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Submarine to Submarine Passive Target Tracking with Ownship S-maneuver

V. Lakshmi Bharathi¹, S. Koteswararao², B. Omkar Lakshmi Jagan³ and M. Kavitha lakshmi⁴

Abstract: This research effort is to analyze the motion of the target in passive mode. Target tracking is done with the help of bearing and elevation measurements using unscented angles-only Kalman filter (UAKF). UAKF is used to process the noise corrupted measurements and to analyze the path of the target. Ownship is S-manuevered for early observability of the process and to obtain the solution fast. Detailed mathematical modeling and its realization in simulation are carried out using UAKF.

Keywords: stochastic theory, statistical signal processing, applied statistics, estimation theory.

1. INTRODUCTION

In underwater, passive target tracking is generally followed to track a submarine target¹. The observer submarine is assumed to be moving at low speed to reduce self-noise for tracking of the targets. In conventional submarines, bearings-only measurements are available and so ownship has to maneuver for observability of the process. These days, submarines with sonar are coming up having the facility to get target elevation measurements also. In this paper, research is towards submarine (observer) tracking another submarine using elevation and bearing measurements. As angle measurements are only available, the process is highly nonlinear and hence unscented angles-only Kalman filter (UAKF), a nonlinear filter is explored for this application, as shown in the Figure 1 [2-6]. Ownship is S-manuevered for early observability of the process and to obtain the solution fast as shown in Figure 2. The estimated target range, course, bearing and speed (RCBS) are utilized in weapon guidance algorithm (which is not discussed here).

Section 2 deals with modeling of measurements, state vector and UAKF. Section 3 describes generalized simulator. Section 4 deals with results obtained for different scenarios in simulation. In section 5, the paper is finally concluded.

2. MATHEMATICAL MODELLING [3,4]

2.1. Measurements and state vector

$X_s(k)$, state vector is defined as

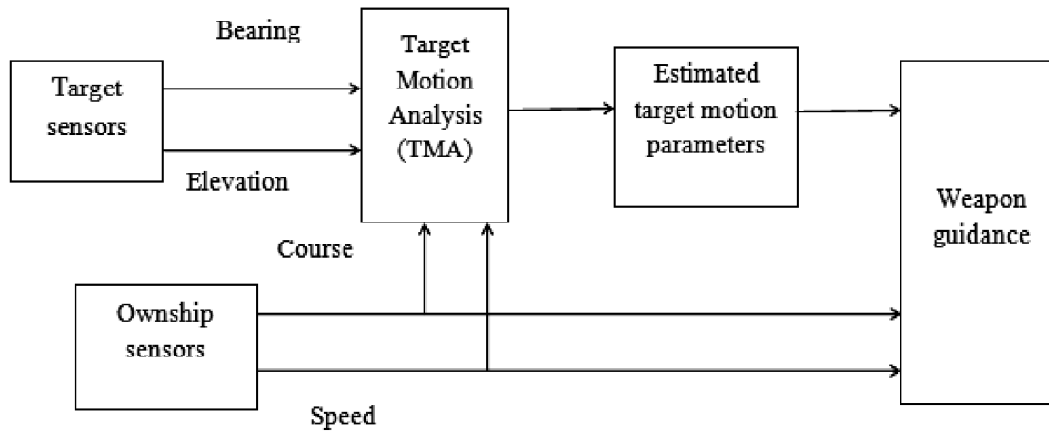


Figure 1: Block diagram of passive target tracking using bearing and elevation measurements

$$X_s(k) = [\dot{x}(k) \dot{y}(k) \dot{z}(k) R_x(k) R_y(k) R_z(k)]^T \quad (1)$$

where $x(k), y(k), z(k), R_x(k), R_y(k)$ and $R_z(k)$ are velocity and range components in x, y and z directions. Azimuth and elevation angles are considered w.r.to True North. B_m is

$$B_m(k+1) = \tan^{-1} \left(\frac{R_x(k+1)}{R_y(k+1)} \right) + \mu(k) \quad (2)$$

Variance of $\mu(k)$ is σ_b^2 . The measurement vector is

$$H(k+1) = \left[0 \ 0 \ 0 \ \hat{r}_z(k+1|k)/R^2(k+1|k) \ \hat{r}_y(k+1|k)/R^2(k+1|k) \ \hat{r}_x(k+1|k)/R^2(k+1|k) \right] \quad (3)$$

