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An Eleven-Term Novel 4-D Hyperchaotic System with Three Quadratic Nonlinearities, Analysis, Control and Synchronization via Adaptive Control Method

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Abstract: First, this paper announces an eleven-term novel 4-D hyperchaotic system with three quadratic nonlinearities. This 4-D hyperchaotic system has been obtained by using a feedback control to the 3-D Zhu chaotic system (2010). The phase portraits of the novel hyperchaotic system are displayed and the mathematical properties are discussed. The Lyapunov exponents of the novel hyperchaotic system are obtained as $L_1 = 0.7365$, $L_2 = 0.0821$, $L_3 = 0$ and $L_3 = -2.9367$. The maximal Lyapunov exponent (MLE) for the novel hyperchaotic system is obtained as $L_1 = 0.7365$ and Lyapunov dimension as $D_L = 3.1779$. Next, we derive new results for the adaptive control design of the novel hyperchaotic system with unknown parameters. The adaptive controller is designed to achieve global exponential stability for the novel hyperchaotic system with unknown system parameters. Next, we derive new results for the adaptive synchronization design of the identical novel hyperchaotic systems with unknown parameters. The adaptive control and synchronization results for the novel hyperchaotic system have been established using Lyapunov stability theory. Numerical simulations with MATLAB have been shown to validate and demonstrate all the new results derived in this paper.

Keywords: Chaos, chaotic systems, hyperchaos, hyperchaotic systems, adaptive control, chaos control, chaos synchronization.

1. INTRODUCTION

A *chaotic system* is commonly defined as a nonlinear dissipative dynamical system that is highly sensitive to even small perturbations in its initial conditions. The sensitivity of a nonlinear chaotic system in response to small changes in the initial conditions is commonly called as *butterfly effect* [1] and this is one of the characterizing features of a chaotic system. The Lyapunov exponent of a dynamical system is a quantitative measure that characterizes the rate of separation of infinitesimally close trajectories of the system. Thus, a chaotic system is also defined mathematically as a dynamical system having at least one positive Lyapunov exponent.

In the last four decades, many chaotic systems have been found in the literature using modelling and other techniques. Some paradigms of chaotic systems can be listed as Lorenz system [2], Rössler system [3], Shimizu-Morioka system [4], Shaw system [5], Chen system [6], Lü system [7], Chen-Lee system [8], Cai system [9], Tigan system [10], Li system [11], etc.

Many new 3-D chaotic systems have been discovered in the recent years such as Sundarapandian systems [12-13], Vaidyanathan systems [14-20], Vaidyanathan-Madhavan system [21], Vaidyanathan-Azar system [22], Vaidyanathan-Volos system [23-24], Pehlivan-Moroz system [25], Pham system [26], etc.

Hyperchaotic systems are the chaotic systems with more than one positive Lyapunov exponent. For continuous-time autonomous dynamical systems, the minimal dimension for a hyperchaotic system is four.

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The first hyperchaotic system was found by Rössler [27]. This was followed by the finding of many hyperchaotic systems such as hyperchaotic Lorenz system [28], hyperchaotic Lü system [29], hyperchaotic Chen system [30], hyperchaotic Wang system [31], etc. Some recently discovered 4-D hyperchaotic systems are hyperchaotic Vaidyanathan systems [32-33], hyperchaotic Vaidyanathan-Azar system [34], etc. A 5-D hyperchaotic system with three positive Lyapunov exponents was also recently found [35].

Hyperchaotic systems have attractive features such as high security, high capacity and high efficiency and they find miscellaneous applications in several areas like neural networks [36-38], oscillators [39-40], circuits [41-44], secure communication [45-46], encryption [47], generator [48], laser [49-50], etc.

The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [51-52]. Some popular methods for chaos control are active control [53-57], adaptive control [58-59], sliding mode control [60-62], etc.

Chaos synchronization problem can be stated as follows. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically. The synchronization of chaotic systems has applications in secure communications [63-65], cryptosystems [66-67], encryption [68-70], etc.

The chaos synchronization problem has been paid great attention in the literature and a variety of impressive approaches have been proposed. Since the pioneering work by Pecora and Carroll [71-72] for the chaos synchronization problem, many different methods have been proposed in the control literature such as active control method [73-80], adaptive control method [81-107], sampled-data feedback control method [108-109], time-delay feedback approach [110], backstepping method [111-122], sliding mode control method [123-132], etc.

In this paper, we have proposed an eleven-term novel 4-D hyperchaotic system with three quadratic nonlinearities. This 4-D hyperchaotic system has been obtained by using a feedback control to the 3-D Zhu chaotic system [133]. The phase portraits of the novel hyperchaotic system are displayed and the mathematical properties are discussed. Next, we derive new results for the adaptive control design of the novel hyperchaotic system with unknown parameters. The adaptive controller is designed to achieve global exponential stability for the novel hyperchaotic system with unknown system parameters. Next, we derive new results for the adaptive synchronization design of the identical novel hyperchaotic systems with unknown parameters. The adaptive control and synchronization results for the novel hyperchaotic system have been established using Lyapunov stability theory. Numerical simulations with MATLAB have been shown to validate and demonstrate all the new results derived in this paper.

2. A NOVEL 4-D HYPERCHAOTIC SYSTEM

In [133], Zhu *et al.* (2010) found a 3-D chaotic system modelled by the dynamics

$$\begin{aligned} \dot{x}_1 &= -x_1 - ax_2 + x_2 x_3 \\ \dot{x}_2 &= bx_2 - x_1 x_3 \\ \dot{x}_3 &= -cx_3 + x_1 x_2 \end{aligned} \tag{1}$$

The Zhu system is found to be *chaotic* when the parameters take the following values [133]:

$$a = 1.5, \quad b = 2.5, \quad c = 4.9 \tag{2}$$

Also, the Lyapunov exponents of the Zhu system (1) for the parameter values in (2) have been obtained as follows:

$$L_1 = 0.6747,\ L_2 = 0,\ L_3 = -4.0738 \tag{3}$$

Since the sum of the Lyapunov exponents of the Zhu system (1) is negative, it is dissipative.

For numerical simulations, we take the initial values of the Zhu system (1) as $x_1(0) = 0.4,\ \ x_2(0) = 0.8$ and $x_3(0) = 0.6$.

The phase portrait of the Zhu chaotic attractor is shown in Figure 1.

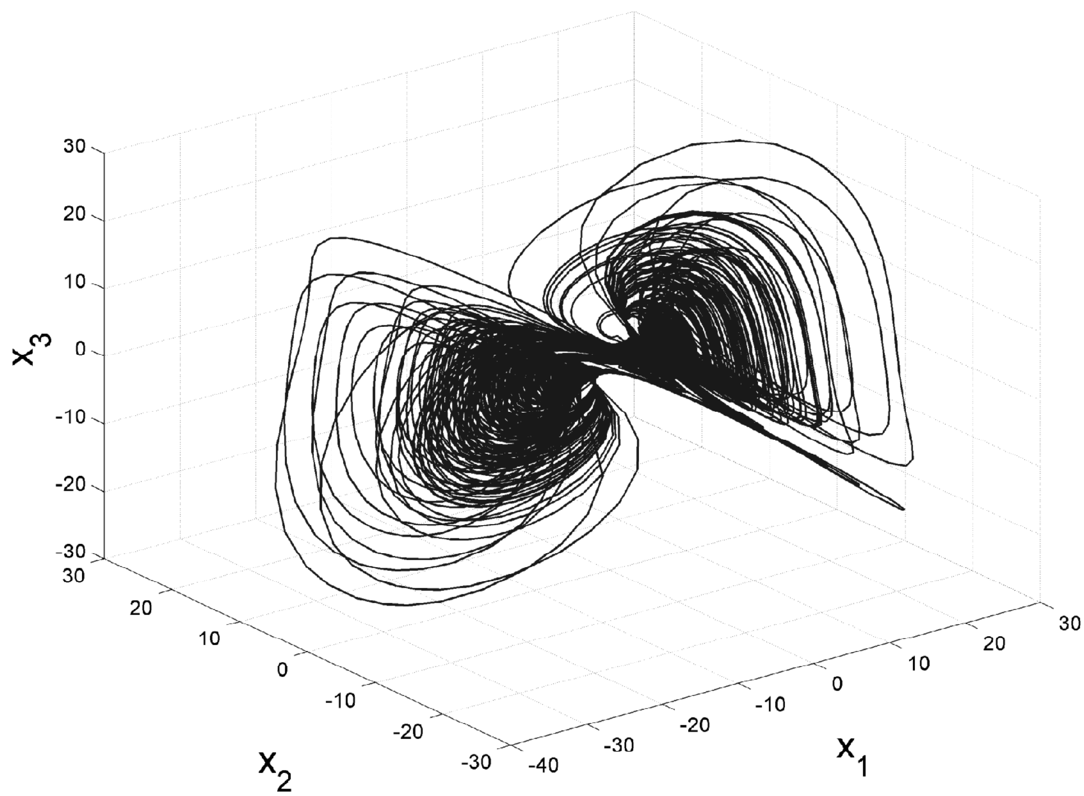

Figure 1: Strange attractor of the Zhu chaotic system

In this paper, we propose a 4-D novel hyperchaotic system by adding state feedback control law to the Zhu system (1) as

$$\begin{aligned} \dot{x}_1 &= -x_1 - ax_2 + x_2x_3 + px_4 \\ \dot{x}_2 &= bx_2 - x_1x_3 + px_4 \\ \dot{x}_3 &= -cx_3 + x_1x_2 \\ \dot{x}_4 &= -qx_1 - px_2 \end{aligned} \tag{4}$$

where x_1, x_2, x_3, x_4 are the states and a, b, c, p, q are constant, positive parameters of the system.

The system (4) exhibits a *hyperchaotic* attractor for the values

$$a = 1.5,\ \ b = 3,\ \ c = 5.8,\ \ p = 1.6,\ \ q = 0.5 \tag{5}$$

For numerical simulations, we take the initial state as $x_1(0) = 0.4,\ \ x_2(0) = 0.8,\ x_3(0) = 0.6,$ and $x_4(0) = 0.2$.

Figures 2-5 show the 3-D view of the hyperchaotic attractor of the system (4) in (x_1, x_2, x_3), (x_1, x_2, x_4), (x_1, x_3, x_4) and (x_2, x_3, x_4) spaces respectively.

