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Optimal Trajectory Generation for the Ascent Phase of Launch Vehicle Using θ-PSO Approach

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Abstract : This paper proposes the application of a recently emerged variant of particle swarm optimization namely theta Particle Swarm Optimization (TH-PSO), in launch vehicle trajectory optimization problem. Constraint handling and accuracy of launch vehicle systems is a challenge for optimization researchers due to highly nonlinear nature .In this paper TH–PSO approach is implemented on the single stage of a multistage liquid propellant rocket, taking angle of attack as the control parameter. The objective is to reduce the terminal error within a constrained boundary by formulating it as a fixed time optimal control problem. It is shown that TH-PSO approach minimizes the shortcomings of other such type of optimization procedures in trajectory optimization scenario by providing better convergence and accuracy. Also to validate the effectiveness of the proposed approach it is compared with Adaptive Particle Swarm Optimization (APSO) in MATLAB simulation environment.

Keywords : Trajectory optimization, Theta-PSO, PSO.

1. INTRODUCTION

Due to the high precision and speed of hypersonic vehicles more and more defense and military agencies are stated doing research in the trajectory optimization arena. This leads to the conclusion that hypersonic vehicles are stepping up to the next level, long range precision strike systems. In order to reduce the operating cost and structural load, good ascent phase guidance is required. But the main focus of the launch vehicle trajectory optimization is to enhance the performance in addition to attain the specific constrained boundaries.

In literature, aerospace trajectory optimization problems have been pursued with various approaches. ‘Discrete approximation’ and ‘parameterization’ are the generally preferred approximation methods to solve such type of problems. One of the standard procedures is the Gradient or Steepest-decent techniques. Implementation of such methods to a hypersonic launch vehicle carrying an arbitrary payload is discussed in [1, 2]. Recently bio inspired techniques such as genetic algorithm (GA), ant colony etc. came into picture. They prove to be more or less complicated and easy to handle as compared to previously mentioned methods.

A new algorithm based on the behavior of social systems such as fish schooling, bird flocking etc is proposed by Kennedy and Eberhart in1995. Since it is based on social behavior the results obtained are more efficient than other stochastic techniques due to the fact that it is taking the best of all the individuals in the group. Kennedy et.al [3] and Trelea[4] proved that the performance of PSO algorithm is parameter dependent and suffers from local optima locking. Many variants of PSO have been developed over the years and the most recent among them is TH-PSO by Zhong et.al [5].Instead of taking velocity as a major parameter, as in standard PSO, TH-PSO considers phase angle and positions are adjusted by phase angle

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mappings rather than velocity. It proves to be more effective in handling non- linear systems as compared to its standard counterpart. This is the motivation behind the application of TH-PSO in a highly nonlinear system like launch vehicle which is explored in this paper.

The rest of the paper is organized as follows. In Section 2, an overview of APSO and TH-PSO is described. Section 3 consolidates the problem formulation and system description whereas simulation results and numerical validation is given in section 4. Conclusions obtained are given in section 5 along with some views and future scope.

2. REVIEW OF TH-PSO METHOD

2.1. Particle swarm optimization algorithm

A new algorithm based on the behavior of social systems such as fish schooling, bird flocking etc is proposed by Kennedy and Eberhart in1995. Since it is based on social behavior the results obtained are more efficient than other stochastic techniques due to the fact that it is taking the best of all the individuals in the group. It has emerged as one of the most popular evolutionary method because of its speed and accuracy.

Consider i is the total number of particles and D is the degree of freedom or dimension of the particle. i^{th} particle in D^{th}- dimension is denoted as, $X_i = (x_{i1}, x_{i2}, ... x_{iD})^T$. The velocity of each particles are represented by, $V_i = (v_{i1}, v_{i2}, ... v_{iD})^T$. In each particle is having individual personal best positions and is denoted as, $P_i = (pi1, p_{i2}, ... , p_{iD})^T$. Also the entire swarm is having its best value in each iteration and it is known as global best position. It is indexed as g in the equation. The global version PSO, the swarm updates are carrying out based on the given equations and the superscripts denote the iteration number [6].

$$v_i^{t+1}(d) = \omega v_i^t(d) + c_1 R(p_i^t(d) - x_i^t(d) + c_2 R(p_g^t(d) - x_i^t(d)) \quad (1)$$

$$x_i^{t+1}(d) = x_i^t(d) + v_i^{t+1}(d) \quad (2)$$

Where $v_i^t(d)$ and $x_i^t(d)$ represents the velocity and current position of the particle. R denotes an uniformly distributed random variable, U $[0, \in 1]$. The acceleration constants are given by c_1 and c_2 which controls the exploration and exploitation rate. ω represents inertia weight.

