HOMOMORPHISM AND ANTI HOMOMORPHISM OF A BI-LEVEL SUB-BIGROUP OF AN INTUITIONISTIC FUZZY SUB-BIGROUP OF A BIGROUP

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ABSTRACT: *In this paper, we introduce the concept of bi-level subset of an intuitionistic fuzzy sub-bigroup of a bigroup and discussed some of its properties under homomorphism and anti homomorphism.*

Keywords: *Fuzzy set , intuitionistic fuzzy subgroup , intuitionistic fuzzy sub-bigroup of a bigroup, bi-level subset, bi-level subgroup, homomorphism , anti homomorphism.*

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INTRODUCTION

The concept of fuzzy sets was initiated by Zadeh. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Rosenfeld gave the idea of fuzzy subgroups and Ranjith Biswas gave the idea of intuitionistic fuzzy subgroups.

The notion of bigroup was first introduced by P.L. Maggu in 1994. W.B. Vasantha Kandasamy and D. Meiyappan introduced concept of fuzzy sub-bigroup of a bigroup and fuzzy sub-bigroup of a group.

In this paper, we introduce the concept of intuitionistic fuzzy sub-bigroup of a bigroup and bi-level subset of an intuitionistic fuzzy sub-bigroup and prove some results under homomorphism and anti homomorphism.

2. PRELIMINARIES

This section contains some definitions and results to be used in the sequel.

Definition 1.1: Let S be a set. A fuzzy subset A of S is a function $A : S \to [0, 1]$.

Definition 1.2: Let G be a group. A fuzzy subset A of G is called a fuzzy subgroup if for $x, y \in G$,

(i) $A(xy) \geq \min \{A(x), A(y)\}$,

(ii) $A(x^{-1}) = A(x)$.

Example 1.1: Let G be the Klein 4-group. Then

$$G = \{e, a, b, ab\},\ a^2 = b^2 = e \text{ with } ab = ba.$$

Define a fuzzy subset A of G by

$$A(e) = 0.8\ A(a) = 0.2,\ A(b) = 0.6,\ A(ab) = 0.6.$$

Clearly A is a fuzzy subgroup of G.

Definition 1.3: An intuitionistic fuzzy subset (IFS) A in a set X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$, where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Definition 1.4: Let G be a group. An intuitionistic fuzzy subset $A = \langle x, \mu_A(x), \nu_A(x)\rangle$ of G is said to be an intuitionistic fuzzy subgroup (IFSG) of G if the following conditions are satisfied:

(i) $\mu_A(xy) \ge \min\{\mu_A(x), \mu_A(y)\}$, $\nu_A(xy) \le \max\{\nu_A(x), \nu_A(y)\}$ for all x and $y \in G$.

(ii) $\mu_A(x^{-1}) \ge \mu_A(x)$, $\nu_A(x^{-1}) \le \nu_A(x)$ for all $x \in G$.

Example 1.2: Let $G = \{1, -1, i, -i\}$ be a group under the operation multiplication. Then $A = \{< 1, 0.4, 0.2 >, < -1, 0.4, 0.2 >, < i, 0.3, 0.3 >, < -i, 0.3, 0.3 >\}$ is an IFSG of G.

Definition 1.5: Let G and G' are any two groups. Then the function $f : G \to G'$ is said to be a homomorphism if $f(xy) = f(x)\,f(y)$ for all x, y in G.

Definition 1.6: Let G and G' be any two groups (not necessarily commutative). Then the function $f : G \to G'$ is said to be an anti-homomorphism if $f(xy) = f(y)\,f(x)$ for all x, y in G.

Definition 1.7: Let A be a fuzzy set in S and f is a function defined on S, then the fuzzy set V in $f(S)$ is defined by

$$V(y) = \sup_{X \in f^{-1}(y)} A(x) \text{ for all } y \text{ in } f(S).$$

V is also called the image of A under f.

Similarly if V is a fuzzy set in $f(S)$,then there exist a fuzzy set A in S such that $A = V \circ f$.

(i.e) the fuzzy set defined by $A(x) = V(f(x))$ for all x in S is called the pre image of V under f.

