

Chaos Suppression for a Fourth Order Memristor Chaotic Oscillator with Uncertain Parameters

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ABSTRACT

The fourth fundamental circuit element- Memristor, was mathematically predicted by Prof. Chua in his seminal research paper in IEEE Transaction on Circuit Theory. After four decades in 2008, researchers at the Hewlett–Packard (HP) laboratories reported the development of a new basic circuit element that completes the missing link between charge and flux linkage, which was postulated by Chua. The new roadmap in the field of circuit designing, soft computing, memory technology and neuromorphic applications are emerged out very quickly in scientific community due to memristor. In this paper we discuss about a fourth order Memristor system with two orthogonal Memristors and its dynamics with implementation in LabVIEW. We have also designed an Adaptive controller to stabilize the states of the Memristor system. The results of the stabilization are established using Lyapunov stability theory. The numerical simulations of the stabilization schema are done using MATLAB.

Keywords: Memristor Chaotic System, Chaos Supression, HP Memristor, Chaos Stabilization, LabVIEW.

1. INTRODUCTION

The circuit theory suggest, there only three; two terminal; passive elements namely resistor, capacitor and inductor are available. These elements are defined in terms of the relation between fundamental circuit variables, such as current (i), voltage (v), charge (q) and flux (ϕ). In 1971, Prof. Leon Chua predicted that there should be a fourth fundamental circuit element to set up the *relation between charge and magnetic flux without an internal power supply Theory* on the symmetric background [1-2]. After four decade in 2008, researchers at the Hewlett–Packard (HP) laboratories published a seminal paper in Nature reporting the development of a new basic circuit element that completes the *missing link between charge and flux linkage*, which was postulated by Chua [3-4].

The nano-device memristor consider as passive element with property of remembrance of last applied state. This unique property make it valuable circuit element for many application such as resistive memories, soft computing, Neurocomputing, FPGAs etc. The memristor is an element (or *class of Memristive element*) that changed its resistance depending on how much charge flowed through it. The memristor behaves like a *linear resistor with memory* but also exhibits many interesting nonlinear characteristics. The several electronic models have been presented to describe the electrical behaviour of memristor devices such as, the linear ion drift model, the nonlinear ion drift model, Simmons tunnel barrier model, and the Threshold Adaptive Memristor (TEAM) model [1, 3, 5-7]. However, the memristor devices are not commercially available, good physical model-to-hardware correlations have not been yet been reported in the published literature [8]. There are also several research groups presented SPICE macro models of memristor [9-13] and MATLAB model [14]. The memristive class consist of the memristive (MrS), memcapacitive (McS) and meminductive (MiS) subsystem. These elements are considering as one port element whose property is

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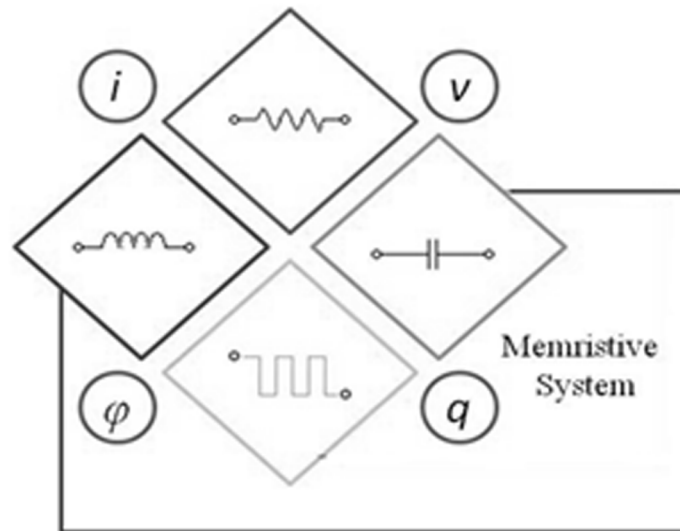


Figure 1: Relationship between Four Fundamental Circuit Elements

depends upon the time derivative of charge and flux linkage [4]. The fig. 1 shows relationship between fundamental circuit elements and also completes the missing link between charge and flux linkage.

The memristor was predicted according to symmetry principles of two of four fundamentals electrical quantities such as current (i), voltage (v), charge (q) and flux linkage (ϕ). In the history few principle are also predicted using symmetry principles e.g. the displacement current in Maxwell's equations, a positron and a magnetic monopole. The first two have been experimentally observed; while the third one remains mysterious [15]. The lot of mathematical and simulative modelling and related work regarding with the memristor is carried out by O. Kavehei [4], Strukov. D. B [3] and Prof. L. Chua [1]. Memristor is a semiconductor thin film sandwiched between two metal contacts with a total length of D of TiO_2 film and it consists of *doped low resistance* and *undoped high resistance* regions [13]. The physical structure with its equivalent circuit model is shown in Fig. 2 [3]. The memristor possess the increases resistance in one direction of current and decreases the resistance in other direction. When applied external potential is removed then memristor remains in the last state i.e. memristor possess resistive memory [1]. In another words, memristor is nothing but an analog resistor which resistance can be change by changing direction of applied voltage or current [13]. Fig.2. shows the basic geometrical structure of a memristor. The present simulations model of memristor is carry out in LTSpice environment. The simulation results are shown in fig. 3. The hysteresis loop represents the coupled equations of motion for current-controlled memristor (equation no. 1 and 2). The current is nonlinear with the applied voltage, resulting in hysteresis loops rather than straight lines. Theoretically, at high frequency the hysteresis loop vanishes and become a straight line. Thickness of the whole component is marked with D , the thickness of the doped layer with w [14]. It is necessary to present a mathematical memristor model to explain the substance of the models and simulation clearly [16].