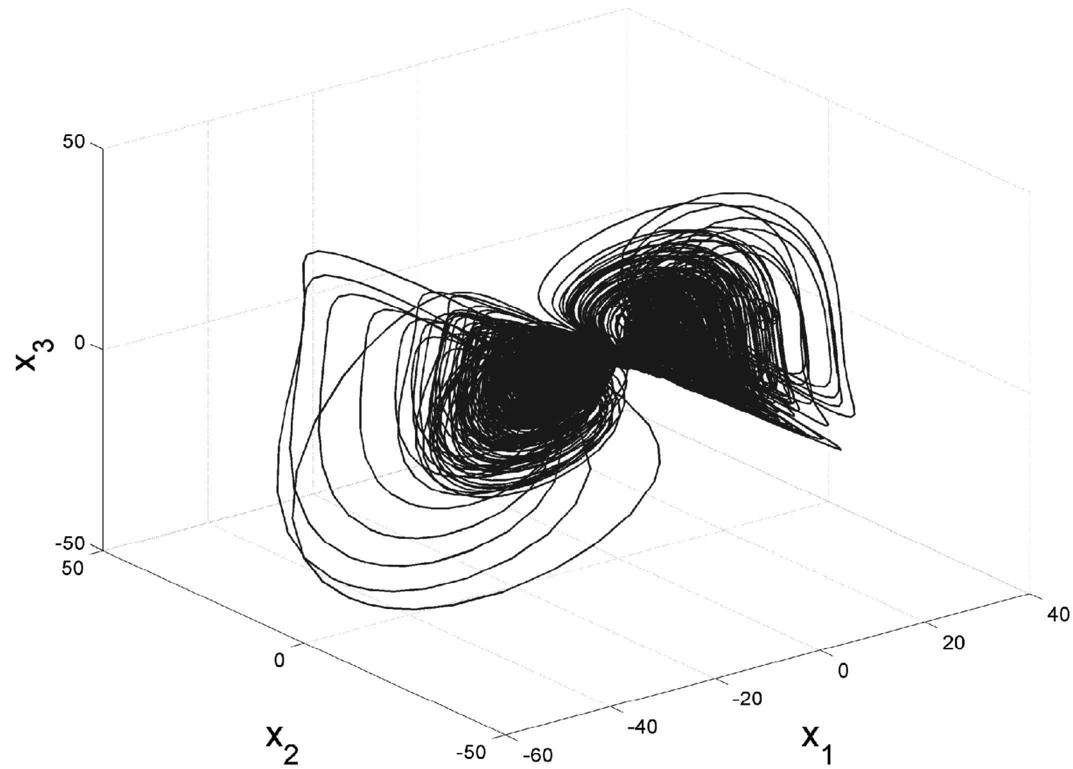

Figure 2: 3-D view of the novel hyperchaotic system in (x_1, x_2, x_3) space

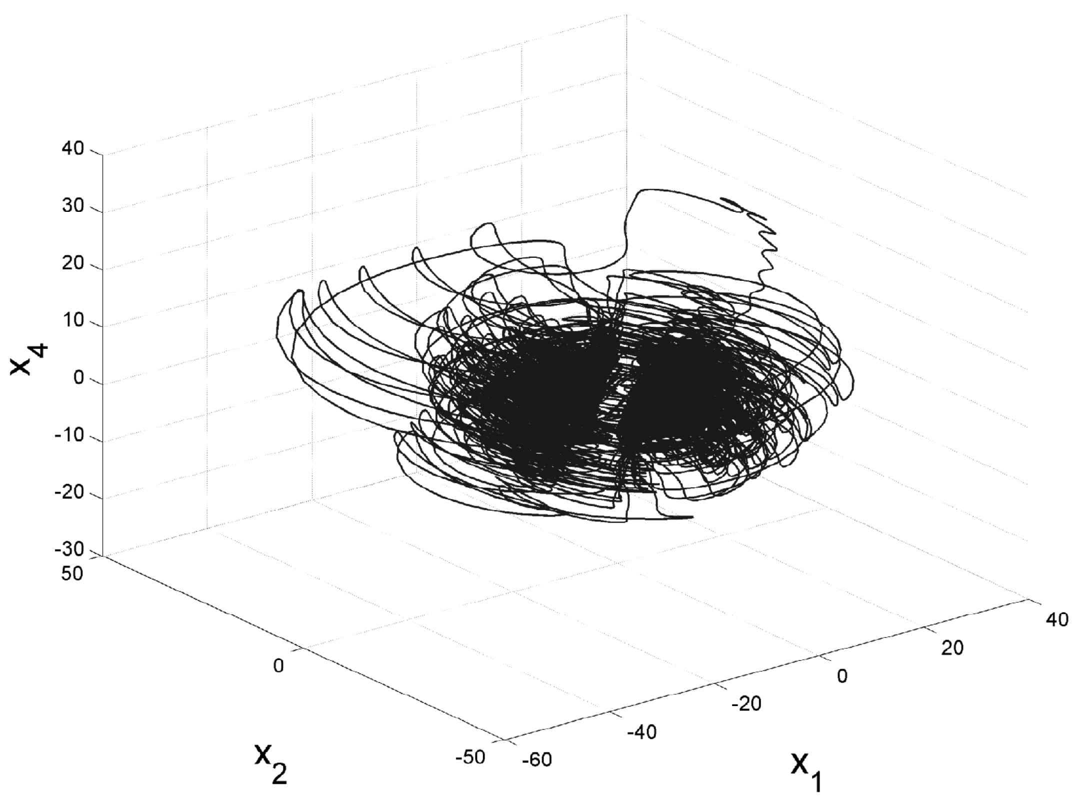

Figure 3: 3-D view of the novel hyperchaotic system in (x_1, x_2, x_4) space

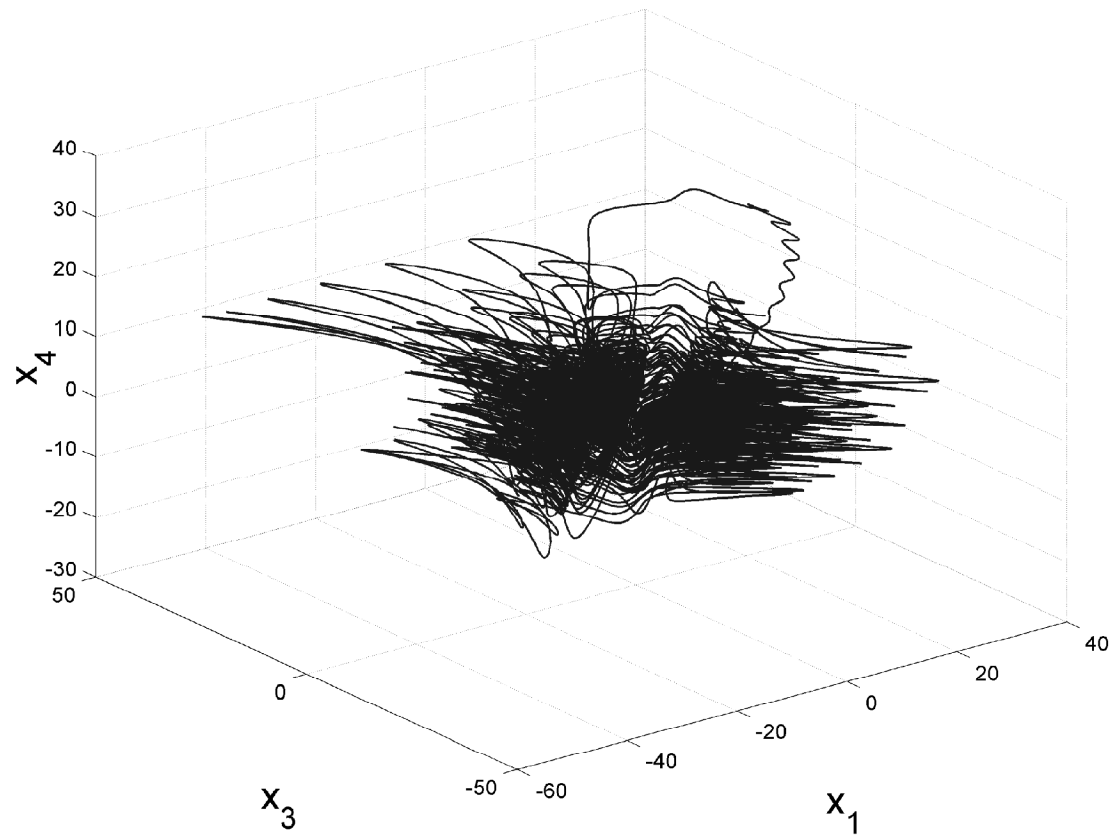

Figure 4: 3-D view of the novel hyperchaotic system in (x_1, x_3, x_4) space

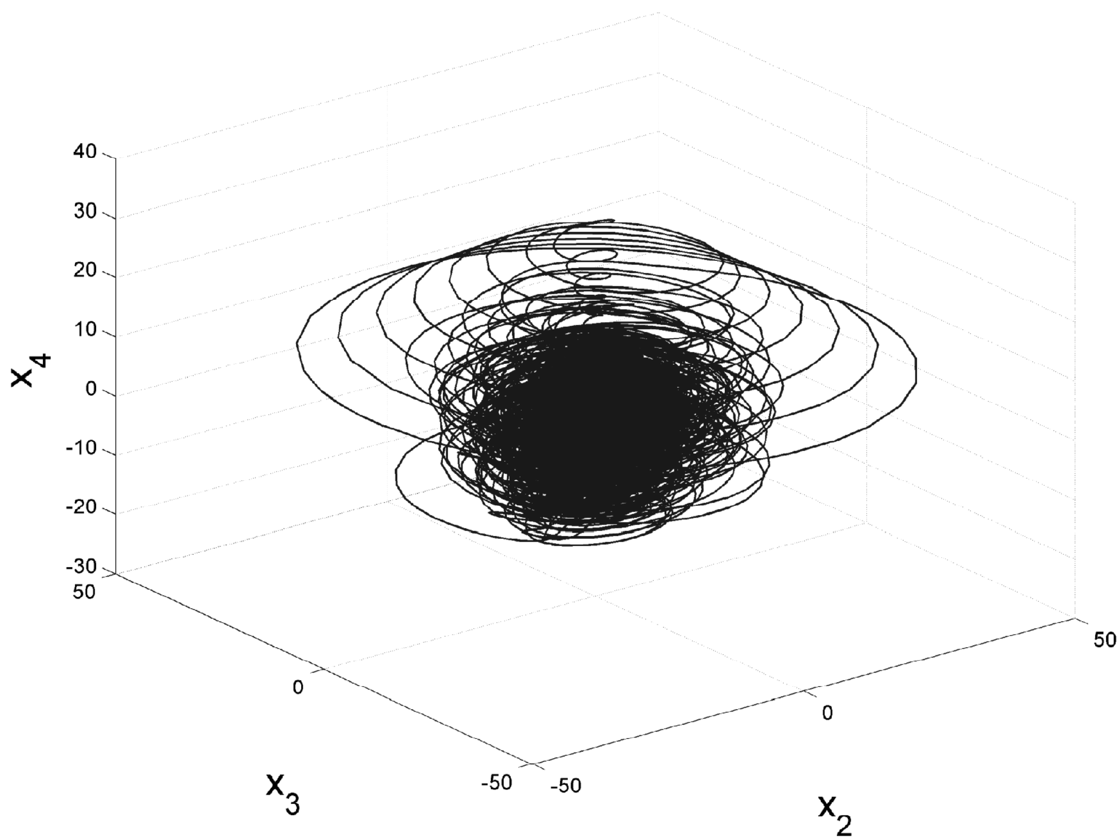

Figure 5: 3-D view of the novel hyperchaotic system in (x_2, x_3, x_4) space

3. PROPERTIES OF THE NOVEL HYPERCHAOTIC SYSTEM

A. Dissipativity

We write the system (4) in vector notation as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{bmatrix} \tag{6}$$

where

$$\begin{aligned} f_1(x) &= -x_1 - ax_2 + x_2x_3 + px_4 \\ f_2(x) &= bx_2 - x_1x_3 + px_4 \\ f_3(x) &= -cx_3 + x_1x_2 \\ f_4(x) &= -qx_1 - px_2 \end{aligned} \tag{7}$$

We take the parameter values as

$$a = 1.5, \quad b = 3, \quad c = 5.8, \quad p = 1.6, \quad q = 0.5 \tag{8}$$

The divergence of the vector field f on R^4 is obtained as

$$\text{div}\, f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} + \frac{\partial f_4}{\partial x_4} = -1 + b - c = -\mu, \tag{9}$$

where

$$\mu = 1 - b + c = 3.8 > 0 \tag{10}$$

Let Ω be any region in R^4 having a smooth boundary.