The state equation is

$$X_s(k+1) = \phi X_s(k) + b(k+1) + \Gamma w(k) \quad (4)$$

where ϕ is

$$\phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ t & 0 & 0 & 1 & 0 & 0 \\ 0 & t & 0 & 0 & 1 & 0 \\ 0 & 0 & t & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

where t is sample time and

$$\Gamma(k) = \begin{bmatrix} t & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & t \\ t^2/2 & 0 & 0 \\ 0 & t^2/2 & 0 \\ 0 & 0 & t^2/2 \end{bmatrix} \quad (6)$$

$$b \text{ is } b(k+1) = [0 \ 0 \ 0 \ -(x_0(k+1)-x_0(k)) \ -(y_0(k+1)-y_0(k)) \ -(z_0(k+1)-z_0(k))] \quad (7)$$

$x_0(k)$ and $y_0(k)$ are observer x and y position components.

$$w(k) = [\omega_x \ \omega_y \ \omega_z]^T \quad (8)$$

$w(k)$, is Gaussian with variance equal to

$$E[\Gamma(k)w(k)w^T(k)\Gamma^T(k)] = S\delta_{ij} \quad (9)$$

where $\delta_{ij} = \sigma_w^2$ if $i=j$

=0 otherwise

$$S = \begin{bmatrix} ts^2 & 0 & ts^3/2 & 0 & 0 & 0 \\ 0 & ts^2 & 0 & 0 & ts^3/2 & 0 \\ 0 & 0 & ts^2 & 0 & 0 & ts^3/2 \\ ts^3/2 & 0 & 0 & ts^3/4 & 0 & 0 \\ 0 & ts^2/2 & 0 & 0 & ts^3/4 & 0 \\ 0 & 0 & ts^2/2 & 0 & 0 & ts^3/4 \end{bmatrix} \quad (10)$$

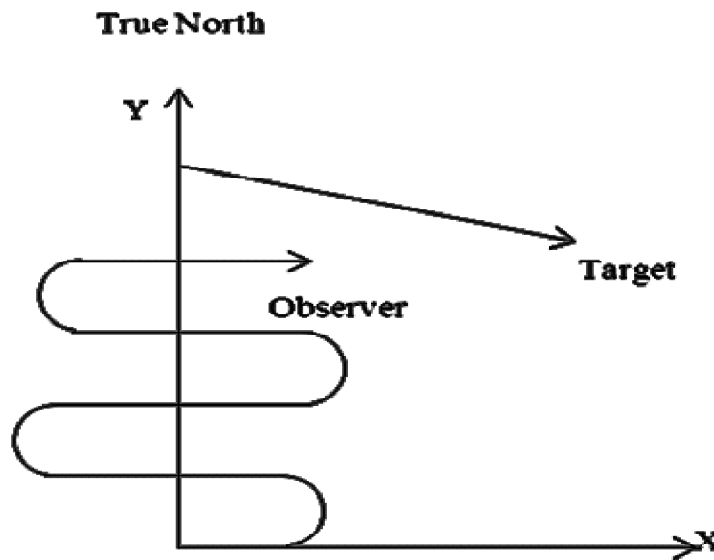


Figure 2: Target Observer Encounter

2.2. UAKF Algorithm

UKF is a combination of a classical filter and an unscented transformation, which is made in order to transmit transformation in the model through a non-linear process. UKF, gives adequately precise solution.

An easy method is adapted to evaluate the statistical properties of a random variable, which endures a non-linear transformation is called unscented transformation. Suppose a random variable x , having an expected value \hat{x} , covariance P_x and dimension L , imparting through $y=g(x)$. $2L+1$ sigma vectors are used to compute statistics of y . UAKF implementation is as follows [3-9].