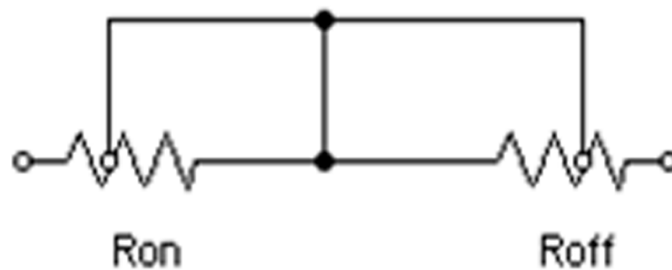


Figure 2: Equivalent Model of Memristor Reported by HP Laboratory

A new scheme of compound synchronization is described by three drive systems (a scaling drive system, two base drive systems) and one response system. The version of synchronization is advantageous in circuit application due to the novel structure [31-32]. Chaotic behaviour, sequence of period-doubling bifurcations, inverse sequence of chaotic band, and intermittent chaos are found in various memristor oscillator systems [17-30].

The memristor acted like a memory resistor, by relating the voltage over the element and the current through it as follows,

$$v = M(w)i \quad (1)$$

The memristance M acts the same as a resistance, except that it depends on a parameter w , which in Chua's derivations was either the charge q or the flux ϕ . Since the charge and current are related as follows,

$$\frac{dq}{dt} = i \quad (2)$$

$$W(\phi(t)) = a + 3b\phi(t)^2 \quad (3)$$

Where a & b are parameters. M depends on the complete history of current passing through the element, which makes the memristor act like a resistor with memory. The nonlinear memristance (M) is a function of charge (q), there is no combination of RLC element which mimics or duplicates such type of property, and hence it is a fundamental circuit element. Chua later showed that memristor are part of a broader class of systems called memristive systems described by,

$$v = M(w, i)i \quad (4)$$

$$\frac{dw}{dt} = f(w, i) \quad (5)$$

Where, w can be any controllable property [16]. Equations (5) will represent the hysteresis loop of memristor. Equation (6) and (7) describe the ideal mathematical Model of memristor [3].

$$V(t) = \left[\left(\frac{RonW(t)}{D} \right) + Roff \left(1 - \frac{W(t)}{D} \right) \right] i(t) \quad (6)$$

$$\frac{dw(t)}{dt} = \frac{\mu v Ron}{D} i(t) \quad (7)$$

2. FOURTH ORDER MEMRISTOR SYSTEM

We considered a fourth order Memristor system with its dynamics described as below

$$\begin{aligned} \dot{\phi}(t) &= v_1(t) \\ \dot{v}_1(t) &= \frac{1}{C_a R_a} v_2(t) - \frac{1}{C_a R_a} v_1(t) + \frac{G}{C_a} v_1(t) - \frac{1}{C_a} v_1(t) W(\phi(t)) \\ \dot{v}_2(t) &= \frac{1}{C_b R_a} v_1(t) - \frac{1}{C_b R_a} v_2(t) + \frac{1}{C_b} i(t) \\ \dot{i}(t) &= -\frac{1}{L} v_2(t) - \frac{R_b}{L} i(t) \end{aligned} \quad (8)$$

Where $v_1(t)$ and $v_2(t)$ are voltages, $i(t)$ the current, C_a, C_b, R_a, R_b represents Capacitance & Resistance respectively. $W(\phi(t))$ denotes the memductance function with $\phi(t)$ as the magnetic flux. L and G represents Inductance and Conductance. Using the mathematical model of a cubic memristor [1, 2], the memductance function is given by

$$W(\phi(t)) = a + 3b\phi(t)^2 \quad (9)$$

where a and b are parameters. From (8) and (9) it follows that

$$\begin{aligned} \dot{\phi}(t) &= v_1(t) \\ \dot{v}_1(t) &= \frac{1}{C_a R_a} v_2(t) - \frac{1}{C_a R_a} v_1(t) + \frac{G}{C_a} v_1(t) - \frac{1}{C_a} v_1(t)(a + 3b\phi(t)^2) \\ \dot{v}_2(t) &= \frac{1}{C_b R_a} v_1(t) - \frac{1}{C_b R_a} v_2(t) + \frac{1}{C_b} i(t) \\ \dot{i}(t) &= -\frac{1}{L} v_2(t) - \frac{R_b}{L} i(t) \end{aligned} \quad (10)$$

Rearranging (10)

$$\begin{aligned} \dot{\phi}(t) &= v_1(t) \\ \dot{v}_1(t) &= \frac{1}{C_a R_a} v_2(t) - \left[\frac{1}{C_a R_a} - \frac{G}{C_a} v_1(t) + \frac{a}{C_a} \right] v_1(t) - \frac{3b}{C_a} b\phi(t)^2 v_1(t) \\ \dot{v}_2(t) &= \frac{1}{C_b R_a} v_1(t) - \frac{1}{C_b R_a} v_2(t) + \frac{1}{C_b} i(t) \\ \dot{i}(t) &= -\frac{1}{L} v_2(t) - \frac{R_b}{L} i(t) \end{aligned} \quad (11)$$

