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THERMO-ELECTRIC EFFECTS ON FLOW OF CHOLESTERIC LIQUID CRYSTALS PAST AN INFINITE HOT PLATE

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ABSTRACT: The theory of cholesteric liquid crystal deals with the coupling of thermal gradients and the motion of molecules. Due to practical utility of thermo-mechanical coupling and electric field effects, we have examined here the effects of electric field on the flow of incompressible cholesteric liquid crystals past an infinite hot plate in the presence of two distinct temperature gradients. The system of governing differential equations has been obtained for this flow. It has been observed that the electric field plays a pivotal role in influencing both the velocity and the orientation of molecules through different angles.

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1. INTRODUCTION

The continuum theory of liquid crystals which was originated by Oseen [1] was subsequently modified by Ericksen [2, 3]. Thereafter, Ericksen [4] also discussed the twisting of liquid crystals. Based on these reformulations, Frank [5] proposed a new form of Helmholtz free energy expression for liquid crystals. Extending this work, Leslie [6] derived the constitutive equations for cholesteric of liquid crystals. On the basis of this Leslie [7] then examined the thermo-mechanical coupling effects in cholesteric liquid crystals. Degennes [8] and Chandrasekhar [9] discussed several mathematical concepts on liquid crystals in their monographs "**The physics of Liquid Crystals**" and "**Liquid Crystals**" respectively. Dreher [10] also discussed the contributions of magnetic field in the mechanical body force and extrinsic director body respectively. In order to examine the effects of external parameters on liquid crystals, Skarp *et al.*, [11] proposed the electromagnetic parts of stress tensor and Helmholtz free energy in the presence of electric field.

The theory of nematic liquid crystals [12] has been successfully applied to a large variety of problems due to simplicity in their structure and constitutive equations. However, less attention was paid to the theory of cholesteric liquid crystals [7] because of its complex helical structure, complicated constitutive equations and also due to less certainity of its appropriate boundary conditions.

Liquid crystals of all types mainly cholesterics are now finding applications in almost every field of life, so there is a strong need of more elaborative research on it. In recent years Bisht *et al*., [13], Joshi *et al*., [14], Chandel *et al*., [15] and Virk [16] have examined the effects of magnetic field and temperature gradients on flows of cholesteric liquid crystals, whereas Martin *et al*., [17], Govindaraju *et al*., [18], Lin [19], Maximovich *et al*., [20], Mao *et al*., [21] and Hajime [22] have discussed different types of applications of liquid crystals.

Using the boundary conditions as suggested by Leslie [7], we have investigated here the effects of electric field in different directions on flow of incompressible cholesteric liquid crystals past an infinite hot plate with two distinct temperature gradients in such a way that the helix axis of liquid crystal remains perpendicular to the plate.

2. BASIC EQUATIONS

The basic conservation laws for incompressible cholesteric liquid crystals in the presence of heat flux vector and temperature as proposed by Leslie [6, 7] are

(i) the equation of continuity:

$$v_{i,i} = 0. \quad [2.1]$$

(ii) the equation of motion of incompressible liquid crystal with respect to mechanical velocity $\vec{v}$:

$$\rho \frac{Dv_i}{Dt} = \rho F_i + \sigma_{ji,j} . \quad [2.2]$$

(iii) the equation of motion of director $\vec{d}$ to describe the motion of liquid crystal molecules:

$$\rho_1 \frac{D^2 d_i}{Dt^2} = \rho_1 G_i + g_i + \pi_{ji,j} . \quad [2.3]$$

(iv) the equation of energy for incompressible liquid crystal:

$$\rho \frac{DU}{Dt} = \sigma_{ji} v_{i,j} + \pi_{ji} \frac{Dd_{i,j}}{Dt} - g_i \frac{Dd_i}{Dt} - q_{i,i} . \quad [2.4]$$

Here the symbols like ρ, $\vec{d}$, $\vec{v}$, F_i, G_i, ρ_1, σ_{ji}, π_{ji}, g_i, $\vec{q}$, U and $\frac{D}{Dt}$ are used to denote the uniform density, the director, the velocity vector, the body force per unit mass, the extrinsic director body force per unit mass, an inertial coefficient, the stress tensor, the director stress tensor, the intrinsic director body force per unit volume, the heat flux vector, the internal energy per unit mass and the material time derivative respectively. The above laws are then supplemented by the following set of constitutive equations due to Skarp *et al*., [11] for the stress tensor σ_{ji}, the director stress tensor π_{ji}, the intrinsic director body force g_i, and the heat flux vector $\vec{q}$ due to coupling effect of temperature and electric fields.

$$\sigma_{ji} = -p\delta_{ij} - \rho\frac{\partial F}{\partial d_{k,j}} d_{k,i} + \alpha e_{jkp}(d_p d_i),_k + \tilde{\sigma}_{ji} + \sigma_{ji}^{em}, \quad [2.5]$$

$$\pi_{ji} = \beta_j d_i + \rho\frac{\partial F}{\partial d_{i,j}} + \alpha e_{ijk} d_k, \quad [2.6]$$

$$g_i = \gamma d_i - \beta_j d_{i,j} - \rho\frac{\partial F}{\partial d_i} - \alpha e_{ijk} d_{k,j} + \tilde{g}_i. \quad [2.7]$$

Here the symbols $F, p, \vec{\beta}, \alpha, \gamma, \tilde{\sigma}_{ji}, \tilde{g}_i$ and σ_{ji}^{em} denotes the Helmholtz free energy per unit mass, the pressure, an arbitrary vector, a material coefficient, the director tension, the non-equilibrium part of stress tensor, the non equilibrium parts of extra intrinsic director body force and electromagnetic part of stress tensor respectively. Furthermore the non-equilibrium parts and heat flux vector are given by

$$\tilde{\sigma}_{ji} = \mu_1 d_k d_p A_{kp} d_i d_j + \mu_2 N_i d_j + \mu_3 N_j d_i + \mu_4 A_{ij} + \mu_5 A_{ik} d_k d_j$$
$$+ \mu_6 A_{jk} d_k d_i + \mu_7 e_{ipq} d_p T,_q d_j + \mu_8 e_{jpq} d_p T,_q d_i, \quad [2.8]$$

$$\tilde{g}_i = \lambda_1 N_i + \lambda_2 A_{ik} d_k + \lambda_3 e_{ipq} d_p T,_q, \quad [2.9]$$

$$q_i = k_1 T,_i + k_2 d_k T,_k d_i + k_3 e_{ipq} d_p N_q + k_4 e_{ipq} d_p A_{qk} d_k. \quad [2.10]$$

