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Implementation for Synchronization of Chen and Liu-Yang Chaotic Systems using SMC and Active Control Schemes

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Abstract : In this paper, new results are devised based on sliding mode control (SMC) using Proportional-Integral switching surface and Active control schemes for synchronization of two different chaotic systems Chen and Liu-Yang chaotic system. Lyapunov stability theory is used to achieve stability of designed switching surface for proposed synchronization scheme. SMC helps in the convergence of error dynamics due to insensitive to parameter uncertainties and disturbances. MATLAB simulation is presented to demonstrate the effectiveness of proposed synchronization strategy. Further, Master-slave systems and synchronization scheme are simulates in MULTISIM to bring the proposed scheme in real world, which reflects novelty of this paper.

1. INTRODUCTION

Chaos theory is used in various ways such as time series prediction [1], system optimization [2], signal processing and communication [3], and many others [4,28]. Motivated by potential applications, chaotic systems are also being used in synchronization because they are sensitive to initial conditions. During synchronization the states of two chaotic systems share common time [4]. Synchronization phenomenon is also being implemented using analog electronics components [5] using different software or hardware.

Pioneer work was developed by Pecora and Carroll in 1990 [6] in the field of chaotic system synchronization. After this many researchers have devised various control schemes for synchronization such as PC method [6], OGY method [7], observer based synchronization [8], adaptive control method [9], backstepping control method [10], time delay feedback method [11], and sliding mode control method [12], etc.. So many techniques are also available related to synchronization like, lag synchronization [13], phase synchronization [14], projective synchronization [15], complete, anti-synchronization and hybrid synchronization [16, 17, 29].

Sliding mode control (SMC) scheme has been applied for control and synchronization, anti-synchronization of identical chaotic or hyperchaotic system as in [18, 19]. Since, SMC is a robust control technique and has many advantages [20].

Circuit implementation of different chaotic system is absorbed in literature such as Lorenz system [21], Chen system [22], Lu system [23], Unified system [24], Rossler chaotic system [25] etc.

In this paper SMC and Active control techniques are for global synchronization of two nonidentical chaotic systems Chen chaotic system [26] and Liu-yang chaotic system [27]. Synchronization of non-identical chaotic system using SMC is the main contribution of this paper. Comparison among performances of controller is shown. Proposed synchronization scheme is simulated using NI MULTISIM circuit

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simulator which is also adding novelty. For global stabilization of error dynamics Lyapunov stability theory [20] is used. Proportional integral (PI) switching surface is designed in terms of error dynamics for sliding mode control, and control law is designed using the relevant variable of master and slave system. Results are verified using MATLAB and MULTISIM environment.

Rest paper discussed like this. Description of Chen and Liu-Yang chaotic system is discussed in Section 2. Section 3 discusses synchronization of Chen and Liu-Yang chaotic systems. In Section 4, design of switching surface and controller is presented. In Section 5, analog circuit implementation of synchronization scheme and including dynamical systems is shown. Simulation results are given for the validation and verification of proposed scheme in Section 6. Finally, in Section 7, summary of paper is derived as conclusions.

2. DESCRIPTION OF CHEN AND LIU-YANG CHAOTIC SYSTEMS

Master system is Chen system [26] which is given in (1):

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= (b-a)x_1 + bx_2 - x_1x_3 \\ \dot{x}_3 &= -cx_3 + x_1x_2 \end{aligned} \tag{1}$$

where $x1$, x_2, x_3 are states and $a = 35$, $b = 28$, $c = 3$ are parameters of system (1) for chaotic behavior. The phase plane behavior of (1) is shown in Fig. 1 (*a*).

The 3D Liu-Yang chaotic system [27] is described as:

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) \\ \dot{y}_2 &= dy_1 - y_1y_3 + u_1 \\ \dot{y}_3 &= -\beta y_3 + y_1y_2 + u_2 \end{aligned} \tag{2}$$

where $y1$, $y2$, y_3 are the states of system (2) and shows a chaotic behavior for parameter values $d = 35$, $\alpha = 35$, $\beta = 3$ and u_1, u_2 are the control input added in (13) for synchronization. The phase plane behavior of (2) is shown in Fig. 1 (*b*).

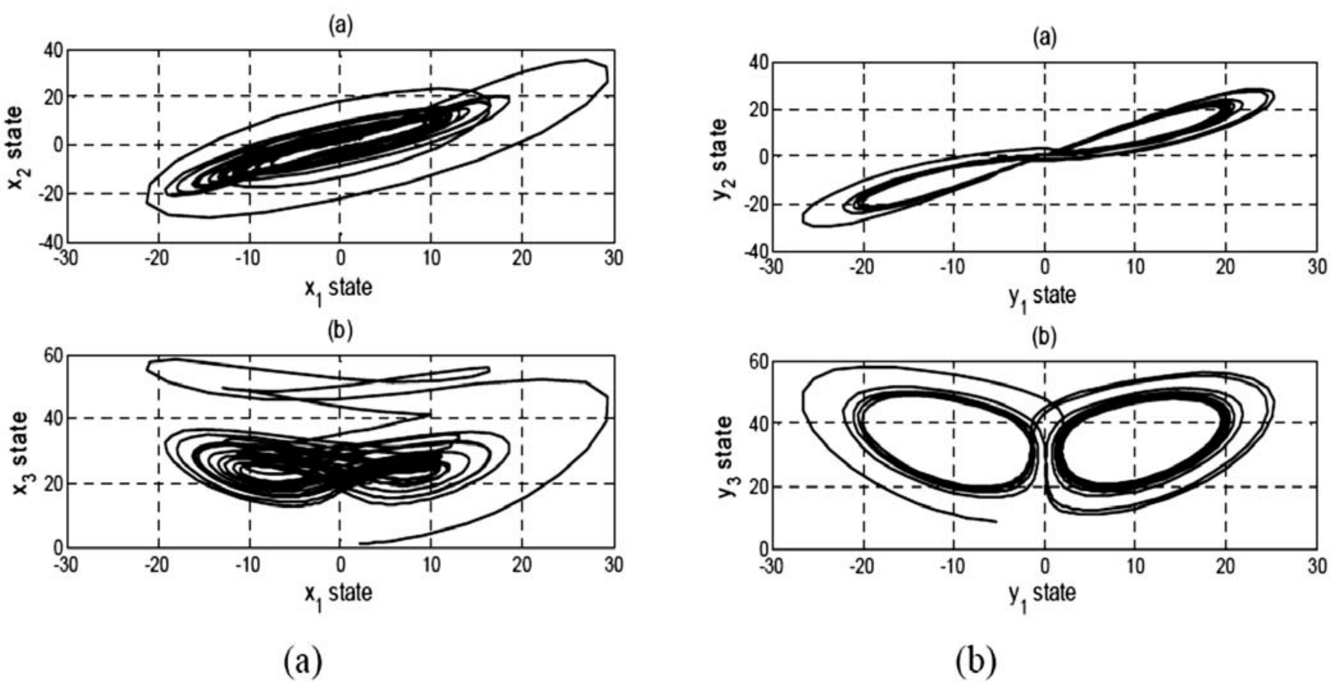


Figure 1: Chaotic attractors of: (a) Chen and (b) Liu-Yang chaotic system

3. SYNCHRONIZATION OF CHEN AND LIU-YANG CHAOTIC SYSTEMS USING SMC

This section discusses the synchronization of two different chaotic systems described in (1) and (2) based on sliding mode control. The master dynamics is equation (1), slave system is (2). Using equation (1) and (2), error dynamics defined as:

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) \text{ since; } a = \alpha \\ \dot{e}_2 &= be_2 - by_2 + dy_1 - (b-a)\,x_1 - y_1y_3 + x_1x_3 + u_1 \\ \dot{e}_3 &= -\beta x_3 - \beta e_3 + cx_3 + y_1y_2 - x_1x_2 + u_2 \end{aligned} \tag{3}$$

Here u_1, u_2 and u_3 are the added control inputs.

3.1. Design of SMC

Here synchronization chaotic systems (1) and (2) using SMC is discussed. It involves two steps: (i) design of switching surfaces and (*ii*) control law. Switching surface as a function of error vector in proportional integral (PI) form is defined as:

$$s_1 = e_2 + \int_0^\tau \{ae_1 + k_2e_2\}d\tau \tag{4}$$

$$s_2 = e_3 + \int_0^\tau \{k_3e_3\}d\tau$$

where, k_1, $k_2 > 0$. The sliding mode exists when it satisfies [20]:

$$\begin{aligned} \dot{s}_1 &= 0 \\ \dot{s}_2 &= 0 \end{aligned} \tag{5}$$

From (4), (5) equivalent sliding mode dynamics is defined as:

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) \\ \dot{e}_2 &= -k_1e_2 - ae_1 \\ \dot{e}_3 &= -k_2e_3 \end{aligned} \tag{6}$$

The existence of sliding mode dynamics (4) is defined using Lyapunov theory [20] by considering the Lyapunov function as

$$\mathrm{V}(e) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \tag{7}$$

Taking derivative of (7) and using (6)

$$\dot{\mathrm{V}}(e) = ae_1^2 - k_1e_2^2 - k_3e_3^2 \leq 0 \tag{8}$$

According to Lyapunov stability theory, sliding motion on the sliding surface is stable and ensure the convergence of error dynamics (3).

Now next step is to design a control law to drive the system trajectories onto the sliding surface. $s_i = 0$. To ensure the occurrence of the sliding motion controller are proposed as follows:

$$\begin{aligned} u_1 &= -be_2 + by_2 - dy_1 + (b-a)\,x_1 + y_1y_3 - x_1x_3 - k_1e_2 - ae_1 - \psi(\text{sign}(s_1)) \\ u_2 &= \beta x_3 + \beta e_3 - cx_3 - \alpha y_2 - y_1y_2 + x_1x_2 - k_2e_3 - \psi(\text{sign}(s_2)) \end{aligned} \tag{9}$$

where, $\psi > 0$.

Theorem : Controller (9) ensures the system trajectory of (1) and (2) to converge to sliding surface $s(t) = 0$.

Proof : Consider another positive definite Lyapunov function as [20];

$$\mathrm{V}(s) = \frac{1}{2}(s_1^2 + s_2^2 + s_3^2)$$

Taking derivative of (10) and using (3), (4), we can write

$$\dot{V}(s) = s_1\dot{s}_1 + s_2\dot{s}_2 = s_1[\dot{e}_2 + ae_1 + k_2e_2] + s_2[\dot{e}_3 + k_3e_3] \quad (11)$$

$$\dot{V}(s) = s_1[be_2 - by_2 + dy_1 - (b-a)x_1 - y_1y_3 + x_1x_3 + u_1 + ae_1 + k_1e_2] + s_2[-\beta x_3 - \beta e_3 + cx_3 + \alpha y_2 + y_1y_2 - x_1x_2 + u_2 + k_2e_3]$$

By, using the control law of (9) and with $\psi > 0$ we can get,

$$\dot{V}(s) = -\psi|s1| - \psi|s_2| \leq 0 \quad (12)$$

Thus, using Lyapunov stability theory it ensure $s(t) = \dot{s}(t) = 0$. Therefore error dynamics on sliding surface is asymptotically stable according to (12), and error dynamics converges to zero.

4. SYNCHRONIZATION OF CHEN AND LIU-YANG CHAOTIC SYSTEMS USING ACTIVE CONTROL TECHNIQUE

Here, we will discuss the synchronization using active control scheme.

Master system is defined in equation (1) and slave system is defined in equation (13).

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= dy_1 - y_1y_3 + u_2 \\ \dot{y}_3 &= -\beta y_3 + y_1y_2 + u_3 \end{aligned} \quad (13)$$

u_1, u_2, u_3 are the control input added for synchronization.

Again the error dynamics are defined as:

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \text{ since; } a = \alpha \\ \dot{e}_2 &= be_2 - by_2 + dy_1 - (b-a)x_1 - y_1y_3 + x_1x_3 + u_2 \\ \dot{e}_3 &= -\beta e_3 + y_1y_2 - x_1x_2 + u_3 \end{aligned} \quad (14)$$

Now, one our aim is to design active control such that error vector (14) converge to zero.