Table 1
Unscented angles-only Kalman filter algorithm

1. Sigma state vectors are presented as

$$X(k) = \left[X_s(k) \quad X_s(k) + \sqrt{(n+\lambda)p(k)} \quad X_s(k) - \sqrt{(n+\lambda)p(k)} \right] \tag{11}$$

2. The same are modified using eqn. (2),

3. The state vector is predicted as

$$X_s(k+1|k) = \sum_{i=0}^{2n} W_i^{(m)} X_s(i, k+1|k) \tag{12}$$

4. The covariance matrix is predicted as

$$P(k+1|k) = \sum_{i=0}^{2n} W_i^{(c)} \left[X_s(i, k+1|k) - X_s(k+1|k) \right] \left[X_s(i, k+1|k) - X_s(k+1|k) \right]^T + S(k) \tag{13}$$

5. The state vectors are updated as

$$X(k+1|k) = \left[X_s(k+1|k) \quad X_s(k+1|k) + \sqrt{(n+\lambda)p(k+1|k)} \quad X_s(k+1|k) - \sqrt{(n+\lambda)p(k+1|k)} \right] \tag{14}$$

6. Then measurement predicted as

$$y(k+1|k) = \sum_{i=0}^{2n} W_i^{(m)} Y(k+1|k) \tag{15}$$

7. Covariance of innovation is

$$P_{yy} = \sum_{i=0}^{2n} W_i^{(c)} \left[Y(i, k+1|k) - y(k+1|k) \right] \left[Y(i, k+1|k) - y(k+1|k) \right]^T + R(k) \tag{16}$$

8. The cross covariance is

$$P_{xy} = \sum_{i=0}^{2n} W_i^{(c)} \left[X(i, k+1|k) - X(k+1|k) \right] \left[Y(i, k+1|k) - y(k+1|k) \right]^T \tag{17}$$

9. Kalman gain is

$$G(k+1) = P_{xy} P_{yy}^{-1} \tag{18}$$

10. The state is estimated as

$$X(k+1|k+1) = X(k+1|k) + G(k+1)(y(k+1|k+1) - y(k+1|k)) \tag{19}$$

11. And its covariance is

$$P(k+1|k+1) = P(k+1|k) - G(k+1)P_{yy}G(k+1)^T \tag{20}$$

Algorithm flow is shown in Figure 3 [3-9].

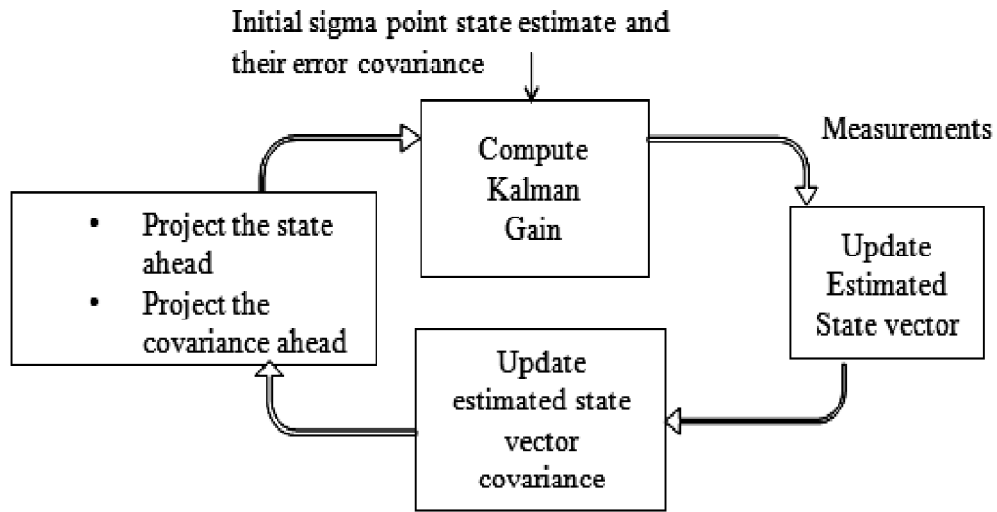


Figure 3: UAKF process

3. GENERALISED SIMULATOR

Let initial position of the target be (x_t, y_t, z_t) and the target moves with velocity v_t . After time t seconds, observer position changes and the change in the observer position is given by

$$dx_0 = v_0 * \sin(ocr) * \sin(oph) * t \quad (21)$$

$$dy_0 = v_0 * \cos(ocr) * \sin(oph) * t \quad (22)$$

$$dz_0 = v_0 * \cos(oph) * t \quad (23)$$

where ocr and oph are observer course and pitch respectively. Now the new observer position becomes