In classical PSO approach the inertia weight parameter is a fixed value in all the iterations done which results in local minima locking. Also there is a chance that the particle goes out of boundary. To alleviate these shortcomings Eberhart *et.al* [7] proposed a technique in which the inertia weight parameter is initially given a higher value and gets reduces linearly with each iteration. This forms the new variant of PSO called APSO, which is used in this paper to verify the TH-PSO algorithm in trajectory optimization problem. In APSO the adaptive inertial weight is updated based on the following equation.

$$\omega^{t+1} = \omega_{max} - \left(\frac{\omega_{max} - \omega_{min}}{\max iter}\right) iteration \quad (3)$$

2.2. Theta-Particle Swarm Optimization Algorithm

Instead of taking velocity as a major parameter, as in standard PSO, TH-PSO considers phase angle and positions are adjusted by phase angle mappings rather than velocity [5,8]. Mathematically θ-PSO can be described using the following equations:

$$\Delta\theta_i^{t+1}(d) = \omega\Delta\theta_i^t(d) + c_1 R_1(\theta_{pi}^t(d) - \theta_i^t(d) + c_2 R_2(\theta_{gi}^t(d) - \theta_i^t(d)) \quad (4)$$

$$\theta_i^{t+1}(d) = \theta_i^t(d) + \Delta\theta_i^{t+1}(d) \quad (5)$$

$$x_i^t = f(\theta_i^t) \quad (6)$$

Where θ_i^t and $\Delta\theta_i^t$ are the phase angle and increment in phase angle respectively. x_i^t is the particle position. $\Delta\theta_i^t \in (\theta_{min}, \theta_{max})$ $\theta_i^t \in (\theta_{min}, \theta_{max})$, $i = 1, 2, \ldots, s$ and $j = 1, 2, \ldots, n$. Where, c_1, c_2, ω, R_1, R_2 and x_i^t are the same as in standard PSO already explained in PSO algorithm. n represents the dimension of the problem; $\theta_i^t(d)$ the phase angle of particle i at time t; $\Delta\theta_i^t(d)$ the increment of particle i's phase angle at time t; $\theta_{pi}^t(d)$ the phase angle of the personal best solution of particle i at time t ; $\theta_{gi}^t(d)$ the phase angle of the global best solution at time t, $f(\theta_i^t)$ is the monotonic mapping function, which is expressed as,

$$f(\theta_i^t) = \frac{x_{max} - x_{min}}{2}\sin\theta + \frac{x_{max} - x_{min}}{2} \tag{7}$$

The working principle of the theta-PSO is similar to PSO and is explained below :

- Initialize the phase angles and increment in phase angle with in the limit $\left[-\pi/2, \pi/2\right]$
- Calculate the particle position based on the mapping function, equation (7)
- Calculate the fitness for each particle in the population.
- Update previous best phase angle of the individual particle from each iteration and it is denoted as θ^t_{pi}
- Update the record of the previous best position of the population and it is denoted as θ^t_{gi}
- The phase angle and it's variations for each particle is updated according to equation (4) and (5) till optimum value is reached.

3. SYSTEM DISCRIPTION AND PROBLEM FORMULATION

3.1. System description

An advanced launch vehicle system model (ALS), developed by NASA is considered in this paper. The same model has been used to study the behavior of optimal and suboptimal control and guidance strategies [9]. The system consists of the core vehicle with liquid rocket boosters having three engines, which produces identical thrust. The point-mass equations of motion governing the vehicle are as below:

$$\dot{r} = V\sin\gamma \tag{8}$$

$$\dot{V} = \frac{1}{mV}(T\cos\alpha - D - mg\sin\gamma) \tag{9}$$

$$\dot{\gamma} = \frac{1}{mV}(T\cos\alpha + L) + \left(\frac{V}{r} - \frac{g}{V}\right)\cos\gamma \tag{10}$$

$$\dot{x} = V\cos\gamma \tag{11}$$

Where, r represents the altitude from earth's center while V and γ represents velocity and the flight path angle of the launch vehicle, which is considered as state variables. The other notations like T, m, α respectively represents thrust, instantaneous mass and angle of attack. The lift, drag and gravitational pull by earth is denoted by L, D and g respectively. The term g varies with altitude and this effect is high at altitudes below 50 Km above sea level. So g can be represented as a function of altitude.

$$g = g_0\left(\frac{r_e}{r}\right)^2 \tag{12}$$

The terms L and D explained earlier and defined by,

$$\left.\begin{aligned} D &= 1/2\,\rho S v^2 c_D \\ L &= 1/2\,\rho S v^2 c_L \end{aligned}\right\} \tag{13}$$

Where, ρ and S represents the aerodynamic density and launch vehicle's aerodynamic reference area respectively. Where r_e the radius of earth.

