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UNSTEADY MHD FREE CONVECTION AND MASS TRANSFER FLOW OF DISSIPATIVE FLUID WITH HEAT GENERATION AND THERMAL DIFFUSION PAST AN INFINITE VERTICAL POROUS PLATE

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ABSTRACT: The problem of MHD free convection and mass transfer flow of viscous conducting fluid past an infinite vertical porous isothermal plate in the presence of heat generation, thermal diffusion and viscous dissipation is studied numerically. The Problem is governed by a coupled nonlinear system of partial differential equations. This system is solved numerically by applying explicit finite difference method. The effects of Heat generation parameter H, Soret number S_0 are examined on Velocity, Temperature and Concentration. It is found that, as Soret number S_0 and Heat source parameter H increase, velocity increases and concentration increases up to certain stage and decreases later.

Keywords: Transient MHD Free Convection flow, heat generation, porous plate, numerical solution

1. INTRODUCTION

In recent years, the problems of free convective and heat transfer flows through porous medium under the influence of a magnetic field have attracted the attention of a number of researchers because of their applications in many branches of science and technology, such as in transportation cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. On the other hand, flow through a porous medium have numerous engineering and geophysical applications, for example, in chemical engineering for filtration and purification process; in agriculture engineering to study the underground water resources; in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs. In view of these applications, the unsteady MHD free convection flows of dissipative fluids past an infinite plate have received much attention. This problem was first solved by Siegel [10] with out taking into account viscous dissipative heat and MHD by integral method. The experimental conformation of these results was presented by Goldstein and Eckert [6]. Other papers in this field are by Gebhart [4], Schetz and Eichhorn [9], Menold and Yang [7], Sparrow and Gregg [13], Chung and Anderson [2],Goldstein and Briggs [5], etc. In all these papers, the effect of viscous dissipative heat and MHD was assumed to be neglected. However, Gebhart [3] has shown that when the temperature difference is small or in high prandtl number fluids or when the gravitational field is of high intensity, viscous dissipative heat should be taken into account in steady free convection flow past a semi-infinite vertical plate. Following this assumption, Soundalgekar, Bhat and Mohiuddin [11] studied the effects of free convection currents on the flow past an impulsively started infinite isothermal vertical plate. Raptis [8] has studied free convection and mass transfer effects on the flow past an infinite moving vertical porous plate with constant suction and heat sources when free stream velocity is an oscillatory function of time. Agarwal *et al*., [1] have discussed the combined buoyancy effects of thermal and mass diffusion on MHD natural convection flows. Vajravelu [14] has studied the problem of free convection heat transfer between two long vertical plates moving in opposite directions. Recently, Soundalgekorl *et al*., [12] have studied free convection flow of an incompressible viscous dissipative fluid.

This paper deals with the numerical study of an unsteady MHD free convection and mass transfer flow of dissipative fluid past an infinite vertical porous isothermal plate with heat generation and thermal diffusion. As the problem is governed by coupled non-linear system of partial differential equations, exact solutions are not possible, explicit finite-difference method is employed.

2. Mathematical Analysis

Unsteady free convection flow of viscous incompressible conducting fluid past an infinite vertical porous isothermal plate is considered. The flow is assumed to be in the x'-direction, which is taken along the vertical plate in the upward direction and the y'-axis is taken normal to the plate. At time $t' > 0$, the surface of the plate is maintained at a uniform constant temperature T'_w and a uniform constant concentration C'_w, of a foreign fluid, which are higher than the corresponding values T'_∞ and C'_∞ respectively, sufficiently faraway from the plate. Initially the temperature of the fluid and plate are assumed to be the same. A uniform transverse magnetic field H_0 is applied in the y'-direction. It is assumed that the induced magnetic field and applied electric fields are neglected.

The equations governing the flow are:

$$\frac{\partial u'}{\partial t'} + V' \frac{\partial u'}{\partial y'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} + g\beta (T' - T'_\infty) + g\beta^* (C' - C'_\infty) - \sigma \frac{\mu_e^2 H_0^2}{\rho} u' \tag{1}$$

$$\rho C_P \left(\frac{\partial T'}{\partial t'} + V' \frac{\partial T'}{\partial y'} \right) = K \frac{\partial^2 T'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'} \right)^2 + q^* (T' - T'_\infty) \tag{2}$$

$$\frac{\partial C'}{\partial t'} + V' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} + D_1 \left(\frac{\partial^2 T'}{\partial y'^2} \right) \tag{3}$$

With the following initial and boundary conditions

$$\begin{aligned} &t' \le 0, \quad u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \text{for all} \quad y' \\ &t' > 0, \quad u' = 0, \quad T' = T'_W, \quad C' = C'_W \quad \text{at} \quad y' = 0 \\ &u' = 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty, \quad \text{as} \quad y' \to \infty \end{aligned} \tag{4}$$

Here u'-Velocity of the fluid in x'-direction, V'-Velocity of the fluid in y'-direction, g-acceleration due to gravity, C'-Concentration in the fluid near the plate, C'_w-Concentration of the plate, C'_∞-Concentration in the fluid far away from the plate, x'-Coordinate axis along the plate, y'-Coordinate axis normal to the plate, T'-Temperature of the fluid near the plate , T'_w-Temperature of the plate, T'_∞-Temperature of the fluid far away from the plate, t'-time, C_p-Specific heat at constant pressure, ρ-density of the fluid, D-Chemical Molecular diffusivity, D_1-Coefficient of thermal diffusivity, K-Thermal conductivity of the fluid, μ_e-Magnetic permeability, H_0-Magnetic field of intensity, q^*-heat generation constant, β-Coefficient of volume expansion, β^*-Coefficient of species expansion, υ-Kinematic viscosity, σ-electrical conductivity of the fluid.