$$x_0 = x_0 + dx_0 \quad (24)$$

$$y_0 = y_0 + dy_0 \quad (25)$$

$$z_0 = z_0 + dz_0 \quad (26)$$

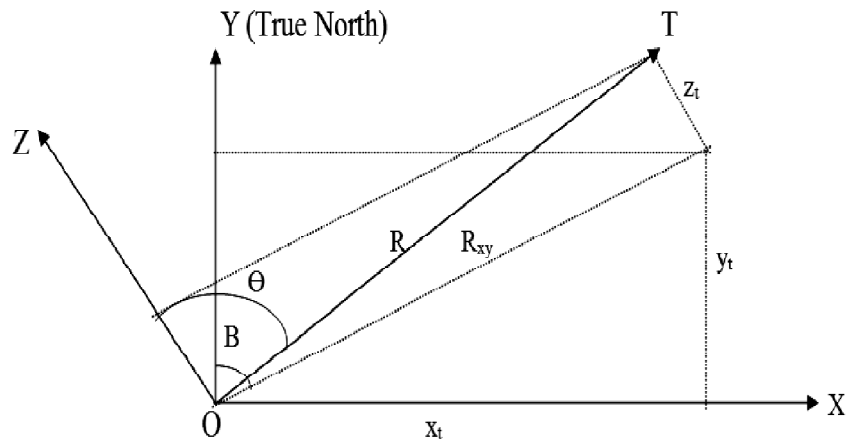


Figure 4: Target and observer positions

From Figure 4

$$x_t = R_{xy} * \sin(B) \quad (27)$$

$$y_t = R_{xy} * \cos(B) \quad (28)$$

$$\sin(\theta) = R_{xy}/R \quad (29)$$

Substituting equations (28) in (26) and (27)

$$x_t = R * \sin(\theta) * \sin(B) \quad (30)$$

$$y_t = R * \sin(\theta) * \cos(B) \quad (31)$$

$$z_t = R * \cos(\theta) \quad (32)$$

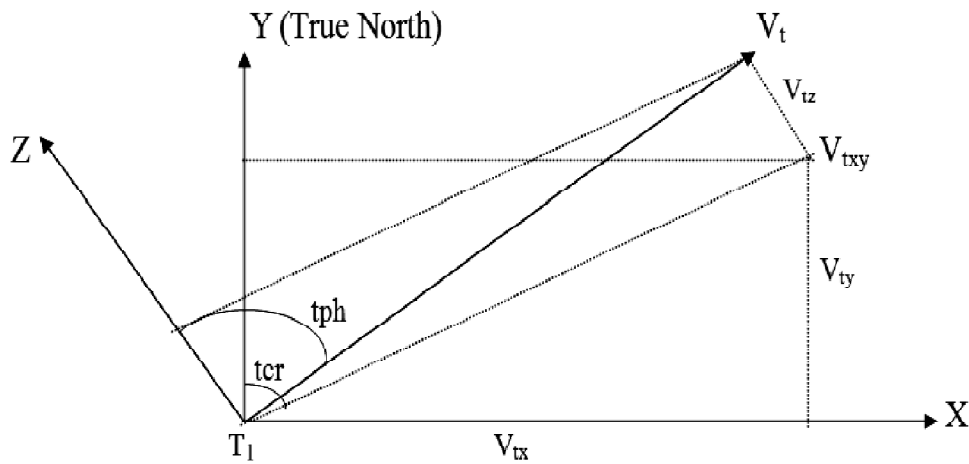


Figure 5: Target and observer velocities

When the target is in motion with velocity v_t , change in target position after t seconds, from Figure 5.

$$dx_t = v_t * \sin(tcr) * \sin(tph) * t \quad (33)$$

$$dy_t = v_t * \cos(tcr) * \sin(tph) * t \quad (34)$$

$$dz_t = v_t * \cos(tph) * t \quad (35)$$

where tcr and tph are target course and pitch respectively.

Now the new target position is

$$x_t = x_t + dx_t \quad (36)$$

$$y_t = y_t + dy_t \quad (37)$$

$$z_t = z_t + dz_t \quad (38)$$

$$true\ bearing = \tan^{-1} \left(\frac{x_t - x_0}{y_t - y_0} \right) \quad (39)$$

$$true\ range = \sqrt{(x_t - x_0)^2 + (y_t - y_0)^2 + (z_t - z_0)^2} \quad (40)$$