3.2. Problem formulation

The problem considered in this paper is to find an optimal trajectory for an initial stage of a multistage liquid propellant launch vehicle. In other words the aim is to estimate the optimum angle of attack α, transferring the system from the given initial to final conditions with minimum terminal error. The thrust (T) is assumed to be constant and the problem is approached as a fixed time problem. In addition to this there are also some boundary conditions to be obeyed in state variables. Here *r*, *v* and γ are the state variables and angle of attack, α is taken as the control variable.

The objective function is formulated such a way that all the objectives should be achieved. The cost function which is a function of final state x_{t_f} alone is given below.

$$J = \sum S_x \left\| x_f - x_{t_f} \right\|_2^2 + \int_{t_0}^{t_f} \alpha dt \tag{14}$$

Where J is called the cost function; x_f is the terminal values for the state variables and x_{t_f} is the values which the state variables achieve at final time. S_x, is the weighting factor for each state variable, which is selected based on the priority of variables.

The boundary conditions and the constant thrust rates are considered as the constraints. The boundary conditions for the velocity and flightpath angle should satisfy during the entire flight time. The constraints are mathematically expressed as,

$$V_{min} \leq V \leq V_{max} \tag{15}$$

$$r_{min} \leq r \leq r_{max} \tag{16}$$

Where, V_{min} and V_{max} are the velocity boundaries, r_{min} and r_{max} are the altitude boundaries m_c fuel consumption rate.