We make use of the following dimensionless quantities in the basic equations and in the initial and boundary conditions to make them dimensionless.

$$u_0 = (\upsilon g \beta \Delta T)^{1/3}, \quad L = \left(\frac{g\beta\Delta T}{\upsilon^2}\right)^{-1/3}, \quad T_R = \frac{(g\beta\Delta T)^{-2/3}}{\upsilon^{-1/3}},$$

$$t = \frac{t'}{T_R}, \quad y = \frac{y'}{L}, \quad u = \frac{u'}{u_0}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty} = \frac{T' - T'_\infty}{\Delta T},$$

$$C = \frac{C' - C'_\infty}{C'_W - C'_\infty}, \quad N = \frac{\beta^*(C'_W - C'_\infty)}{\beta(T'_W - T'_\infty)}, \quad P_r = \frac{\mu C_P}{K},$$

$$E = \frac{u_0^2}{C_P \Delta T}, \quad M = \frac{\sigma\mu_e^2 H_0^2 T_R}{\rho}, \quad H = \frac{q^* L}{\rho C_P u_0},$$

$$S_0 = \frac{D_1(T'_W - T'_\infty)}{\upsilon(C'_W - C'_\infty)}, \quad S_C = \frac{L^2}{T_R.D}, \quad \gamma = \frac{-V'T_R}{L}, \tag{5}$$

Where C-Dimensionless concentration, N-ratio of mass transformation, P_r-prandtl number, S_C-Schmidt number, E-Eckert number, t-dimensionless time, u_0-Velocity of the plate, u-dimensionless velocity, T_R-Reference time, L-reference length, M-Magnetic field parameter, H-Heat source parameter, S_0-Soret number, y-dimensionless coordinate axis normal to the plate, γ-suction parameter, θ-dimensionless temperature.

We get the following governing equations

$$\frac{\partial u}{\partial t} - \gamma \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta + NC - Mu \tag{6}$$

$$P_r \frac{\partial \theta}{\partial t} - \gamma P_r \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + P_r E\left(\frac{\partial u}{\partial y}\right)^2 + P_r H.\theta \tag{7}$$

$$S_C \frac{\partial C}{\partial t} - \gamma S_C \frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} + S_C S_0 \frac{\partial^2 \theta}{\partial y^2} \tag{8}$$

The initial and boundary conditions are

$$\begin{array}{llllll} t \le 0, & u = 0, & \theta = 0, & C = 0 & \text{for all} & y \\ t > 0, & u = 0, & \theta = 1, & C = 1 & \text{at} & y = 0 \\ u = 0, & \theta \to 0, & C = 0 & & \text{as} & y \to \infty \end{array} \tag{9}$$

Equations (6), (7) and (8) are coupled non linear Partial differential equations and are to be solved by using initial and boundary conditions of equation (9). However exact or approximate solutions are not possible for this set of equations. And hence we solve these equations by explicit finite difference Method.

The equivalent finite difference scheme of equations for (6), (7) and (8) are as follows.

$$\left[\frac{u_{i,j+1} - u_{i,j}}{\Delta t}\right] - \gamma\left[\frac{u_{i+1,j} - u_{i,j}}{\Delta y}\right] = \left[\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{(\Delta y)^2}\right] + \theta_{i,j} + NC_{i,j} - Mu_{i,j} \tag{10}$$

$$P_r\left[\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t}\right] - \gamma P_r\left[\frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta y}\right]$$
$$= \left[\frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta y)^2}\right] + P_r E\left[\frac{u_{i+1,j} - u_{i,j}}{\Delta y}\right]^2 + P_r H\theta_{i,j} \tag{11}$$

$$S_C\left[\frac{C_{i,j+1} - C_{i,j}}{\Delta t}\right] - \gamma S_C\left[\frac{C_{i+1,j} - C_{i,j}}{\Delta y}\right]$$
$$= \left[\frac{C_{i+1,j} - 2C_{i,j} + C_{i-1,j}}{(\Delta y)^2}\right] + S_C S_0\left[\frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta y)^2}\right] \tag{12}$$

Here, index i refers to y and j to time t.