4.1. Design of Active Control law and stabilization of error

We consider the active nonlinear controller for above error dynamics (14) are as follows:

$$\begin{aligned} u_1 &= ae_2 \\ u_2 &= -be_2 + by_2 - dy_1 + (b-a)x_1 + y_1y_3 - x_1x_3 - ke_2 \\ u_3 &= -\beta e_3 - y_1y_2 + x_1x_2 \end{aligned} \quad (15)$$

Now, using (15) in (14) we obtained the derivative of another Lyapunov function as

$$\dot{v}(e) = -e_1^2 - ke_2^2 - \beta e_3^2 \quad (16)$$

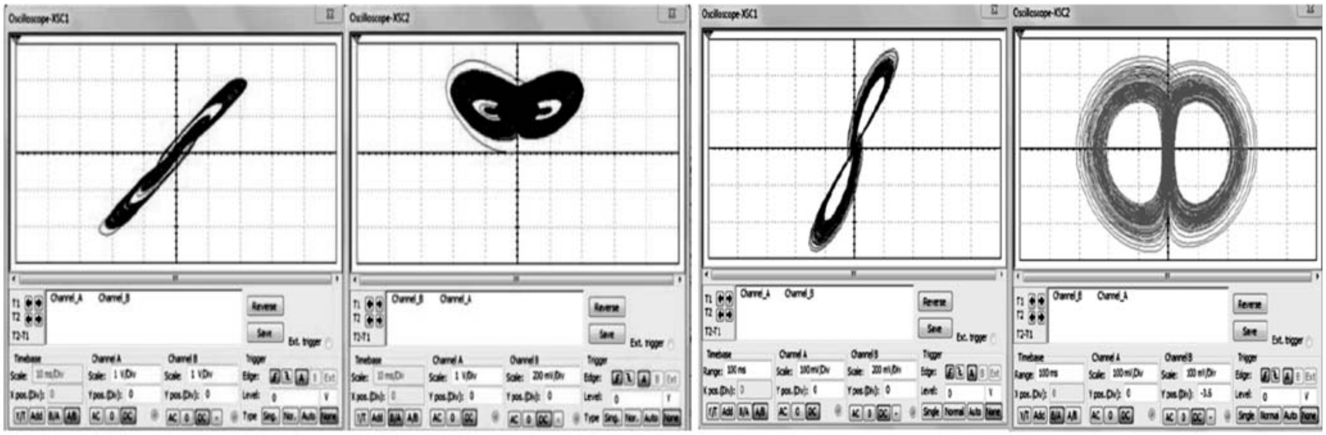
which is a negative definite function with $w < 0$.

Now, we can say that according to Lyapunov stability theory error dynamics (14) globally asymptotically stable for equilibrium state at origin *i.e*. error dynamics will converge to zero as $t \to \infty$. We obtained the following result.

Remarks : The chaotic Chen (1) and chaotic Liu-yang (2) system are exponentially and globally synchronized for any value of initial conditions with the nonlinear controller U defined by (15).

5. ANALOG CIRCUIT SIMULATION

Here, circuit simulation of two chaotic systems (Chen chaotic system, & Liu-Yang Chaotic system), and for complete synchronization scheme is shown using NI MULTISIM 11.0. Fig. 2 shows the phase portraits of Chen system using circuit simulation.



(*a*) (*b*)

Figure 2: Phase Plane of: (*a*) Chen and (b) Liu-Yang chaotic system with respect to plane in oscilloscope (1), with respect to plane in oscilloscope (2)

A. Circuit Design for Synchronization using SMC scheme

Here, sliding surface, control law describes in equation (4) and (4) respectively for synchronization scheme is simulated using analog components resistance, capacitance, Op-Amp, of different values. Components values used in circuit shown in Fig. 3 according to equation in (28), (29).

$$s_1 = \left(\frac{1}{R_{48}c_8}\right)\left[\left(\frac{R_{47}}{R_{46}}\right)\dot{e}_2 + \left(\frac{R_{47}}{R_{45}}\right)e_2 + \left(\frac{R_{47}}{R_{44}}\right)e_1\right]$$

$$s_2 = \left(\frac{1}{R_{57}c_{10}}\right)\left[\left(\frac{R_{56}}{R_{55}}\right)\dot{e}_3 + \left(\frac{R_{56}}{R_{55}}\right)e_3\right] \tag{17}$$

$$u_1 = -\left(\frac{R_{29}}{R_{26}}\right)y_1 - \left(\frac{R_{29}}{R_{65}}\right)x_1 - \left(\frac{R_{29}}{R_{26}}\right)y_1 + \left(\frac{R_{29}}{R_{71}}\right)\frac{y_1 y_2}{10} + \left(\frac{R_{29}}{R_{84}}\right)x_2 - \left(\frac{R_{29}}{R_{72}}\right)s_1 - \left(\frac{R_{29}}{R_{70}}\right)e_2 - \left(\frac{R_{29}}{R_{69}}\right)e_1 \tag{18}$$

$$u_2 = -\left(\frac{R_{35}}{R_{74}}\right)x_3 - \left(\frac{R_{35}}{R_{73}}\right)\frac{y_1 y_2}{10} + \left(\frac{R_{35}}{R_{75}}\right)\frac{x_1 x_2}{10} - \left(\frac{R_{35}}{R_{77}}\right)e_3 - \left(\frac{R_{35}}{R_{78}}\right)s_2$$

B. Circuit Design for Synchronization using Active control Scheme

Here, control law describes in (15) for synchronization using NAC is simulated using analog components resistance, capacitance, Op-Amp, of different values. Components values used in circuit shown in Fig. 4 according to equation in (18).

$$u_2 = -\left(\frac{R_{39}}{R_{49}}\right)e_2,\ e_2 = \left(\frac{R_{37}}{R_{38}}\right)y_2 - \left(\frac{R_{37}}{R_{36}}\right)x_2$$

$$u_2 = -\left(\frac{R_{21}}{R_{47}}\right)y_1 - \left(\frac{R_{21}}{R_{51}}\right)x_1 - \left(\frac{R_{21}}{R_{59}}\right)\frac{x_1 x_3}{10} + \left(\frac{R_{21}}{R_{58}}\right)x_2 - \left(\frac{R_{29}}{R_{72}}\right)s_1 - \left(\frac{R_{21}}{R_{52}}\right)e_2$$

$$u_2 = -\left(\frac{R_{25}}{R_{57}}\right)\frac{y_1 y_2}{10} - \left(\frac{R_{25}}{R_{55}}\right)x_1 x_2$$