Here λ_i, μ_i and k_i are the material coefficients related to each other by the set of relations

$$\lambda_1 = \mu_2 - \mu_3, \quad \lambda_2 = \mu_5 - \mu_6, \quad \lambda_3 = \mu_7 - \mu_8 \quad \text{and} \quad \left.\begin{aligned} 2A_{ij} &= v_{i,j} + v_{j,i}, \\ 2W_{ij} &= v_{i,j} - v_{j,i}, \\ N_i &= \frac{Dd_i}{Dt} - W_{ij} d_j. \end{aligned}\right\} \quad [2.11]$$

The electromagnetic part of stress tensor σ_{ji}^{em} occurring in (2.5) as given by Skarp *et al.*, [11] is given by

$$\sigma_{ji}^{em} = \left\{\frac{D_i E_j + D_j E_i - (D_k E_k)\delta_{ij}}{2}\right\} \quad [2.12]$$

where D_i represents the electric displacement and E_i the electric field and both satisfy the Poisson's equation i.e.

$$D_i = (\varepsilon_\perp \delta_{ij} + \varepsilon_a d_i d_j) E_j. \quad [2.13]$$

The symbols ε_a and $\varepsilon_\perp$ denote dielectric anisotropies along and perpendicular to the molecules.

Since the couple stress l_{ji} plays an important role in the boundary conditions of cholesteric type of liquid crystals, it is defined by [7]

$$l_{ji} = e_{ipq} d_p \pi_{jq} \qquad [2.14]$$

where e_{ijk} is the alternating tensor.

The Helmholtz form of free energy F for cholesteric liquid crystal in the presence of electric field proposed by Skarp *et al.*, [11] is

$$2\rho F = \alpha_1 (d_{i,i})^2 + \alpha_2 (\tau + d_i e_{ijk} d_{k,j})^2 + \alpha_3 d_i d_j d_{k,i} d_{k,j} + (\alpha_2 + \alpha_4)[d_{i,j} d_{j,i} - (d_{i,i})^2] - D_i E_i, \qquad [2.15]$$

here α_i and τ are the material coefficients.

All the material coefficients occurring in the above equations will be regarded as constants for the present study.

3. Statement of Problem

We choose a right handed system of cartesian co-ordinates (x, y, z) in such a way that the incompressible cholesteric liquid crystal film of thickness h lying on an infinite horizontal hot plate occupies the plane $z = 0$ with its helix axis normal to the plate. The electric field $\vec{E}$ acting on the fixed plate has the components E_x, E_y and E_z in x, y and z directions respectively.

Following Leslie [7] we seek the solutions of equations (2.1) to (2.4) in the form

$$\left.\begin{aligned} &d_x = \cos\theta(z)\cos\phi(z),\ d_y = \cos\theta(z)\sin\phi(z),\ d_z = \sin\theta(z) \\ &v_x = u(z),\ v_y = v(z), v_z = 0 \\ &T = ax + by + f(\mathrm{z}) \end{aligned}\right\} \qquad [3.1]$$

where θ and Φ determine the orientation of molecules and u, v and f are all unknown functions of z.

At a solid boundary liquid adheres to the surface, the boundary conditions for the orientations θ and ϕ may be taken as

$$\theta = 0, \quad \phi = \phi_0, \quad \text{at} \quad z = 0. \qquad [3.2]$$

Also at a free surface [7] the couple stress components l_{zx}, l_{zy} and l_{zz} should be taken as zero. Therefore, the appropriate boundary conditions for θ and ϕ at free surface will be

$$\theta' = 0, \qquad F_3(\theta)\frac{d\phi}{dz} = \alpha_2 \tau_0 \qquad \text{at} \quad z = h \tag{3.3}$$

where $F_3(\theta) = \{\alpha_2 \cos^2\theta + \alpha_3 \sin^2\theta\}$.

The boundary conditions for u, v and f on the plate are

$$\left.\begin{array}{llll} u = 0, & v = 0 & \text{at} & z = 0 \\ f = T_0 & & \text{at} & z = 0. \end{array}\right\} \tag{3.4}$$

4. FORMULATION OF DIFFERENTIAL EQUATIONS

We observe that the equation of continuity (2.1) is fully satisfied by the choice of velocity components given in (3.1). [4.1]. Furthermore, the equations (2.2) to (2.4) with the help of (3.1) and (2.10) lead to

$$\frac{d(\sigma_{zx})}{dz} = 0, \qquad \frac{d(\sigma_{zy})}{dz} = 0, \qquad \frac{d(\sigma_{zz})}{dz} = 0, \tag{4.1}$$

$$\frac{d(\pi_{zx})}{dz} + g_x = 0, \tag{4.2}$$

$$\frac{d(\pi_{zy})}{dz} + g_y = 0, \tag{4.3}$$

$$\frac{d(\pi_{zz})}{dz} + g_z = 0, \tag{4.4}$$

$$2(\tilde{\sigma}_{zx}\xi + \tilde{\sigma}_{zy}\eta) - \frac{d}{dz}(q_z) = 0. \tag{4.5}$$

Here, $\quad 2\xi = \dfrac{du}{dz} \quad \text{and} \quad 2\eta = \dfrac{dv}{dz}.$

The first two terms of equation (4.1) has the general solution

$$\sigma_{zx}, \quad \sigma_{zy} = l \tag{4.6}$$

where k and l are the constants of integration.