The mesh system is divided by taking $\Delta y = 0.1$

From the initial condition in (9), we have the following equivalent

$$u(0, 0) = 0, \quad \theta(0, 0) = 1, \; u(i, 0) = 0, \; \theta(i, 0) = 0 \quad \text{for all } i \text{ except } i = 0 \tag{13}$$

The boundary conditions from (9) are expressed in finite difference form as follows

$$u(0, j) = 0, \quad \theta(0, j) = 1, \quad C(0, j) = 1, \quad \text{for all} \quad j$$
$$u(4.1, j) = 0, \quad \theta(4.1, j) = 1, \quad C(4.1, j) = 1, \quad \text{for all} \quad j \tag{14}$$

Here infinity is taken as $y = 4.1$ because from the exact solution of equations (6), (7) and (8) for $E = 0$, it has been observed that u, θ and C tend to zero around $y \sim 4$ (Soundalgekar [12]).

First the velocity at the end of time step viz $u(i, j + 1)$, $(i = 1, 40)$ is computed from equation (10) and temperature $\theta(i, j + 1)$, $(i = 1, 40)$ from (11) and concentration $C(i, j + 1)$, $(i = 1, 40)$ from (12). The Procedure is repeated until $t = 1$ (i.e. $j = 800$). During computation Δt was chosen as 0.00125. These computations are carried out for $P_r = 0.71$, $E = 0.1$, $S_c = 0.3$, $N = 0.2$, $\gamma = 0.4$, $M = 5$, $H = 0, 0.5, 1.2, 2, 5$, and $S_0 = 0, 0.2, 1.2, 2, 5$ and $t = 0.2, 0.4$. To judge the accuracy of the convergence of the finite difference scheme, the same program was run with smaller values of Δt i.e., $\Delta t = 0.0009, 0.001$ and no significant change was observed. Hence we conclude the finite difference scheme is stable and convergent.

3. RESULTS

The velocity profiles are drawn in Figures 1 to 5. From Fig. 1 for fixed values of $P_r = 0.71$, $\gamma = 0.4$, $E = 0.1$, $S_c = 0.3$, $N = 0.2$, $M = 5$, $H = 0$, $t = 0.2$ we notice that as S_o