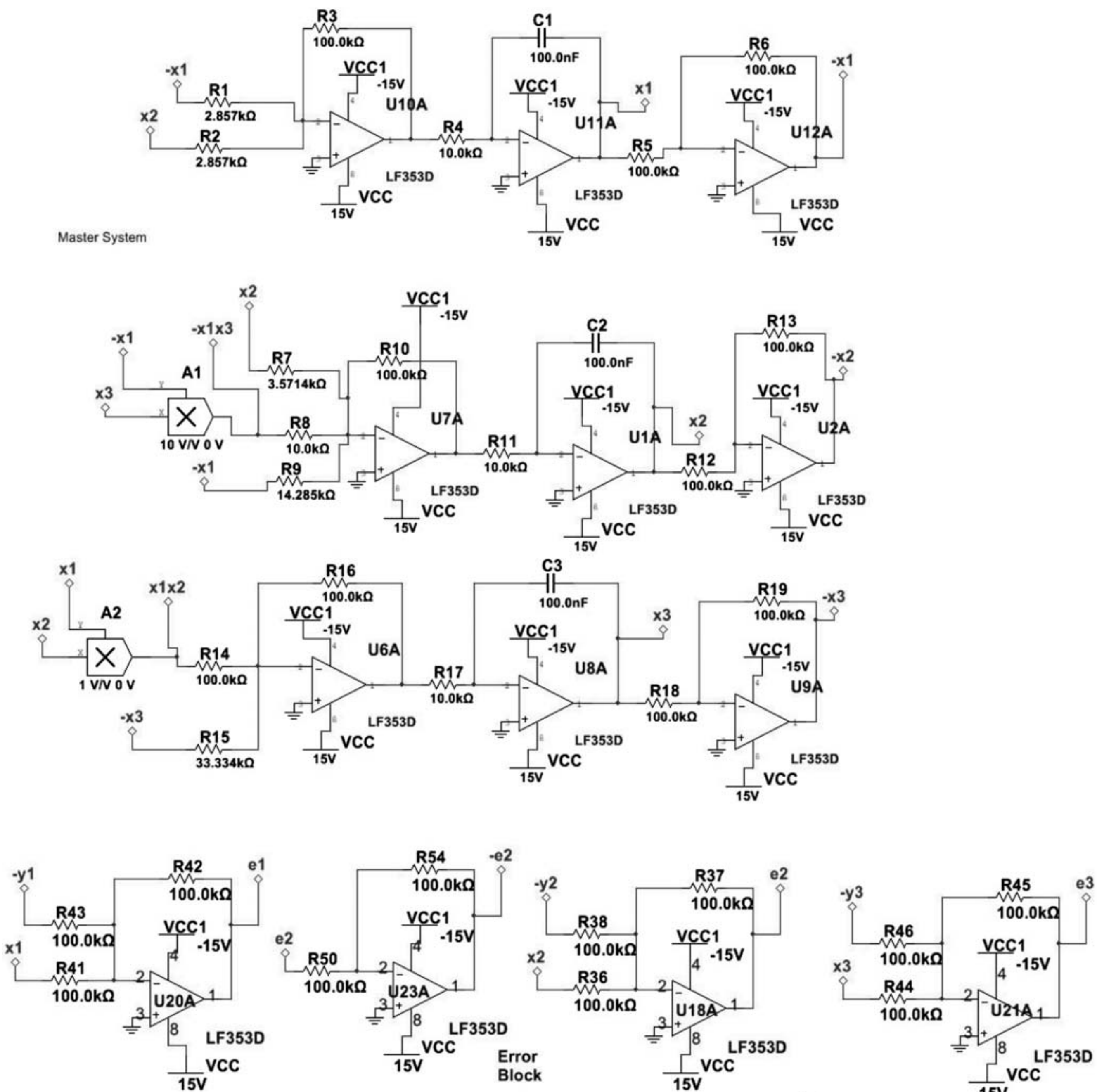
6. RESULTS AND DISCUSSION

A. MATLAB Simulation Results

The initial condition for simulating master (1) and slave system (2) are as $x(0) = (2, 2, 1)$, and $y(0) = (–5.2, – 7.5, 8.8)$ for both the control schemes. The response of the sliding surfaces is given in Fig. 5. Figure 6 show the synchronization errors between Chen (master) and Liu-Yang (slave) chaotic systems using SMC and NAC, respectively.

B. MULTISIM Simulation Results for synchronizationm using Active control scheme

The initial condition for simulating master (1) and slave system (2) are as $x(0) = (0.1, 0.2, 0.1)$, and $y(0) = (0,0,0)$. Figs. 7 and 8 shows the time responses of synchronized master and slave systems using NAC in multisim simulation.



(a)