Definition 1.8: Let A be a fuzzy subset of S. For $\alpha, \beta \in [0, 1]$, the level subset of A is the set,

$$A_{<\alpha,\beta>} = \{x \in S : \mu_A(x) \geq \alpha \text{ and } \nu_A(x) \leq \beta\}.$$

Definition 1.9: A set $(G, +, \bullet)$ with two binary operation + and • is called a bigroup if there exist two proper subsets G_1 and G_2 of G such that

(i) $G = G_1 \cup G_2$

(ii) $(G_1, +)$ is a group.

(iii) $(G_2, \bullet)$ is a group.

A non-empty subset H of a bigroup $(G, +, \bullet)$ is called a sub-bigroup if H itself is a bigroup under + and • operations defined on G.

Definition 1.10: Let $G = (G, +, \bullet)$ be a bigroup. Then $A : G \to [0, 1]$ is said to be an intuitionistic fuzzy sub-bigroup of the bigroup G if there exist two fuzzy subsets A_1 of G_1 and A_2 of G_2 such that

(i) $A = A_1 \cup A_2$.

(ii) $(A_1, +)$ is an IFSG of $(G_1, +)$.

(iii) $(A_2, \bullet)$ is a an IFSG of $(G_2, \bullet)$.

Example 1.3: Let $G_1 = \{0, \pm 1, \pm 2, \pm 3 \ldots\}$ be a group under the operation +.

Let $G_2 = \{1, -1, i, -i\}$ be a group under the operation •.

Define $\mu_{A1} : G_1 \to [0, 1]$, $\nu_{A1} : G_2 \to [0, 1]$ by

$$\mu_{A1}(x) = \begin{cases} 0.9 \text{ if } x \in \{0, \pm 2, \pm 4, \ldots\} \\ 0.25 \text{ if } x \in \{\pm 1, \pm 3, \ldots\}, \end{cases}$$

$$\nu_{A1}(x) = \begin{cases} 0.1 \text{ if } x \in \{0, \pm 2, \pm 4, \ldots\} \\ 0.6 \text{ if } x \in \{\pm 1, \pm 3, \ldots\}, \end{cases}$$

and $\mu_{A2} : G_2 \to [0, 1]$ and $\nu_{A2} : G_2 \to [0, 1]$ by

$$\mu_{A2}(x) = \begin{cases} 0.25 \text{ if } x \in \{\pm 1\} \\ 0.2 \text{ if } x \in \{\pm i\}, \end{cases}$$

$$\nu_{A2}(x) = \begin{cases} 0.6 \text{ if } x \in \{\pm 1\} \\ 0.6 \text{ if } x \in \{\pm i\} \end{cases}$$

Then

$$\mu_A(x) = \begin{cases} 0.9 \text{ if } x \in \{0, \pm 2, \pm 4, ...\} \\ 0.6 \text{ if } x \in \{\pm 1, \pm 3, \pm 5...\}, \\ 0.2 \text{ if } x \in \{\pm i\} \end{cases}$$

and

$$\nu_A(x) = \begin{cases} 0.1 \text{ if } x \in \{0, \pm 2, \pm 4, ...\} \\ 0.6 \text{ if } x \in \{\pm 1, \pm 3, ...\} \\ 0.6 \text{ if } x \in \{\pm i\} \end{cases}$$

Clearly $(A_1,+)$ is an IFSG of $(G_1, +)$ and $(A_2, \bullet)$ is an IFSG of $(G_2, \bullet)$ and $A = A_1 \cup A_2$. It is easy to verify that A is an intuitionistic fuzzy sub-bigroup of the bigroup G

We introduce the notion of bi-level subset of an intuitionistic fuzzy sub-bigroup.

Definition 1.11: Let $G = (G_1 \cup G_2, +, \bullet)$ be a bigroup and $A = (A_1 \cup A_2, +, \bullet)$ be an intuitionistic fuzzy sub-bigroup of the bigroup G. The bi-level subset of the intuitionistic fuzzy sub-bigroup A of the bigroup G is defined as

$A_{<\alpha, \beta>} = A_{1<\alpha, \beta>} \cup A_{2<\alpha, \beta>}$ for every $\alpha \in [0, \min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\}]$ and $\beta \in [\max\{\nu_{A1}(e_1), \nu_{A2}(e_2)\}, 1]$, Where e_1 denotes the identity element of the group $(G_1, +)$ and e_2 denotes the identity element of the group $(G_2, \bullet)$.