Let

$$\begin{aligned} x(t) &= \phi(t), y(t) = v_1(t), z(t) = v_2(t), w(t) = i(t), \alpha = \frac{1}{C_a R_a}, \beta = \frac{1}{C_a R_a} - \frac{G}{C_a} v_1(t) + \frac{a}{C_a}, \\ \varepsilon &= \frac{3b}{C_a}, \lambda = \frac{1}{C_b R_a}, \mu = \frac{1}{C_b R_a}, \sigma = \frac{1}{C_b}, \rho = \frac{1}{L}, \tau = \frac{R_b}{L} \end{aligned}$$

Equation (11) becomes,

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= \alpha z - \beta y - \varepsilon x^2 y \\ \dot{z} &= \lambda y - \mu z + \sigma w \\ \dot{w} &= -\rho z - \tau w \end{aligned} \quad (12)$$

The parameters of the above equation are chosen as follows for the system to exhibit chaos.

$$\alpha = 16.4, \beta = -3.28, \varepsilon = 19.68, \lambda = 1, \mu = 1, \sigma = 1, \rho = 15, \tau = 0.5$$

2.1. Lyapunov Exponents

The Initial conditions are chosen as $x(t) = 0.01$, $y(t) = 0.01$, $z(t) = 0.01$, $w(t) = 0.01$. The Lyapunov exponents of the system (12) are 0.334022, 0.007606, -0.008434, -7.832555.

The Numerical results of the simulation are shown in Figure 3.

b. Bifurcation

By fixing and varying, the new system (12) is investigated. The bifurcation diagram is shown in figure 4(a).

By fixing and varying the bifurcation is investigated and shown in figure 4(b). By fixing and varying the bifurcation is investigated and shown in figure 4(c). Generally speaking, when the system's biggest Lyapunov exponents is large than zero, and the points in the corresponding bifurcation diagram are dense, the chaotic attractor will be found to exit in this system. Therefore, From the Lyapunov exponents and bifurcation diagrams in figure 4(a), 4(b) and 4(c) a conclusion can be obtained that chaos exit in the fourth order memristor system when selecting a certain range of parameters.

3. SPICE MODEL OF THE FOURTH ORDER MEMRISTOR SYSTEM

The fourth order memristor system discussed in the paper is implemented as a Spice Model. The Spice circuit model of the fourth order Memristor chaotic circuit is shown in Figure 5a.