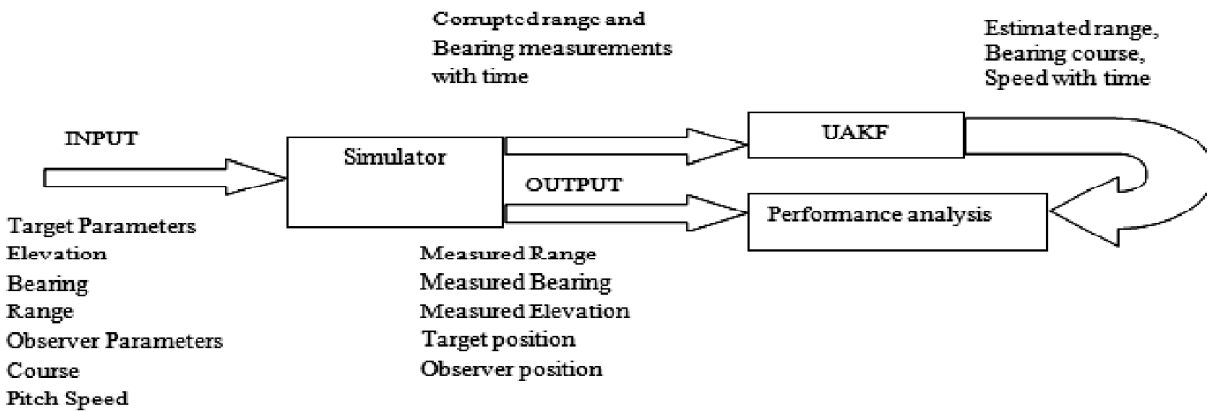


Figure 6: Block diagram of TMA in simulation mode Target true bearing, range and elevation are

$$true\ elevation = \tan^{-1} \left(\frac{R_{xy}}{z_t - z_0} \right) \quad (41)$$

Since the measurements are affected by noise in real situations, noise is added to these measurements.

Measured bearing = true bearing + sigma b

Measured range = true range + sigma r

Measured elevation = true elevation + sigma e

where sigma b, sigma r and sigma e are 16 values of white Gaussian process. The details are shown in Figure 6.

4. SIMULATION AND RESULTS

It is assumed that experiment is conducted at favorable environmental conditions and hence the angle measurements are available continuously. Simulation is realized on a personal computer using Matlab. The scenarios chosen for evaluation of algorithm are shown in Table 2. For example, scenario1 describes a target moving with bearing of 45° with course and speeds of 255° and 10m/s respectively. The elevation angle is 45°. The bearing and elevation measurements are corrupted with 0.5° and 0.33° respectively.

The ownship carries out S-maneuver for observability of the process^{3,4}, as shown in Figure 2. Ownship moves initially at 90°, perpendicular to line of sight for a period of 120 seconds and then turns to 270° with a turning rate of 0.5deg/s towards target. Then it moves in straight line for a period of 240 seconds. Afterwards again it turns to 90° towards target for a period of 240 seconds and so on.

In simulation, estimated and actual values are available and hence the validity of the solution based on certain acceptance criterion is possible. The following acceptance criterion is chosen. The solution is converged when error in estimated course, speed and range are ≤ 3°, ≤ 5m/s and ≤ 8% respectively. The solution is converged when the course, speed and range are converged.

For scenario 1, it is observed that the estimated course, speed and range of the target are converged at 152nd, 28th and 299th sample respectively for scenario1. So, for scenario 1, the total solution is obtained at 299th sample. Similarly for other scenarios the convergence time is shown in Table 3.

True and estimated paths of target and ownship path are shown in Figure 7. The errors in estimated range, speed and course for scenario 1 are presented in Figure 8-9 respectively.

Table 2
Input parameters chosen for the algorithm

Scenario	Initial range (m)	Bearing (deg)	Elevation (deg)	Pitch (deg)	Course (deg)	Speed (m/s)
1	3000	45	45	10	225	10
2	3000	45	135	45	30	20
3	3000	60	60	70	135	18

Table 3
Convergence time in samples for the chosen scenarios

Scenario	Course	Elevation	Range	Speed	Total solution
1	152	2	299	28	299
2	96	2	248	28	248
3	354	2	361	13	361

