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ON THE STABILITY OF A FOUR SPECIES: A PREY-PREDATOR-HOST-COMMENSAL-SYN ECO-SYSTEM-VII (Host of the Prey Washed Out States)

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ABSTRACT: This paper deals with an investigation on a Four Species Syn-Ecological System (Host of the Prey washed out states). The System comprises of a Prey (S_1), a Predator (S_2) that survives upon S_1, two Hosts S_3 and S_4 for which S_1, S_2 are commensal respectively i.e., S_3 and S_4 benefit S_1 and S_2 respectively, without getting effected either positively or adversely. Further S_3 and S_4 are neutral. The model equations of the system constitute a set of four first order non-linear ordinary differential coupled equations. In all, there are sixteen equilibrium points. Criteria for the asymptotic stability of four of these sixteen equilibrium points: the Host of the Prey washed out states only are established in this paper. The linearized equations for the perturbations over the equilibrium points are analyzed to establish the criteria for stability and the trajectories are illustrated.

Keywords: Equillibrium point, Host, Prey, Predator, Trajectories, Unstable.

1. INTRODUCTION

Mathematical modeling of Eco-System was initiated in 1925 by Lotka [11] and in 1931 by Volterra [15]. The general concepts of modeling have been presented in the treatises of Meyer [12], Kushing [8], Kapur J. N. [6, 7] and several others. The ecological interactions can be broadly classified as Prey-Predation, Commensalism, Competition, Neutralism, Mutualism and so on. N. C. Srinivas [14] studied competitive eco-systems of two species and three species with limited and unlimited resources. Later Lakshminarayan [9], Lakshminarayan and Pattabhi Ramacharyulu [10] studied Prey Preadtor ecological models with partial cover for the Prey and alternate food for the predator. Recently, Archana Reddy [1] and Bhaskara Rama Sharma [2] investigated diverse problems related to two species competitive systems with time delay, employing analytical and numerical techniques. Further Phani Kumar, Seshagiri Rao and Pattabhi Ramacharyulu [13] studied the stability of a Host-A flourishing commensal species pair with limited resources. The present authors Hari Prasad and Pattabhi Ramacharyulu studied the stability of the fully washed out state [3], Prey and Predator washed out states [4] and co-existent state [5]. Continuation of this criteria for the stability of only the Host of the Prey of the system is presented in this paper.

Figure1 A Schematic Sketch of the system under investigation is shown here under Fig. 1.

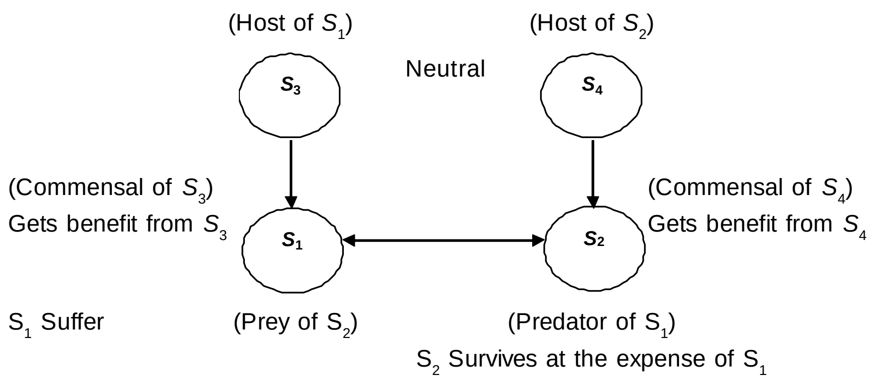

Figure 1: Schematic Sketch of the Syn Eco-System

2. BASIC EQUATIONS OF THE MODEL

Notation Adopted:

S_1	: Prey for S_2 and commensal for S_3.
S_2	: Predator surviving upon S_1 and commonsal for S_4.
S_3	: Host for the commonsal – Prey (S_1).
S_4	: Host of the commonsal – Predator (S_2).
$N_1(t)$	: The Population of the Prey (S_1).
$N_2(t)$	: The Population of the Predator (S_2).
$N_3(t)$	: The Population of the Host (S_3) of the Prey (S_1).
$N_4(t)$	: The Population of the Host (S_4) of the Predator (S_2)
t	: Time instant
a_1, a_2, a_3, a_4	: Natural growth rates of S_1, S_2, S_3, S_4
$a_{11}, a_{22}, a_{33}, a_{44}$	: Self inhibition coefficients of S_1, S_2, S_3, S_4
a_{12}, a_{21}	: Interaction (Prey-Predator) coefficients of S_1 due to S_2 and S_2 due to S_1
a_{13}	: Coefficient for commensal for S_1 due to the Host S_3
a_{24}	: Coefficient for commensal for S_2 due to the Host S_4
$\frac{a_1}{a_{11}}, \frac{a_2}{a_{22}}, \frac{a_3}{a_{33}}, \frac{a_4}{a_{44}}$	: Carrying capacities of S_1, S_2, S_3, S_4