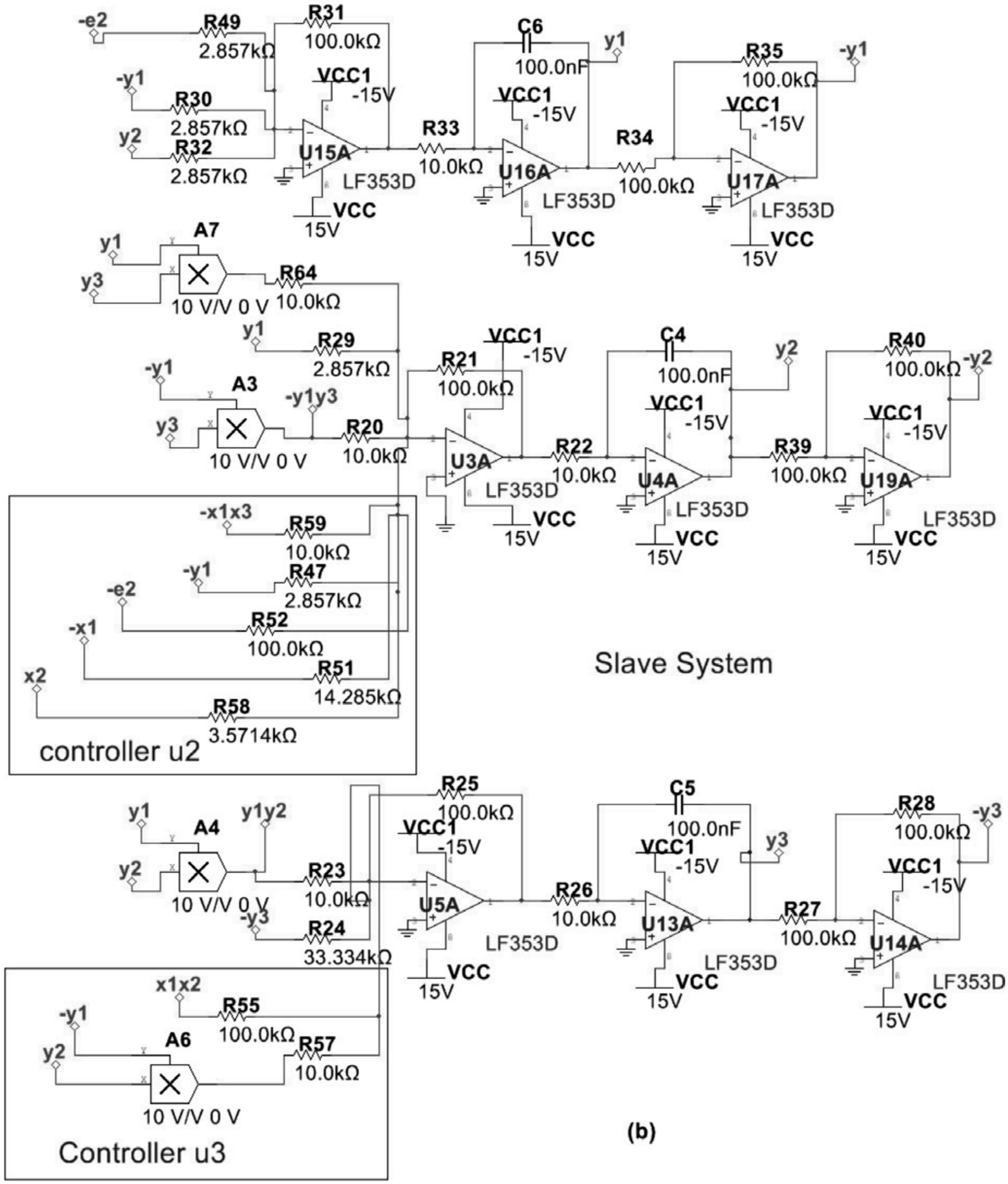
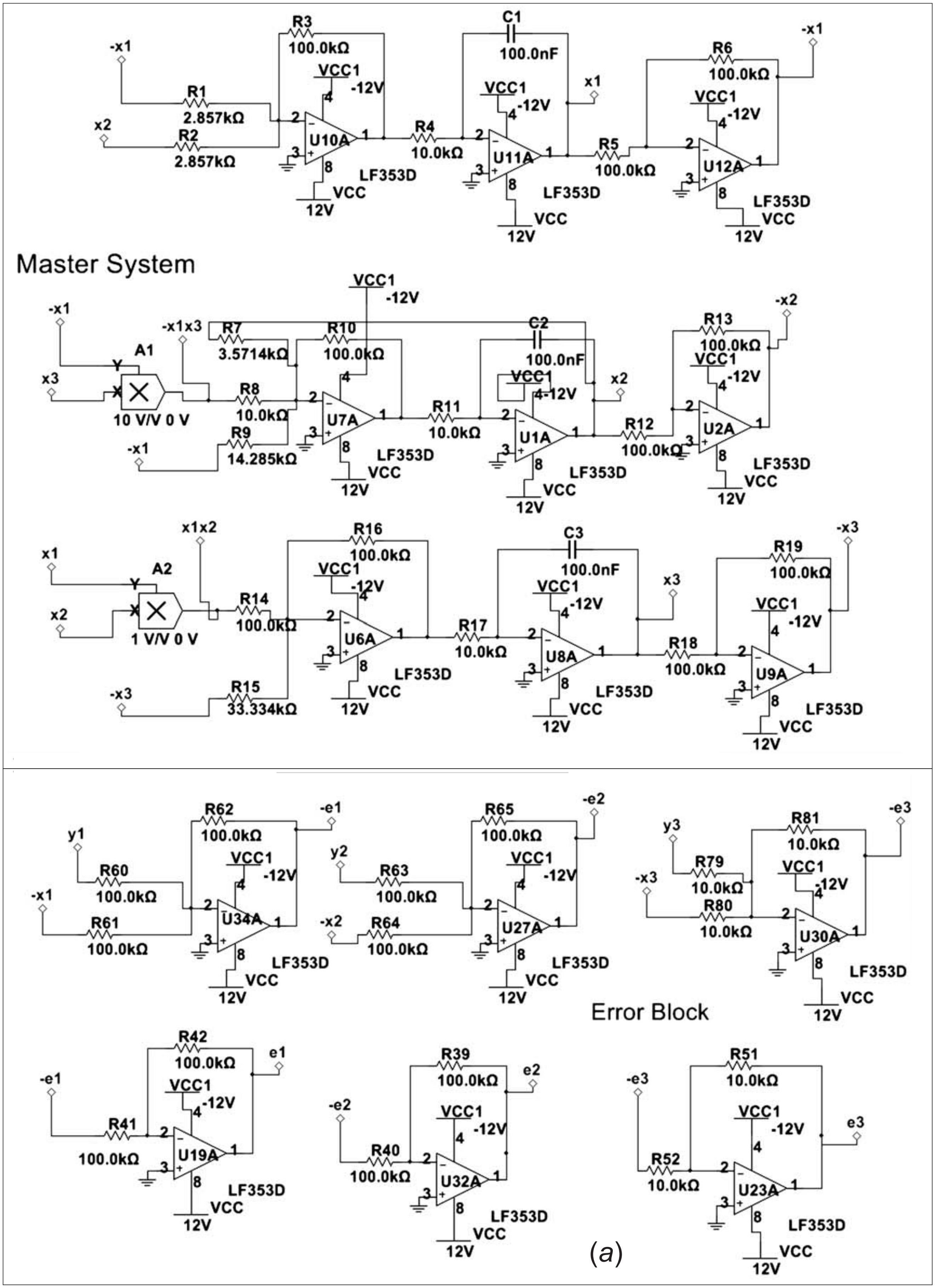


Figure 3: Circuit simulation of synchronization between Chen and Liu-Yang chaotic systems using Active control

C. MULTISIM Simulation Results for synchronizationm using SMC scheme

The initial condition for simulating master (1) and slave system (2) are as $x(0) = (0.1, 0.1, 0.1)$, and $y(0) = (0,0,0)$. Fig. 9 and Fig. 10 shows the time response plot of the each state for synchronized master and slave systems using SMCC in multisim simulation.


Master System
-x1
x2
R1
2.857kΩ
R2
2.857kΩ
R3
100.0kΩ
VCC1
-12V
U10A
LF353D
VCC
12V
R4
10.0kΩ
C1
100.0nF
x1
U11A
R5
100.0kΩ
R6
100.0kΩ
U12A
-x1
x3
A1
10 V/V 0 V
-x1x3
R7
3.5714kΩ
R8
10.0kΩ
R9
14.285kΩ
R10
100.0kΩ
U7A
R11
10.0kΩ
C2
100.0nF
U1A
R12
100.0kΩ
R13
100.0kΩ
U2A
-x2
A2
1 V/V 0 V
x1x2
R14
100.0kΩ
R15
33.334kΩ
-x3
R16
100.0kΩ
U6A
R17
10.0kΩ
C3
100.0nF
x3
U8A
R18
100.0kΩ
R19
100.0kΩ
U9A
Error Block
y1
R60
100.0kΩ
R61
100.0kΩ
R62
100.0kΩ
U34A
-e1
y2
-x2
R63
100.0kΩ
R64
100.0kΩ
R65
100.0kΩ
U27A
-e2
y3
R79
10.0kΩ
R80
10.0kΩ
R81
10.0kΩ
U30A
-e3
R41
100.0kΩ
R42
100.0kΩ
U19A
e1
R40
100.0kΩ
R39
100.0kΩ
U32A
e2
R52
10.0kΩ
R51
10.0kΩ
U23A
e3