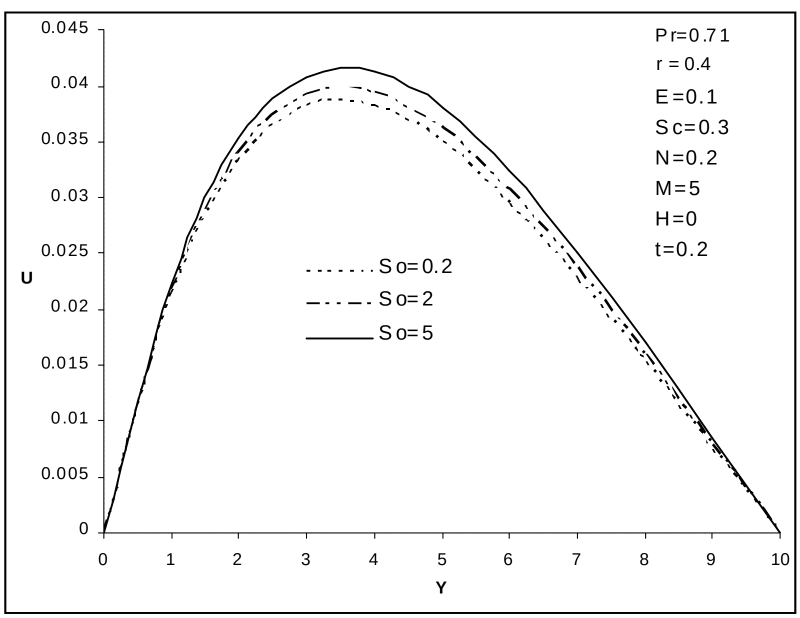

Figure 1: Velocity Profiles for Different Values of S_0

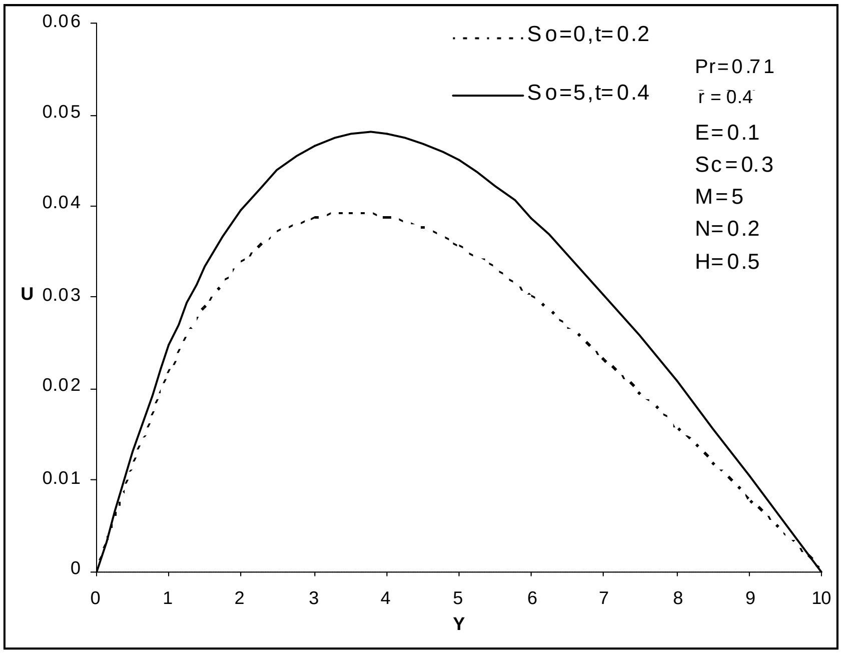

Figure 2: Velocity Profiles for Different Values of S_0, t

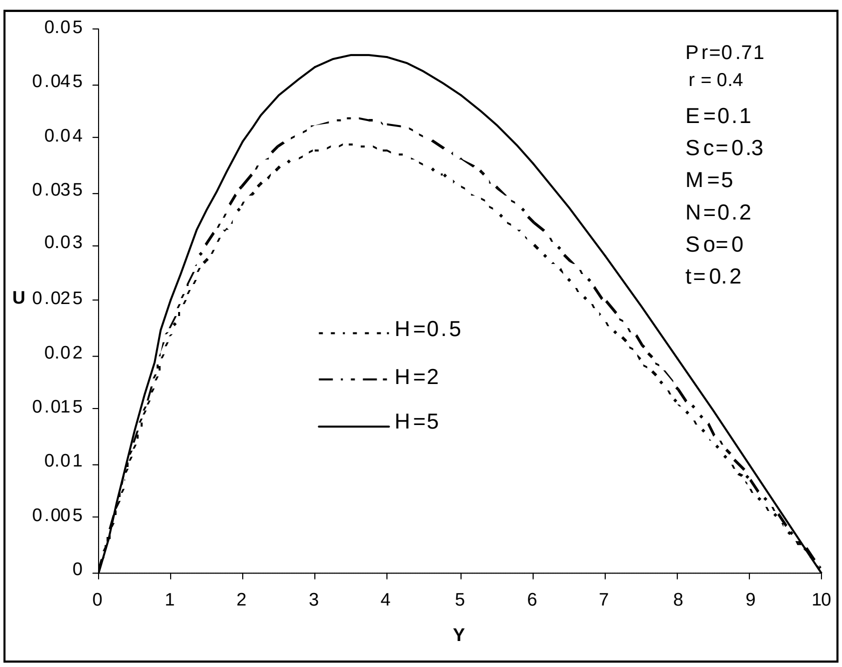

Figure 3: Velocity Profiles for Different Values of H

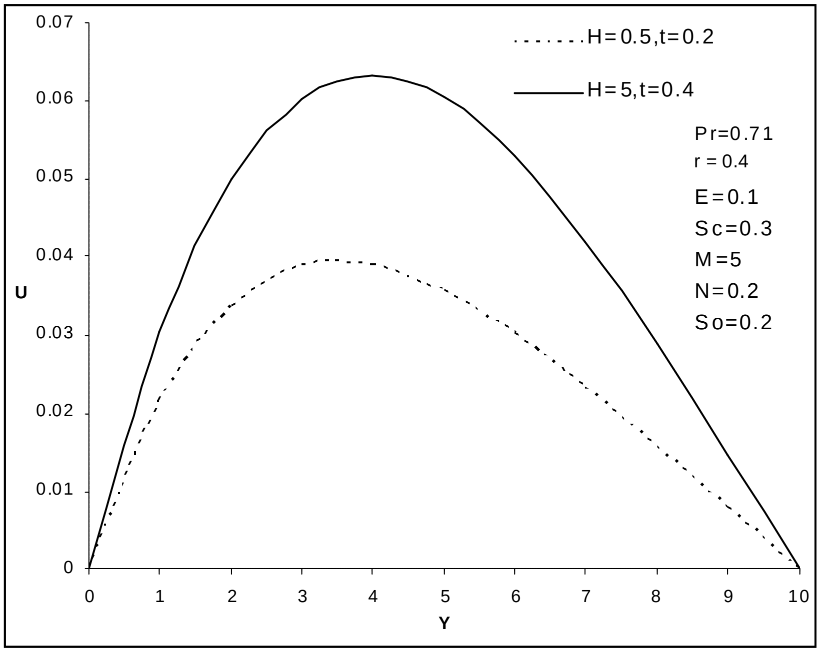

Figure 4: Velocity Profiles for Different Values of H, t

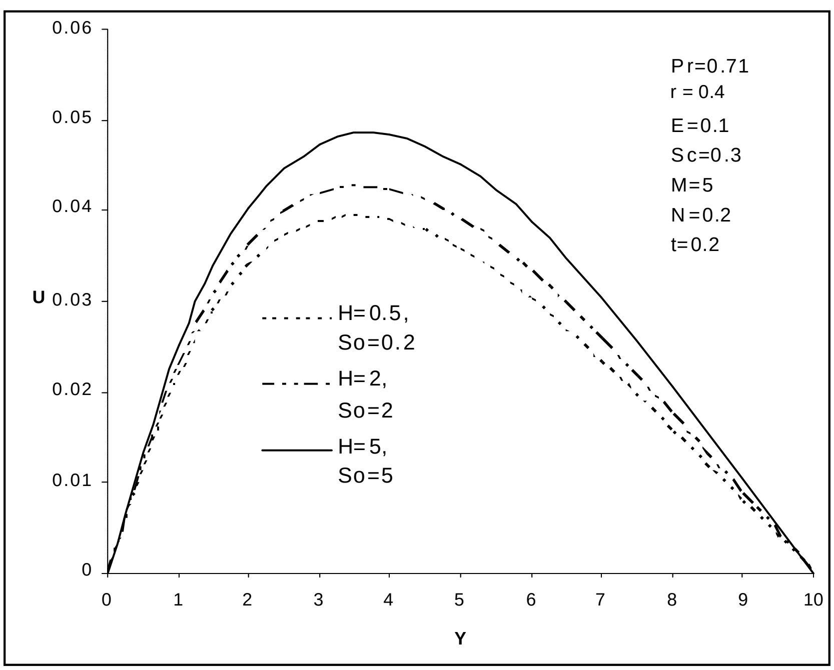

Figure 5: Velocity Profiles for Different Values of H, S_0

increases, Velocity increases. From Fig. 2 for fixed values of $P_n = 0.71$, $\gamma = 0.4$, $E = 0.1$, $S_c = 0.3$, $N = 0.2$, $M = 5$, $H = 0.5$ we observe that increase S_o in and t correspond to increase in Velocity. From Fig. 3 for fixed values of $P_r = 0.71$, $\gamma = 0.4$, $E = 0.1$, $S_c = 0.3$, $N = 0.2$, $M = 5$, $S_0 = 0$, $t = 0.2$ we notice that as H increases, Velocity increases. From Fig. 4 for fixed values of $P_r = 0.71$, $\gamma = 0.4$, $E = 0.1$, $S_c = 0.3$, $N = 0.2$, $M = 5$, $S_0 = 0.2$ we observe