The first two terms in equation (4.5) which represent viscous heating can be neglected [7] particularly in case of small temperature gradients a and b and weak electric field as these terms become quadratic in ξ and η whereas q_z is linear in these quantities. Hence on neglecting these terms, we get on integration

$$q_z = r \tag{4.7}$$

where r is the constant of integration.

Also at a free surface, the stress components σ_{zx}, σ_{zy} and q_z are zero [7], therefore, the equations (4.6) and (4.7) give

$$\sigma_{zx} = 0, \quad \sigma_{zy} = 0 \quad \text{and} \quad q_z = 0. \qquad [4.8]$$

These equations (4.8) with the help of (2.5), (2.8), (2.12), (2.13) and (2.15) lead to

$$\{H_1(\theta) + H_2(\theta)\cos^2\phi\}\xi + \{H_2(\theta)\sin\phi\cos\phi\}\eta + \{H_3(\theta)\sin\phi\}\zeta - b\{H_4(\theta) - H_5(\theta)\cos^2\phi\} - a\{H_5(\theta)\sin\phi\cos\phi\} + E_1 = 0, \qquad [4.9]$$

$$\{H_2(\theta)\sin\phi\cos\phi\}\xi + \{H_1(\theta) + H_2(\theta)\sin^2\phi\}\eta - \{H_3(\theta)\cos\phi\}\zeta + a\{H_4(\theta) - H_5(\theta)\sin^2\phi\} + b\{H_5(\theta)\sin\phi\cos\phi\} + E_2 = 0, \qquad [4.10]$$

$$K_2(\theta)\sin\phi\xi - K_2(\theta)\cos\phi\eta + K_1(\theta)\zeta + K_3(\theta)(a\cos\phi + b\sin\phi) = 0, \qquad [4.11]$$

where $\zeta = \dfrac{df}{dz}$,

$H_1(\theta) = \mu_4 + (\mu_5 - \mu_2)\sin^2\theta$,

$H_2(\theta) = (2\mu_1\sin^2\theta + \mu_3 + \mu_6)\cos^2\theta$,

$H_3(\theta) = \mu_7\sin\theta\cos\theta$,

$H_4(\theta) = \mu_7\sin^2\theta$,

$H_5(\theta) = \mu_8\cos^2\theta$,

$K_1(\theta) = k_1 + k_2\sin^2\theta$,

$K_2(\theta) = (k_3 - k_4)\sin\theta\cos\theta$,

$K_3(\theta) = k_2\sin\theta\cos\theta$,

$F_2(\theta) = \alpha_1\cos^2\theta + \alpha_3\sin^2\theta$,

$F_1(\theta) = (\alpha_2\cos^2\theta + \alpha_3\sin^2\theta)\cos^2\theta$,

$$E_1 = \frac{1}{2}\left\{\begin{array}{l} \varepsilon_a \sin\theta\cos\theta\cos\phi(E_z^2 + E_x^2) + \varepsilon_a\cos^2\theta\sin\phi\cos\phi E_y E_z \\ + \varepsilon_a\sin\phi\sin\theta\cos\theta E_x E_y + (\varepsilon_\perp + \varepsilon_a\cos^2\theta\cos^2\phi)E_x E_z \\ + (\varepsilon_\perp + \varepsilon_a\sin^2\theta)E_x E_z \end{array}\right\},$$

$$E_2 = \frac{1}{2}\left\{\begin{array}{l} \varepsilon_a \sin\theta\cos\theta\sin\phi(E_y^2 + E_z^2) + (\varepsilon_\perp + \varepsilon_a\cos^2\theta\sin^2\phi)E_y E_z \\ + (\varepsilon_\perp + \varepsilon_a\sin^2\theta)E_y E_z + \varepsilon_a\sin\phi\cos\phi\cos^2\theta E_x E_z \\ + \varepsilon_a\sin\theta\cos\theta\cos\phi E_x E_y \end{array}\right\}.$$

On solving the equations (4.9), (4.10) and (4.11) for ξ, η and ζ

$$\xi = a \sin\phi \cos\phi\{G_1(\theta) + G_2(\theta)\} + b\{G_2(\theta)\sin^2\phi - G_1(\theta)\cos^2\phi\} - E_1\{G_3(\theta)\cos^2\phi + G_4(\theta)\sin^2\phi\} + E_2[\{G_4(\theta) - G_3(\theta)\}\sin\phi\cos\phi]. \quad [4.12]$$

$$\eta = a\{G_1(\theta)\sin^2\phi - G_2(\theta)\cos^2\phi\} - b\sin\phi\cos\phi\{G_1(\theta) + G_2(\theta)\} - E_2\{G_3(\theta)\sin^2\phi + G_4(\theta)\cos^2\phi\} + E_1[\{G_4(\theta) - G_3(\theta)\}\sin\phi\cos\phi] = 0. \quad [4.13]$$

$$\zeta = -G_5(\theta)\{a\cos\phi + b\sin\phi\} - G_6(\theta)[\{\mathrm{a}\cos\phi + b\sin\phi\}G_2(\theta) - G_4(\theta)\{E_1\sin\phi - E_2\cos\phi\}]. \quad [4.14]$$

Here

$$G_1(\theta) = \frac{\{H_5(\theta) - H_4(\theta)\}}{\{H_1(\theta) + H_2(\theta)\}}, \qquad G_2(\theta) = \frac{\{K_1(\theta)H_4(\theta) + K_3(\theta)H_3(\theta)\}}{\{K_1(\theta)H_1(\theta) - K_2(\theta)H_3(\theta)\}},$$

$$G_3(\theta) = \frac{1}{\{H_1(\theta) + H_2(\theta)\}}, \qquad G_4(\theta) = \frac{K_1(\theta)}{\{K_1(\theta)H_1(\theta) - K_2(\theta)H_3(\theta)\}},$$

$$G_5(\theta) = \frac{K_3(\theta)}{K_1(\theta)}, \qquad G_6(\theta) = \frac{K_2(\theta)}{K_1(\theta)}.$$