Let $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f.

Let $V(t)$ denote the hypervolume of $\Omega(t)$.

By Liouville's theorem, it follows that

$$\frac{dV(t)}{dt} = \int_{\Omega(t)} (\text{div}\, f)\, dx_1dx_2dx_3dx_4 = -\mu \int_{\Omega(t)} dx_1dx_2dx_3dx_4 = -\mu V(t) \tag{11}$$

Integrating the linear differential equation (12), we get the solution as

$$V(t) = V(0)\exp(-\mu t) \tag{12}$$

From Eq. (9), it follows that the volume $V(t)$ shrinks to zero exponentially as $t \to \infty$.

Thus, the novel hyperchaotic system (4) is dissipative. Hence, the asymptotic motion of the system (4) settles exponentially onto a set of measure zero, *i.e.* a strange attractor.

B. Symmetry

The 4-D novel hyperchaotic system (4) is invariant under the coordinates transformation

$$(x_1, x_2, x_3, x_4) \mapsto (-x_1, -x_2, x_3, -x_4) \tag{13}$$

Since the transformation (13) persists for all values of the system parameters, the novel hyperchaotic system (4) has rotation symmetry about the x_3– axis and that any non-trivial trajectory must have a twin trajectory.

C. Invariance

The x_3– axis ($x_1 = 0,\ x_2 = 0,\ x_4 = 0$) is invariant for the system (4). Hence, all orbits of the system (4) starting on the x_3– axis stay in the x_3– axis for all values of time. Also, this invariant motion is globally exponentially stable as $c > 0$.

D. Equilibrium Points

The equilibrium points of the novel hyperchaotic system (4) are obtained by solving the following nonlinear system of equations

$$\begin{aligned} f_1(x) &= -x_1 - ax_2 + x_2x_3 + px_4 = 0 \\ f_2(x) &= bx_2 - x_1x_3 + px_4 = 0 \\ f_3(x) &= -cx_3 + x_1x_2 = 0 \\ f_4(x) &= -qx_1 - px_2 = 0 \end{aligned} \tag{14}$$

We take the parameter values as in the hyperchaotic case, *viz.*

$$a = 1.5,\quad b = 3,\quad c = 5.8,\quad p = 1.6,\quad q = 0.5 \tag{15}$$

Solving the equations (14) using the values (15), we obtain the unique equilibrium point:

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

The Jacobian matrix of the novel hyperchaotic system (4) at E_0 is obtained as

$$J_0 = J(E_0) = \begin{bmatrix} -1 & -1.5 & 0 & 1.6 \\ 0 & 3 & 0 & 1.6 \\ 0 & 0 & -5.8 & 0 \\ -0.5 & -1.6 & 0 & 0 \end{bmatrix} \tag{17}$$

which has the eigenvalues

$$\lambda_{1,2} = -0.0345 \pm 0.7082i,\quad \lambda_3 = 2.0690,\quad \lambda_4 = -5.8 \tag{18}$$

This shows that the equilibrium E_0 is a saddle-focus, which is unstable.

E. Lyapunov Exponents

We take the parameter values of the novel system (4) as

$$a = 1.5,\quad b = 3,\quad c = 5.8,\quad p = 1.6,\quad q = 0.5 \tag{19}$$

The Lyapunov exponents of the system (4) are numerically obtained with MATLAB as

$$\begin{cases} L_1 & = & 0.7365 \\ L_2 & = & 0.0821 \\ L_3 & = & 0 \\ L_4 & = & -4.6005 \end{cases} \tag{20}$$

Thus, the system (4) is hyperchaotic, since it has two positive Lyapunov exponents.

Also, the maximal Lyapunov exponent (MLE) of the system (4) is obtained as $L_1 = 0.7365,$ which is greater than the MLE of the Zhu system, viz. $L_1 = 0.6747.$

Since $L_1 + L_2 + L_3 + L_4 = -3.7819 < 0,$ it is immediate that the system (4) is dissipative.

The dynamics of the Lyapunov exponents of the novel hyperchaotic system (4) is depicted in Figure 6.

F. Lyapunov Dimension

The Lyapunov dimension of the hyperchaotic system (4) is determined as

$$D_L = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.1779 \tag{21}$$

which is fractional.

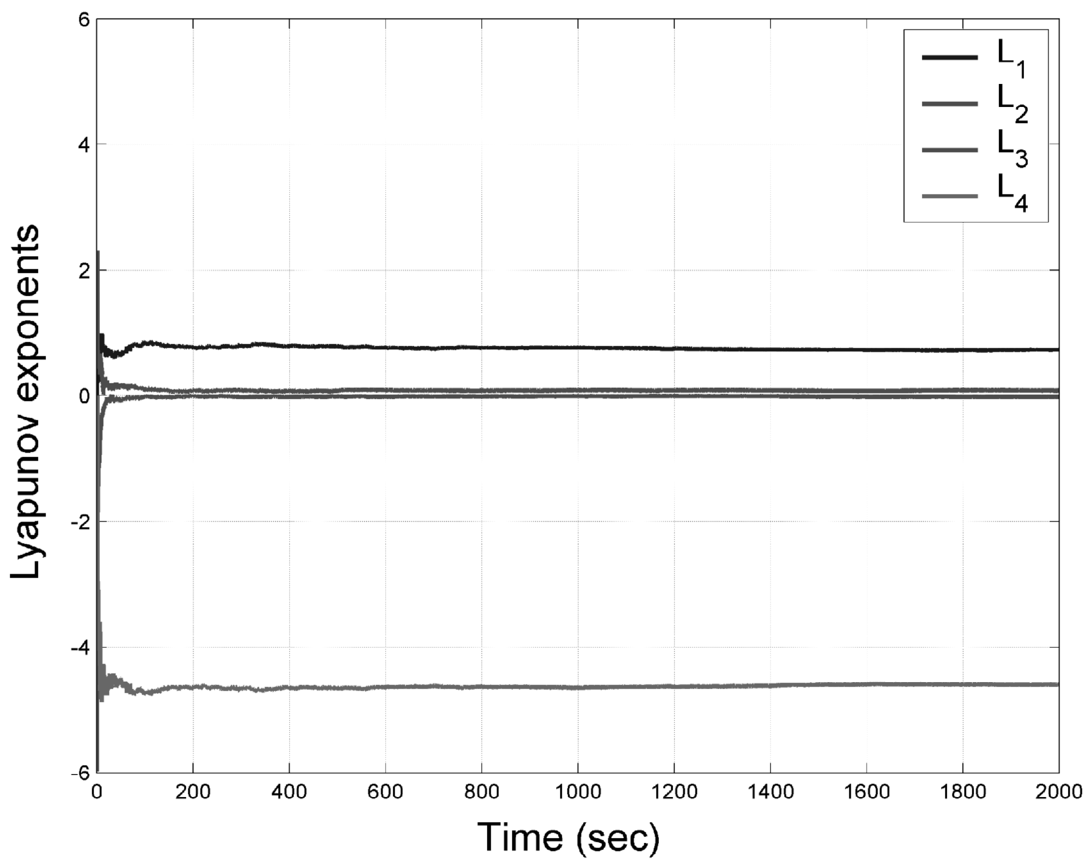

Figure 6: Dynamics of the Lyapunov exponents of the novel hyperchaotic system

4. ADAPTIVE CONTROL OF THE THREE-SCROLL CHAOTIC SYSTEM

In this section, we design new results for the adaptive controller to stabilize the 4-D novel hyperchaotic system with unknown parameters for all initial conditions.

Thus, we consider the controlled novel 4-D hyperchaotic system

$$\begin{aligned} \dot{x}_1 &= -x_1 - ax_2 + x_2x_3 + px_4 + u_1 \\ \dot{x}_2 &= bx_2 - x_1x_3 + px_4 + u_2 \\ \dot{x}_3 &= -cx_3 + x_1x_2 + u_3 \\ \dot{x}_4 &= -qx_1 - px_2 + u_4 \end{aligned} \tag{22}$$

where x_1, x_2, x_3, x_4 are state variables, a, b, c, p, q are constant, unknown, parameters of the system and u_1, u_2, u_3, u_4 are adaptive controls to be designed.

We aim to solve the adaptive control problem by considering the adaptive feedback control law

$$\begin{aligned} u_1 &= x_1 + \hat{a}(t)x_2 - x_2x_3 - \hat{p}(t)x_4 - k_1x_1 \\ u_2 &= -\hat{b}(t)x_2 + x_1x_3 - \hat{p}(t)x_4 - k_2x_2 \\ u_3 &= \hat{c}(t)x_3 - x_1x_2 - k_3x_3 \\ u_4 &= \hat{q}(t)x_1 + \hat{p}(t)x_2 - k_4x_4 \end{aligned} \tag{23}$$

where $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t), \hat{q}(t)$ are estimates for the unknown system parameters a, b, c, p, q, respectively, and k_1, k_2, k_3, k_4 are positive gain constants.

The closed-loop system is obtained by substituting (23) into (22) as

$$\begin{aligned} \dot{x}_1 &= -(a - \hat{a}(t))x_2 + (p - \hat{p}(t))x_4 - k_1x_1 \\ \dot{x}_2 &= (b - \hat{b}(t))x_2 + (p - \hat{p}(t)x_4 - k_2x_2 \\ \dot{x}_3 &= -(c - \hat{c}(t))x_3 - k_3x_3 \\ \dot{x}_4 &= -(q - \hat{q}(t))x_1 - (p - \hat{p}(t)x_2 - k_4x_4 \end{aligned} \tag{24}$$

To simplify (24), we define the parameter estimation error as

$$\begin{aligned} e_a(t) &= a - \hat{a}(t) \\ e_b(t) &= b - \hat{b}(t) \\ e_c(t) &= c - \hat{c}(t) \\ e_p(t) &= p - \hat{p}(t) \\ e_q(t) &= q - \hat{q}(t) \end{aligned} \tag{25}$$

Substituting (25) into (24), we obtain

$$\begin{aligned} \dot{x}_1 &= -e_ax_2 + e_px_4 - k_1x_1 \\ \dot{x}_2 &= e_bx_2 + e_px_4 - k_2x_2 \\ \dot{x}_3 &= -e_cx_3 - k_3x_3 \\ \dot{x}_4 &= -e_qx_1 - e_px_2 - k_4x_4 \end{aligned} \tag{26}$$

Differentiating the parameter estimation error (25) with respect to t, we get

$$\begin{aligned}\dot{e}_a(t) &= -\dot{\hat{a}}(t)\\ \dot{e}_b(t) &= -\dot{\hat{b}}(t)\\ \dot{e}_c(t) &= -\dot{\hat{c}}(t)\\ \dot{e}_p(t) &= -\dot{\hat{p}}(t)\\ \dot{e}_q(t) &= -\dot{\hat{q}}(t)\end{aligned} \tag{27}$$

Next, we find an update law for parameter estimates using Lyapunov stability theory.

Consider the quadratic Lyapunov function defined by

$$V(x_1,x_2,x_3,x_4,e_a,e_b,e_c,e_p,e_q)=\frac{1}{2}\left(x_1^2+x_2^2+x_3^2+x_4^2+e_a^2+e_b^2+e_c^2+e_p^2+e_q^2\right), \tag{28}$$

which is positive definite on R^9.