4. RESULTS AND DISCUSSIONS

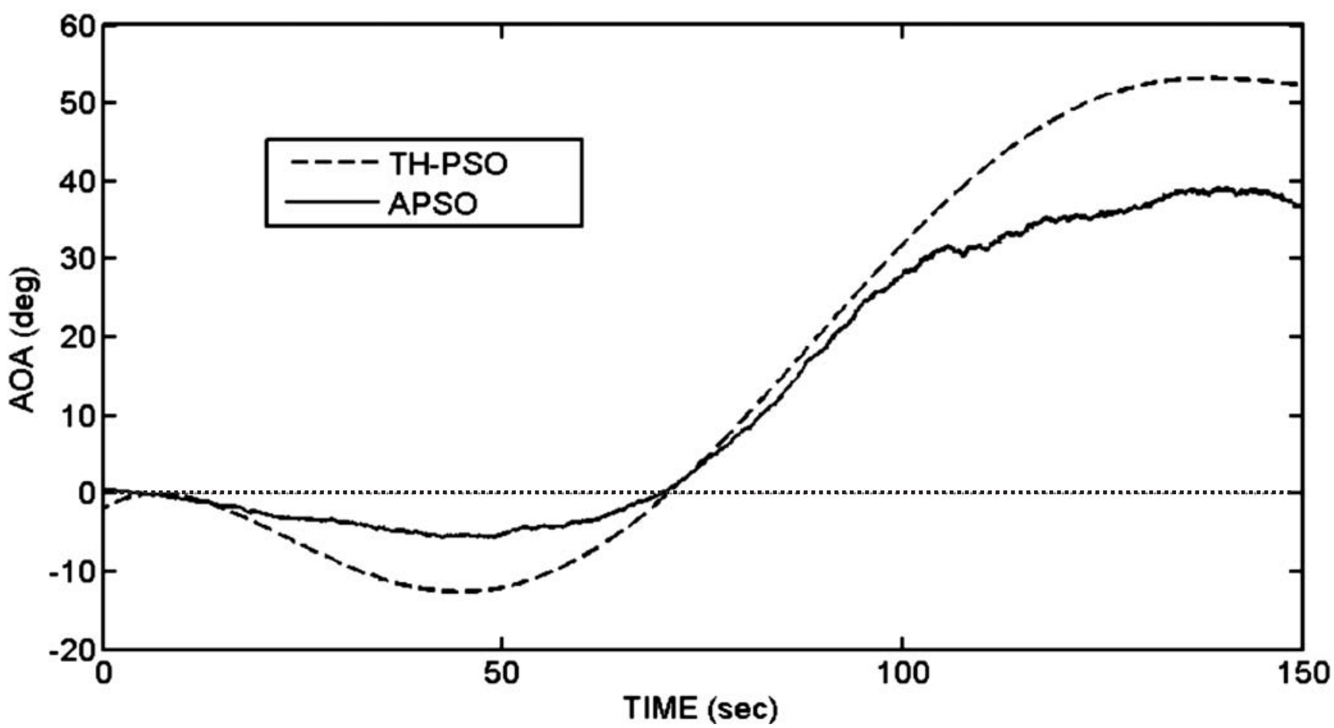

Figure 1: Angle of attack profile

The problem of optimizing launch vehicle trajectory is solved by minimizing the terminal error under certain path constraints and boundary conditions as explained earlier. Constant engine burning time is considered so as to convert the given problem to a fixed time optimal control approach. The angle of attack is considered as control parameter and it is forced to a maximum allowable value so as not to increase the structural load above the maximum allowable limit during the implementation. An advanced variety of PSO called TH-PSO which proved to be very effective in non-linear scenario, is used to optimize the given system. For validation of the proposed approach, TH-PSO is compared with APSO.