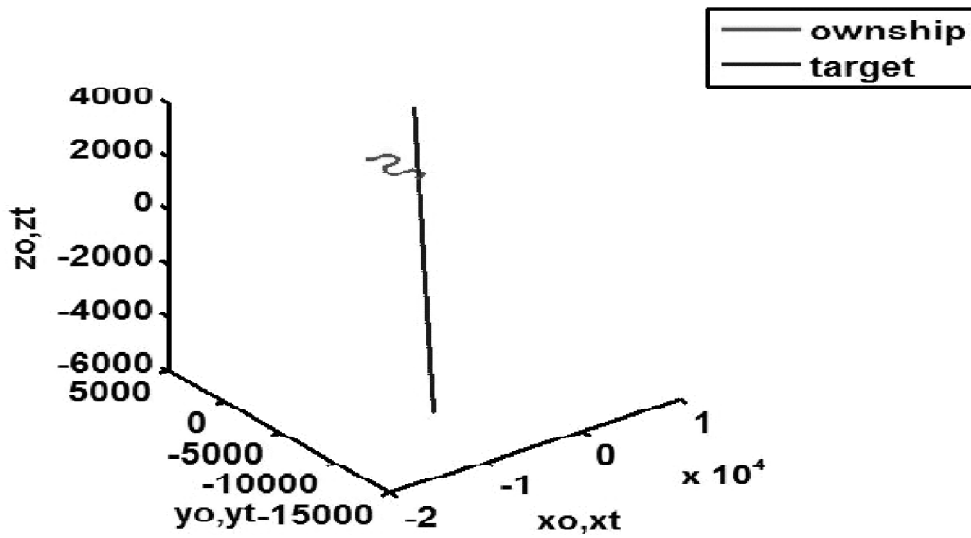


Figure 7: Ownship and target paths of scenario 1

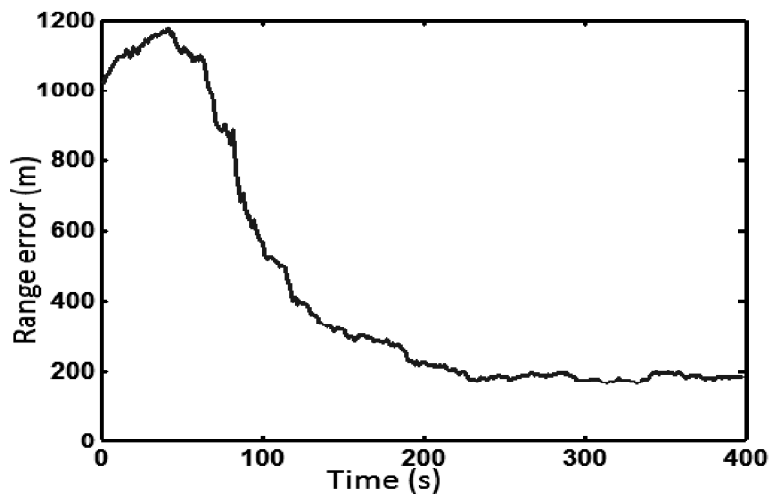


Figure 8: Error in estimated range

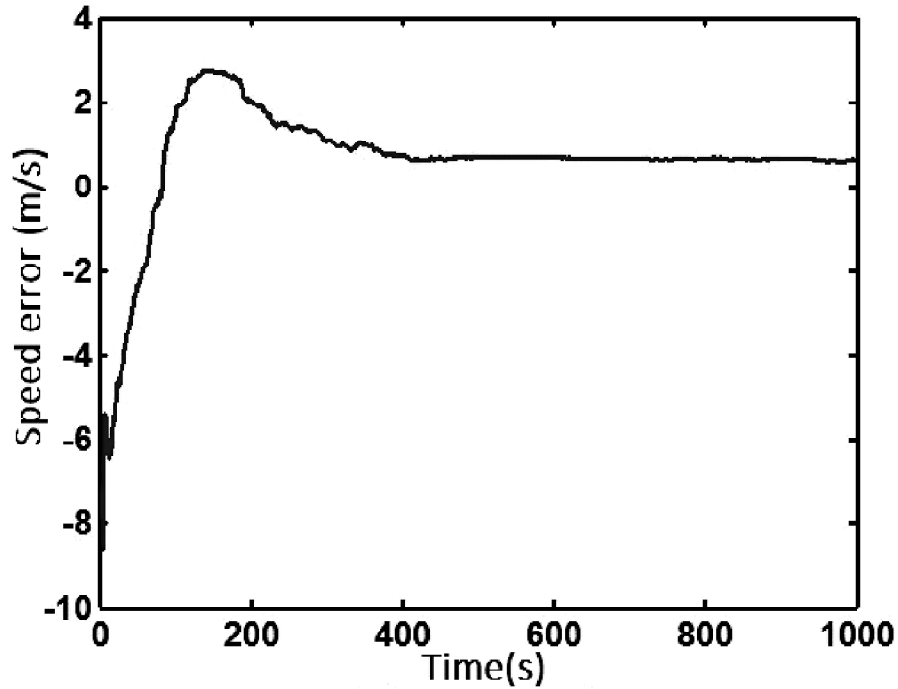


Figure 9: Error in estimated speed

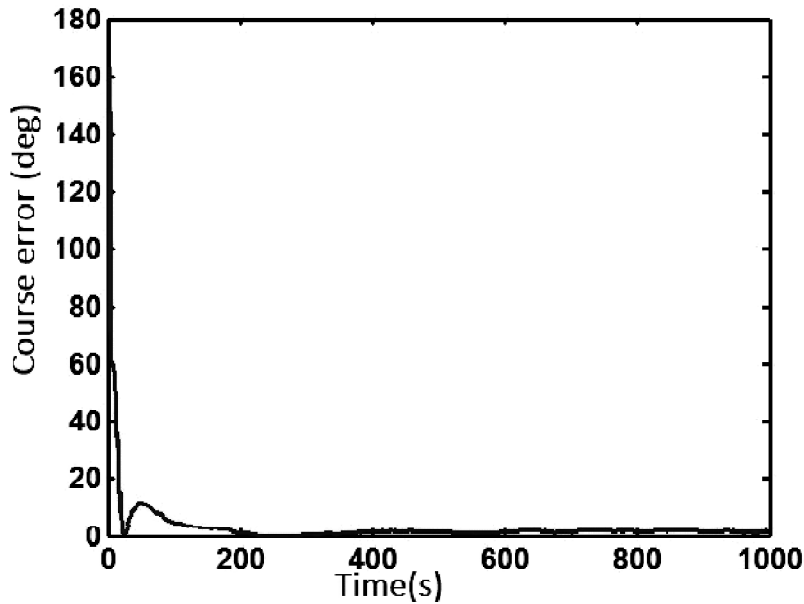


Figure 10: Error in estimated course

5. CONCLUSION

Ownship carries out S-maneuver for the observability of the process. Based on these results, UAKF is recommended for passive target tracking and in particular, submarine to submarine scenario, when elevation measurements are also available along with bearing measurements.

REFERENCES

- [1] V.J. Aidala, S.E.Hammel, "Utilization of modified polar coordinates for bearings only tracking", IEEE trans. Automatic control, Vol AC-28, No.3, pp. 283-294, March 1983.
- [2] E.A.Wan and R.Van Der Merwe, "The unscented Kalman filter for non-linear estimation", in Proceedings of Symposiums 2000 on Adaptive System for Signal Processing, Communication and Control, Canada, Oct. 2000.
- [3] S. Koteswara Rao, K. Raja Rajeswari, K. S. Lingamurthy, "Unscented Kalman Filter with Application to Bearings-Only Target Tracking", IETE Journal of Research Vol. 55, No. 2, pp. 63-67, Mar-Apr 2009.
