Higher Order Sliding Mode Control of Electromagnetic Suspension System

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Abstract: This paper presents design and analysis of sliding mode controller for stabilization and reduction of oscillations of the Electromagnetic suspension ball system. The Electromagnetic suspension system (EMS) is a highly nonlinear time varying and unstable system. The sliding mode controller (SMC) is a classical alternative to the controllers which are existing for designing the tricky non-linear control systems. It's design does not depend on system modeling or complex dynamic equations which are governing the association between inputs and outputs. The main purpose of the higher order sliding mode Controller (HOSMC) is to maintain the balance between the developed electromagnetic force and the suspending ball's weight which is due to gravitational force. The proposed controller guarantees the asymptotic stability and regulation of the system state variables to their reference values. The state space model of the electromagnetic suspension system having subsystems of electrical and mechanical is developed and its simulation is carried out using Matlab/Simulink. The transient and steady state performance of the proposed sliding mode controller (SMC) is substantiated by comparing with the conventional PID controller.

Keywords: Electromagnetic Suspension System – HOSMC – Suspension and Stability.

1. INTRODUCTION

The contact-free and wear-free suspension is most essential for the applications involving precision motion control in the fields of mechatronics, automatic control, biomedical and mechanical engineering [1]-[5] and the Electromagnetic suspension systems are becoming popular solutions in numerous applications, such as tool machines high-speed maglev trains, bearingless drives, conveyance system, and isolation vibration tables. In many applications the Electromagnetic suspension systems operate in air gap, its dynamic performance is greatly influenced by the fundamental characteristic of nonlinearities and disturbance rejection capability can be achieved with a suitable controller.

The classical controller development strategy relies on the linearised model of the system dynamics and the most well-known approach is the PID compensation which stabilises the system close to its nominal operating point. In recent years sliding mode controllers for variable structural systems have become popular because of their non requirement of plant mathematical model which is to be controlled [6].

This paper explains the design of variable structure higher order sliding mode controller to improve control dynamics in aspects like disturbance rejection, robustness, stability, and transient and steady-state characteristics with a comparative analysis of the conventional PID controller.

The electromagnetic suspension system basically contains a simple electromagnet and a ferromagnetic ball whose suspension height from equilibrium position is controlled by electromagnetic field strength through ampere turns. The main objective is to maintain the suspending balls position stable at the desired height and minimisation of oscillations by controlling the current through the magnetic coils.

The organization of this paper begins with introduction of the electromagnetic suspension system structure and dynamic model. The paper then presents the design of switching surface for higher order sliding mode control method and ends with performance analysis of the simulation results by comparing with PID controller with MATLAB.

2. ELECTROMAGNETIC SUSPENSION SYSTEM

The considered electromagnetic suspension system consists of a voltage-controlled electromagnetic coil and a ferromagnetic ball suspended under it. The suspension in the vertical direction with respect to the equilibrium point is considered. The objective is to maintain the ball's suspension position at a prescribed reference position with reduced oscillations. The schematic representation of the electromagnetic suspension system is shown in Figure 1. The electromagnetic force, generated in the electromagnet always acts opposite to gravitational force on the suspending ball and maintains the stable levitatation [7].

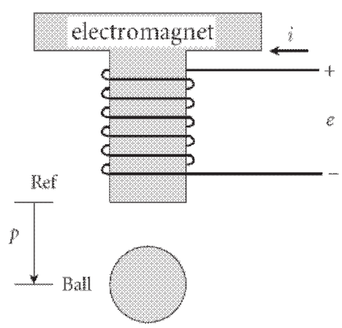

Figure 1: Simplified electromagnetic suspension system

The system dynamic model can be expressed as,

$$\frac{dp}{dt} = v \tag{1}$$

$$Ri + \frac{d(L(p), i)}{dt} = e \tag{2}$$

where, p is the position of suspended ball,

v is the rate of change of displacement of ball,

i is the excitation current in the electromagnet,

e is the voltage applied across electromagnet,

R is the resistance offered by the electromagnetic coil,

L is the inductance offered by the electromagnetic coil,

g is the constant of gravitational force, and

m is the mass of the suspended ball

The inductance offered by the electromagnetic coil L is a nonlinear function, and is highly influenced by the location of the ferromagnetic ball "*p*" with respect to the equilibrium point is defined as:

$$L(p) = L_1 + \frac{L_0 P_0}{p} \tag{3}$$

where, L_1 is the electromagnetic coil inductivity in free space (in the absence of the ball),

L_0 is the additional inductivity developed in the electromagnetic coil by presence of the ball and

P_0 is the equilibrium position of the ball.