(*a*)

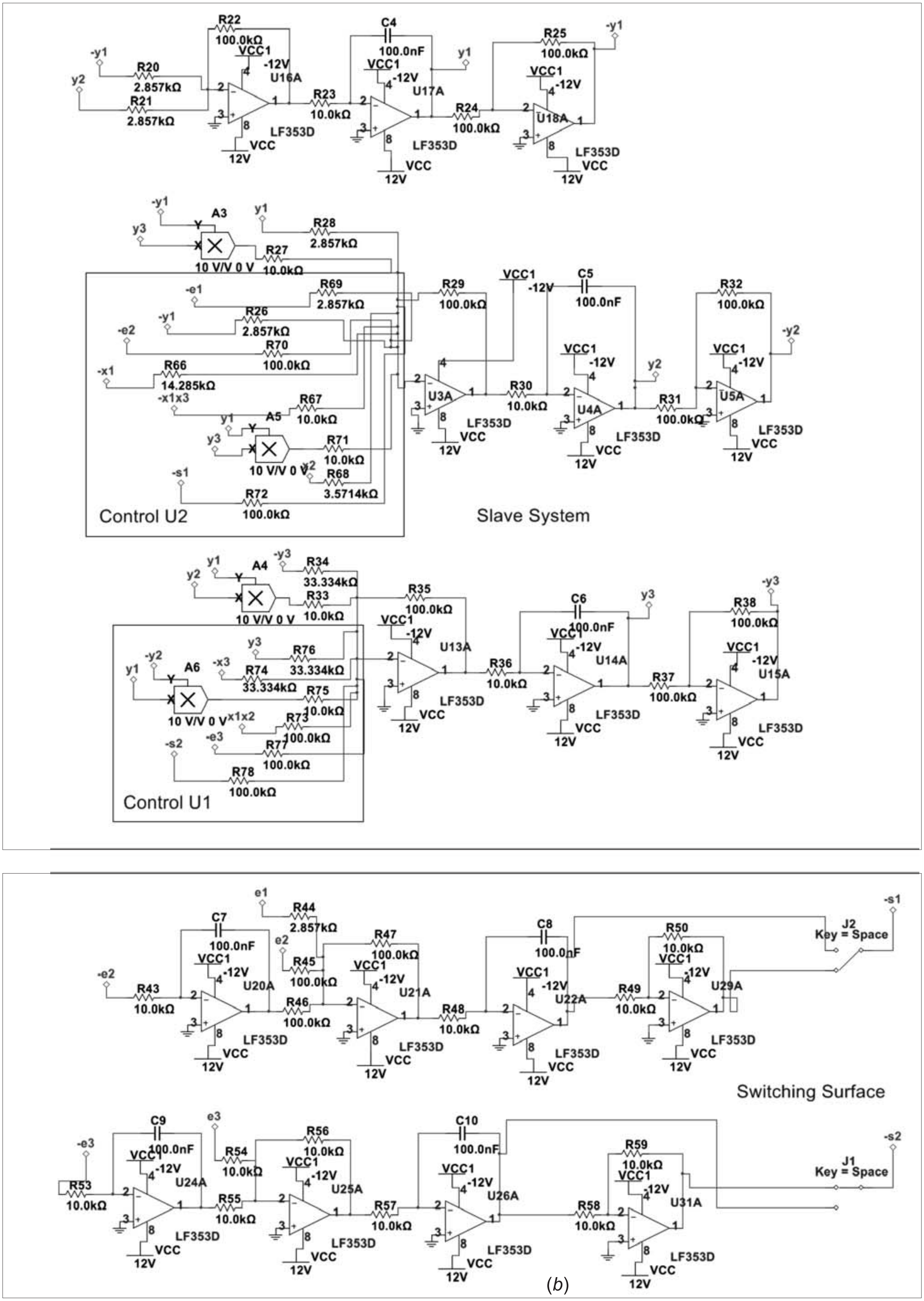


Figure 4: Circuit simulation of synchronization between Chen and Liu-Yang chaotic systems using SMC