Differentiating V along the trajectories of (26) and (27), we obtain

$$\begin{aligned}\dot{V} = -k_1x_1^2-k_2x_2^2-k_3x_3^2-k_4x_4^2+e_a\left[-x_1x_2-\dot{\hat{a}}\right]+e_b\left[x_2^2-\dot{\hat{b}}\right]+e_c\left[-x_3^2-\dot{\hat{c}}\right]\\ +e_p\left[x_1x_4-\dot{\hat{p}}\right]+e_q\left[-x_1x_4-\dot{\hat{q}}\right]\end{aligned} \tag{29}$$

In view of (29), we define an update law for the parameter estimates as

$$\begin{aligned}\dot{\hat{a}} &= -x_1x_2\\ \dot{\hat{b}} &= x_2^2\\ \dot{\hat{c}} &= -x_3^2\\ \dot{\hat{p}} &= x_1x_4\\ \dot{\hat{q}} &= -x_1x_4\end{aligned} \tag{30}$$

Theorem 1. The 4-D novel hyperchaotic system (22) with unknown parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law (23) and the parameter update law (30), where k_i, $(i=1,2,3,4)$ are positive gain constants.

Proof. The result is proved using Lyapunov stability theory [134]. We consider the quadratic Lyapunov function V defined by (19), which is a positive definite function on R^9.

Substituting the parameter update law (21) into (20), we obtain $\dot{V}$ as

$$\dot{V} = -k_1x_1^2-k_2x_2^2-k_3x_3^2-k_4x_4^2 \tag{31}$$

which is a negative semi-definite function on R^9.

Therefore, it can be concluded that the state vector $x(t)$ and the parameter estimation error are globally bounded, i.e.

$$\begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & x_4(t) & e_a(t) & e_b(t) & e_c(t) & e_p(t) & e_q(t) \end{bmatrix}^T \in L_\infty. \tag{32}$$

We define

$$k = \min\{k_1, k_2, k_3, k_4\}. \tag{33}$$

Then it follows from (21) that

$$\dot{V} \le -k\|x\|^2 \text{ or } k\|x\|^2 \le -\dot{V}. \tag{34}$$

Integrating the inequality (34) from 0 to t, we get

$$k\int_o^t \|x(\tau)\|^2 \, d\tau \;\le\; -\int_0^t \dot{V}(\tau)d\tau = V(0) - V(t) \tag{35}$$

From (35), it follows that $x(t) \in L_2$.

Using (26), we can conclude that $\dot{x}(t) \in L_\infty$.

Hence, using Barbalat's lemma, we can conclude that $x(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0) \in R^3$.

This completes the proof.

Numerical Results

For the system (22), the parameter values are taken as in the hyperchaotic case, *viz.*

$$a = 1.5, \quad b = 3, \quad c = 5.8, \quad p = 1.6, \quad q = 0.5 \tag{36}$$

We take the feedback gains as $k_i = 6$ for $i = 1,2,3,4$.

The initial values of the hyperchaotic system (22) are taken as

$$x_1(0) = 4.2, \; x_2(0) = 2.7, \; x_3(0) = -3.8, \;\; x_4(0) = 7.4 \tag{37}$$

The initial values of the parameter estimates are taken as

$$\hat{a}(0) = 6, \; \hat{b}(0) = 5.1, \; \hat{c}(0) = 2.5, \; \hat{d}(0) = 3.4, \; \hat{p}(0) = 4.2 \tag{38}$$

Figure 7 depicts the time history of the controlled novel hyperchaotic system.

5. ADAPTIVE SYNCHRONIZATION OF THE IDENTICAL NOVEL HYPERCHAOTIC SYSTEMS

In this section, we derive new results for the adaptive synchronization of the identical novel hyperchaotic systems with unknown parameters.

As the master system, we take the novel 4-D hyperchaotic system

$$\begin{aligned} \dot{x}_1 &= -x_1 - ax_2 + x_2x_3 + px_4 \\ \dot{x}_2 &= bx_2 - x_1x_3 + px_4 \\ \dot{x}_3 &= -cx_3 + x_1x_2 \\ \dot{x}_4 &= -qx_1 - px_2 \end{aligned} \tag{39}$$

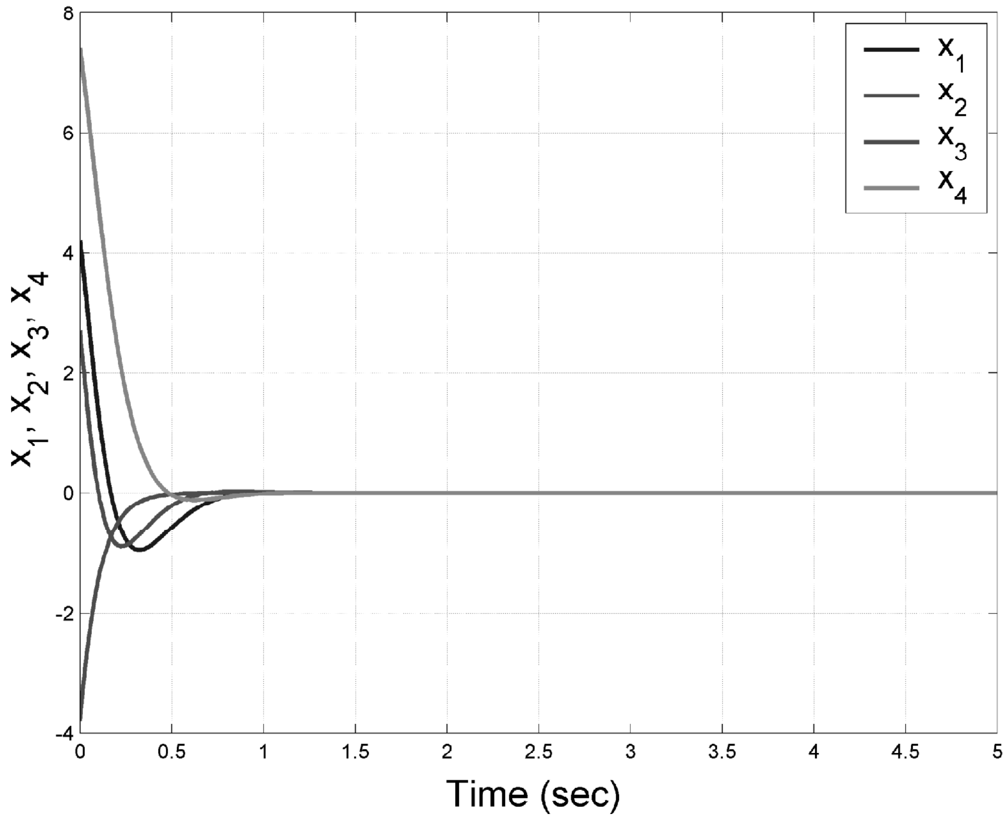

Figure 7: Time history of the controlled novel hyperchaotic system

where x_1, x_2, x_3, x_4 are state variables and a, b, c, p, q are constant, unknown, parameters of the system.

As the slave system, we take the controlled novel 4-D hyperchaotic system

$$
\begin{aligned}
\dot{y}_1 &= -y_1 - ay_2 + y_2 y_3 + py_4 + u_1 \\
\dot{y}_2 &= by_2 - y_1 y_3 + py_4 + u_2 \\
\dot{y}_3 &= -cy_3 + y_1 y_2 + u_3 \\
\dot{y}_4 &= -qy_1 - py_2 + u_4
\end{aligned} \tag{40}
$$

where y_1, y_2, y_3, y_4 are state variables and u_1, u_2, u_3, u_4 are adaptive controllers to be designed.

The synchronization error is defined by

$$
e_i = y_i - x_i, \quad (i = 1, 2, 3, 4) \tag{41}
$$

The error dynamics is easily obtained as

$$
\begin{aligned}
\dot{e}_1 &= -e_1 - ae_2 + pe_4 + y_2 y_3 - x_2 x_3 + u_1 \\
\dot{e}_2 &= be_2 + pe_4 - y_1 y_3 + x_1 x_3 + u_2 \\
\dot{e}_3 &= -ce_3 + y_1 y_2 - x_1 x_2 + u_3 \\
\dot{e}_4 &= -qe_1 - pe_2 + u_4
\end{aligned} \tag{42}
$$

We consider the adaptive control law defined by

$$
\begin{aligned}
u_1 &= e_1 + \hat{a}(t)e_2 - \hat{p}(t)e_4 - y_2 y_3 + x_2 x_3 - k_1 e_1 \\
u_2 &= -\hat{b}(t)e_2 - \hat{p}(t)e_4 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\
u_3 &= \hat{c}(t)e_3 - y_1 y_2 + x_1 x_2 - k_3 e_3 \\
u_4 &= \hat{q}(t)e_1 + \hat{p}(t)e_2 - k_4 e_4
\end{aligned}
\tag{43}
$$

where k_1, k_2, k_3, k_4 are positive gain constants.

Substituting (43) into (42), we get the closed-loop error dynamics as

$$
\begin{aligned}
\dot{e}_1 &= -[a - \hat{a}(t)]e_2 + [p - \hat{p}(t)]e_4 - k_1 e_1 \\
\dot{e}_2 &= [b - \hat{b}(t)]e_2 + [p - \hat{p}(t)]e_4 - k_2 e_2 \\
\dot{e}_3 &= -[c - \hat{c}(t)]e_3 - k_3 e_3 \\
\dot{e}_4 &= -[q - \hat{q}(t)]e_1 - [p - \hat{p}(t)]e_2 - k_4 e_4
\end{aligned}
\tag{44}
$$

To simplify the error dynamics (44), we define the parameter estimation error as

$$
\begin{aligned}
e_a(t) &= a - \hat{a}(t) \\
e_b(t) &= b - \hat{b}(t) \\
e_c(t) &= c - \hat{c}(t) \\
e_p(t) &= p - \hat{p}(t) \\
e_q(t) &= q - \hat{q}(t)
\end{aligned}
\tag{45}
$$

Using (45), we can simplify the error dynamics (44) as

$$
\begin{aligned}
\dot{e}_1 &= -e_a e_2 + e_p e_4 - k_1 e_1 \\
\dot{e}_2 &= e_b e_2 + e_p e_4 - k_2 e_2 \\
\dot{e}_3 &= -e_c e_3 - k_3 e_3 \\
\dot{e}_4 &= -e_q e_1 - e_p e_2 - k_4 e_4
\end{aligned}
\tag{46}
$$

Differentiating the parameter estimation error (45) with respect to t, we get

$$
\begin{aligned}
\dot{e}_a(t) &= -\dot{\hat{a}}(t) \\
\dot{e}_b(t) &= -\dot{\hat{b}}(t) \\
\dot{e}_c(t) &= -\dot{\hat{c}}(t) \\
\dot{e}_p(t) &= -\dot{\hat{p}}(t) \\
\dot{e}_q(t) &= -\dot{\hat{q}}(t)
\end{aligned}
\tag{47}
$$

Next, we find an update law for parameter estimates using Lyapunov stability theory.

Consider the quadratic Lyapunov function defined by

$$V(e_1,e_2,e_3,e_4,e_a,e_b,e_c,e_p,e_q)=\frac{1}{2}\left(e_1^2+e_2^2+e_3^2+e_4^2+e_a^2+e_b^2+e_c^2+e_p^2+e_q^2\right), \tag{48}$$

which is positive definite on R^9.