The electromagnetic levitation force developed $F(p, i, t)$ can be expressed using the laws of the generalized forces.

$$\begin{aligned} F(p,i,t) &= -\frac{i^2}{2}\frac{\partial L(p)}{\partial p} \\ &= -\frac{i^2}{2}\frac{\partial}{\partial p}\left(L_1 + \frac{L_0 P_0}{p}\right) = k\left(\frac{i}{p}\right)^2 \end{aligned} \tag{4}$$

Defining the magnetic force constant as $k = \frac{L_0 P_0}{2}$

The mathematical model of the EMS system expressing in terms of force balancing equation as

$$m\frac{d^2 p}{dt^2} = mg - F(p,i,t) \tag{5}$$

Let, $v = \frac{dp}{dt}$ velocity of the ball

$$m\frac{dv}{dt} = mg - F(p,i,t) = mg - k\left(\frac{i}{p}\right)^2 \tag{6}$$

Considering the state variables and the control input as $x_1 = p,\ x_2 = v,\ x_3 = i,\ u = e$.

Then the state vector becomes

$$X = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$$

From equations (1) and (6)

$$\frac{dx_1}{dt} = \frac{dp}{dt} = v = x_2 \tag{7}$$

$$\frac{dx_2}{dt} = \frac{dv}{dt} = g - \frac{k}{m}\left(\frac{i}{p}\right)^2 = g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2 \tag{8}$$

From equation (2)

$$\begin{aligned} e &= Ri + \frac{\partial(L(p),i)}{\partial L}\frac{\partial L}{\partial t} + \frac{\partial(L(p),i)}{\partial t}\frac{\partial i}{\partial t} \\ &= Ri + i\frac{dL}{dt} + L\frac{di}{dt} = Ri + i\frac{\partial L}{\partial p}\frac{\partial p}{\partial t} + L\frac{di}{dt} \end{aligned} \tag{9}$$

where, $\frac{\partial L}{\partial p} = -\frac{2k}{p^2} = -\frac{2k}{x_1^2}$

$$e = R\, x_3 - 2k\left(\frac{x_2 x_3}{x_1^2}\right) + L\frac{dx_3}{dt}$$

$$\frac{dx_3}{dt} = -\frac{R}{L}x_3 + \frac{2k}{L}\left(\frac{x_2 x_3}{x_1^2}\right) + \frac{1}{L}u \tag{10}$$

Then, the state variable representation of the electromagnetic Suspension system becomes

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2 \\ \frac{dx_3}{dt} &= -\frac{R}{L}x_3 + \frac{2k}{L}\left(\frac{x_2 x_3}{x_1^2}\right) + \frac{1}{L}u \end{aligned} \tag{11}$$

Let x_{d1}, x_{d2} and x_{d3} be the desired values of x_1, x_2 and x_3, respectively and

$$x_{d2} = \frac{dx_{d1}}{dt} = 0\ (x_{d1} = \text{constant}).$$

From (11), the equilibrium point for the system is $X_d = [x_{d1} \quad 0 \quad x_{d3}]^T$

and $$g - \frac{k}{m}\left(\frac{x_{d3}}{x_{d1}}\right)^2 = 0$$

hence $$x_{d3} = g_c\left(\frac{g_c m}{k}\right)^{1/2} x_{d1}$$

The objective of the proposed controller is to drive the state vector $X = [x_1 \quad x_2 \quad x_3]^T$ to their desired constant value state $X_d = [x_{d1} \quad x_{d2} \quad x_{d3}]^T$.

3. HIGH ORDER SLIDING MODE CONTROLLER

The Higher order sliding mode controller (HOSMC) can be designed for the proposed electromagnetic suspension system using the state-space model given in (11).

The switching manifold integrated with integral operation for the higher order sliding-mode position controller is designed as [8].