Remark: The conditions $\alpha \in [0, \min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\}]$ and $\beta \in [\max\{\nu_{A1}(e_1), \nu_{A2}(e_2)\}, 1]$ are essential for the bi-level subset to be a sub-bigroup, for if $\alpha \notin [0, \min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\}]$ and $\beta \notin [\max \{\nu_{A1}(e_1), \nu_{A2}(e_2)\}, 1]$, the bi-level subset need not in general be a sub-bigroup of the bigroup G which is evident from the following example.

Example 1.4: Consider Example 1.3, the bi-level subset $A_{<\alpha, \beta>}$ for $\alpha = 0.7$ and $\beta = 0.5$ of the intuitionistic fuzzy sub-bigroup A is given by $A_{<\alpha, \beta>} = \{0, \pm 2, \pm 4, ...\}$ which is not a sub-bigroup of the bigroup G. Therefore the bi-level subset $A_{<\alpha, \beta>}$ for $\alpha = 0.7$ and $\beta = 0.5$ is not a sub-bigroup of the bigroup G.

Theorem 1.1: Every bi-level subset of an intuitionistic fuzzy sub-bigroup A of a bigroup G is a sub-bigroup of the bigroup G.

Proof: Let $A = (A_1 \cup A_2, +, \bullet)$ be an intuitionistic fuzzy sub-bigroup of a bigroup $G = (G_1 \cup G_2, +, \bullet)$. Consider the bi-level subset $A_{<\alpha, \beta>}$ of an intuitionistic fuzzy sub-bigroup A, for every $\alpha \in [0, \min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\}]$ and $\beta \in [\max\{\nu_{A1}(e_1), \nu_{A2}(e_2)\}, 1]$, where e_1

denotes the identity element of the group $(G_1, +)$ and e_2 denotes the identity element of the group $(G_2, \bullet)$. Then $A_{<\alpha, \beta>} = A_{1<\alpha, \beta>} \cup A_{2<\alpha, \beta>}$ where $A_{1<\alpha, \beta>}$ and $A_{2<\alpha, \beta>}$ are subgroups of G_1 and G_2 respectively. Hence by the definition of sub-bigroup $A_{<\alpha, \beta>}$ is a sub-bigroup of the bigroup $(G, +, \bullet)$.

We illustrate the following by an example.

Example 1.5: Let $G = \{0, \pm 1, \pm \mathrm{i}\}$ is a bigroup with respect to addition modulo 2 and multiplication. Clearly $G_1 = \{0, 1\}$ and $G_2 = \{\pm 1, \pm \mathrm{i}\}$ are groups with respect to addition modulo 2 and multiplication respectively.

Define $\mu_A : G \to [0, 1]$ and $\nu_A : G \to [0, 1]$ by

$$\mu_A(x) = \begin{cases} 1 & \text{for } x = 0 \\ 0.5 & \text{for } x = \pm 1 \\ 0.4 & \text{for } x = \pm i \end{cases} \qquad \nu_A(x) = \begin{cases} 0 & \text{for } x = 0 \\ 0.5 & \text{for } x = \pm 1 \\ 0.6 & \text{for } x = \pm i \end{cases}$$

Define $\mu_{A1} : G_1 \to [0, 1]$ and $\nu_{A1} : G_1 \to [0, 1]$ by,

$$\mu_{A1}(x) = \begin{cases} 1 & \text{for } x = 0 \\ 0.5 & \text{for } x = 1 \end{cases} \qquad \nu_{A1}(x) = \begin{cases} 0 & \text{for } x = 0 \\ 0.5 & \text{for } x = 1 \end{cases}$$

Define $\mu_{A2} : G_2 \to [0, 1]$ and $\nu_{A2} : G_1 \to [0, 1]$ by,

$$\mu_{A2}(x) = \begin{cases} 0.5 & \text{for } x = \pm 1 \\ 0.4 & \text{for } x = \pm i \end{cases} \qquad \nu_{A2}(x) = \begin{cases} 0.5 & \text{for } x = \pm 1 \\ 0.6 & \text{for } x = \pm i \end{cases}$$

It is easy to verify that A is an intuitionistic fuzzy sub-bigroup of the bigroup G, since there exist two intuitionistic fuzzy subgroups A_1 and A_2 such that $A = A_1 \cup A_2$.