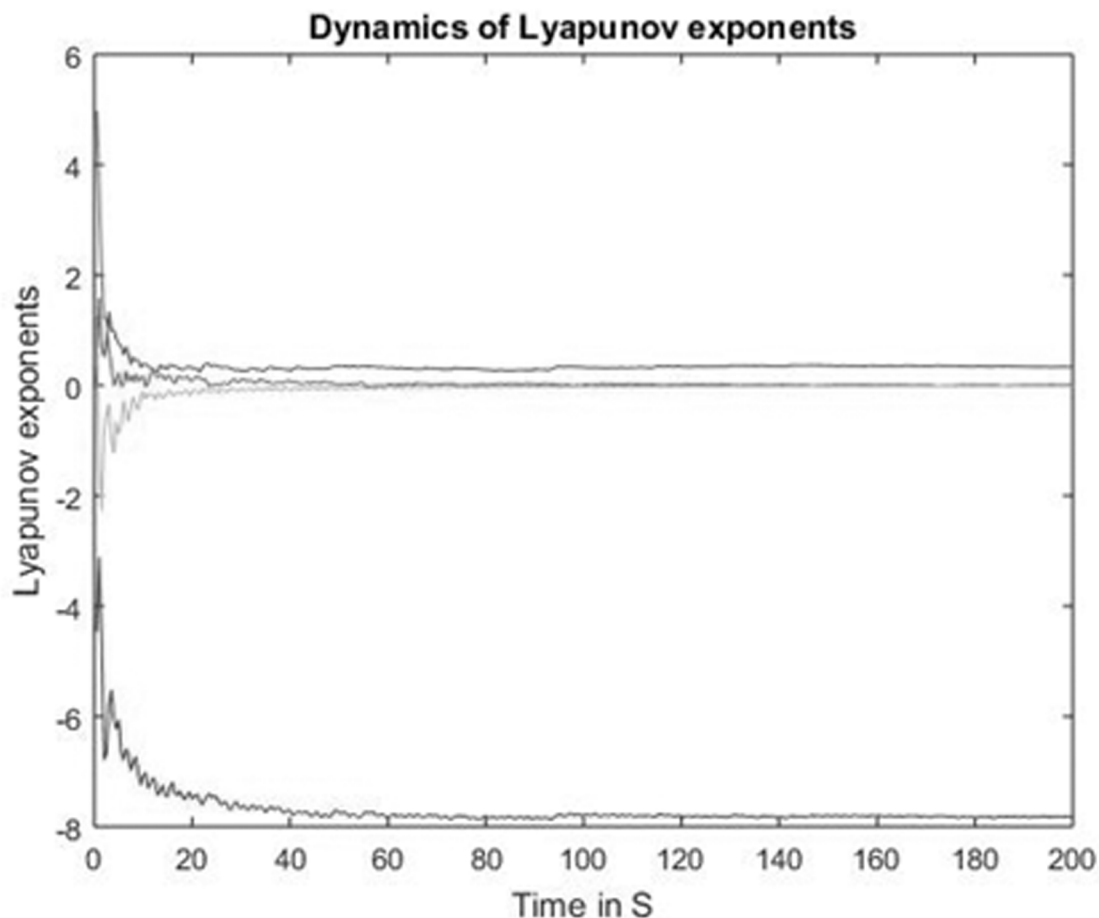
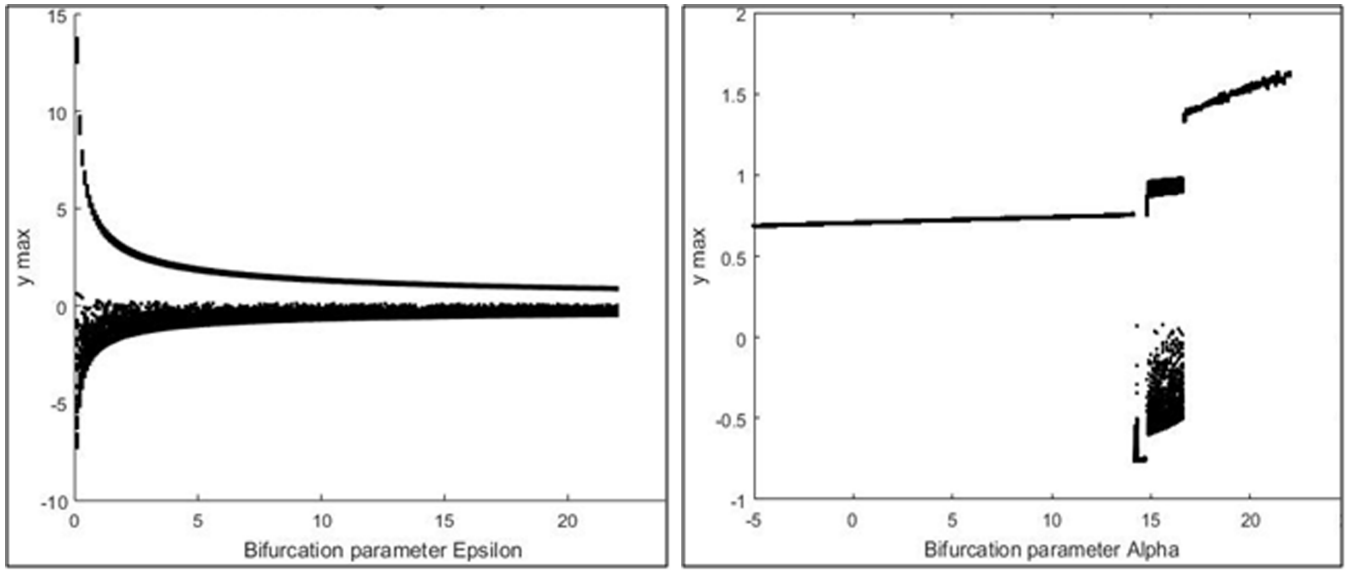
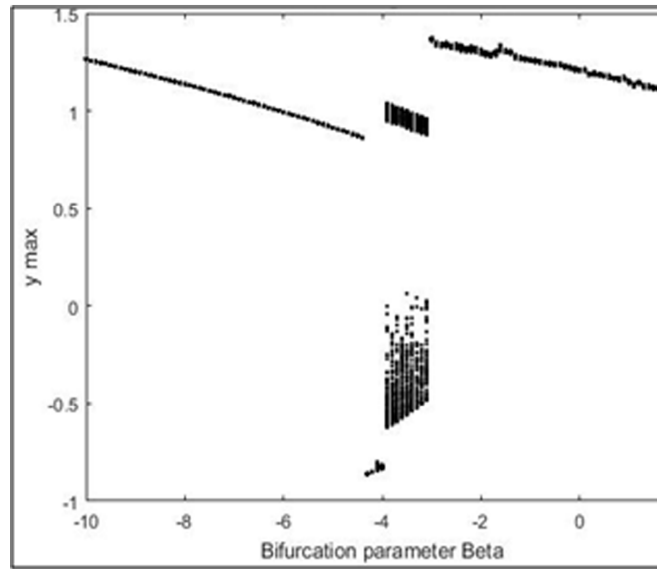


Figure 3: Lyapunov exponents of the System (12).



(a)

(a)



(c)

Figure 4: (a) Bifurcation plot versus. (b) Bifurcation plot versus. (c) Bifurcation plot versus.