Differentiating V along the trajectories of (46) and (47), we obtain

$$\begin{aligned}\dot{V}=-k_1e_1^2-k_2e_2^2-k_3e_3^2-k_4e_4^2+e_a\left[-e_1e_2-\dot{\hat{a}}\right]+e_b\left[e_2^2-\dot{\hat{b}}\right]+e_c\left[-e_3^2-\dot{\hat{c}}\right]\\ +e_p\left[e_1e_4-\dot{\hat{p}}\right]+e_q\left[-e_1e_4-\dot{\hat{q}}\right]\end{aligned} \tag{49}$$

In view of (49), we define an update law for the parameter estimates as

$$\begin{aligned}\dot{\hat{a}}&=-e_1e_2\\ \dot{\hat{b}}&=e_2^2\\ \dot{\hat{c}}&=-e_3^2\\ \dot{\hat{p}}&=e_1e_4\\ \dot{\hat{q}}&=-e_1e_4\end{aligned} \tag{50}$$

Theorem 2. The identical novel hyperchaotic systems (39) and (40) with unknown system parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (43) and the parameter update law (50), where $k_i,\ (i=1,2,3,4)$ are positive constants.

Proof. The result is proved using Lyapunov stability theory [134]. We consider the quadratic Lyapunov function V defined by (39), which is a positive definite function on R^9.

Substituting the parameter update law (50) into (49), we obtain $\dot{V}$ as

$$\dot{V}=-k_1e_1^2-k_2e_2^2-k_3e_3^2-k_4e_4^2 \tag{51}$$

which is a negative semi-definite function on R^8.

Thus, it can be concluded that the synchronization vector $e(t)$ and the parameter estimation error are globally bounded, i.e.

$$\left[e_1(t)\quad e_2(t)\quad e_3(t)\quad e_4(t)\quad e_a(t)\quad e_b(t)\quad e_c(t)\quad e_p(t)\quad e_q(t)\right]^T\in L_\infty. \tag{52}$$

We define

$$k=\min\{k_1,k_2,k_3,k_4\}. \tag{53}$$

Then it follows from (51) that

$$\dot{V}\le -k\|e\|^2 \text{ or } k\|e\|^2\le -\dot{V}. \tag{54}$$

Integrating the inequality (54) from 0 to t, we get

$$k\int_o^t\|e(\tau)\|^2\,d\tau\ \le\ -\int_0^t\dot{V}(\tau)d\tau=V(0)-V(t) \tag{55}$$

Therefore, we can conclude that $e(t)\in L_2$.

Using (37), we can conclude that $\dot{e}(t) \in L_\infty$.

Hence, using Barbalat's lemma, we can conclude that $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in R^3$.

This completes the proof.

Numerical Results

For the novel chaotic systems, the parameter values are taken as in the chaotic case, *viz.*

$$a = 1.5, \quad b = 3, \quad c = 5.8, \quad p = 1.6, \quad q = 0.5 \tag{56}$$

We take the feedback gains as $k_i = 6$ for $i = 1, 2, 3, 4$.

The initial values of the master system (39) are taken as

$$x_1(0) = 5.2, \; x_2(0) = 2.7, \; x_3(0) = 6.2, \quad x_4(0) = 4.9 \tag{57}$$

The initial values of the slave system (40) are taken as

$$y_1(0) = 2.8, \; y_2(0) = 6.3, \; y_3(0) = 3.7, \quad y_4(0) = 2.5 \tag{58}$$

The initial values of the parameter estimates are taken as

$$\hat{a}(0) = 8, \; \hat{b}(0) = 4.3, \; \hat{c}(0) = 7.5, \; \hat{d}(0) = 2.1, \; \hat{p}(0) = 5.3 \tag{59}$$

Figures 8-11 depicts the complete synchronization of the identical novel chaotic systems.

Figure 12 depicts the time-history of the synchronization errors.