Upon substituting the values of π_{zx}, π_{zy}, π_{zz}, g_x, g_y and g_z in equations (4.2), (4.3) and (4.4) and then eliminating γ between them we obtain the differential equations in θ and ϕ as

$$F_2(\theta)\frac{d^2\theta}{dz^2} + \frac{1}{2}\frac{d}{d\theta}F_2(\theta)\left(\frac{d\theta}{dz}\right)^2 - \frac{1}{2}\frac{d}{d\theta}F_1(\theta)\left(\frac{d\phi}{dz}\right)^2 - 2\alpha_2\tau\sin\theta\cos\theta\frac{d\phi}{dz} + (\lambda_1 + \lambda_2\cos 2\theta)(\xi\cos\phi + \eta\sin\phi) - \lambda_3(a\sin\phi - b\cos\phi) + \varepsilon_a F(\cos^2\theta\cos\phi - \sin^2\theta\cos\phi) + \varepsilon_a E(\cos^2\theta\sin\phi - \sin^2\theta\sin\phi) + \varepsilon_a C\sin\theta\cos\theta - \varepsilon_a A\sin\theta\cos\theta\cos^2\phi - 2\varepsilon_a D\sin\theta\cos\theta\sin\phi\cos\phi - \varepsilon_a B\sin\theta\cos\theta\sin^2\phi = 0, \quad [4.15]$$

$$F_1(\theta)\frac{d^2\phi}{dz^2} + \frac{d}{d\theta}F_1(\theta)\frac{d\theta}{dz}\frac{d\phi}{dz} + 2\alpha_2\tau\sin\theta\cos\theta\frac{d\theta}{dz} + (\lambda_1 - \lambda_2)\sin\theta\cos\theta(\xi\sin\phi - \eta\cos\phi) - \lambda_3\sin\theta\cos\theta(a\cos\phi + b\sin\phi) - \cos\theta\sin\phi(\varepsilon_a\cos\theta\cos\phi A + \varepsilon_a\cos\theta\sin\phi D + \varepsilon_a\sin\theta F) + \cos\theta\cos\phi(\varepsilon_a\cos\theta\sin\phi B + \varepsilon_a\cos\theta\cos\phi D + \varepsilon_a\sin\theta E) - \lambda_3\zeta\cos^2\theta = 0, \quad [4.16]$$

where $A = E_x^2, \quad B = E_y^2, \quad C = E_z^2, \quad D = E_x E_y, \quad E = E_y E_z, \quad F = E_x E_z$.

From the above equations it is observed that the electric field influences all the unknown functions u, v, θ, ϕ and f.

5. Solution of Differential Equations at Low Shear Rates

Since the differential equations derived above are coupled and nonlinear so in order to get a insight into the picture of the behavior of velocities and orientations we linearise them. Accordingly, we can assume their solutions in case of weak electric field and small temperature gradients as:

$$u(z) = au_1(z) + bu_2(z) + Au_3(z) + Bu_4(z) + Cu_5(r) + Du_6(z) + Eu_7(z) + Fu_8(z) + o(a^2, b^2, A^2, B^2, C^2, D^2, E^2, F^2, ..., ab, ...), \tag{5.1}$$

$$v(z) = av_1(z) + bv_2(z) + Av_3(z) + Bv_4(z) + Cv_5(z) + Dv_6(z) + Ev_7(z) + Fv_8(z) + o(a^2, b^2, A^2, B^2, C^2, D^2, E^2, F^2, ..., ab, ...), \tag{5.2}$$

$$\theta(z) = a\theta_1(z) + b\theta_2(z) + A\theta_3(z) + B\theta_4(z) + C\theta_5(z) + D\theta_6(z) + E\theta_7(z) + F\theta_8(z) + o(a^2, b^2, A^2, B^2, C^2, D^2, E^2, F^2, ..., ab, ...), \tag{5.3}$$

$$\phi(z) = \tau_0(z - z_0) + a\phi_1(z) + b\phi_2(z) + A\phi_3(z) + B\phi_4(z) + C\phi_5(z) + D\phi_6(z) + E\phi_7(z) + F\phi_8(z) + o(a^2, E^2, A^2, B^2, D^2, F^2, ..., ab, ...), \tag{5.4}$$

$$f(z) = T_0 + af_1(z) + bf_2(z) + Af_3(z) + Bf_4(z) + Cf_5(z) + Df_6(z) + Ef_7(z) + Ff_8(z) + o(a^2, b^2, A^2, B^2, C^2, D^2, E^2, F^2, ..., ab, ...). \tag{5.5}$$

Here u_i, v_i, θ_i, ϕ_i and f_i ($i = 1, ..., 8$) are to be determined from (4.12), (4.13), (4.14), (4.15) and (4.16) by making use of boundary conditions (3.2), (3.3) and (3.4). Whence we obtain

$$\begin{aligned} u &= ar \sin d \sin (d - 2\tau_0 z_0) - br\{d + \sin d \cos (d - 2\tau_0 z_0)\} \\ &\quad + E_1[- \sin d \sin (d - 2\tau_0 z_0) + 2cr_1 \sin d \sin (d - 2\tau_0 z_0)] \\ &\quad + F_1\begin{bmatrix} - \{d + \sin d \cos (d - 2\tau_0 z_0)\} - 4c(d) \\ - 2cr_1\{d - \sin d \cos (d - 2\tau_0 z_0)\} \end{bmatrix}, \end{aligned} \tag{5.6}$$

$$\begin{aligned} v &= - ar \{- (d) + \sin d \cos (d - 2\tau_0 z_0)\} - br \sin d \sin (d - 2\tau_0 z_0) \\ &\quad + E_1\begin{bmatrix} - \{(d) - \sin d \cos (d - 2\tau_0 z_0)\} - 4c(d) \\ + 2cr_1\{- (d) - \sin d \cos (d - 2\tau_0 z_0)\} \end{bmatrix} \\ &\quad + F_1[-\sin d \sin (d - 2\tau_0 z_0) + 2cr_1 \sin d \sin (d - 2\tau_0 z_0)], \end{aligned} \tag{5.7}$$