The nonlinear state transformation can be achieved from $X = [x_1 \quad x_2 \quad x_3]^T$ to $Z = [z_1 \quad z_2 \quad z_3]^T$ by assuming.

$$\begin{aligned} z_1 &= x_1 - x_{d1} \\ z_2 &= x_2 \\ z_3 &= g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2 \end{aligned} \tag{12}$$

As $t \rightarrow \infty$, the $Z = [z_1 \quad z_2 \quad z_3]^T$, vector is forced to zero then $X = [x_1 \quad x_2 \quad x_3]^T$ will converge to $\begin{bmatrix} x_{d1} & 0 & g\left(\frac{gm}{k}\right)^{\frac{1}{2}} x_{d1} \end{bmatrix}^T$ as $t \rightarrow \infty$.

where, x_{d1} is reference variable and the dynamic behavior of the electromagnetic suspension system in the transformed coordinate system becomes

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = z_3$$
$$\dot{z}_3 = \frac{d}{dt}\left[g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2\right] = -\frac{2k}{m}\left(\frac{x_3 \dot{x}_3}{x_1^2}\right) + \frac{2k}{m}\left(\frac{x_3^2 \dot{x}_1}{x_1^3}\right)$$
$$= -\frac{2k}{m}\left[\frac{x_3}{x_1^2}\left(-\frac{R}{L}x_3 + \frac{2k}{L}\left(\frac{x_2 x_3}{x_1^2}\right) + \frac{u}{L}\right) + \left(\frac{x_3^2 x_2}{x_1^3}\right)\right]$$
$$= f(z) + g(z)u \tag{13}$$

The functions $f(z)$ and $g(z)$ are in terms of original coordinates $X = [x_1 \quad x_2 \quad x_3]^T$ expressed as follows:

$$f(z) = \frac{2k}{m}\left[\left(1 - \frac{2k}{Lx_1}\right)\frac{x_3^2 x_2}{x_1^3} + \frac{Rx_3^2}{Lx_1^2}\right]$$
$$g(z) = -\frac{2kx_3}{Lmx_1^2} \quad (14)$$

Let the response of the electromagnetic suspension system be

$$y = z_1 = x_1 - x_{d1} \tag{15}$$

The complete design of HOSMC for the system requires the selection of suitable the switching surface function.

The general form of the switching surface *S is*

$$\begin{aligned} S &= \ddot{y} + s_1\dot{y} + s_2 y \\ &= \ddot{z}_1 + s_1\dot{z}_1 + s_2 z_1 \\ &= z_3 + s_1 z_2 + s_2 z_1 \end{aligned} \tag{16}$$

where, s_1 and s_2 are positive scalar feedback gains to be designed such that the error dynamics will have the desired response while the system becomes with negligible uncertainties and reduced disturbances [9].

Using equation (12), the sliding mode switching surface:

$$S = g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2 + s_1 x_2 + s_2(x_1 - x_{d1}) \tag{17}$$

This switching surface guarantees during sliding phase (i.e., S = 0), the output $y = z_1 = (x_1 - x_{d1})$ converges to 0 as $t \rightarrow \infty$.

The adopted sliding mode control strategy guarantees that the system trajectory always shift toward and reside on the sliding surface S = 0 for any initial state if the following condition meets (the condition needed to guarantee switching).

$$S\dot{S} \le -\gamma |S| \tag{18}$$

where, γ is a constant (positive) that ensures the system phase trajectories always strike the sliding surface in finite time and let $\dot{S}$ is defined as[10]

$$\dot{S} = \gamma \,\mathrm{sign}(S) \tag{19}$$

where,

$$\begin{aligned} \dot{S} &= \dddot{x}_1 + s_1\ddot{x}_1 + s_2\dot{x}_1 = \dot{x}_3 + s_1\dot{x}_2 + s_2\dot{x}_1 \\ &= f(x) + g(x)u + s_1 x_3 + s_2 x_2 \\ &= f(x) + g(x)u + s_1\left(g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2\right) + s_2 x_2 \end{aligned} \tag{20}$$

From (19) and (20) the control law obtained as:

$$u = \frac{1}{g}\left[-f - s_1\left(g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2\right) - s_2 x_2 - \gamma\,\mathrm{sign}(S)\right]$$

The system parameters and their specifications of the proposed electromagnetic suspension system are given in Table 1.