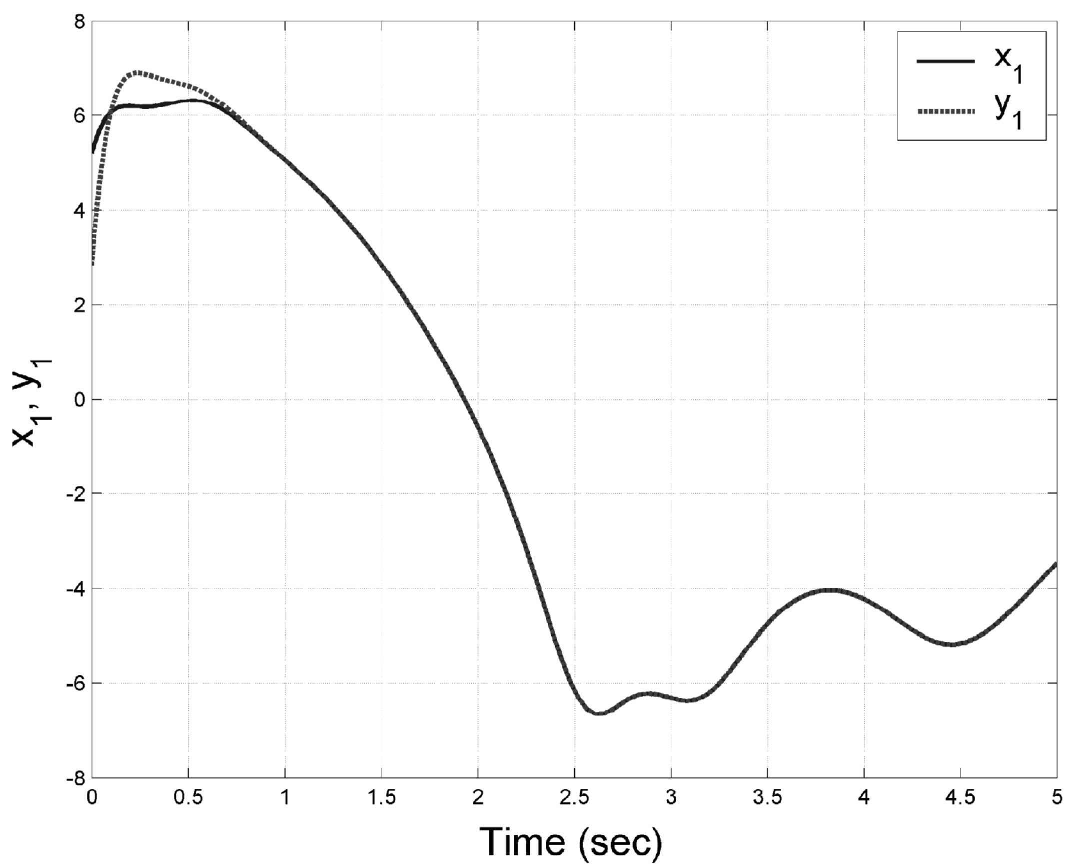

Figure 8: Complete synchronization of the states x_1 and y_1

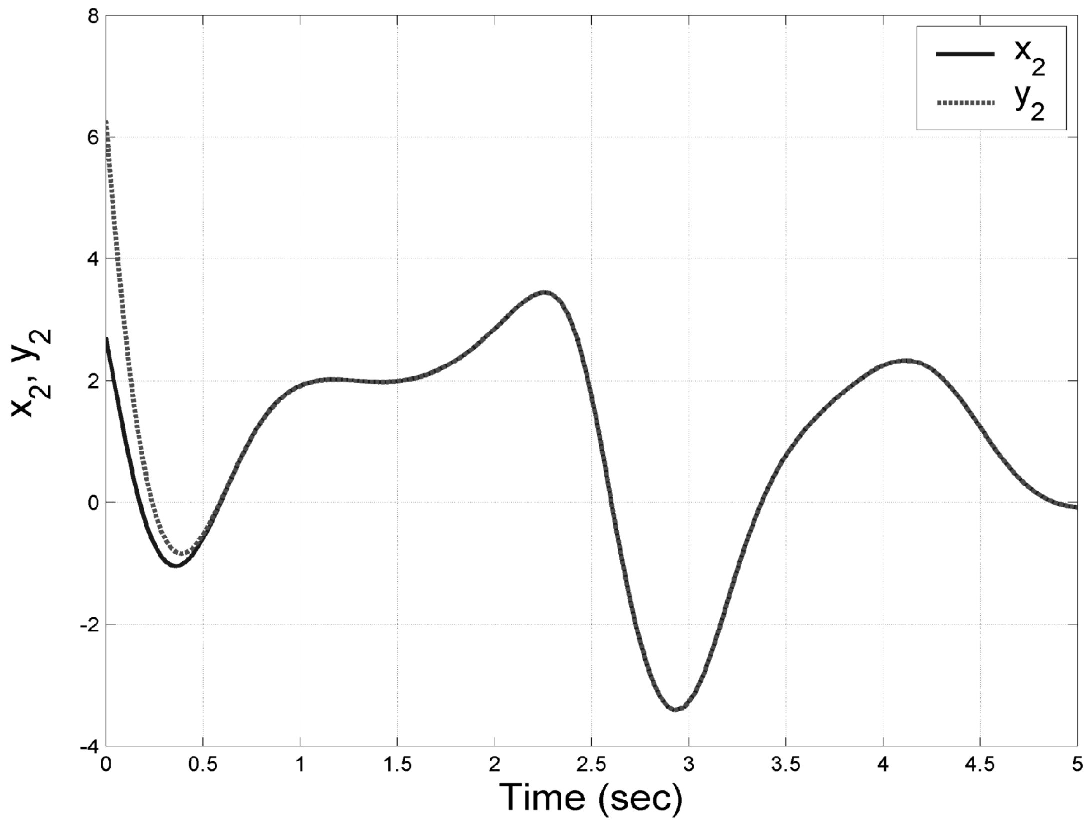

Figure 9: Complete synchronization of the states x_2 and y_2

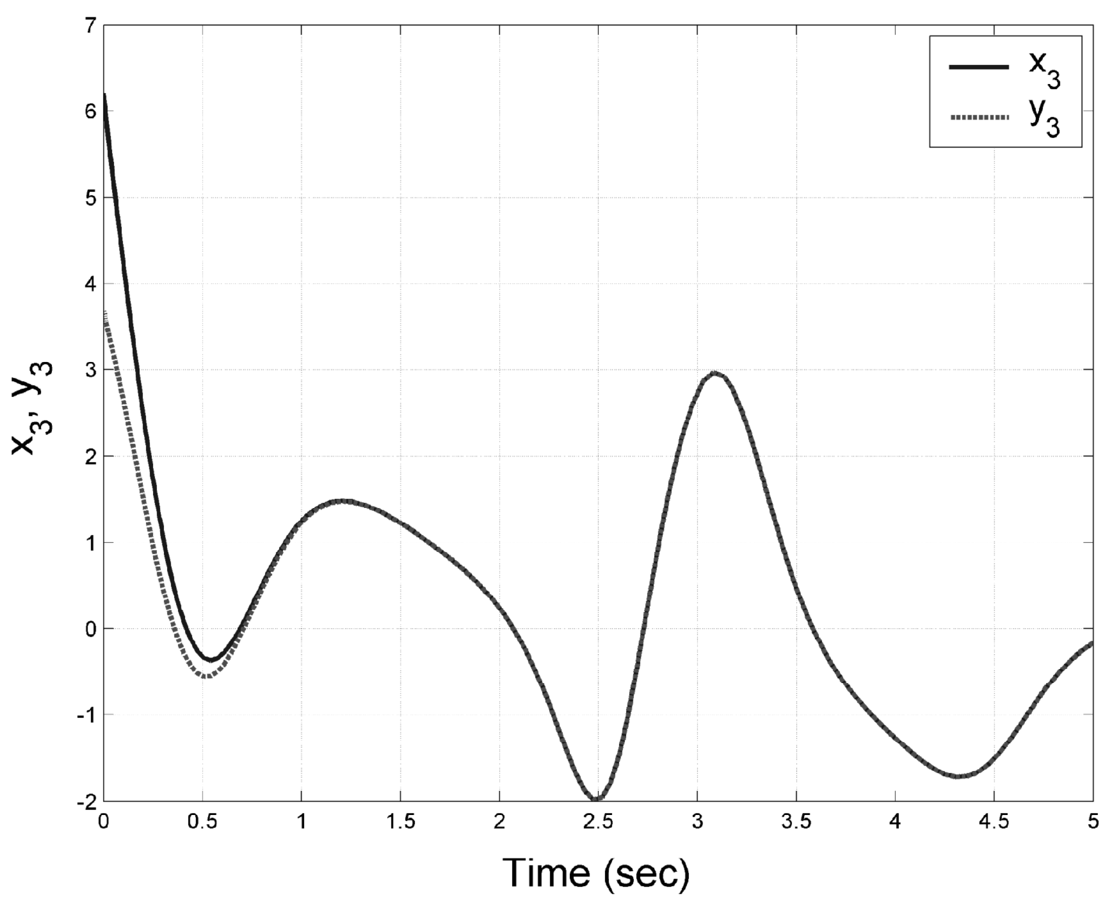

Figure 10: Complete synchronization of the states x_3 and y_3

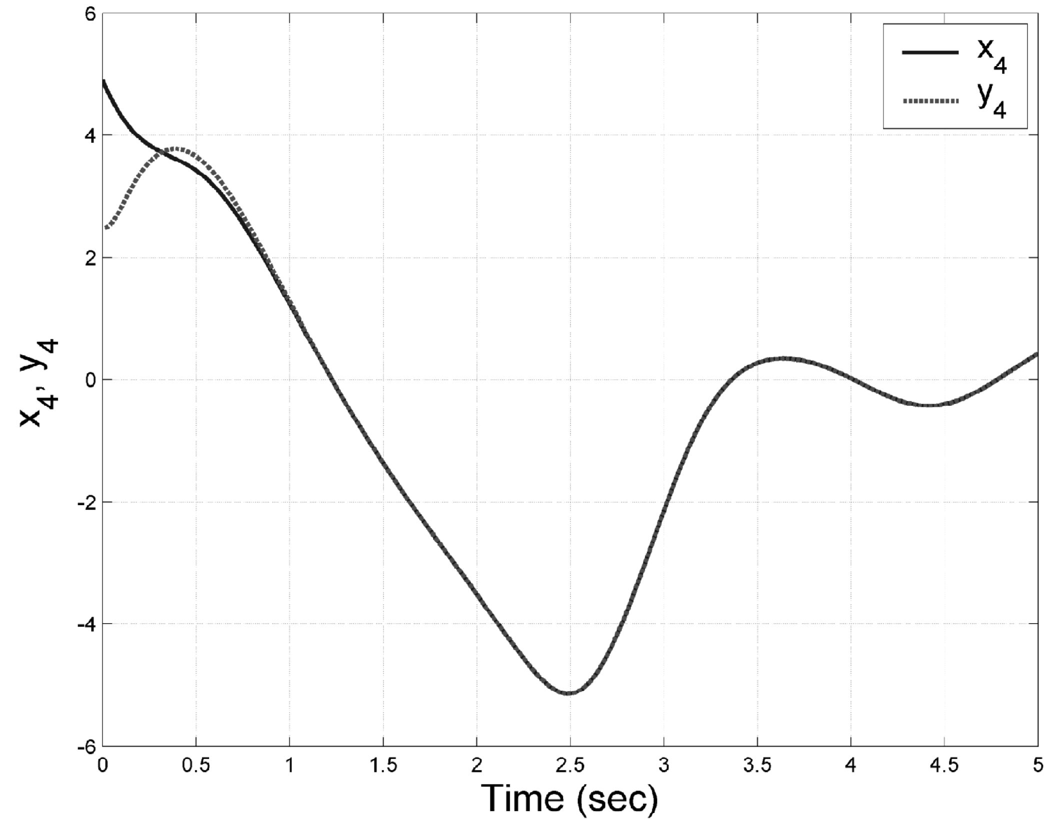

Figure 11: Complete synchronization of the states x_4 and y_4

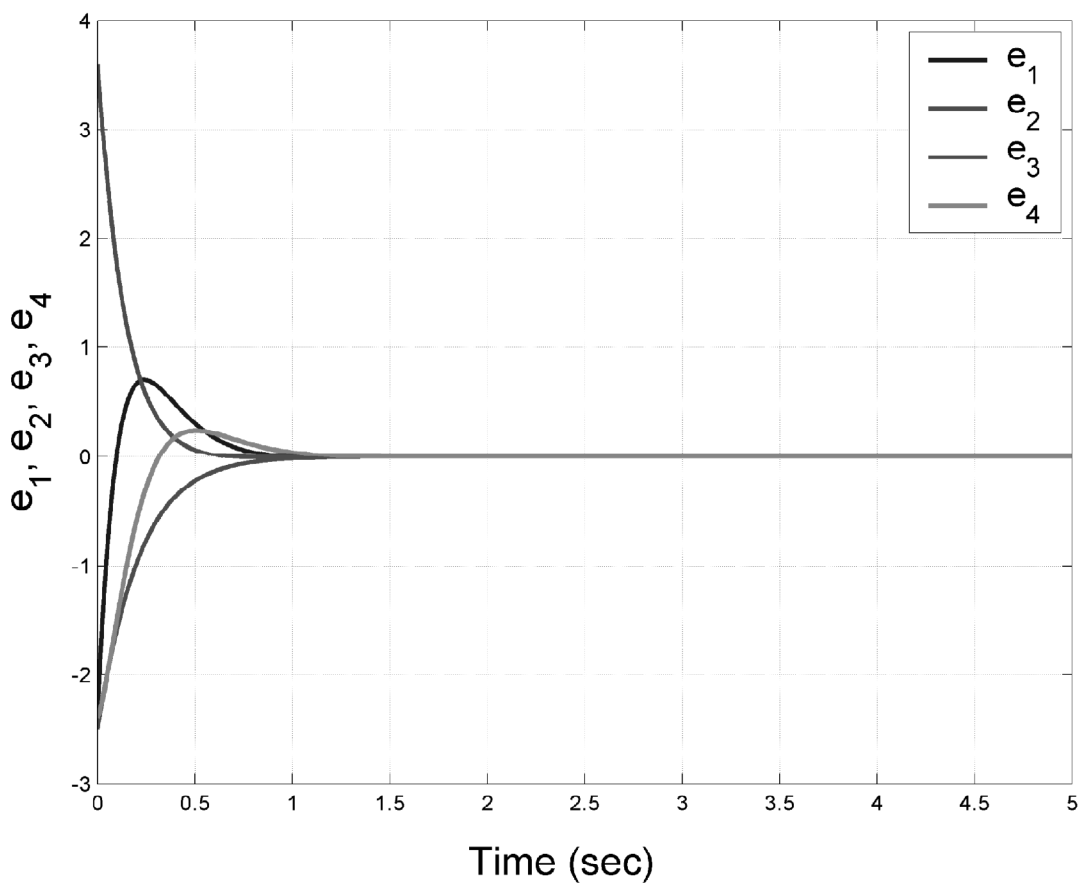

Figure 12: Time history of the chaos synchronization errors e_1, e_2, e_3

6. CONCLUSIONS

In this paper, we have derived an eleven-term 4-D novel hyperchaotic system with two positive Lyapunov exponents. We gave a qualitative analysis of the mathematical properties of the novel 4-D hyperchaotic system. We determined the Lyapunov exponents and Lyapunov dimension of the 4-D hyperchaotic system. Next, we have derived adaptive control and synchronization results for the novel hyperchaotic system with unknown parameters, which have been established using Lyapunov stability theory. Numerical simulations with MATLAB were exhibited to demonstrate the phase portraits of the novel hyperchaotic system and the adaptive results derived in this paper for adaptive control and adaptive synchronization of the 4-D novel hyperchaotic system.

References

[1] A.T. Azar and S. Vaidyanathan, *Chaos Modeling and Control Systems Design*, Springer, New York, USA, 2015.

[2] E.N. Lorenz, "Deterministic nonperiodic flow," *Journal of Atmospheric Sciences*, **20**, 130-141, 1963.

[3] O.E. Rössler, "An equation for continuous chaos," *Physics Letters*, **57A**, 397-398, 1976.

[4] T. Shimizu and N. Morioka, "On the bifurcation of a symmetric limit cycle to an assymetric one in a simple model," *Physics Letters A*, **76**, 201-204, 1980.

[5] R. Shaw, "Strange attractors, chaotic behavior and information flow," *Z. Naturforsch*, **36A**, 80-112, 1981.

[6] G. Chen and T. Ueta, "Yet another chaotic oscillator," *International Journal of Bifurcation and Chaos*, **9**, 1465-1466, 1999.

[7] J. Lü and G. Chen, "A new chaotic attractor coined," *International Journal of Bifurcation and Chaos*, **12**, 659-661, 2002.

[8] H.K. Chen and C.I. Lee, "Anti-control of chaos in rigid body motion," *Chaos, Solitons and Fractals*, **21**, 957-965, 2004.

[9] G. Cai and Z. Tan, "Chaos synchronization of a new chaotic system via nonlinear control," *Journal of Uncertain Systems*, **1**, 235-240, 2007.

[10] G. Tigan and D. Opris, "Analyis of a 3D chaotic system," *Chaos, Solitons and Fractals*, **36**, 1315-1319, 2008.

[11] D. Li, "A three-scroll chaotic attractor," *Physics Letters A*, **372**, 387-393, 2008.

[12] V. Sundarapandian and I. Pehlivan, "Analysis, control, synchronization and circuit design of a novel chaotic system," *Mathematics and Computer Modelling*, **55**, 1904-1915, 2012.

[13] V. Sundarapandian, "Analysis and anti-synchronization of a novel chaotic system via active and adaptive controllers," *Journal of Engineering Science and Technology Review*, **6**, 45-52, 2013.

[14] S. Vaidyanathan, "A new six-term 3-D chaotic system with an exponential nonlinearity," *Far East Journal of Mathematical Sciences*, **79**, 135-143, 2013.

[15] S. Vaidyanathan, "Analysis and adaptive synchronization of two novel chaotic systems with hyperbolic sinusoidal and cosinusoidal nonlinearity and unknown parameters," Journal of Engienering Science and Technology Review, **6**, 53-65, 2013.

[16] S. Vaidyanathan, "A new eight-term 3-D polynomial chaotic system with three quadratic nonlinearities," *Far East Journal of Mathematical Sciences*, **84**, 219-226, 2014.

[17] S. Vaidyanathan, "Analysis, control and synchronisation of a six-term novel chaotic system with three quadratic nonlinearities," *International Journal of Modelling, Identification and Control*, **22**, 41-53, 2014.

[18] S. Vaidyanathan, "Analysis and adaptive synchronisation of eight-term 3-D polynomial chaotic systems with three quadratic nonlinearities," *European Physical Journal: Special Topics*, **223**, 1519-1529, 2014.

[19] S. Vaidyanathan, "Generalised projective synchronisation of novel 3-D chaotic systems with an exponential non-linearity via active and adaptive control," *International Journal of Modelling, Identification and Control*, **22**, 207-217, 2014.