$$f = T_0, \tag{5.8}$$

$$\begin{aligned}
\theta = r_2 & \begin{bmatrix} -2a \sin\{(d)/2\} \cos\{(d - 2\tau_0 z_0)/2\} \\ -2b \sin\{(d)/2\} \sin\{(d - 2\tau_0 z_0)/2\} \\ +(d)\{a \cos(\tau_0 h - \tau_0 z_0) + b \sin(\tau_0 h - \tau_0 z_0)\} \end{bmatrix} \\
& + E_2 S_1 \begin{bmatrix} -\sin\{(d)/2\} \cos\{(d - 2\tau_0 z_0)/2\} \\ -2c \sin\{(d)/2\} \cos\{(d - 2\tau_0 z_0)/2\} \\ +\dfrac{d}{2}\{\cos(\tau_0 h - \tau_0 z_0) + 2c \cos(\tau_0 h - \tau_0 z_0)\} \end{bmatrix} \\
& + E_2[2 \sin\{(d)/2\} \cos\{(d - 2\tau_0 z_0)/2\} - d \cos(\tau_0 h - \tau_0 z_0)] \\
& + F_2 S_1 \begin{bmatrix} \sin\{(d)/2\} \sin\{(d - 2\tau_0 z_0)/2\} \\ +2c \sin\{(d)/2\} \sin\{(d - 2\tau_0 z_0)/2\} \\ -\dfrac{d}{2}\{\sin(\tau_0 h - \tau_0 z_0) + 2c \sin(\tau_0 h - \tau_0 z_0)\} \end{bmatrix} \\
& + F_2[-2 \sin\{(d)/2\} \sin\{(d - 2\tau_0 z_0)/2\} + d \sin(\tau_0 h - \tau_0 z_0)],
\end{aligned} \tag{5.9}$$

$$\begin{aligned}
\phi = \phi_0 + (d) + A_1 & \begin{bmatrix} -\{\sin(2d - 2\tau_0 z_0) + \sin 2\tau_0 z_0\} \\ +(d)\{2 \cos 2\tau_0 (h - z_0)\} \end{bmatrix} \\
& + D_1 \begin{bmatrix} \{\cos(2d - 2\tau_0 z_0) - \cos 2\tau_0 z_0\} \\ +(d)\{2 \sin 2\tau_0 (h - z_0)\} \end{bmatrix} \\
& + B_1 \begin{bmatrix} \{\sin(2d - 2\tau_0 z_0) + \sin 2\tau_0 z_0\} \\ -(d)\{2 \cos 2\tau_0 (h - z_0)\} \end{bmatrix},
\end{aligned} \tag{5.10}$$

where

$$\mu_1 = \left\{ \frac{\lambda_3}{\alpha_1} - \frac{(\lambda_1 + \lambda_2)\mu'}{\alpha_1} \right\}, \quad c = \frac{\varepsilon_\perp}{\varepsilon_a}, \quad r_1 = \frac{\mu'''}{\mu''}, \quad d = \tau_0 z, \quad E_1 = \frac{\varepsilon_a \mu'' E}{2\tau_0},$$

$$F_1 = \frac{\varepsilon_a \mu'' F}{2\tau_0}, \qquad r = \frac{\mu'}{\tau_0}, \qquad \mu' = \frac{\mu_8}{\mu_4 + \mu_3 + \mu_6}, \qquad \mu'' = \frac{1}{\mu_4 + \mu_3 + \mu_6},$$

$$\mu''' = \frac{(\mu_3 + \mu_6)}{\mu_4(\mu_4 + \mu_3 + \mu_6)}, \qquad r_2 = \frac{\mu_1}{{\tau_0}^2}, \quad S_1 = (\lambda_1 + \lambda_2)\mu'', \quad E_2 = \frac{\varepsilon_a E}{\alpha_1 {\tau_0}^2},$$

$$F_2 = \frac{\varepsilon_a F}{\alpha_1 {\tau_0}^2}, \qquad A_1 = \frac{\varepsilon_a A}{8\alpha_2 {\tau_0}^2}, \qquad B_1 = \frac{\varepsilon_a B}{8\alpha_2 {\tau_0}^2}, \qquad D_1 = \frac{\varepsilon_a D}{4\alpha_2 {\tau_0}^2}.$$

From these solutions it is observed that electric parameters E and F occur in the expressions for θ, u and v whereas expressions for Φ and f do not contain terms involving these parameters.