Table 1
Parameters of The Electromagnetic Suspension System

Symbol	*Parameter*	*Optimum values*
m	Mass of the Ball	0.014 kg
g	Gravity acceleration	9.81 m/s^2
p	Equilibrium distance	0.009 m
i	Equilibrium current	1.5 A
k	Force constant	4.94×10^{-6} Nm^2/A^2
R	Coil resistance	5.2 ohm
L	Coil inductance	0.027 H
L^o	Coil inductance in the absence ball	0.0011 H

4. SIMULATION RESULTS OF THE HIGHER ORDER SLIDING MODE CONTROLLER

The simulated model of proposed electromagnetic suspension system incorporating with the higher order sliding mode controller for ball's suspension position control is shown in Figure 2 and the complete control scheme shown in Figure 3.

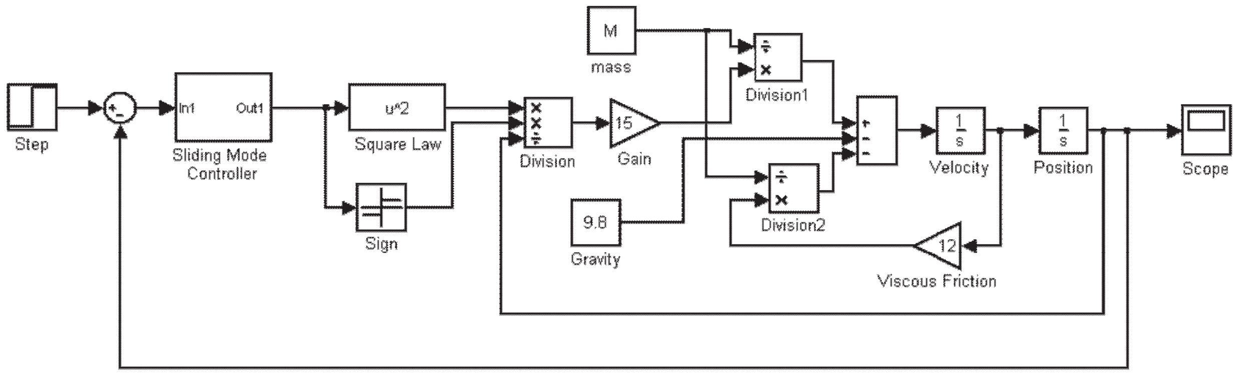

Figure 2: Electromagnetic suspension system with HOSMC Controller

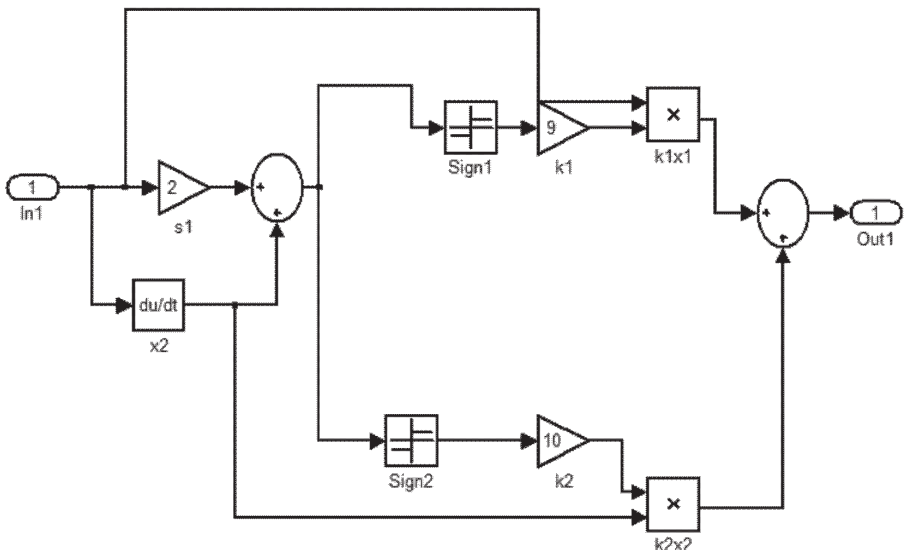

Figure 3: HOSMC Controller

The suspended ferromagnetic ball's position using Higher order sliding mode controller and classical PID controller for the proposed magnetic suspension system is shown in Figure 4. The designed PID control parameters are $K_P = 23$, $K_I = 3$ and $K_D = 0.8$.