Further the variables N_1, N_2, N_3, N_4 are non-negative and the model parameters $a_1, a_2, a_3, a_4; a_{11}, a_{22}, a_{33}, a_{44}; a_{12}, a_{21}, a_{13}, a_{24}$ are assumed to be non-negative constants.

The model equations for the growth rates of S_1, S_2, S_3, S_4 are

$$\frac{dN_1}{dt} = a_1 N_1 - a_{11} N_1^2 - a_{12} N_1 N_2 + a_{13} N_1 N_3 \tag{2.1}$$

$$\frac{dN_2}{dt} = a_2 N_2 - a_{22} N_2^2 + a_{21} N_1 N_2 + a_{24} N_2 N_4 \tag{2.2}$$

$$\frac{dN_3}{dt} = a_3 N_3 - a_{33} N_3^2, \quad \frac{dN_4}{dt} = a_4 N_4 - a_{44} N_4^2 \tag{2.3}$$

3. EQUILIBRIUM STATES

The system under investigation has sixteen equilibrium states defined by

$$\frac{dN_i}{dt} = 0, \qquad i = 1, 2, 3, 4 \tag{3.1}$$

are given in the following Table 1.

Table 1

S.No.	*Equilibrium State*	*Equilibrium Point*
1	Fully Washed out state	$\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$
2	Only the Host (S_4) of S_2 survives	$\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$
3	Only the Host (S_3) of S_1 survives	$\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$
4	Only the Predator S_2 survives	$\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = 0, \overline{N_4} = 0$
5	Only the Prey S_1 survives	$\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$
6	Prey (S_1) and Predator (S_2) washed out	$\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = \frac{a_4}{a_{44}}$
7	Prey (S_1) and Host (S_3) of S_1 washed out	$\overline{N_1} = 0, \overline{N_2} = \frac{a_2 a_{44} + a_4 a_{24}}{a_{22} a_{44}}, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$
8	Prey (S_1) and Host (S_4) of S_2 washed out	$\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$
9	Predator (S_2) and Host (S_3) of S_1 washed out	$\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$

(Table Contd...)

S.No.	Equilibrium State	Equilibrium Point
10	Predator (S_2) and Host (S_4) of S_2 washed out	$\overline{N_1}=\frac{a_1a_{33}+a_3a_{13}}{a_{11}a_{13}}, \overline{N_2}=0, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=0$
11	Prey (S_1) and Predator (S_2)survives	$\overline{N_1}=\frac{a_1a_{22}+a_2a_{12}}{a_{11}a_{13}+a_{12}a_{21}}, \overline{N_2}=\frac{a_1a_{21}+a_2a_{11}}{a_{11}a_{22}+a_{12}a_{21}},$ $\overline{N_3}=0, \overline{N_4}=0$
12	Only the Prey (S_1) washed out	$\overline{N_1}=0, \overline{N_2}=\frac{a_2a_{44}+a_4a_{24}}{a_{22}a_{44}}, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=\frac{a_4}{a_{44}}$
13	Only the predator (S_2) washed out	$\overline{N_1}=\frac{a_1a_{23}+a_3a_{13}}{a_{11}a_{13}}, \overline{N_2}=0, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=\frac{a_4}{a_{44}}$
14	Only the Host (S_3) of S_1 washed out	$\overline{N_1}=\frac{\delta_2}{\delta_1}, \overline{N_2}=\frac{\delta_3}{\delta_1}, \overline{N_3}=0, \overline{N_4}=\frac{a_4}{a_{44}}$ where $\delta_1 = a_{44}(a_{11}a_{22}+a_{12}a_{21})>0$ $\delta_2 = a_{22}(a_1a_{33}+a_3a_{13})-a_2a_{12}a_{33}$ $\delta_3 = a_{21}(a_1a_{33}+a_3a_{13})+a_2a_{12}a_{33}>0$
15	Only the Host (S_4) of S_2 washed out	$\overline{N_1}=\frac{\delta_2}{\delta_1}, \overline{N_2}=\frac{\delta_2}{\delta_1}, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=0$ where $\sigma_1 = a_{33}(a_{11}a_{22}+a_{12}a_{21})>0$ $\sigma_2 = a_{22}(a_1a_{33}+a_3a_{13})-a_2a_{12}a_{33}$ $\sigma_3 = a_{21}(a_1a_{33}+a_3a_{13})+a_2a_{12}a_{33}>0$
16	The co-existent state (or) Normal steady state	$\overline{N_1}=\frac{a_{22}a_{44}\psi_1+a_{12}a_{33}\psi_2}{\psi_3}, \overline{N_2}=\frac{a_{21}a_{44}\psi_1+a_{22}a_{33}\psi_2}{\psi_3},$ $\overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=\frac{a_4}{a_{44}}$ where $\psi_1 = a_1a_{33}+a_3a_{13}>0$ $\psi_2 = a_2a_{44}+a_4a_{24}>0$ $\psi_3 = a_{33}a_{44}(a_{11}a_{22}+a_{12}a_{21})>0$