6. RESULT AND DISCUSSION

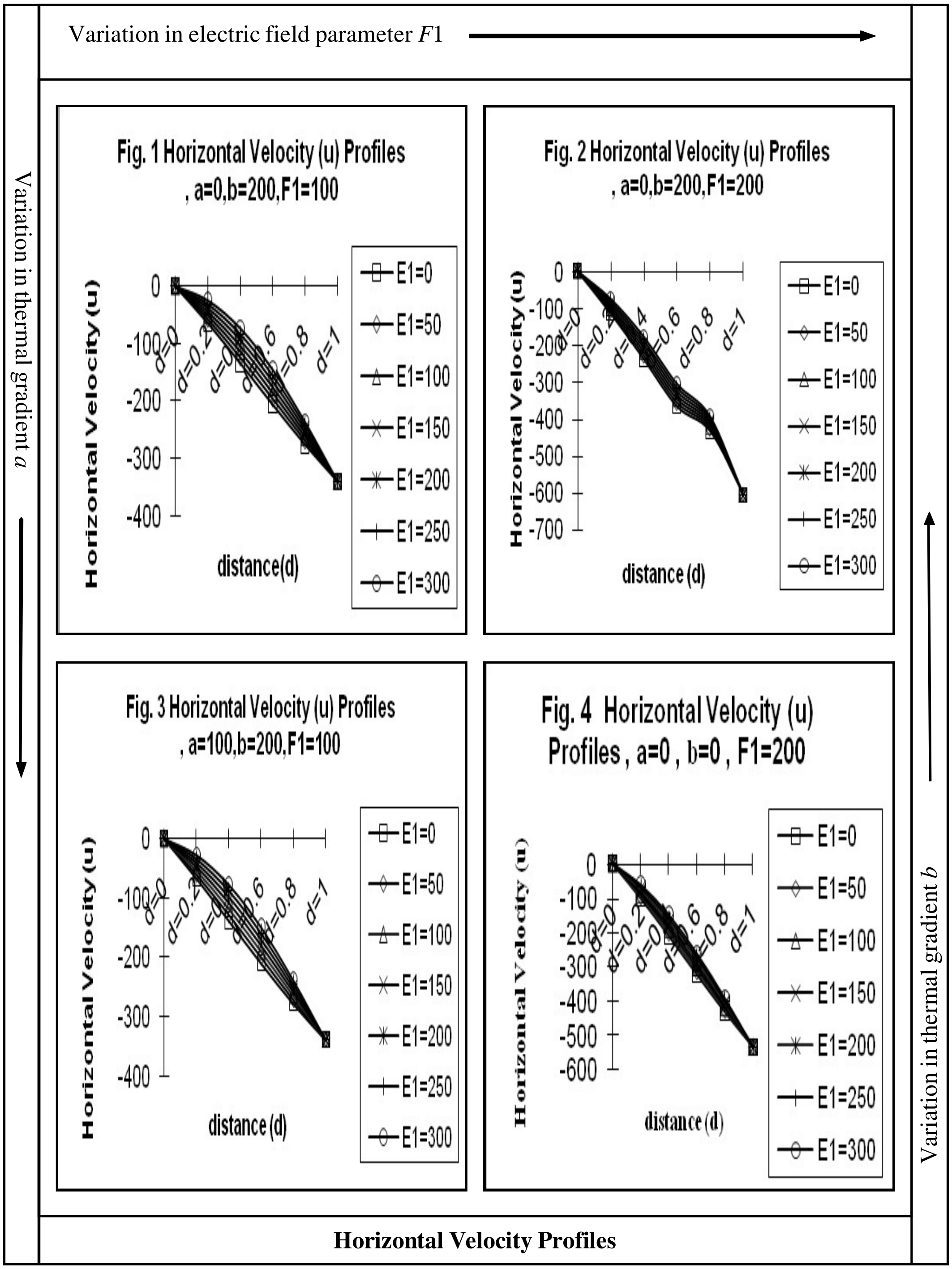
In this section we shall present the numerical solutions to the problem which support and illustrate the detailed analysis given in the previous section. The computation procedure for this problem strongly depends on the equations (5.6), (5.7), (5.8), (5.9) and (5.10). By using these equations the numerical values of the velocities and orientations have been obtained for the varying intensities of the electric field and thermal gradients. Corresponding to these variations , different values of the unknowns have been plotted versus the distance (d), where the distance (d) depicting the diffusion of molecules of cholesteric liquid crystals past the plate is being varied between 0 and 1 with the increment of 0.2. The purpose of this study is to assess the impact of thermo-electric effects on the flow.

Figs. (1) to (4) illustrate the horizontal velocity (u) profiles of cholesteric liquid crystals for different values of electric field parameters E_1 and F_1 and also for the thermal gradients a and b. It has been observed that the horizontal velocity (u) increases with the increase in E_1 and decrease in F_1, whereas it decreases with the increase in temperature gradients. Furthermore it also gets decreased with the increase of distance.

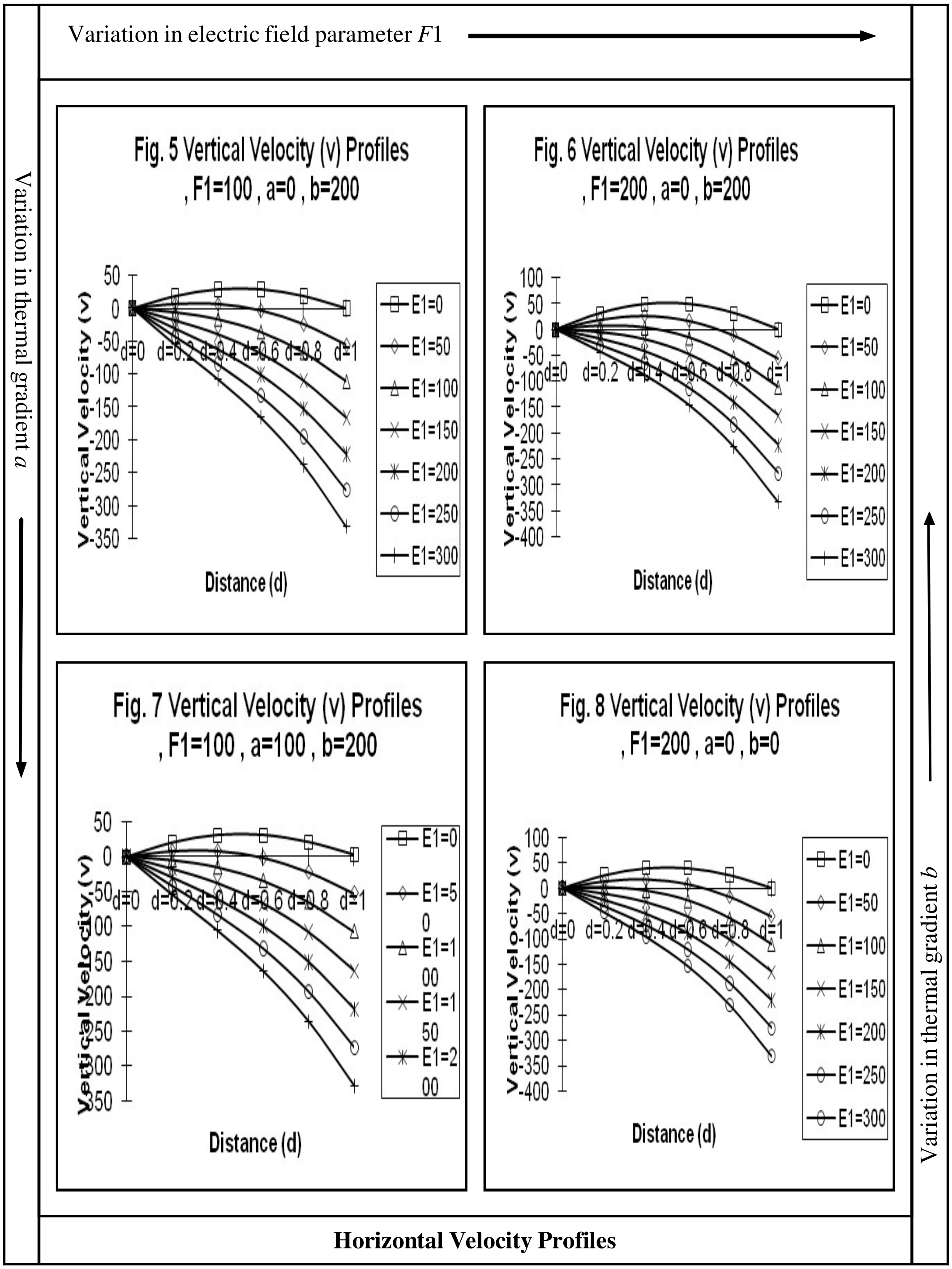
Figs. (5) to (8) illustrate the vertical velocity (v) profiles of an incompressible cholesteric liquid crystal for different values of electric field parameters and thermal gradient parameters. It has been observed that as the electric field parameter E_1 increases the vertical velocity (v) component decreases , whereas this component increases with the increase in intensity of electric field parameter F_1. Also the vertical velocity (v) increases with the increase in temperature gradients a and b as well. However, it decreases with the increase of distance.