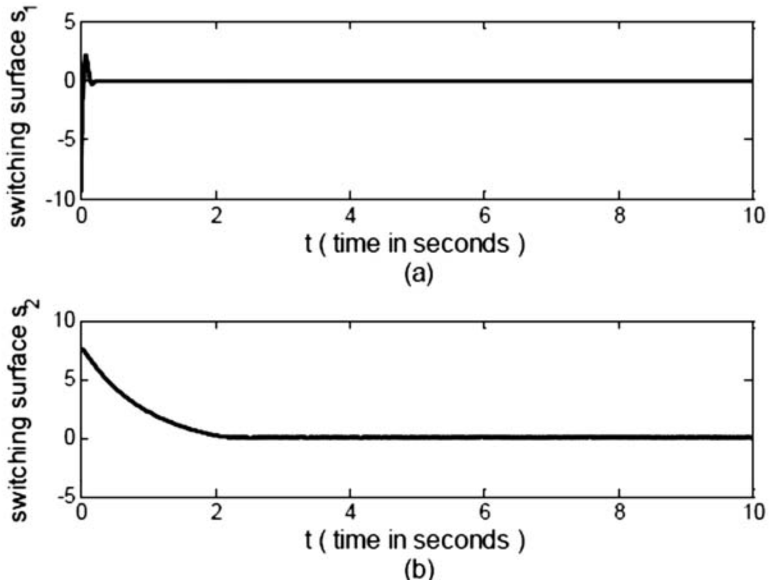


Figure 5: Response of sliding surfaces during synchronization

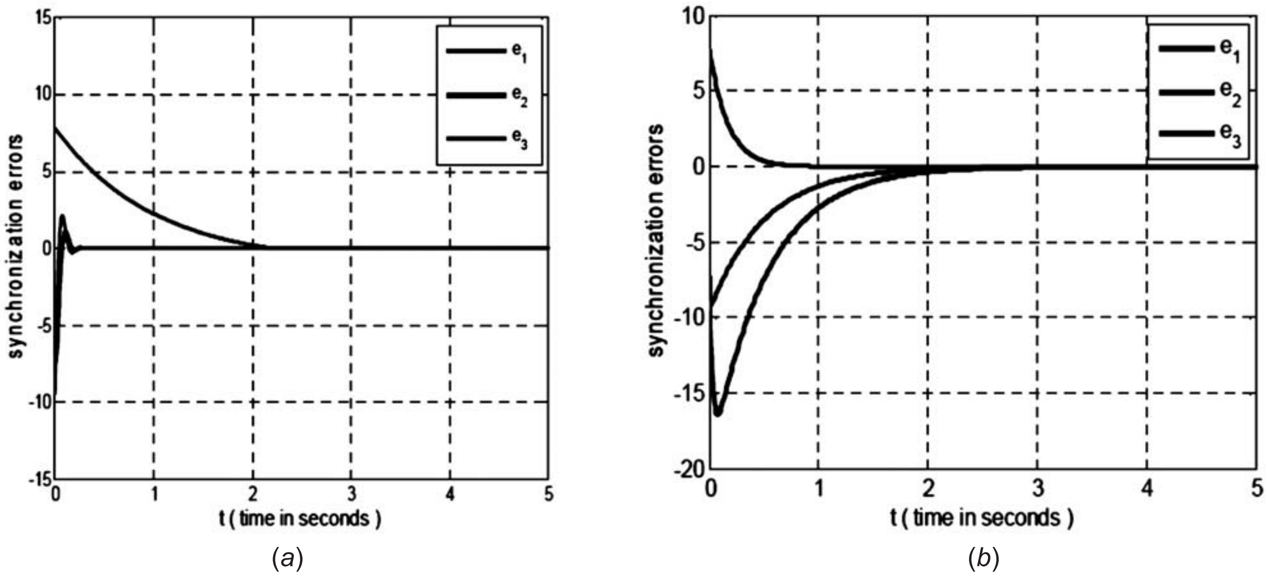


Figure 6: Synchronized errors: (*a*) using SMC and (*b*) using NAC

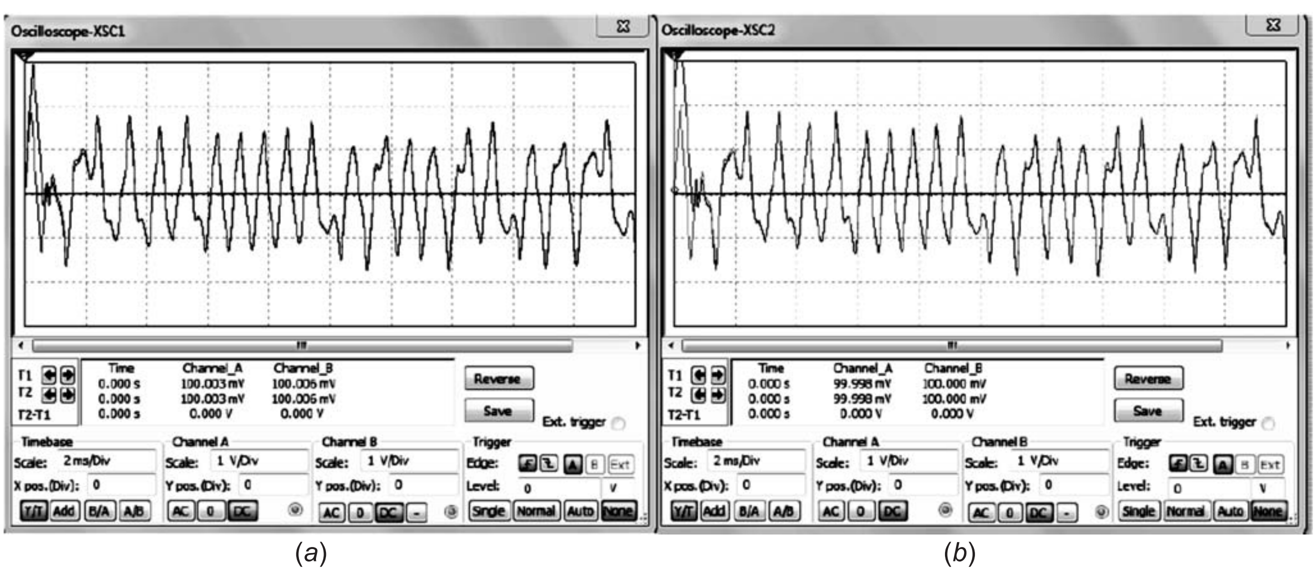


Figure 7: Synchronization between: (*a*) $x_1 - y_1$ and (*b*) $x_2 - y_2$ states using SMC