[20] S. Vaidyanathan, "Qualitative analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system with a quartic nonlinearity," *International Journal of Control Theory and Applications*, **7**, 1-20, 2014.

[21] S. Vaidyanathan and K. Madhavan, "Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system," *International Journal of Control Theory and Applications*, **6**, 121-137, 2013.

[22] S. Vaidyanathan and A.T. Azar, "Analysis, control and synchronization of a nine-term 3-D novel chaotic system," *Studies in Computational Intelligence*, **581**, 19-38, 2015.

[23] S. Vaidyanathan, Ch. K. Volos, V.T. Pham, K. Madhavan and B.A. Idowu, "Adaptive backstepping control, synchronization and circuit simulation of a 3-D novel jerk chaotic system with two hyperbolic sinusoidal nonlinearities," *Archives of Control Sciences*, **24**, 257-285, 2014.

[24] S. Vaidyanathan, Ch. K. Volos and V.T Pham, "Global chaos control of a novel nine-term chaotic system via sliding mode control," *Studies in Computational Intelligence*, **576**, 571-589, 2015.

[25] I. Pehlivan, I.M. Moroz and S. Vaidyanathan, "Analysis, synchronization and circuit design of a novel butterfly attractor," *Journal of Sound and Vibration*, **333**, 5077-5096, 2014.

[26] V.T. Pham, Ch.K. Volos and S. Vaidyanathan, "Multi-scroll chaotic oscillator based on a first-order delay differential equation," *Studies in Computational Intelligence*, **581**, 59-72, 2015.

[27] O.E. Rössler, "An equation for hyperchaos," *Physics Letters A*, **71**, 155-157, 1979.

[28] Q. Jia, "Hyperchaos generated from the Lorenz chaotic system and its control," *Physics Letters A*, **366**, 217-222, 2007.

[29] A. Chen, J. Lu, J. Lü and S. Yu, "Generating hyperchaotic Lü attractor via state feedback control," *Physica A*, **364**, 103-110, 2006.

[30] X. Li, "Modified projective synchronizatoin of a new hyperchaotic system via nonlinear control," *Communications in Theoretical Physics*, **52**, 274-278, 2009.

[31] J. Wang and Z. Chen, "A novel hyperchaotic system and its complex dynamics," *International Journal of Bifurcation and Chaos*, **18**, 3309-3324, 2008.

[32] S. Vaidyanathan, "A ten-term novel 4-D hyperchaotic system with three quadratic nonlinearities and its control," *International Journal of Control Theory and Applications*, **6**, 97-109, 2013.

[33] S. Vaidyanathan, "Qualitative analysis and control of an eleven-term novel 4-D hyperchaotic system with two quadratic nonlinearities," *International Journal of Control Theory and Applications*, **7**, 35-47, 2014.

[34] S. Vaidyanathan and A.T. Azar, "Analysis and control of a 4-D novel hyperchaotic system," *Studies in Computational Intelligence*, **581**, 3-17, 2015.

[35] S. Vaidyanathan, Ch. K. Volos and V.T. Pham, "Hyperchaos, adaptive control and synchronization of a novel 5-D hyperchaotic system with three positive Lyapunov exponents and its SPICE implementation," *Archives of Control Sciences*, **24**, 409-446, 2014.

[36] Q. Li, X.S. Yang and F. Yang, "Hyperchaos in Hopfield-type neural networks," *Neurocomputing*, **67**, 275-280, 2004.

[37] Y. Huang and X.S. Yang, "Hyperchaos and bifurcation in a new class of four-dimensional Hopfield neural networks," *Neurocomputing*, **69**, 1787-1795, 2006.

[38] P.C. Rech, "Chaos and hyperchaos in a Hopfield neural network," *Neurocomputing*, **74**, 3361-3364, 2011.

[39] K. Grygiel and P. Szlachetka, "Chaos and hyperchaos in coupled Kerr oscillators," *Optics Communications*, **177**, 425-431, 2000.

[40] Y.L. Zou, J. Zhu, G. Chen and X.S. Luo, "Synchronization of hyperchaotic oscillators via single unidirectional chaotic coupling," *Chaos, Solitons and Fractals*, **25**, 1245-1253, 2005.

[41] R. Riganti and M.G. Zavattaro, "Transition to hyperchaos in the dynamics of a nonlinear vibration absorber," *Mathematical and Computer Modelling*, **22**, 55-61, 1995.

[42] K. Thamilmaran, M. Lakshmanan and A. Venkatesan, "Hyperchaos in a modified canonical Chua's circuit," *International Journal of Bifurcation and Chaos*, **14**, 221-243, 2004.

[43] N. Yujun, W. Xingyuan, W. Mingjun and Z. Huaguang, "A new hyperchaotic system and its circuit implementation," *Communications in Nonlinear Science and Numerical Simulation*, **15**, 3518-3524, 2010.

[44] T. Banerjee, D. Biswas and B.C. Sarkar, "Design of chaotic and hyperchaotic time-delayed electronic circuit," *Bonfring International Journal of Power Systems and Integrated Circuits*, **2**, 13-17, 2012.

[45] C. Li, X. Liao and K. Wong, "Lag synchronization of hyperchaos with application to secure communications," *Chaos, Solitons and Fractals*, **23**, 183-193, 2005.

[46] X. Wu, H. Wang and H. Lu, "Modified generalized projective synchronization of a new fractional-order hyperchaotic system and its application to secure communication," *Nonlinear Analysis: Real World Applications*, **13**, 1441-1450, 2012.

[47] H. Liu, X. Wang and A. Kadir, "Color image encryption using Choquet fuzzy integral and hyper chaotic system," *Optik*, **124**, 3527-3533, 2013.

[48] X. Wu, C. Bai and H. Kan, "A new color image cryptosystem via hyperchaos synchronization," *Communications in Nonlinear Science and Numerical Simulation*, **19**, 1884-1897, 2014.

[49] A.S. Elwakil and M.P. Kennedy, "Inductorless hyperchaos generator," *Microelectronics Journal*, **30**, 739-743, 1999.

[50] X. Pu, X.J. Tian, H.Y. Zhai, L. Qiao, C.Y. Liu and Y.Q. Cui, "Simulation study on hyperchaos analysis of reforming system based on single-ring erbium-doped fiber laser," *The Journal of China Universities of Posts and Telecommunications*, **20**, 117-121, 2013.

[51] E. Ott, C. Grebogi and J.A. Yorke, "Controlling chaos," *Physical Review Letters*, **64**, 1196-1199, 1990.

[52] J. Wang, T. Zhang and Y. Che, "Chaos control and synchronization of two neurons exposed to ELF external electric field," *Chaos, Solitons and Fractals*, **34**, 839-850, 2007.

[53] V. Sundarapandian, "Output regulation of the Van der Pol oscillator," *Journal of the Institution of Engineers (India): Electrical Engineering Division*, **88**, 20-14, 2007.

[54] V. Sundarapandian, "Output regulation of the Lorenz attractor," *Far East Journal of Mathematical Sciences*, **42**, 289-299, 2010.

[55] S. Vaidyanathan, "Output regulation of the unified chaotic system," *Communications in Computer and Information Science*, **198**, 1-9, 2011.

[56] S. Vaidyanathan, "Output regulation of Arneodo-Coullet chaotic system," *Communications in Computer and Information Science*, **133**, 98-107, 2011.

[57] S. Vaidyanathan, "Output regulation of the Liu chaotic system," *Applied Mechanics and Materials*, **110**, 3982-3989, 2012.

[58] V. Sundarapandian, "Adaptive control and synchronization of uncertain Liu-Chen-Liu system," *International Journal of Computer Information Systems*, **3**, 1-6, 2011.

[59] V. Sundarapandian, "Adaptive control and synchronization of the Shaw chaotic system," *International Journal in Foundations of Computer Science and Technology*, **1**, 1-11, 2011.

[60] S. Vaidyanathan, "Sliding mode control based global chaos control of Liu-Liu-Liu-Su chaotic system," *International Journal of Control Theory and Applications*, **5**, 15-20, 2012.

[61] S. Vaidyanathan, "Global chaos control of hyperchaotic Liu system via sliding control method," *International Journal of Control Theory and Applications*, **5**, 117-123, 2012.

[62] S. Vaidyanathan, "Global chaos synchronization of identical Li-Wu chaotic systems via sliding mode control," *International Journal of Web and Grid Services*, **22**, 170-177, 2014.

[63] M. Feki, "An adaptive chaos synchronization scheme applied to secure communication," *Chaos, Solitons and Fractals*, **18**, 141-148, 2003.

[64] L. Kocarev and U. Parlitz, "General approach for chaos synchronization with applications to communications," *Physical Review Letters*, **74**, 5028-5030, 1995.

[65] K. Murali and M. Lakshmanan, "Secure communication using a compound signal using sampled-data feedback," *Applied Math. Mech.*, **11**, 1309-1315, 2003.

[66] J. Yang and F. Zhu, "Synchronization for chaotic systems and chaos-based secure communications via both reduced-order and step-by-step sliding mode observers," *Communications in Nonlinear Science and Numerical Simulation*, **18**, 926-937, 2013.

[67] L. Kocarev, "Chaos-based cryptography: a brief overview," *IEEE Circuits and Systems*, **1**, 6-21, 2001.

[68] H. Gao, Y. Zhang, S. Liang and D. Li, "A new chaotic algorithm for image encryption," *Chaos, Solitons and Fractals*, **29**, 393-399, 2006.

[69] Y. Wang, K.W. Wang, X. Liao and G. Chen, "A new chaos-based fast image encryption," *Applied Soft Computing*, **11**, 514-522, 2011.

[70] X. Zhang, Z. Zhao and J. Wang, "Chaotic image encryption based on circular substitution box and key stream buffer," *Signal Processing: Image Communication*, **29**, 902-913, 2014.

[71] L.M. Pecora and T.I. Carroll, "Synchronization in chaotic systems," Phys. Rev. Lett., 64, 821-824, 1990.

[72] L.M. Pecora and T.L. Carroll, "Synchronizing in chaotic circuits," IEEE Trans. Circ. Sys., 38, 453-456, 1991.

[73] L. Huang, R. Feng and M. Wang, "Synchronization of chaotic systems via nonlinear control," *Physics Letters A*, **320**, 271-275, 2004.

[74] V. Sundarapandian and R. Karthikeyan, "Global chaos synchronization of hyperchaotic Liu and hyperchaotic Lorenz systems by active nonlinear control", *International Journal of Control Theory and Applications*, **3**, 79-91, 2010.

[75] S. Vaidyanathan and S. Rasappan, "New results on the global chaos synchronization for Liu-Chen-Liu and Lü chaotic systems," *Communications in Computer and Information Science*, **102**, 20-27, 2010.

[76] S. Vaidyanathan and K. Rajagopal, "Anti-synchronization of Li and T chaotic systems by active nonlinear control," *Communications in Computer and Information Science*, **198**, 175-184, 2011.

[77] S. Vaidyanathan and S. Rasappan, "Global chaos synchronization of hyperchaotic Bao and Xu systems by active nonlinear control," *Communications in Computer and Information Science*, **198**, 10-17, 2011.

[78] S. Vaidyanathan and K. Rajagopal, "Global chaos synchronization of hyperchaotic Pang and Wang systems by active nonlinear control," *Communications in Computer and Information Science*, **204**, 84-93, 2011.