We now find the bi-level subset $A_{<\alpha, \beta>}$ for $\alpha = \beta = 0.5$,

$$\begin{aligned} A_{<\alpha, \beta>} &= A_{1<\alpha, \beta>} \cup A_{2<\alpha, \beta>} \\ &= \{x \in G_1 : \mu_{A1}(x) \geq 0.5 \text{ and } \upsilon_{A1}(x\} \leq 0.5\} \cup \\ &\quad \{x \in G_2 : \mu_{A2}(x) \geq 0.5 \text{ and } \mu_{A1}(x\} \leq 0.5\} \\ &= \{0, 1\} \cup \{\pm 1\} \\ &= \{0, \pm 1\} \\ A_{<\alpha, \beta>} &= \{0, \pm 1\} \end{aligned}$$

It is easily verified that $A_{<\alpha, \beta>}$ is a sub-bigroup of the bigroup G.

Theorem 1.2: The homomorphic image of an intuitionistic fuzzy sub-bigroup A of a bigroup G is an intuitionistic fuzzy sub-bigroup of a bigroup G′.

Proof: Let $G = (G_1 \cup G_2, +, \bullet)$ and $G' = (G'_1 \cup G'_2, +, \bullet)$ are bigroups.

Let $A = (A_1 \cup A_2, +, \bullet)$ be an intuitionistic fuzzy sub-bigroup of a bigroup G.

Then $(A_1, +)$ in an IFSG of $(G_1, +)$ and $(A_2, \bullet)$ in an IFSG of $(G_2, \bullet)$.

Let $f : G \to G'$ be a homomorphism. (i.e) $f(xy) = f(x)\, f(y)$ for all x, y in G.

We have to prove that $f(A) = V$ is an intuitionistic fuzzy sub-bigroup of G.

$$V = f(A) = f(A_1 \cup A_2) = f(A_1) \cup f(A_2).$$

Since f is a homomorphism, $(f(A_1), +)$ is an intuitionistic fuzzy subgroup of $(G'_1, +)$ and $(f(A_2), \bullet)$ is an intuitionistic fuzzy subgroup of $(G'_2, \bullet)$.

Hence $V = f(A_1) \cup f(A_2)$ is an intuitionistic fuzzy sub-bigroup of G'.

Theorem 1.3: The homomorphic pre-image of an intuitionistic fuzzy sub-bigroup V of a bigroup G' is an intuitionistic fuzzy sub-bigroup of a bigroup G.

Proof: Let $G = (G_1 \cup G_2, +, \bullet)$ and $G' = (G'_1 \cup G'_2, +, \bullet)$ are bigroups.

Let $V = (V_1 \cup V_2, +, \bullet)$ be an intuitionistic fuzzy sub-bigroup of a bigroup G'.

Then $(V_1, +)$ in an IFSG of $(G'_1, +)$ and $(V_2, \bullet)$ in an IFSG of $(G'_2, \bullet)$

Let $f : G \to G'$ be a homomorphism. (i.e) $f(xy) = f(x)\, f(y)$ for all x, y in G.

We have to prove that $f^{-1}(V) = A$ is an intuitionistic fuzzy sub-bigroup of G.

$$A = f^{-1}(V) = f^{-1}(V_1 \cup V_2) = f^{-1}(V_1) \cup f^{-1}(V_2).$$

Since f is a homomorphism, $(f^{-1}(V_1), +)$ is an intuitionistic fuzzy subgroup of $(G_1, +)$ and $(f^{-1}(V_2), \bullet)$ is an intuitionistic fuzzy subgroup of $(G_2, \bullet)$.

Hence $A = f^{-1}(V_1) \cup f^{-1}(V_2)$ is an intuitionistic fuzzy sub-bigroup of G.

Theorem 1.4: The anti homomorphic image of an intuitionistic fuzzy sub-bigroup A of bigroup G is an intuitionistic fuzzy sub-bigroup of a bigroup G'.