- [4] S. Koteswara Rao, V. Sunanda Babu, "Unscented Kalman Filter with Application to Bearings-Only Passive Maneuvering Target Tracking", IEEE-International Conference on Signal processing, Communications and Networking, pp.219-224, Jan'2008.
- [5] MeiliangWuz, Andrew W. Smyth, "Application of the unscented Kalman filter for real-timenonlinear structural system identification", Structural Control and Health Monitoring, pp.971-990, Oct'2006.
- [6] Rudolph van der Merwe and Eric A. Wan, "The Square-Root Unscented Kalman Filter for State and Parameter-Estimation", IEEE Trans, pp. 3461-3464, 2001.
- [7] BrankoRistic, Sanjeev Arulampalam and Neil Gordon, "Beyond the Kalman Filter: Particle Filters for Tracking Applications", Artech House, 2004.
- [8] Dan Simon, "Optimal State Estimation: Kalman, H"and Nonlinear Approaches", John Wiley & Sons, Inc. 2006.
- [9] B. Omkar Lakshmi Jagan, S. KoteswaraRao, A. Jawahar and SK. B.Karishma,"Unscented Kalman Filter with Application to Bearings-only Passive Target Tracking", Indian Journal of Science and Technology, Vol 9(19), pp. 1-10, May 2016.