4. STABILITY OF THE HOST (S_3) OF THE PREY (S_1) ONLY IS WASHED OUT STATES (SL. NOS. 2, 7, 9, 14 IN THE ABOVE TABLE)

The equilibrium point $\bar{N}_1=0, \bar{N}_2=0, \bar{N}_3=0, \bar{N}_4=\frac{a_4}{a_{44}}$ (Sl. No. 2) was already discussed in the paper "On the Stability of a four Species: a Prey Predator-Host-Commensal-Syn Eco-System-II", and published in "International eJournal of Mathematics and Engineering." 5, (2010), 60-74. Also the equilibrium points $\bar{N}_1=0, \bar{N}_2=\frac{a_2a_{44}+a_4a_{24}}{a_{22}a_{44}}, \bar{N}_3=0, \bar{N}_4=\frac{a_4}{a_{44}}$ and $\bar{N}_1=\frac{a_1}{a_{11}}, \bar{N}_2=0, \bar{N}_3=0, \bar{N}_4=\frac{a_4}{a_{44}}$ (Sl. No.s 7 and 9) were discussed in the papers "On the Stability of a four Species: a Prey-Predator-Host-Commensal-Syn Eco-System-V and VI", communicated to "International eJournal of Mathematics and Engineering".

4.1 Equilibrium Point

$$\bar{N}_1 = \frac{\delta_2}{\delta_1}, \bar{N}_2 = \frac{\delta_3}{\delta_1}, \bar{N}_3 = 0, \bar{N}_4 = \frac{a_4}{a_{44}}$$

where $$\delta_1 = a_{44}(a_{11}a_{22} + a_{12}a_{21}) > 0 \quad (4.1.1)$$

$$\delta_2 = a_1 a_{22} a_{44} - a_{12}(a_2 a_{44} + a_4 a_{24}), \delta_3 = a_1 a_{21} a_{44} - a_{11}(a_2 a_{44} + a_4 a_{24}) \quad (4.1.2)$$

Let us consider small deviations from the steady state

$$\text{i.e,} \quad N_i(t) = \bar{N}_i + u_i(t), \qquad i = 1, 2, 3, 4 \quad (4.1.3)$$

where $u_i(t)$ is a small perturabations in the species S_i.

Substituting (4.13) in (2.1), (2.2), (2.3) and neglecting products and higher power of u_1, u_2, u_3, u_4.

We get,

$$\frac{du_1}{dt} = \phi_1 u_1 - \frac{a_{12}\delta_2}{\delta_1} u_2 + \frac{a_{12}\delta_{12}}{\delta_1} u_3 \quad (4.1.4)$$

$$\frac{du_2}{dt} = \frac{a_{12}\delta_3}{\delta_1} u_1 + \phi_2 u_2 + \frac{a_{24}\delta_3}{\delta_1} u_4 \quad (4.1.5)$$

$$\frac{du_3}{dt} = a_3 u_3, \quad \frac{du_4}{dt} = -a_4 u_4 \quad (4.1.6)$$

where

$$\phi_1 = a_1 - \frac{2a_{11}\delta_2}{\delta_1} - \frac{a_{12}\delta_3}{\delta_1}, \quad \phi_2 = a_2 - \frac{2a_{22}\delta_3}{\delta_1} + \frac{a_{21}\delta_2}{\delta_1} + \frac{a_4 a_{24}}{a_{44}}. \quad (4.1.7)$$