Proof: Let $= (G_1 \cup G_2, +, \bullet)$ and $G' = (G'_1 \cup G'_2, +, \bullet)$ are bigroups.

Let $A = (A_1 \cup A_2, +, \bullet)$ be an intuitionistic fuzzy sub-bigroup of a bigroup G.

Then $(A_1, +)$ in an IFSG of $(G_1, +)$ and $(A_2, \bullet)$ in an IFSG of $(G_2, \bullet)$

Let $f : G \to G'$ be an anti homomorphism. (i.e) $f(xy) = f(y)\, f(x)$ for all x, y in G.

We have to prove that $f(A) = V$ is an intuitionistic fuzzy sub-bigroup of G.

$$V = f(A) = f(A_1 \cup A_2) = f(A_1) \cup f(A_2).$$

Since f is an anti homomorphism, $(f(A_1), +)$ is an intuitionistic fuzzy subgroup of $(G_1', +)$ and $(f(A_2), \bullet)$ is an intuitionistic fuzzy subgroup of $(G_2', \bullet)$.

Hence $V = f(A_1) \cup f(A_2)$ is an intuitionistic fuzzy sub-bigroup of G'.

Theorem 1.5: The anti homomorphic pre-image an intuitionistic fuzzy sub-bigroup V of a bigroup G' is an intuitionistic fuzzy sub-bigroup of a bigroup G.

Proof: Let = $(G_1 \cup G_2, +, \bullet)$ and $G' = (G_1' \cup G_2', +, \bullet)$ are bigroups.

Let $V = (V_1 \cup V_2, +, \bullet)$ be an intuitionistic fuzzy sub-bigroup of a bigroup G'.

Then $(V_1, +)$ in an IFSG of $(G_1', +)$ and $(V_2, \bullet)$ in an IFSG of $(G_2', \bullet)$

Let $f : G \to G'$ be an anti homomorphism. (i.e) $f(xy) = f(y) f(x)$ for all x, y in G.

We have to prove that $f^{-1}(V) = A$ is an intuitionistic fuzzy sub-bigroup of G.

$$A = f^{-1}(V) = f^{-1}(V_1 \cup V_2) = f^{-1}(V_1) \cup f^{-1}(V_2).$$

Since f is an anti homomorphism, $(f^{-1}(V_1), +)$ is an intuitionistic fuzzy subgroup of $(G_1, +)$ and $(f^{-1}(V_2), \bullet)$ is an intuitionistic fuzzy subgroup of $(G_2, \bullet)$.

Hence $A = f^{-1}(V_1) \cup f^{-1}(V_2)$ is an intuitionistic fuzzy sub-bigroup of G.

2. WE NOW DISCUSS THE PROPERTIES OF BI-LEVEL SUB-BIGROUP OF AN INTUITIONISTIC FUZZY SUB-BIGROUP OF A BIGROUP UNDER HOMOMORPHISM AND ANTI HOMOMORPHISM.

Theorem 2.1: The homomorphic image of a bi-level sub-bigroup of an intuitionistic fuzzy sub-bigroup A of a bigroup G is a bi-level sub-bigroup of an intuitionistic fuzzy sub-bigroup $f(A)$ of a bigroup G'.

Proof: Let $G = (G_1 \cup G_2, +, \bullet)$ and $G' = (G_1' \cup G_2', +, \bullet)$ are bigroups.

Let $f : G \to G'$ be a homomorphism.

(i.e.) $f(xy) = f(x) f(y)$ for all x, y in G.

Let A be an intuitionistic fuzzy sub-bigroup of the bigroup G, then $A = A_1 \cup A_2$

Consider the bi-level sub-bigroup $A_{<\alpha, \beta>}$ of an intuitionistic fuzzy sub-bigroup A, for every $\alpha \in [0, \min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\}]$ and $\beta \in [\max\{\nu_{A1}(e_1), \nu_{A2}(e_2)\}, 1]$, where e_1 denotes the identity element of the group $(G_1, +)$ and e_2 denotes the identity element of the group $(G_2, \bullet)$. Then $A_{<\alpha, \beta>} = A_{1<\alpha, \beta>} \cup A_{2<\alpha, \beta>}$ where $A_{1<\alpha, \beta>}$ and $A_{2<\alpha, \beta>}$ are subgroups of G_1 and G_2 respectively.