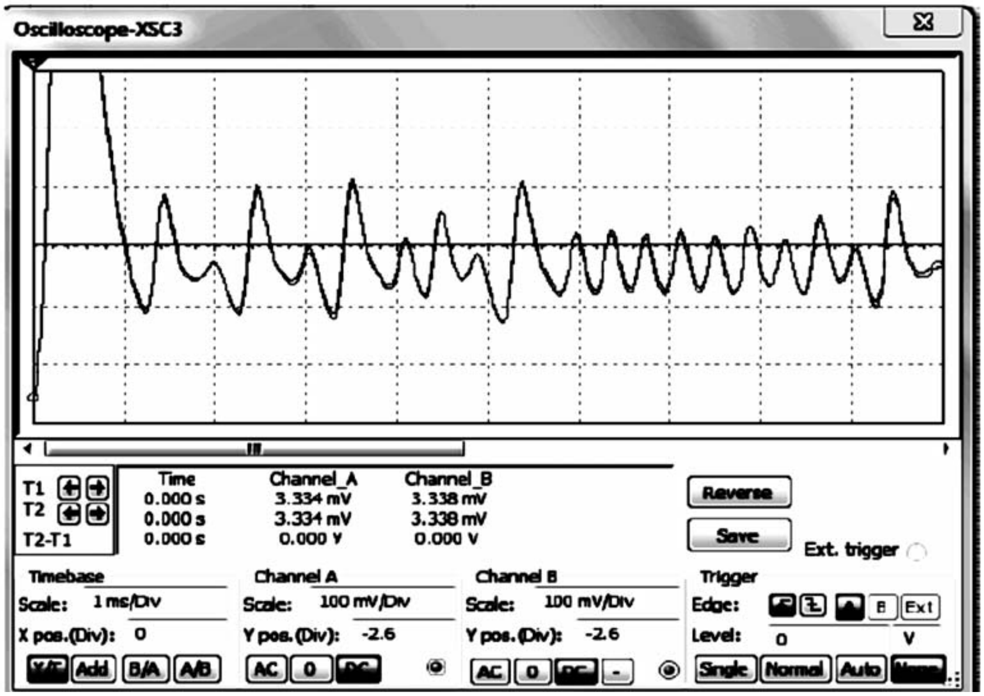
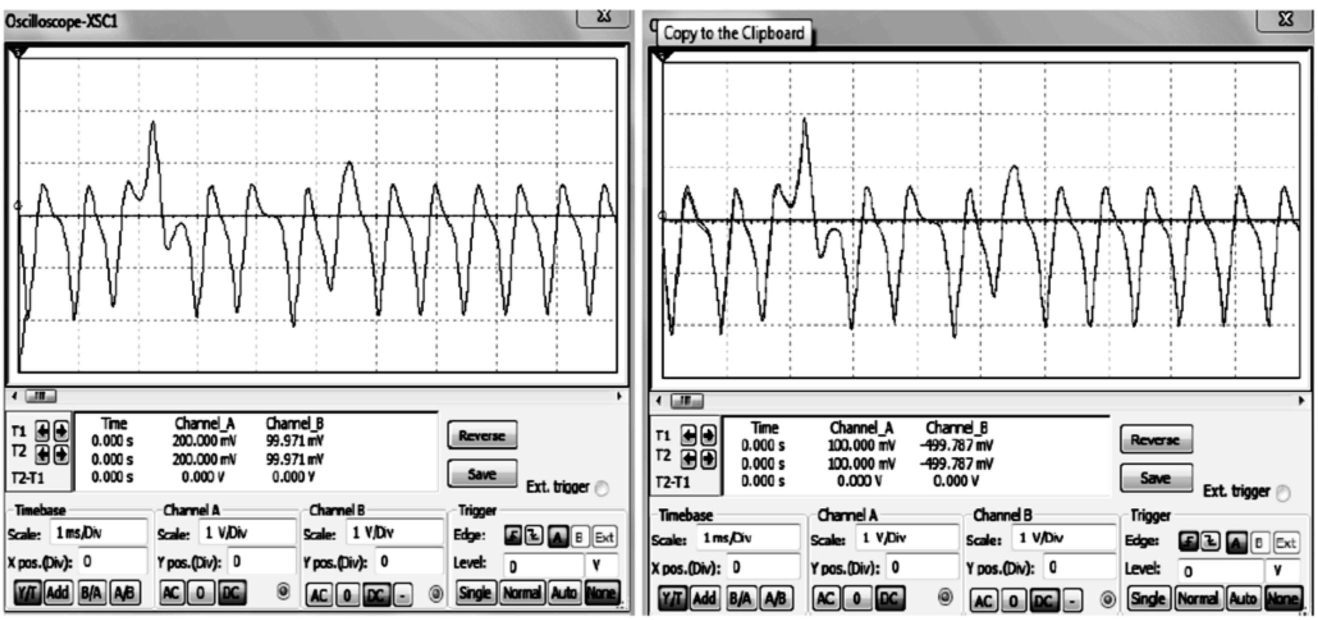


Figure 8: Synchronization between: $x_3 - y_3$ states using SMC



(*a*) (*b*)

Figure 9: Synchronization between: (*a*) $x_1 - y_1$ and (*b*) $x_2 - y_2$ states using NAC

7. CONCLUSION

This paper presented the global chaotic synchronization between two different chaotic systems (Chen and Liu-Yang) using sliding mode control (SMC) technique is achieved. PI switching surfaces of SMC are designed for the synchronization of states. Lyapunov stability theory is used for stabilizing the synchronization error. Here it is proved that SMC is very effective and convenient technique to achieve global chaos synchronization for Chen and Liu-Yang chaotic system. Synchronization of non-identical systems along with circuit simulation presented here is the novelty of this paper. Circuit simulation is shown for proposed synchronization scheme as well as master and slave system using NI MULTISIM. This master and slave pair can be used as transmitter receiver pair for secure communication as application of proposed synchronization scheme.

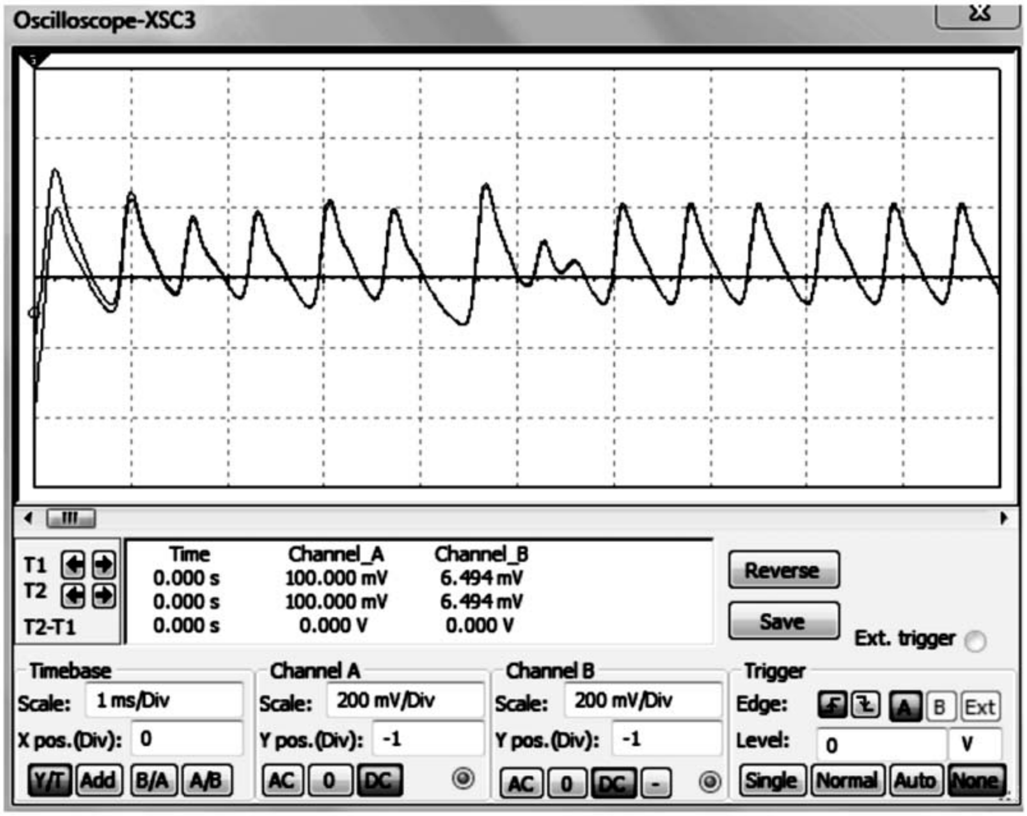


Figure 10: Synchronization between: states using NAC

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