S}_u(h(z))-z^b)}, \tag{27}$$

which coincides with Jeyakumar et al. (2012).

Case 2: When there is no breakdown then equation (26) becomes

$$P(z) = \frac{(\tilde{S}_u(h(z))-1)\sum_{i=a}^{b-1}(z^b-z^i)m_i + [\tilde{R}(h(z))(\tilde{V}(h(z))-1)\sum_{n=0}^{a-1}v_n z^n + (\tilde{R}(h(z))\tilde{V}(h(z))-1)\sum_{n=0}^{a-1}m_n z^n](z^b-1)}{(h(z))(\tilde{S}_u(h(z))-z^b)}, \tag{28}$$

which coincides with Jeyakumar et al. (2014).

SOME PERFORMANCE MEASURES

4.1 Expected queue length

The expected queue length $E(Q)$ is

$$E(Q) = \frac{\begin{aligned}&f_1(X,S_u,S_v)[\textstyle\sum_{i=a}^{b-1}[b(b-1)-i(i-1)](m_i+(q_i+\sum_{k=0}^{a-1}S_k\lambda g_{i-k}))] \\ &+f_2(X,S_u,S_v)[\textstyle\sum_{i=a}^{b-1}(b-i)m_i] \\ &+f_3(X,S_u,S_v)\textstyle\sum_{i=a}^{b-1}(b-i)(q_i+\sum_{k=0}^{a-1}S_k\lambda g_{i-k}) \\ &+f_4(X,S_u,S_v,R,V)\textstyle\sum_{n=0}^{a-1}m_n+f_5(X,S_u,S_v,R,V)\sum_{n=0}^{N-1}v_n \\ &+f_6(X,S_u,S_v,R,V)\textstyle\sum_{n=0}^{a-1}S_n+f_7(X,S_u,S_v,R,V)\sum_{n=0}^{a-1}nm_n \\ &+f_8(X,S_u,S_v,V)\textstyle\sum_{n=0}^{N-1}nv_n+f_9(X,S_u,S_vR,V)\sum_{n=0}^{a-1}nS_n\end{aligned}}{3(T_{10})^2}, \tag{29}$$

the expressions for f_i(i=1,2,...,9) are given in Appendix-(II).

4.2 Main server's expected length of idle period

Let I be the idle random variable. Then, the expected length of idle E(I) is:

$$E(I) = E(R) + \frac{E(V)}{1-P(B=1)}. \tag{30}$$

where E(R) is main server's expected length of setup time.

4.3 Expected waiting time

The Expected waiting time is of the customers in queue E(W) is:

$$E(W) = \frac{E(Q)}{\lambda E(X)}, \tag{31}$$

5 NUMERICAL ILLUSTRATION

This section deals with the numerical illustration of the proposed queueing model through variations in the parameters using MATLAB software by considering the service time distribution follows Erlang-2 for both server(main server and stand-by server). Vacation time and setup time of main server follows exponential distribution.

Let us consider main server and stand-by server's service rate be μ_1 and μ_2 respectively. Vacation and setup rate of main server be γ and ζ respectively.

The results have been analysed in tabular forms and two dimensional graphs. The arbitrary chosen values satisfy the stability condition.

Let us consider the values be $a = 3, b = 5, N = 7 \mu_1 = 10, \mu_2 = 8, \alpha = 1,$ $\eta = 2, \gamma = 10 \text{and} \zeta = 8.$

Table 1: Arrival rate (vs) Performance measures

λ	ρ	$E(Q)$	$E(W)$
2.00	0.0888	5.32231	1.33058
2.25	0.0998	6.22889	1.38420
2.50	0.1109	7.33701	1.46740
2.75	0.1220	8.67171	1.57668
3.00	0.1331	10.2621	1.71035
3.25	0.1442	12.1453	1.86851
3.50	0.1553	14.3298	2.04712
3.75	0.1664	16.8563	2.24750
4.00	0.1775	19.7748	2.47185

For the arbitrary chosen values $a = 3, b = 5, N = 7, \lambda = 3, \mu_2 = 5, \alpha = 1,$ $\eta = 2, \gamma = 10 \text{and} \zeta = 8.$

Table 2: Main server's service rate (vs) Performance measures

μ_1	ρ	$E(Q)$	$E(W)$
6.00	0.264641	16.16468	2.694113
6.25	0.260242	15.69838	2.616396
6.50	0.254984	15.26758	2.544596
6.75	0.249240	14.86563	2.477605
7.00	0.243255	14.49199	2.415250
7.25	0.237189	14.14206	2.357011
7.50	0.231150	13.81161	2.301934
7.75	0.225207	13.50425	2.250709
8.00	0.219405	13.21369	2.202282

Take a=3,b=5,N=7, $\lambda = 7, \mu_1 = 10, \mu_2 = 8, \alpha = 1, \gamma = 10$ and $\zeta = 8$

Table 3: Main server's repair rate (vs)Performance measures

η	ρ	$E(Q)$	$E(W)$
1.0	0.319750	113.580016	8.112858
1.1	0.318426	110.047697	7.860550
1.2	0.317229	107.354598	7.668186
1.3	0.316142	105.200748	7.514339
1.4	0.315150	103.544280	7.396020
1.5	0.314240	102.197617	7.299830
1.6	0.313404	101.083783	7.220270
1.7	0.312632	100.967520	7.211966
1.8	0.311916	99.472494	7.105178
1.9	0.311252	98.903395	7.064528
2.0	0.310632	98.371045	7.026503

CONCLUSION

In this paper, we have studied a bulk service queueing system with unreliable server, stand-by server, multiple vacation, setup and N-policy. According to the author's best of learning, there is no one has been made in the queueing literature with these combination. The notable contribution in the proposed model is that the stand-by server is utilized during the main server's repair period, which reduce the waiting time of the customers. Using supplementary variable technique the PGF of the number of customers in the queue is found. The performance measures like, the mean number of customers in the queue, the mean waiting time of customers in the queue and main server's expected idle period are obtained.

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