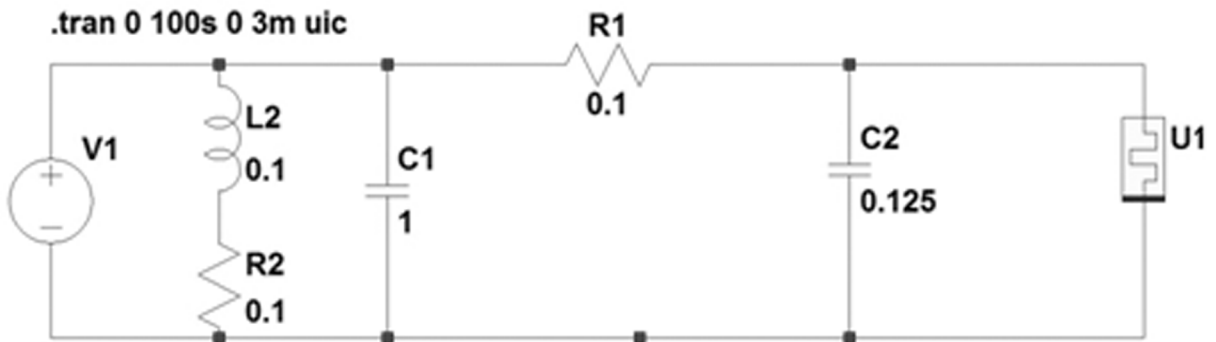


Figure 5 a: Spice Model of the Fourth-Order memristor System.

The transient analysis of the V-I characteristics of the System (11) is described in Figure 5b.

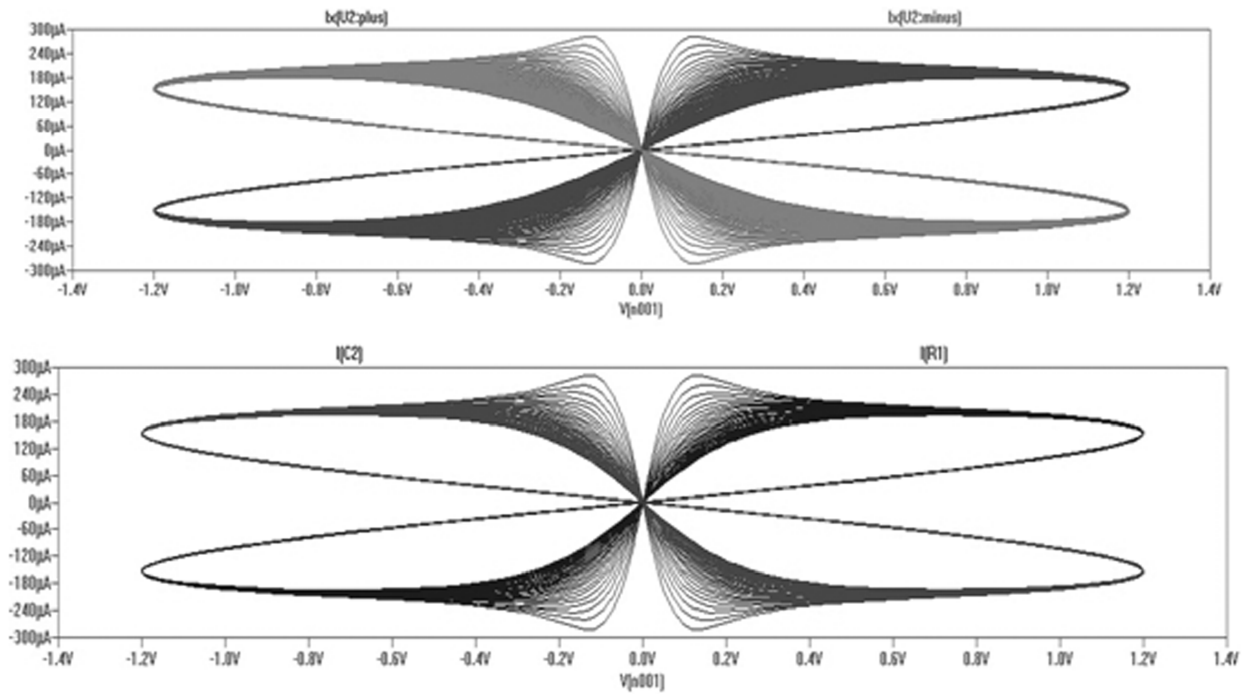


Figure 5b: Plot of Current V/S Voltage at $\dot{u} = 10$ rad/s. The M-efficiency factor (R_{OFF}/R_{ON}) is considered as 160.

4. LABVIEW IMPLEMENTATION OF THE FOURTH ORDER MEMRISTOR SYSTEM

The Fourth order memristor system (11) is implemented in LabVIEW using the control and simulation tools. The control and simulation loop is used to implement the memristor system.