Figs. (9) to (12) depict the effect of electric field and temperature gradient parameters on the orientation profiles. It may be observed here that as the parameters E_2, a and b are increased, the orientation angle increases whereas the electric field parameter F_2 corresponds to the decrease in the orientation angle. Fig. (9) shows the spinning of the molecules along with the translational motion which ultimately change the path of as shown in Fig. (10) when the parameter F_2 is increased. Here the orientation first decreases than slightly it increases in its negative direction with the increase in the value of (d).

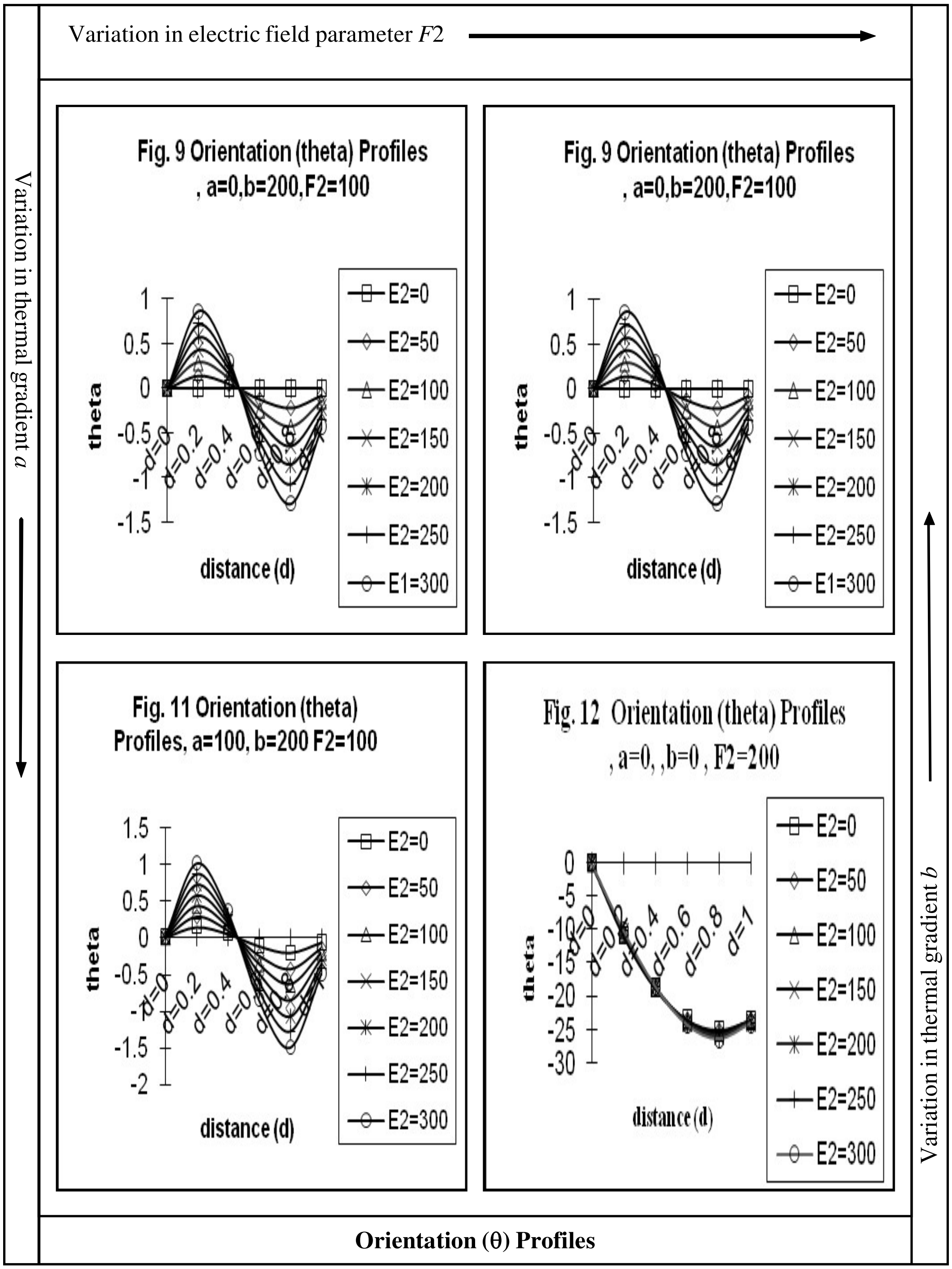
Figs. (13) to (16) correspond to the variation of orientational angle Φ versus the distance (d) for different values of the electric field parameters A_1, B_1 and D_1. It may be observed that the orientation angle Φ increases when the parameters A_1 and D_1 are increased. Also it decreases with the increase in electric field parameter B_1. The orientation angle Φ first increases than decreases until the free surface is reached.