Now we have to prove that $f(A_{<\alpha, \beta>})$ is a sub-bigroup of G'.

Since f is a homomorphism $f(A)$ is an intuitionistic fuzzy sub-bigroup of the bigroup G' and $f(A) = f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$.

Since f is a homomorphism,

$f(A_{1<\alpha, \beta>})$ and $f(A_{2<\alpha, \beta>})$ are subgroups of G'_1 and G'_2 respectively.

$$f(A_{<\alpha, \beta>}) = f(A_{1<\alpha, \beta>} \cup A_{2<\alpha, \beta>}) = f(A_{1<\alpha, \beta>}) \cup f(A_{2<\alpha, \beta>}).$$

Hence,

$f(A_{<\alpha, \beta>})$ is a bi-level sub-bigroup of an intuitionistic fuzzy sub-bigroup $f(A)$ of G'.

Theorem 2.2: The homomorphic pre-image of a bi-level sub-bigroup of an intuitionistic fuzzy sub-bigroup V of a bigroup G' is a bi-level sub-bigroup of an intuitionistic fuzzy sub-bigroup A of a bigroup G.

Proof: Let $G = (G_1 \cup G_2, +, \bullet)$ and $G' = (G'_1 \cup G'_2, +, \bullet)$ are bigroups.

Let $f : G \to G'$ be a homomorphism.

(i.e.) $f(xy) = f(x) f(y)$ for all x, y in G.

Let V be an intuitionistic fuzzy sub-bigroup of the bigroup G', then $V = V_1 \cup V_2$.

Consider the bi-level sub-bigroup $V_{<\alpha, \beta>}$ of an intuitionistic fuzzy sub-bigroup V, for every $\alpha \in [0, \min\{\mu_{V_1}(e'_1), \mu_{V_2}(e'_2)\}]$ and $\beta \in [\max\{\nu_{V_1}(e'_1), \nu_{V_2}(e'_2)\}, 1]$, where e'_1 denotes the identity element of the group $(G'_1, +)$ and e'_2 denotes the identity element of the group $(G'_2, \bullet)$. Then $V_{<\alpha, \beta>} = V_{1<\alpha, \beta>} \cup V_{2<\alpha, \beta>}$ where $V_{1<\alpha, \beta>}$ and $V_{2<\alpha, \beta>}$ are subgroups of G'_1 and G'_2 respectively.

Now we have to prove that $f^{-1}(V_{<\alpha, \beta>})$ is a sub-bigroup of G.

Since f is a homomorphism, $f^{-1}(V) = A$ is an intuitionistic fuzzy sub-bigroup of the bigroup G and $A = f^{-1}(V) = f^{-1}(V_1 \cup V_2) = f^{-1}(V_1) \cup f^{-1}(V_2)$.

Since f is a homomorphism,

$f^{-1}(V_{1<\alpha, \beta>})$ and $f^{-1}(V_{2<\alpha, \beta>})$ are subgroups of G_1 and G_2 respectively.

$f^{-1}(V_{<\alpha, \beta>}) = f^{-1}(V_{1<\alpha, \beta>} \cup V_{2<\alpha, \beta>}) = f^{-1}(V_{1<\alpha, \beta>}) \cup f^{-1}(V_{2<\alpha, \beta>})$.

Hence,

$f^{-1}(V_{<\alpha, \beta>})$ is a bi-level sub-bigroup of an intuitionistic fuzzy sub-bigroup A of G.

Theorem 2.3: The anti homomorphic image of a bi-level sub-bigroup of an intuitionistic fuzzy sub-bigroup A of a bigroup G is a bi-level sub-bigroup of an intuitionistic fuzzy sub-bigroup $f(A)$ of a bigroup G'.

Proof: Let $G = (G_1 \cup G_2, +, \bullet)$ and $G' = (G'_1 \cup G'_2, +, \bullet)$ are bigroups.

Let $f : G \to G'$ be an anti homomorphism.

(i.e.) $f(xy) = f(y) f(x)$ for all x, y in G.

Let A be an intuitionistic fuzzy sub-bigroup of the bigroup G, then $A = A_1 \cup A_2$.