that increase in H and t correspond to increase in Velocity. From Fig. 5 for fixed values of $P_r = 0.71$, $\gamma = 0.4$, $E = 0.1$, $S_c = 0.3$, $N = 0.2$, $M = 5$, $t = 0.2$ we notice that as H and S_0 increase, Velocity increases.

The Temperature profiles are drawn in Figures 6 to 9. From Fig. 6 for fixed values of $P_r = 0.71$, $\gamma = 0.4$, $E = 0.1$, $S_c = 0.3$, $M = 5$, $N = 0.2$, $H = 0$, $t = 0.2$ we observe that increase in S_o shows no change in Temperature. From Fig. 7 for fixed values of $P_r = 0.71$, $\gamma = 0.4$, $E = 0.1$, $S_c = 0.3$, $M = 5$, $N = 0.2$, $S_0 = 0$, $t = 0.2$ we observe that increase in H leads to increase in Temperature. From Fig. 8 for fixed values of $P_r = 0.71$, $\gamma = 0.4$, $E = 0.1$, $S_c = 0.3$, $M = 5$, $N = 0.2$, $S_0 = 0.2$ we notice that as H and t increase, Temperature increases. From Fig. 9 for fixed values of $P_r = 0.71$, $\gamma = 0.4$, $E = 0.1$, $S_c = 0.3$, $M = 5$, $N = 0.2$, $t = 0.2$ we notice that as H and S_o increase, Temperature increases.