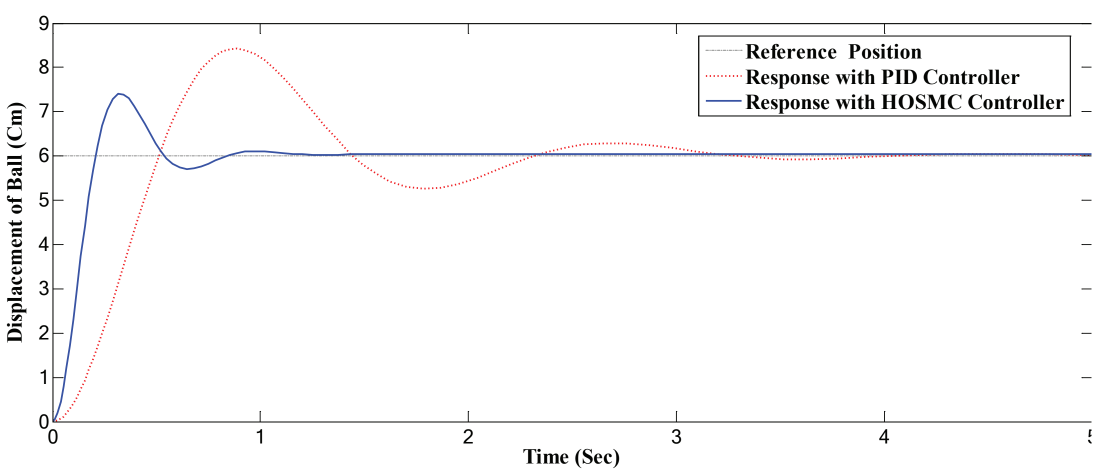

Figure 4: The displacement versus time with the HOSMC and PID Controllers

From the Figure 4 it can be observed that with the sliding mode controller the displacement of the ball converges quickly to the desired step value with less oscillation than the PID controller. Hence, the higher order sliding mode controlled system is robust to changes in the reference value.

The suspended ferromagnetic ball's position tracking for sudden changes in the reference position different with higher order sliding mode controller and classical PID controllers is shown in Figure 5 and can be observed that the sliding mode controller responds quickly when the reference position is varied suddenly.

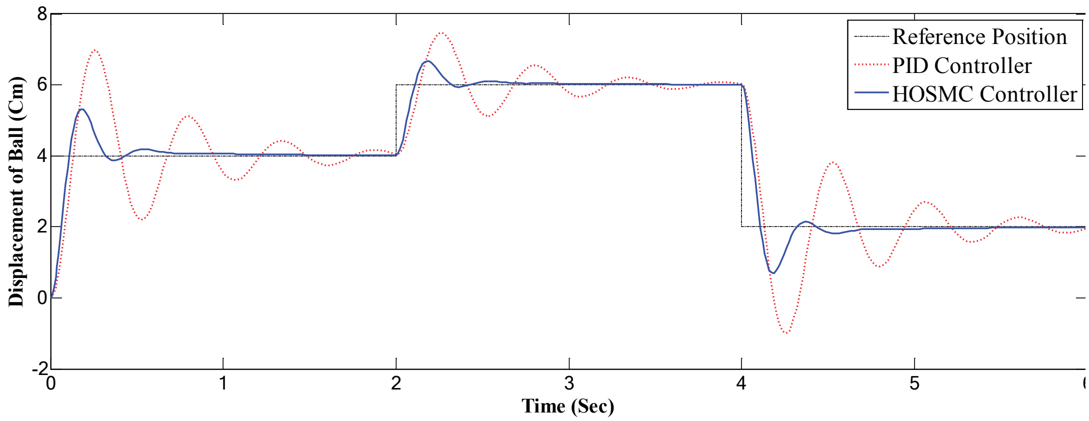

Figure 5: The displacement versus time for sudden changes in the reference position with the HOSMC and PID Controllers

The simulation results indicate that the proposed higher order sliding mode control scheme work well for the highly nonlinear time varying and unstable electromagnetic levitation system.

5. CONCLUSIONS

Sliding mode controller with increased order approach for a highly unstable, nonlinear electromagnetic suspension system has been presented. The simplified model in state space form has been formulated from the basic laws of electromagnetism. The performance of the HOSMC controller in tracking the desired position trajectory of the suspended ball is superior compared to conventional PID controller. The time domain specifications settling time and Steady state error are improved with the proposed controller. The proposed higher order sliding mode control can be successfully applied for achieving robustness and reduced chattering in the sliding surface for the highly nonlinear and unstable systems.

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