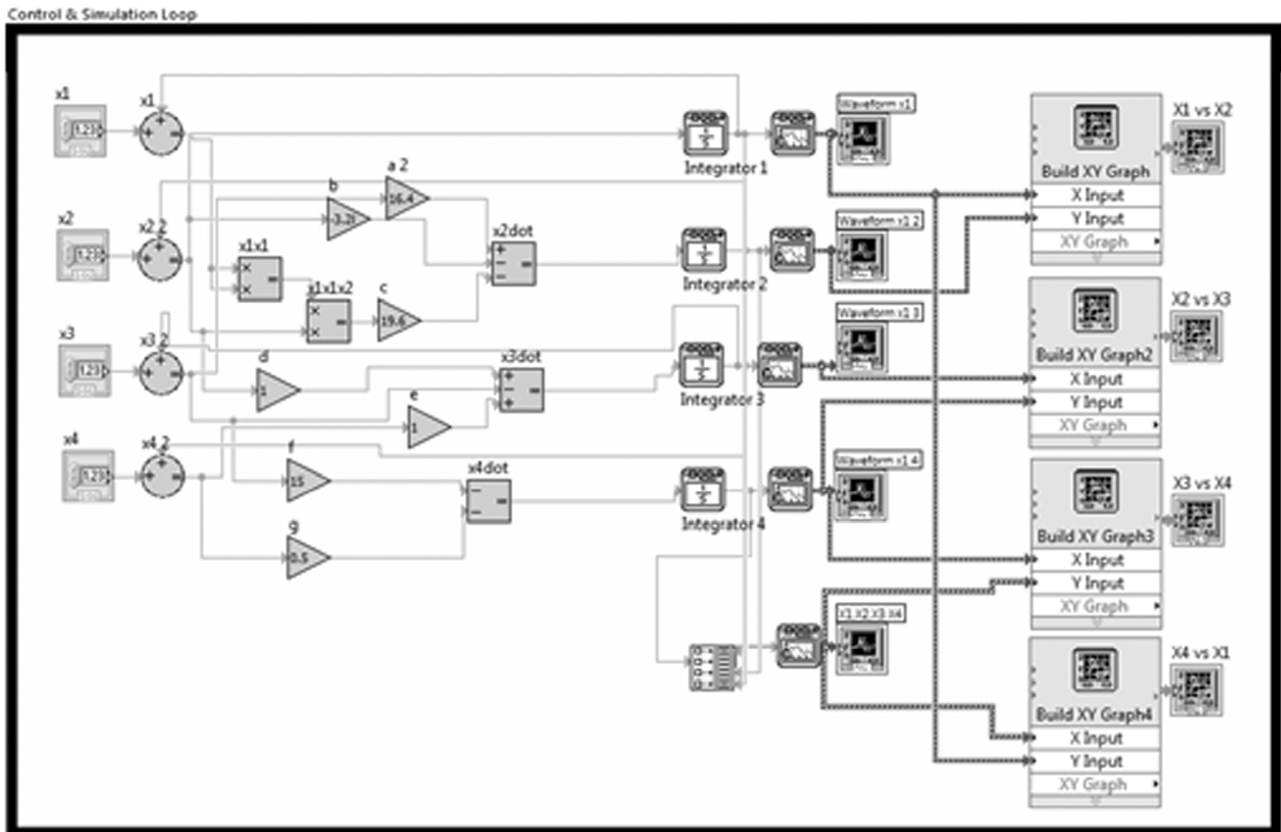


Figure 6: Block Diagram of the Fourth order Memristor in LabVIEW

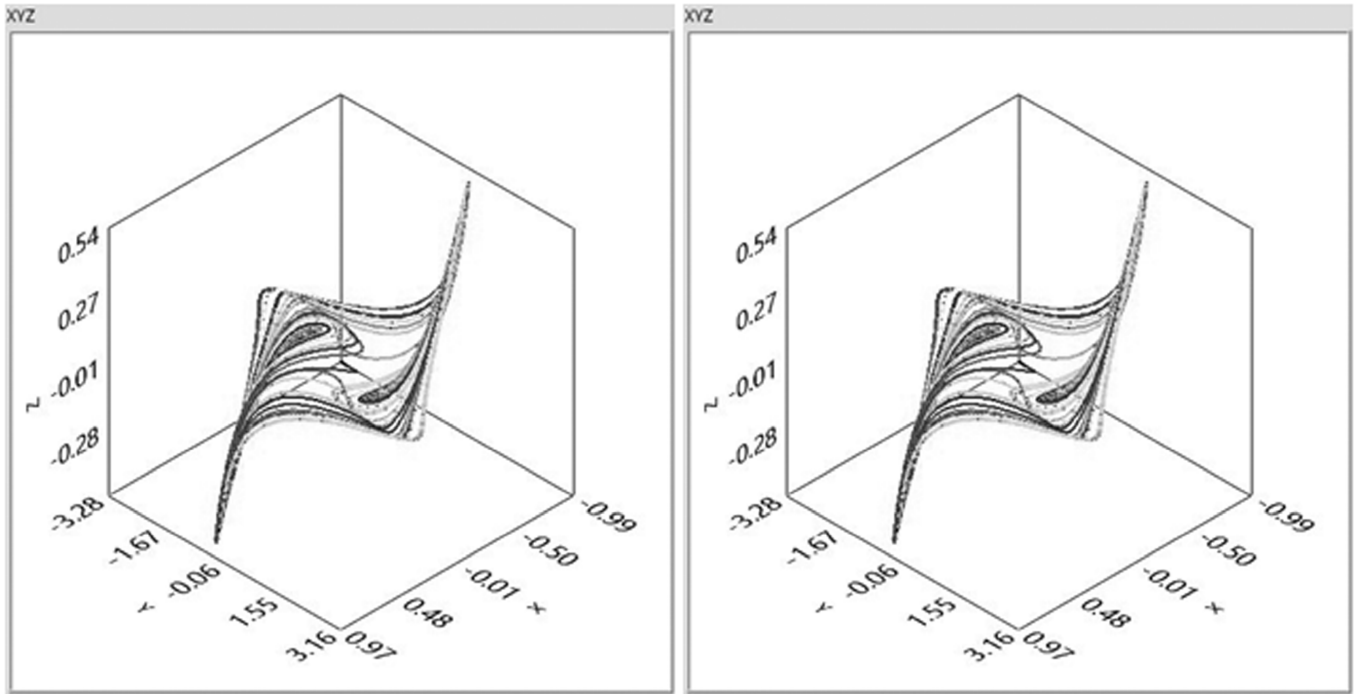


Figure 7: 3D State portraits of the Fourth order Memristor System in LabVIEW

Figure 6 shows the block diagram of the fourth order memristor system in LabVIEW. Figure 7 shows the 3D state space analysis of the fourth order memristor system.

5. ADAPTIVE STABILIZATION OF THE FOURTH ORDER MEMRISTOR DYNAMICS

In this section, we use adaptive control method to derive an adaptive feedback control law for globally and exponentially stabilizing the fourth order memristor system with unknown parameters.

$$\begin{aligned}
 \dot{x} &= y + u_x \\
 \dot{y} &= \alpha z - \beta y - \varepsilon x^2 y + u_y \\
 \dot{z} &= \lambda y - \mu z + \sigma w + u_z \\
 \dot{w} &= -\rho z - \tau w + u_w
 \end{aligned} \tag{13}$$

In (13), x, y, z, w are the states and u_x, u_y, u_z, u_w are adaptive controls to be determined using estimates $\hat{\alpha}(t), \hat{\beta}(t), \hat{\lambda}(t), \hat{\mu}(t), \hat{\tau}(t), \hat{\varepsilon}(t), \hat{\sigma}(t)$ and $\hat{\rho}(t)$ for the unknown parameters.

We consider the adaptive control law defined by

$$\begin{aligned}
 u_x &= -y - k_1 x \\
 u_y &= -\hat{\alpha} z + \hat{\beta} y + \hat{\varepsilon} x^2 y - k_2 y \\
 u_z &= -\hat{\lambda} y + \hat{\mu} z - \hat{\sigma} w - k_3 z \\
 u_w &= \hat{\rho} z + \hat{\tau} w - k_4 w
 \end{aligned} \tag{14}$$

where k_1, k_2, k_3, k_4 are positive gain constants.

Substituting (14) into (13), we get the closed-loop plant dynamics as

$$\begin{aligned}
 \dot{x} &= -k_1 x \\
 \dot{y} &= (\alpha - \hat{\alpha})z - (\beta - \hat{\beta})y - (\varepsilon - \hat{\varepsilon})x^2 y - k_2 y \\
 \dot{z} &= (\lambda - \hat{\lambda})y - (\mu - \hat{\mu})z + (\sigma - \hat{\sigma})w - k_3 z \\