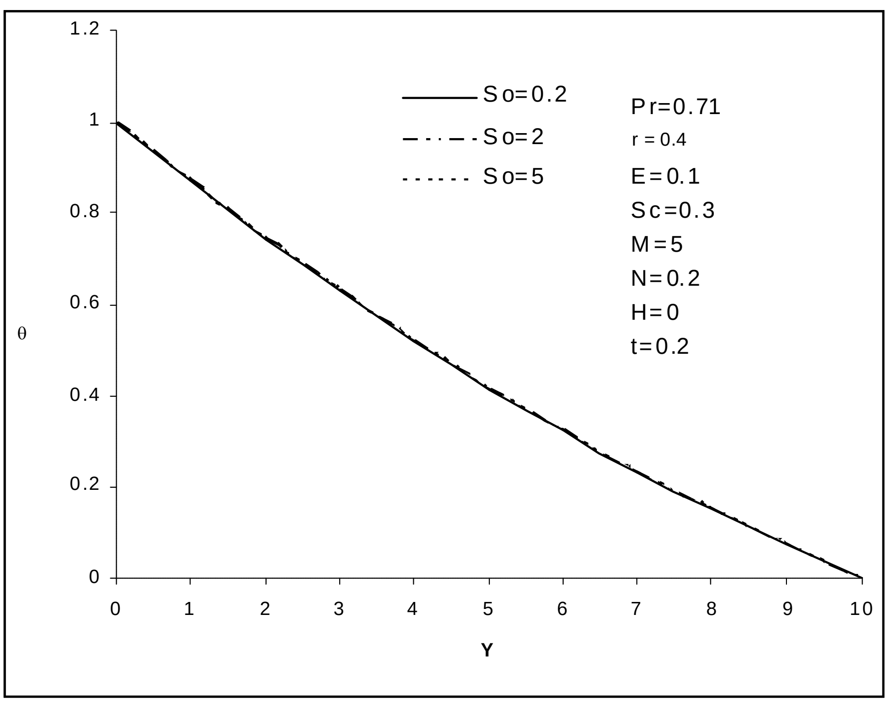

Figure 6: Temperature Profiles for Different Values of S_0

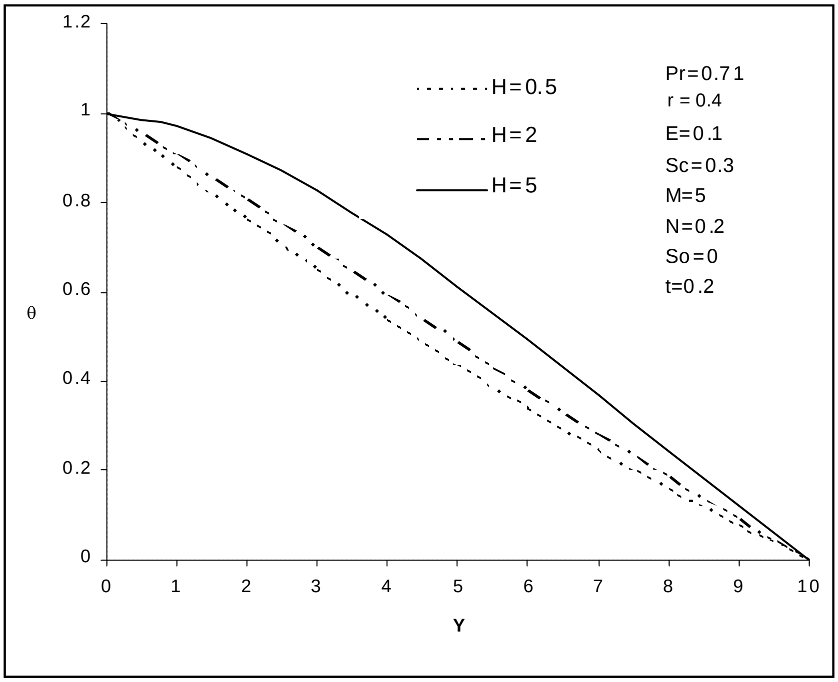

Figure 7: Temperature Profiles for Different Values of *H*

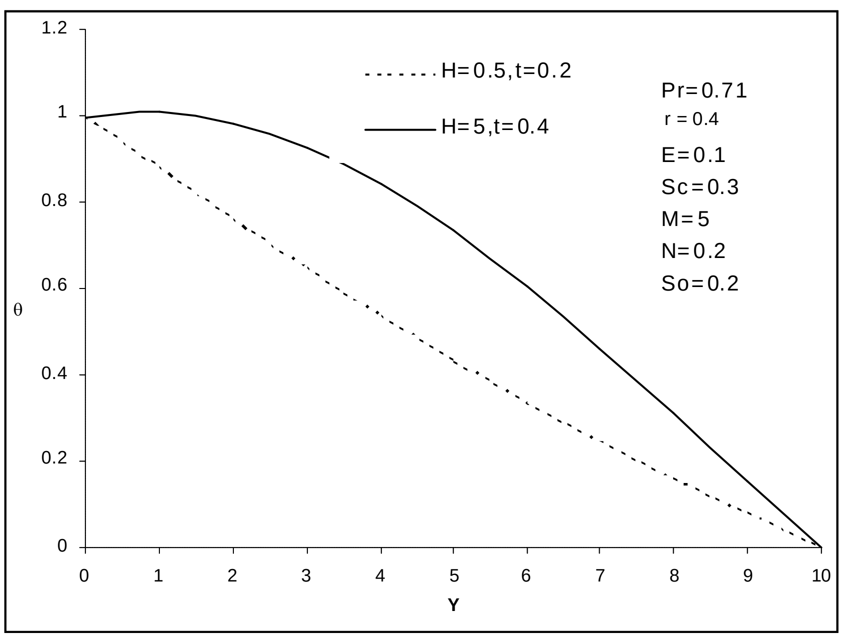

Figure 8: Temperature Profiles for Different Values of *H*, *t*

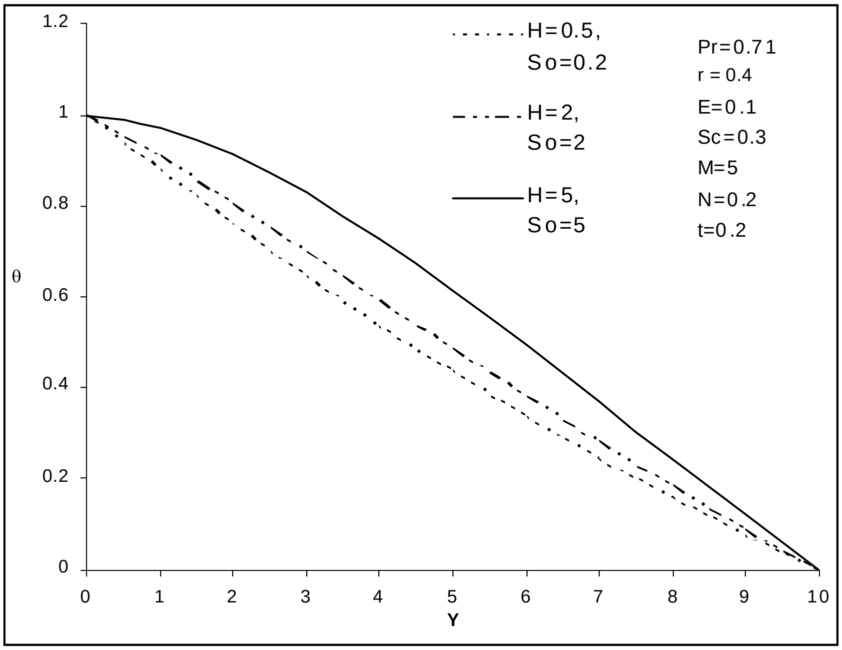

Figure 9: Temperature Profiles for Different Values of H, S_0

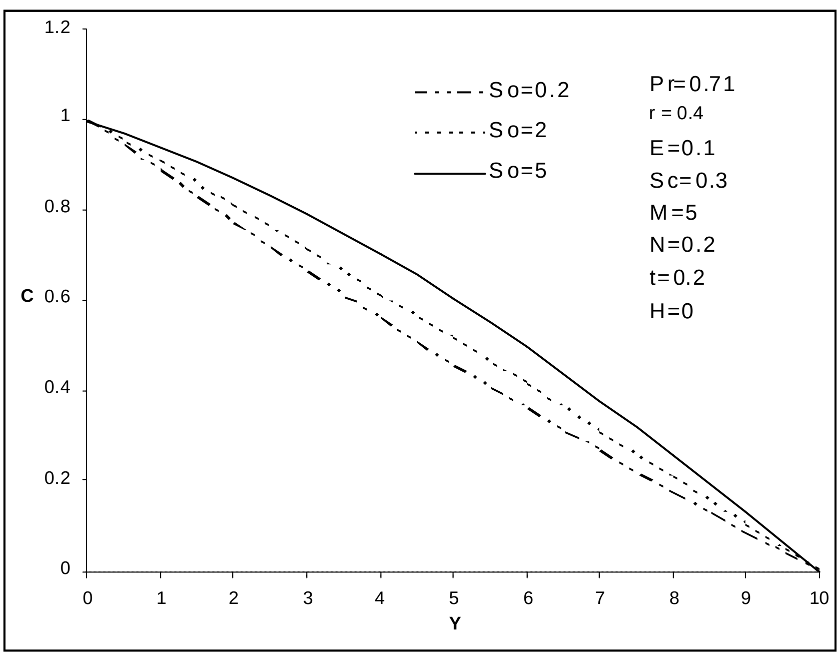

Figure 10: Concentration Profiles for Different Values of S_0

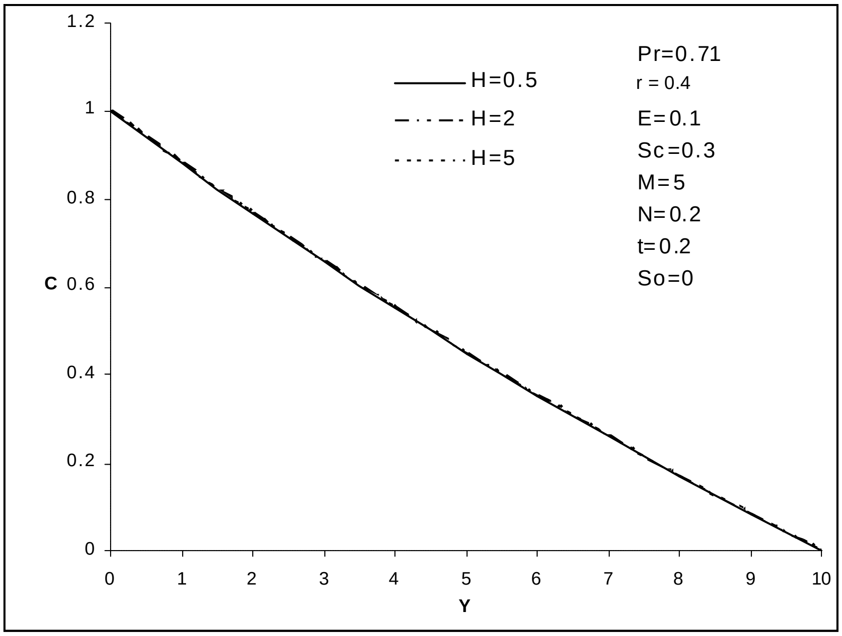

Figure 11: Concentration Profiles for Different Values of *H*

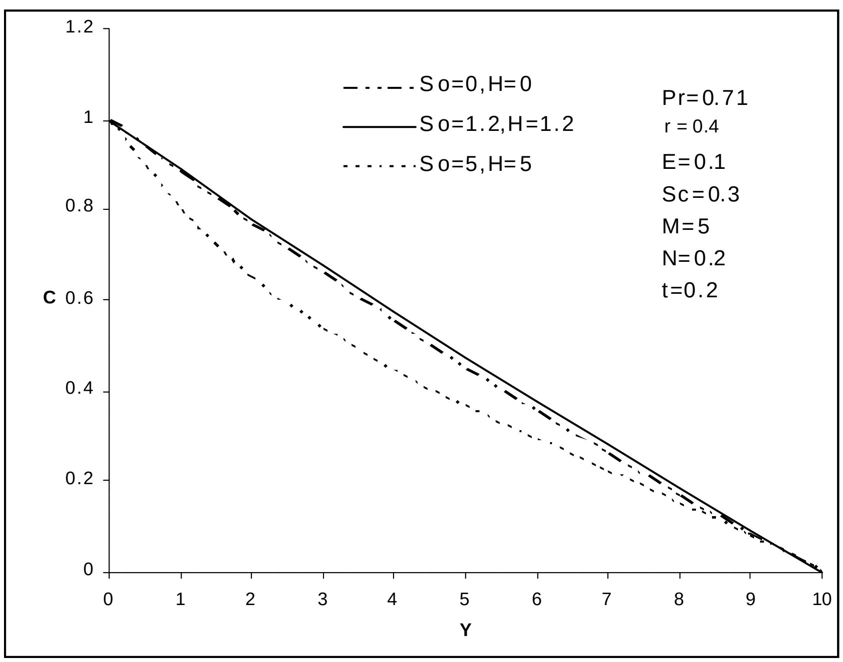

Figure 12: Concentration Profiles for Different Values of H, S_0

The Concentration profiles are drawn in Figures 10 to 12. From Fig. 10 for fixed values of $P_r = 0.71$, $\gamma = 0.4$, $E = 0.1$, $S_c = 0.3$, $N = 0.2$, $M = 5$, $H = 0$, $t = 0.2$ we notice that as So increases, Concentration increases. From Fig. 11 for fixed values of $P_r = 0.71$, $\gamma = 0.4$, $E = 0.1$, $S_c = 0.3$, $N = 0.2$, $M = 5$, $S_0 = 0$, $t = 0.2$ we observe that increase in H leads to no change in Concentration. From Fig. 12 for fixed values of $P_r = 0.71$, $\gamma = 0.4$, $E = 0.1$, $S_c = 0.3$, $N = 0.2$, $M = 5$, $t = 0.2$ we observe that when both S_o and H increase, Concentration increases up to certain stage and decreases later.