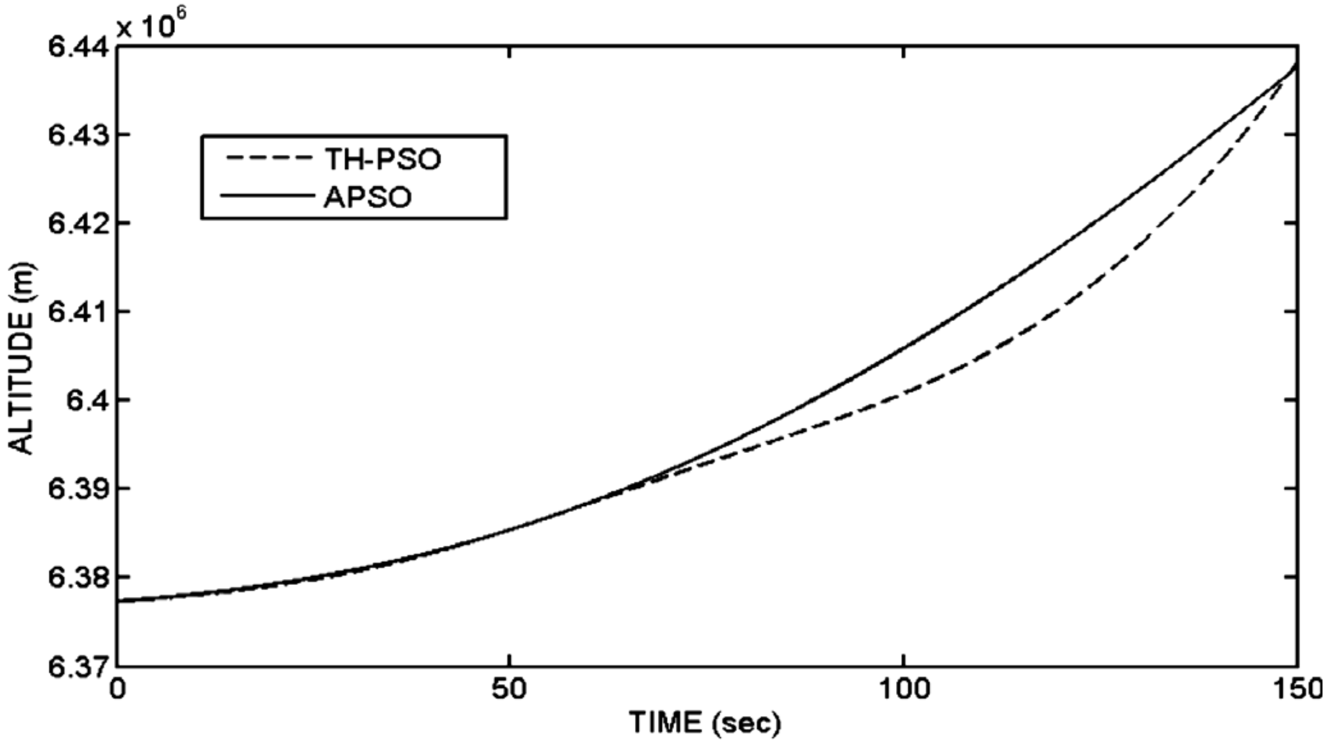

Figure 2: Altitude variation from earth center

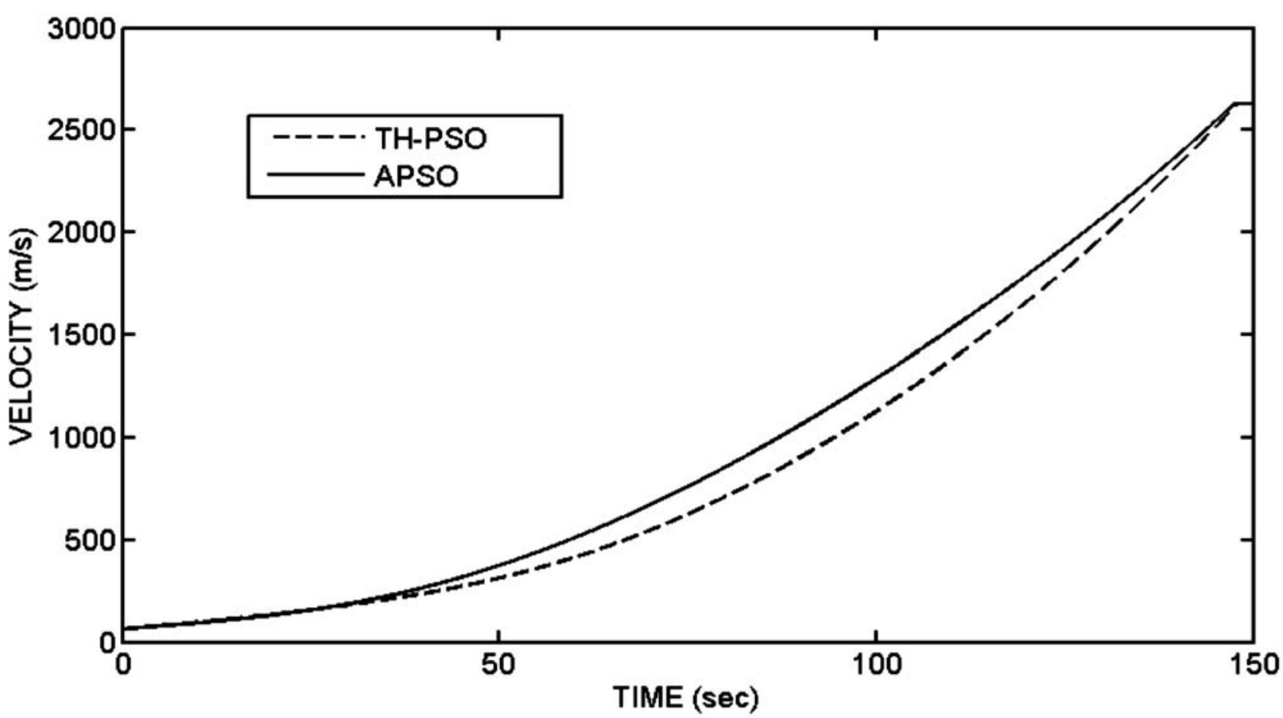

Figure 3: Velocity variation of the vehicle

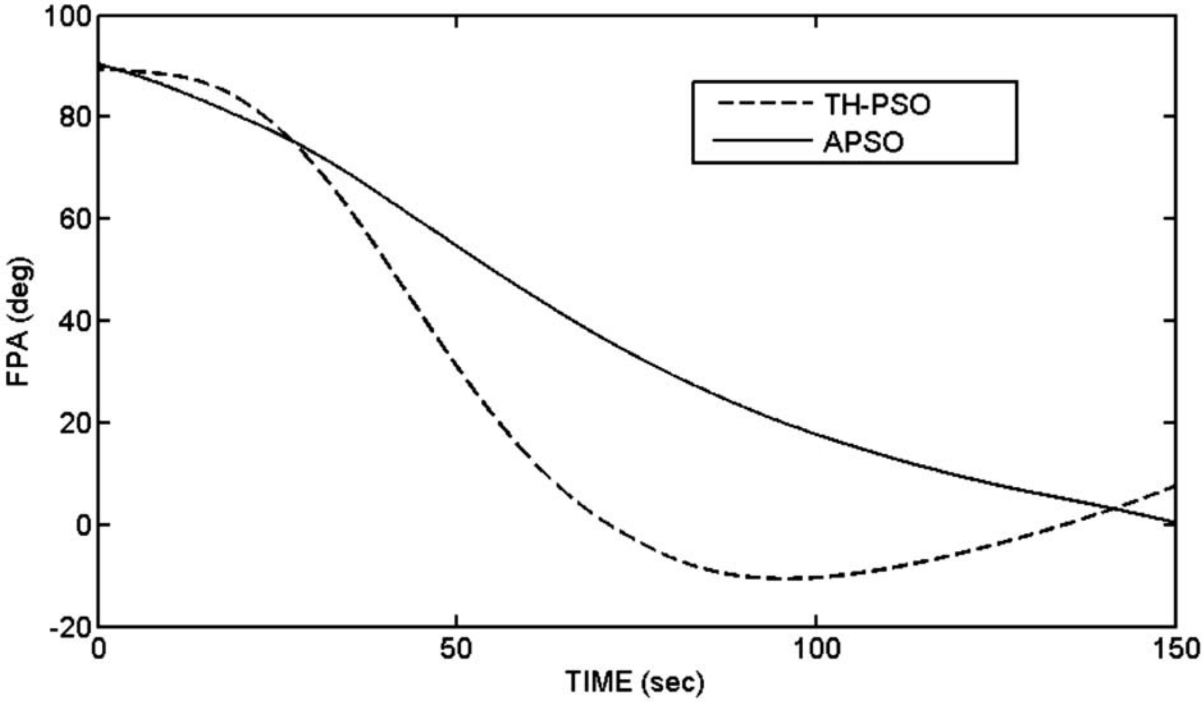

Figure 4: Flightpath angle variation of the vehicle

Figure 1-4 shows the variation of control and state variables with time. From the graphs it is clear that state variables are achieving their terminal conditions with minimum error. From figure -1 it is clear that the smoothness in the control variable is very high in TH-PSO as compared to APSO which results in the reduction of the control effort. This will result in the smooth movement of actuators which controls the tilting of thrusters of the launch vehicle. But the drawback is that TH-PSO algorithm the control boundary is high with respect to the other that may cause physical constraint violation in practical cases.

Table 1

Terminal Conditions and Simulations Results

Methods	*APSO*				*THETA-PSO*			
Parameters	r(km)	v(m/s)	γ(deg)	J	r(km)	v(m/s)	γ(deg)	J
x_0	6377.353	65	89.5	--	6377.353	65	89.5	--
x_f	6438.553	2630	0	--	6438.553	2630	0	--
Δx_f	± 5	± 5	± 100	--	± 5	± 5	±10	--
Δx_{if}	0.00845	–4.4704	2.14	5.664 *e*-4	–0.00543	–0.3240	6.48	1.983 *e*-4
x_{t_f}(best)	6438.561	2625.53	–2.14	5.664 *e*-4	6438.548	2629.68	–6.48	1.983 *e*-4
x_{t_f}(worst)	6439.437	2621.73	–8.29	6.967 *e*-3	6438.003	2628.88	–9.39	6.735 *e*-4