 \dot{w} &= -(\rho - \hat{\rho})z - (\tau - \hat{\tau})w - k_4 w
 \end{aligned}
 \tag{15}$$

The parameter estimation errors are defined as

$$\begin{cases}
 e_\alpha(t) = \alpha - \hat{\alpha}(t), e_\mu(t) = \mu - \hat{\mu}(t) \\
 e_\beta(t) = \beta - \hat{\beta}(t), e_\sigma(t) = \sigma - \hat{\sigma}(t) \\
 e_\varepsilon(t) = \varepsilon - \hat{\varepsilon}(t), e_\rho(t) = \rho - \hat{\rho}(t) \\
 e_\lambda(t) = \lambda - \hat{\lambda}(t), e_\tau(t) = \tau - \hat{\tau}(t)
 \end{cases}
 \tag{16}$$

we can simplify the plant dynamics (16) as

$$\begin{aligned}
 \dot{x} &= -k_1 x \\
 \dot{y} &= e_\alpha z - e_\beta y - e_\varepsilon x^2 y - k_2 y \\
 \dot{z} &= e_\lambda y - e_\mu z + e_\sigma w - k_3 z \\
 \dot{w} &= -e_\rho z - e_\tau w - k_4 w
 \end{aligned}
 \tag{17}$$

We use adaptive control theory to find an update law for the parameter estimates.

We consider the quadratic candidate Lyapunov function defined by

$$\begin{aligned}
 &V(x, y, z, w, \alpha, \beta, \varepsilon, \rho, \sigma, \lambda, \mu, \tau) \\
 &= \frac{1}{2} \left(x^2 + y^2 + z^2 + w^2 + e_\alpha^2 + e_\beta^2 + e_\varepsilon^2 + e_\rho^2 + e_\sigma^2 + e_\lambda^2 + e_\mu^2 + e_\tau^2 \right)
 \end{aligned}
 \tag{18}$$

In view of (18) the parameter update law is defined as follows.

$$\begin{cases}
 \dot{\hat{\alpha}}(t) = -zy, \dot{\hat{\beta}}(t) = y^2 \\
 \dot{\hat{\varepsilon}}(t) = x^2 y^2, \dot{\hat{\lambda}}(t) = -zy \\
 \dot{\hat{\mu}}(t) = z^2, \dot{\hat{\sigma}}(t) = -wz \\
 \dot{\hat{\rho}}(t) = zw, \dot{\hat{\tau}}(t) = w^2
 \end{cases}
 \tag{19}$$

Theorem. The novel 3-D chaotic system (13) with unknown system parameters is globally and exponentially stabilized for all initial conditions $\mathbf{x}(0) \in R^3$ by the adaptive control law (14) and the parameter update law (19), where k_1, k_2, k_3 are positive gain constants.