CONCLUSIONS

A detailed numerical study has been carried out for the unsteady MHD free convection and mass transfer flow of dissipative fluid with heat generation and thermal diffusion past an infinite vertical porous isothermal plate. The dimensionless governing equations are solved by an explicit finite-difference method. Conclusions of the study are as follows.

1. As Soret number S_0, Heat source parameter H increase, the velocity is observed to increase.
2. When there is no Heat source ($H = 0$), it is observed that temperature remained unaffected even if Soret number S_0 increases.
3. When Heat source parameter H increases, temperature increased. Soret number S_0 has least effect on increase of temperature.
4. Regarding concentration, it is observed that concentration increases as Soret number S_0 increases, in the absence of Heat source parameter ($H = 0$).
5. It is also observed that there is no change in the concentration when Soret number S_0 is zero, even if Heat source parameter H increases.
6. But, when both Soret number S_0 and Heat source parameter H increase, it is observed that Concentration increases up to certain stage and decreases later.

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REFERENCES

[1] H. L. Agrawal, P. C. Ram, and S. S. Singh, (1980), *Can. J. Chem Engng.*, **58**, 131.

[2] P. M. Chung, and A. D. Anderson, (1961), *Trans ASME J. Heat Transfer*, **83C**, 473-78.

[3] B. Gebhart, (1962), *J. Fluid Mech*, **14**, 225-32.

[4] B. Gebhart, (1961), (Trans. ASME) *J. Heat Transfer,* **83C**, 61-70.

[5] R. J. Goldstein, and D. G. Briggs, (1964), (Trans ASME) *J. Heat. Transfer*, **86C**, 490-500.

[6] R. J. Goldstein, and E. G. G. Eckert, (1960), *Int. J. Heat Mass Transfer*, **1**, 208-18.

[7] E. R. Menold, and K. T. Yang, (1962), (Trans. ASME) *J. appl. Mech.*, **29E**, 124-26.

[8] A. A. Raptis, (1982), *Astrophys. Space Sci*., **86**, 43-48.

[9] J. A. Schetz, and R. Eichhorn, (1962), (Trans. ASME) *J. Heat Transfer*, **84**, 334-38.

[10] R. Siegel, (1958), Trans. Amer. Soc. Mech Engng, **80**, 347-59.

[11] V. M. Soundalgekar, J. P. Bhatt, and M. Mohiuddin, (1979), *Int. J. Engng. sci*., **17**, 1283-88.

[12] V. M. Soundalgekar, R. M. Lahurikar, and S. G. Pohanerkar, (1997), *Int. J. Heat Mass Transfer*, **32**, 301-305.

[13] E. M. Sparrow, and J. L. Gregg, (1960), (Tr. ASME) *J. Heat. Tr.*, **82C**, 258-60.

[14] K. Vajravelu, (1978), *Indian J. Tech*., **16**, 397-40

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