The parameters of both PSO variants are selected from reference [10] and 50 iterations are performed. Table. 1 shows the best and worst solutions obtained from the iterations of both TH-PSO and APSO with standard tolerance values. First two rows of the table represents the initial and terminal conditions of the launch vehicle. From the numerical results it can be analyzed that TH-PSO is having more accurate solution than APSO.

The cost function is chosen in such a way so as to minimize the terminal error as, $J = S_r(r_f - r_{t_f})^2 + S_v (v_f - v_{t_f})^2 S_\gamma (\gamma_f - \gamma_{t_f})^2$ and the parameters S_r, S_v and S_γ are the weighting factors used to indicate the priority given to each state variables.

5. CONCLUTION

The problem considered in this paper is to find an optimal trajectory for an initial stage of a multistage liquid propellant launch vehicle. It is solved by minimizing the terminal error under certain path constraints and boundary conditions by considering angle of attack as the control variable. A new variant of PSO, TH-PSO is implemented to solve the problem, due to its effectiveness in solving nonlinear scenario. For validating the effectiveness of the proposed method, optimization problem is also solved using a variant of PSO namely APSO. It is shown that the TH-PSO has better efficiency, higher convergence rate and less accuracy compared to general PSO and APSO techniques. Time taken for the simulation is almost same for all algorithms.

6. ACKNOWLEDGEMENT

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