[79] P. Sarasu and V. Sundarapandian, "Active controller design for generalized projective synchronization of four-scroll chaotic systems," *International Journal of Systems Signal Control and Engineering Application*, **4**, 26-33, 2011.

[80] S. Vaidyanathan, "Hybrid chaos synchronization of Liu and Lü systems by active nonlinear control," *Communications in Computer and Information Science*, **204**, 1-10, 2011.

[81] P. Sarasu and V. Sundarapandian, "The generalized projective synchronization of hyperchaotic Lorenz and hyperchaotic Qi systems via active control," *International Journal of Soft Computing*, **6**, 216-223, 2011.

[82] S. Vaidyanathan and S. Rasappan, "Hybrid synchronization of hyperchaotic Qi and Lü systems by nonlinear control," *Communications in Computer and Information Science*, **131**, 585-593, 2011.

[83] S. Vaidyanathan and S. Pakiriswamy, "The design of active feedback controllers for the generalized projective synchronization of hyperchaotic Qi and hyperchaotic Lorenz systems," *Communications in Computer and Information Science*, **245**, 231-238, 2011.

[84] S. Vaidyanathan and K. Rajagopal, "Hybrid synchronization of hyperchaotic Wang-Chen and hyperchaotic Lorenz systems by active non-linear control," *International Journal of Systems Signal Control and Engineering Application*, **4**, 55-61, 2011.

[85] V. Sundarapandian and R. Karthikeyan, "Hybrid synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems via active control," *Journal of Engineering and Applied Sciences*, **7**, 254-264, 2012.

[86] S. Pakiriswamy and S. Vaidyanathan, "Generalized projective synchronization of three-scroll chaotic systems via active control," *Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering*, **85**, 146-155, 2012.

[87] S. Vaidyanathan and S. Pakiriswamy, "Generalized projective synchronization of double-scroll chaotic systems using active feedback control," *Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering*, **84**, 111-118, 2012.

[88] S. Pakiriswamy and S. Vaidyanathan, "Generalized projective synchronization of hyperchaotic Lü and hyperchaotic Cai systems via active control," *Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering*, **84**, 53-62, 2012.

[89] S. Vaidyanathan, "Complete chaos synchronization of six-term Sundarapandian chaotic systems with exponential nonlinearity via active and adaptive control," *Proceedings of the 2013 International Conference on Green Computing, Communication and Conservation of Energy*, **ICGCE 2013**, 608-613, 2013.

[90] R. Karthikeyan and V. Sundarapandian, "Hybrid chaos synchronization of four-scroll systems via active control," *Journal of Electrical Engineering*, **65**, 97-103, 2014.

[91] B. Samuel, "Adaptive synchronization between two different chaotic dynamical systems," *Adaptive Commun. Nonlinear Sci. Num. Simul.*, **12**, 976-985, 2007.

[92] J.H. Park, S.M. Lee and O.M. Kwon, "Adaptive synchronization of Genesio-Tesi system via a novel feedback control," *Physics Letters A*, **371**, 263-270, 2007.

[93] V. Sundarapandian and R. Karthikeyan, "Anti-synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems by adaptive control," *International Journal of System Signal Control and Engineering Applications*, **4**, 18-25, 2011.

[94] S. Vaidyanathan and K. Rajagopal, "Global chaos synchronization of Lü and Pan systems by adaptive nonlinear control," *Communications in Computer and Information Science*, **205**, 193-202, 2011.

[95] V. Sundarapandian and R. Karthikeyan, "Anti-synchronization of Lü and Pan chaotic systems by adaptive nonlinear control," *European Journal of Scientific Research*, **64**, 94-106, 2011.

[96] V. Sundarapandian and R. Karthikeyan, "Anti-synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems by adaptive control," *International Journal of Systems Signal Control and Engineering Application*, **4**, 18-25, 2011.

[97] P. Sarasu and V. Sundarapandian, "Generalized projective synchronization of three-scroll chaotic systems via adaptive control," *European Journal of Scientific Research*, **72**, 504-522, 2012.

[98] V. Sundarapandian and R. Karthikeyan, "Adaptive anti-synchronization of uncertain Tigan and Li systems," *Journal of Engineering and Applied Sciences*, **7**, 45-52, 2012.

[99] S. Vaidyanathan and K. Rajagopal, "Global chaos synchronization of hyperchaotic Pang and hyperchaotic Wang systems via adaptive control," *International Journal of Soft Computing*, **7**, 28-37, 2012.

[100] P. Sarasu and V. Sundarapandian, "Generalized projective synchronization of two-scroll systems via adaptive control," *International Journal of Soft Computing*, **7**, 146-156, 2012.

[101] P. Sarasu and V. Sundarapandian, "Adaptive controller design for the generalized projective synchronization of 4-scroll systems," *International Journal of Systems Signal Control and Engineering Application*, **5**, 21-30, 2012.

[102] S. Vaidyanathan, "Adaptive controller and synchronizer design for the Qi-Chen chaotic system," *Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering*, **85**, 124-133, 2012.

[103] S. Vaidyanathan, "Anti-synchronization of Sprott-L and Sprott-M chaotic systems via adaptive control," *International Journal of Control Theory and Applications*, **5**, 41-59, 2012.

[104] V. Sundarapandian, "Adaptive control and synchronization design for the Lu-Xiao chaotic system," *Springer-Verlag Lecture Notes in Electrical Engineering*, **131**, 319-327, 2013.

[105] S. Vaidyanathan and S. Pakiriswamy, "Generalized projective synchronization of six-term Sundarapandian chaotic systems by adaptive control," *International Journal of Control Theory and Applications*, **6**, 153-163, 2013.

[106] S. Vaidyanathan and S. Pakiriswamy, "Generalized projective synchronization of Elhadj chaotic systems via adpative control," *Proceedings of the 2013 International Conference on Green Computing, Communication and Conservation of Energy*, **ICGCE 2013**, 614-618, 2013.

[107] S. Vaidyanathan, "Anaysis, control and synchronization of hyperchaotic Zhou system via adaptive control," *Advances in Intelligent Systems and Computing*, **177**, 1-10, 2013.

[108] T. Yang and L.O. Chua, "Control of chaos using sampled-data feedback control," *International Journal of Bifurcation and Chaos*, **9**, 215-219, 1999.

[109] N. Li, Y. Zhang, J. Hu and Z. Nie, "Synchronization for general complex dynamical networks with sampled-data", *Neurocomputing*, **74**, 805-811, 2011.

[110] J.H. Park and O.M. Kwon, "A novel criterion for delayed feedback control of time-delay chaotic systems," *Chaos, Solitons and Fractals*, **17**, 709-716, 2003.

[111] X. Wu and J. Lü, "Parameter identification and backstepping control of uncertain Lü system," *Chaos, Solitons and Fractals*, **18**, 721-729, 2003.

[112] Y.G. Yu and S.C. Zhang, "Adaptive backstepping synchronization of uncertain chaotic systems," *Chaos, Solitons and Fractals*, **27**, 1369-1375, 2006.

[113] S. Vaidyanathan and S. Rasappan, "Global chaos synchronization of Chen-Lee systems via backstepping control," *IEEE-International Conference on Advances in Engineering, Science and Management*, **ICAESM-2012**, 73-77, 2012.

[114] R. Suresh and V. Sundarapandian, "Global chaos synchronization of WINDMI and Coullet chaotic systems by backstepping control," *Far East Journal of Mathematical Sciences*, **67**, 265-287, 2012.

[115] S. Rasappan and S. Vaidyanathan, "Hybrid synchronization of n-scroll Chua and Lur'e chaotic systems via backstepping control with novel feedback," *Archives of Control Sciences*, **22**, 343-365, 2012.

[116] S. Rasappan and S. Vaidyanathan, "Synchronization of hyperchaotic Liu via backstepping control with recursive feedback," *Communications in Computer and Information Science*, **305**, 212-221, 2012.

[117] S. Vaidyanathan, "Global chaos synchronization of Arneodo chaotic system via backstepping controller design", *ACM International Conference Proceeding Series*, **CCSEIT-12**, 1-6, 2012.

[118] R. Suresh and V. Sundarapandian, "Global chaos synchronization of a family of n-scroll hyperchaotic Chua circuits using backstepping controller with recursive feedback," *Far East Journal of Mathematical Sciences*, **73**, 73-95, 2013.

[119] S. Rasappan and S. Vaidyanathan, "Hybrid synchronization of n-scroll chaotic Chua circuits using adaptive backstepping control design with recursive feedback," *Malaysian Journal of Mathematical Sciences*, **7**, 219-246, 2013.

[120] S. Rasappan and S. Vaidyanathan, "Global chaos synchronization of WINDMI and Coullet chaotic systems using adaptive backstepping control design," *Kyungpook Mathematical Journal*, **54**, 293-320, 2014.

[121] S. Vaidyanathan and S. Rasappan, "Global chaos synchronization of n-scroll Chua circuit and Lur'e system using backstepping control design with recursive feedback," *Arabian Journal for Science and Engineering*, **39**, 3351-3364, 2014.

[122] S. Vaidyanathan, B.A. Idowu and A.T. Azar, "Backstepping controller design for the global chaos synchronization of Sprott's jerk systems," Studies in Computational Intelligence, **581**, 39-58, 2015.

[123] S. Vaidyanathan, "Global chaos synchronization of Lorenz-Stenflo and Qi chaotic systems by sliding mode control," *International Journal of Control Theory and Applications*, **4**, 161-172, 2011.

[124] S. Vaidyanathan and S. Sampath, "Global chaos synchronization of hyperchaotic Lorenz systems by sliding mode control," *Communications in Computer and Information Science*, **205**, 156-164, 2011.

[125] V. Sundarapandian and S. Sivaperumal, "Sliding controller design of hybrid synchronization of four-wing chaotic systems", *International Journal of Soft Computing*, **6**, 224-231, 2011.

[126] S. Vaidyanathan and S. Sampath, "Anti-synchronization of four-wing chaotic systems via sliding mode control", *International Journal of Automation and Computing*, **9**, 274-279, 2012.

[127] S. Vaidyanathan and S. Sampath, "Sliding mode controller design for the global chaos synchronization of Coullet systems," *Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering*, **84**, 103-110, 2012.

[128] S. Vaidyanathan, "Global chaos synchronization of identical Li-Wu chaotic systems via sliding mode control," *International Journal of Modelling, Identification and Control*, **22**, 170-177, 2014.

[129] C.H. Lien, L. Zhang, S. Vaidyanathan and H.R. Karimi, "Switched dynamics with its applications," *Abstract and Applied Analysis*, **2014**, art. no. 528532, 2014.

[130] S. Vaidyanathan and A.T. Azar, "Anti-synchronization of identical chaotic systems using sliding mode control and an application to Vaidyanathan-Madhavan chaotic systems," *Studies in Computational Intelligence*, **576**, 527-545, 2015.

[131] S. Vaidyanathan and A.T. Azar, "Hybrid synchronization of identical chaotic systems using sliding mode control and an application to Vaidyanathan chaotic systems," *Studies in Computational Intelligence*, **576**, 549-569, 2015.

[132] A.T. Azar and Q. Zhu, *Advances and Applications in Sliding Mode Control Systems*, Springer, New York, USA, 2015.

[133] C. Zhu, Y. Liu and Y. Guo, "Theoretic and numerical study of a new chaotic system," *Intelligent Information Management*, **2**, 104-109, 2010.

[134] H.K. Khalil, *Nonlinear Systems*, Prentice Hall of India, New Jersey, USA, 2002.