Consider the bi-level sub-bigroup $A_{<\alpha, \beta>}$ of an intuitionistic fuzzy sub-bigroup A, for every $\alpha \in [0, \min\{\mu_{A_1}(e_1), \mu_{A_2}(e_2)\}]$ and $\beta \in [\max\{\nu_{A_1}(e_1), \nu_{A_2}(e_2)\}, 1]$, where e_1 denotes the identity element of the group $(G_1, +)$ and e_2 denotes the identity element of the group $(G_2, \bullet)$. Then $A_{<\alpha, \beta>} = A_{1<\alpha, \beta>} A_{2<\alpha, \beta>}$ where $A_{1<\alpha, \beta>}$ and $A_{2<\alpha, \beta>}$ are subgroups of G_1 and G_2 respectively.

Now we have to prove that $f(A_{<\alpha, \beta>})$ is a sub-bigroup of G'.

Since f is an anti homomorphism $f(A)$ is an intuitionistic fuzzy sub-bigroup of the bigroup G' and $f(A) = f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$.

Since f is an anti homomorphism,

$f(A_{1<\alpha, \beta>})$ and $f(A_{2<\alpha, \beta>})$ are subgroups of G'_1 and G'_2 respectively.

$f(A_{<\alpha, \beta>}) = f(A_{1<\alpha, \beta>} \cup A_{2<\alpha, \beta>}) = f(A_{1<\alpha, \beta>}) \cup f(A_{2<\alpha, \beta>})$.

Hence,

$f(A_{<\alpha, \beta>})$ is a bi-level sub-bigroup of an intuitionistic fuzzy sub-bigroup $f(A)$ of G'.

Theorem 2.4: The anti homomorphic pre-image of a bi-level sub-bigroup of an intuitionistic fuzzy sub-bigroup V of a bigroup G' is a bi-level sub-bigroup of an intuitionistic fuzzy sub-bigroup A of a bigroup G.

Proof: Let $G = (G_1 \cup G_2, +, \bullet)$ and $G' = (G'_1 \cup G'_2, +, \bullet)$ are bigroups.

Let $f : G \to G'$ be an anti homomorphism.

(i.e.) $f(xy) = f(y) f(x)$ for all x, y in G.

Let V be an intuitionistic fuzzy sub-bigroup of the bigroup G', then $V = V_1 \cup V_2$.

Consider the bi-level sub-bigroup $V_{<\alpha, \beta>}$ of an intuitionistic fuzzy sub-bigroup V, for every $\alpha \in [0, \min\{\mu_{V_1}(e'_1), \mu_{V_2}(e'_2)\}]$ and $\beta \in [\max\{\nu_{V_1}(e'_1), \nu_{V_2}(e'_2)\}, 1]$, where e'_1 denotes the identity element of the group $(G'_1, +)$ and e'_2 denotes the identity element of the group $(G'_2, \bullet)$. Then $V_{<\alpha, \beta>} = V_{1<\alpha, \beta>} \cup V_{2<\alpha, \beta>}$ where $V_{1<\alpha, \beta>}$ and $V_{2<\alpha, \beta>}$ are subgroups of G'_1 and G'_2 respectively.

Now we have to prove that $f^{-1}(V_{<\alpha, \beta>})$ is a sub-bigroup of G.

Since f is an anti homomorphism, $f^{-1}(V) = A$ is an intuitionistic fuzzy sub-bigroup of the bigroup G and $A = f^{-1}(V) = f^{-1}(V_1 \cup V_2) = f^{-1}(V_1) \cup f^{-1}(V_2)$.

Since f is an anti homomorphism,

$f^{-1}(V_{1<\alpha, \beta>})$ and $f^{-1}(V_{2<\alpha, \beta>})$ are subgroups of G_1 and G_2 respectively.

$f^{-1}(V_{<\alpha, \beta>}) = f^{-1}(V_{1<\alpha, \beta>} \cup V_{2<\alpha, \beta>}) = f^{-1}(V_{1<\alpha, \beta>}) \cup f^{-1}(V_{2<\alpha, \beta>})$.

Hence,

$f^{-1}(V_{<\alpha, \beta>})$ is a bi-level sub-bigroup of an intuitionistic fuzzy sub-bigroup A of G.

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