Proof. We prove this result by using Lyapunov stability theory [73].

We consider the quadratic Lyapunov function defined by (18), which is positive definite on R^7 .

By substituting the parameter update law (19) into (18), we obtain the time derivative of V as

$$\dot{V} = -k_1 x^2 - k_2 y^2 - k_3 z^2 - k_4 w^2 \quad (20)$$

From (20), it is clear that \dot{V} is a negative semi-definite function on R^7 .

Thus, we conclude that the state vector $\mathbf{x}(t)$ and the parameter estimation error are globally bounded *i.e.*

$$[x, y, z, w, \alpha, \beta, \varepsilon, \rho, \sigma, \lambda, \mu, \tau]^T \in L_\infty$$

We define $k = \min\{k_1, k_2, k_3, k_4\}$. Thus, it follows from (20) that

$$\dot{V} \leq -k \|\mathbf{x}(t)\|^2 \quad (21)$$

Thus, we have

$$k \|\mathbf{x}(t)\|^2 \leq -\dot{V} \quad (22)$$

Integrating the inequality (22) from 0 to t , we get

$$k \int_0^t \|\mathbf{x}(\tau)\|^2 d\tau \leq V(0) - V(t) \quad (23)$$

From (23), it follows that $\mathbf{x} \in L_2$. Using (28), we can conclude that $\dot{\mathbf{x}} \in L_\infty$.

Using Barbalat's lemma [73], we can conclude that $\mathbf{x}(t) \rightarrow 0$ exponentially as $t \rightarrow \infty$ for all initial conditions $\mathbf{x}(0) \in R^3$.

This completes the proof. ■

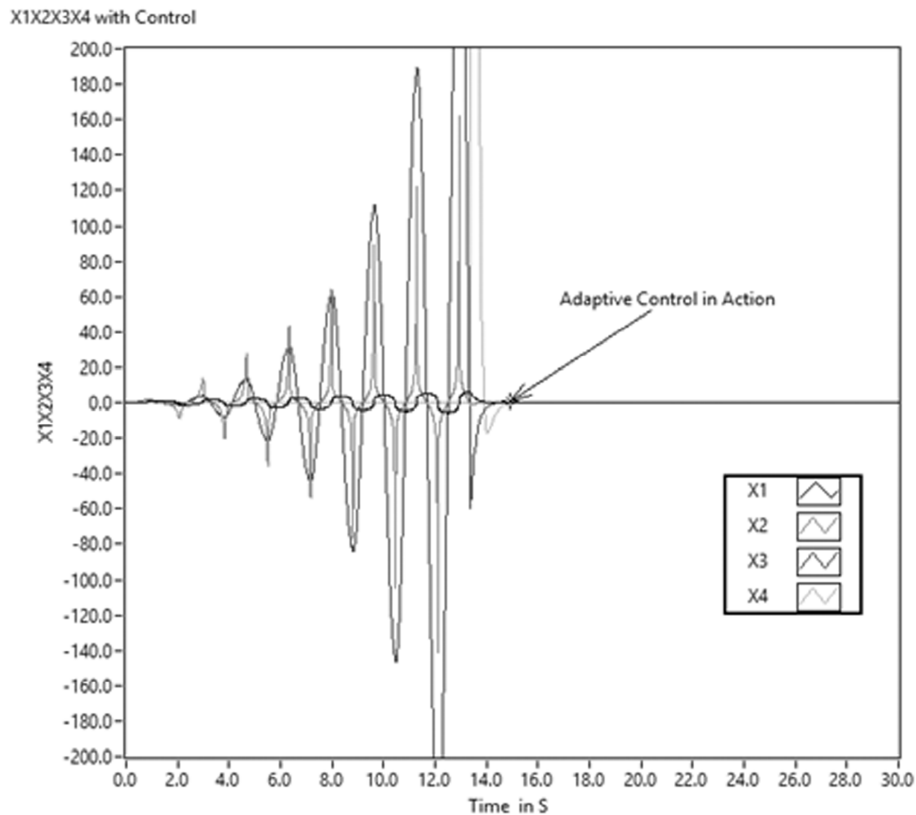


Figure 8: State Oscillations without and with adaptive control

Figure 8 shows the state oscillations without and with adaptive controller. The controller is introduced at $t = 14s$ and it can be clearly seen from Figure 8 that after the introduction of the controller, the chaotic oscillations of the states are suppressed.

6. CONCLUSION

This paper has introduced a hyperchaotic memristor oscillator system and presented a novel control method using adaptive scheme to drive two memristor oscillator systems to synchronize the first response of the oscillator with its second version. The resulting hyperchaos synchronization via adaptive combination scheme is also verified by SPICE modelling and LabVIEW Simulations. It is believed that the derived results and analytical techniques have great potential in controlling various hyperchaotic systems and hyperchaotic circuits, which open up a wide area for further research of chaos and hyperchaos memristive dynamics.

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