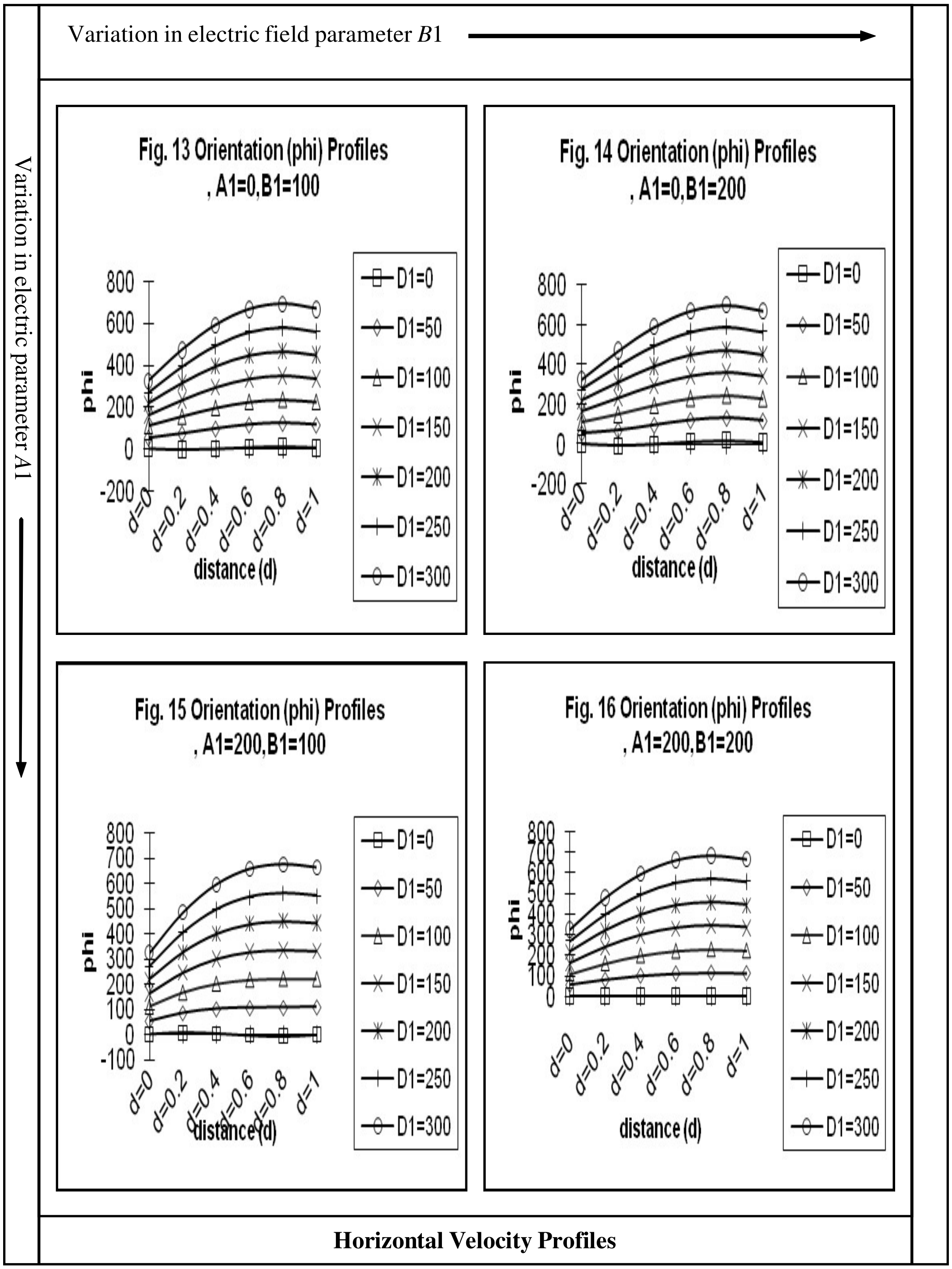
Figures: (1) to (4) for Horizontal Velocity Profiles



Figures: (5) to (8) for Vertical Velocity Profiles



Figures: (9) to (12) for Orientation (θ) Profiles



Figures: (1) to (4) for Horizontal Velocity Profiles

7. Conclusion

As discussed in the preceding sections Joshi *et al.*, [14], Chandel *et al.*, [15] and Bisht *et al.*, [13] have discussed mathematical models to observe the thermo-magnetic and magnetic field effects on flow of incompressible cholesteric liquid crystals between two parallel plates as well as on and inside the two co-axial circular cylinders of infinite extent. Here this paper is devoted to study the impact of electric field and temperature gradients on flow of incompressible cholesteric liquid crystals past an infinite hot plate by using the free surface boundary conditions as suggested by Leslie [7]. From the above discussion it is being concluded that electric field has influenced the orientation angle (Φ) of the molecules in a significant way, whereas the remaining unknowns the horizontal velocity (u), the vertical velocity (v) and the orientation angle (Φ) of the molecules show only small deflections in the presence of electric field. Thus the molecules of the cholesteric liquid crystals undergoes translational motion along with the spinning motion. While in the presence of linear temperature gradients, the velocities and both type of orientations of the molecules of cholesteric liquid crystals are slightly deviated from their initial positions.

Today Scientists and engineers are able to use liquid crystals in a variety of applications. The unique optical properties of liquid crystals enable them to be used in a variety of applications. Birefringence can lead to multicoloured images in the examination of liquid crystals under polarized white light. The Freedericksz transition can be produced by the application of a magnetic or electric field of sufficient strength. This transition is of fundamental importance in the operation of many liquid crystal displays. So the results of the present study may be of use in industrial applications.

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