The characteristic equation for which is

$$\left[\lambda^2 - (\phi_1 + \phi_2)\lambda + \phi_1\phi_2 - \frac{a_{12}a_{21}\delta_2\delta_3}{\delta_1}\right](\lambda - a_3)(\lambda + a_4) = 0. \quad (4.1.8)$$

One of the four roots a_3 is positive and $-a_4$ is negative. Hence the steady state is **unstable**.

Let λ_1, λ_2 be the zeros of the quadratic polynomial on the L.H.S. of the above equation (4.1.8).

Case (A): When the roots λ_1 and λ_2 have opposite signs.

The solutions of the equations (4.1.4), (4.1.5), (4.1.6) are:

$$u_1 = \left[\frac{a_{12}\delta_2(\beta - u_{20}) - (\alpha - u_{10})(\phi_1 - \lambda_2)\delta_1}{\delta_1(\lambda_1 - \lambda_2)}\right] e^{\lambda_1 t}$$

$$+ \left[\frac{a_{12}\delta_2(\beta - u_{20}) - (\alpha - u_{10})(\phi_1 - \lambda_1)\delta_1}{\delta_1(\lambda_2 - \lambda_1)}\right] e^{\lambda_2 t} + A_2 e^{a_3 t} + B_2 e^{-a_4 t} \quad (4.1.9)$$

$$u_2 = \left[\frac{a_{12}\delta_2(\beta - u_{20}) - (\alpha - u_{10})(\phi_1 - \lambda_2)\delta_1}{a_{12}\delta_2(\lambda_1 - \lambda_2)}\right](\phi_1 - \lambda_1)\, e^{\lambda_1 t}$$

$$+ \left[\frac{a_{12}\delta_2(\beta - u_{20}) - (\alpha - u_{10})(\phi_1 - \lambda_1)\delta_1}{a_{12}\delta_2(\lambda_2 - \lambda_1)}\right](\phi_1 - \lambda_2)\, e^{\lambda_2 t}$$

$$+ \left[\frac{A_2\delta_1(\phi_1 - a_3)}{a_{12}\delta_2} + \frac{a_{13}u_{30}}{a_{12}}\right] e^{a_3 t} + \frac{B_2\delta_1}{a_{12}\delta_2}(\phi_1 + a_4)\, e^{-a_4 t} \quad (4.1.10)$$

$$u_3 = u_{30}\, e^{a_3 t}, \quad u_4 = u_{40}\, e^{-a_4 t} \quad (4.1.11)$$

where

$$A_1 = \frac{a_{12}\delta_2 u_{30}}{\delta_1}(a_3 - \phi_2), \quad B_1 = \frac{a_{12}a_{24}\delta_2\delta_3}{\delta_1^2} u_{40} \quad (4.1.12)$$

$$\alpha = A_2 + B_2, \quad \phi_3 = \phi_1\phi_2 + \frac{a_{12}a_{21}\delta_2\delta_3}{\delta_1^2} \quad (4.1.13)$$

$$\beta = \left[A_2(\phi_1 - a_3) + B_2(\phi_1 + a_4) + \frac{\delta_{13}\delta_2}{\delta_1} u_{30}\right]\frac{\delta_1}{a_{12}\delta_2} \quad (4.1.14)$$

$$A_2 = \frac{A_1}{a_3^2 - (\phi_1 + \phi_2)a_3 + \phi_3}, \quad B_2 = \frac{B_1}{a_4^2 + (\phi_1 + \phi_2)a_4 + \phi_3} \quad (4.1.15)$$

and u_{10}, u_{20}, u_{30}, u_{40} are the initial values of u_1, u_2, u_3, u_4 respectively.

There would arise in all 576 cases depending upon the ordering the magnitudes of the growth rates a_1, a_2, a_3, a_4 and the initial values of the perturbations $u_{10}(t)$, $u_{20}(t)$, $u_{30}(t)$, $u_{40}(t)$ of the spaces S_1, S_2, S_3, S_4. Of these 576 situations some typical variations are illustrated through respective solution curves that would facilitate to make some reasonable observations. And the solution curves are illustrated in the Figs. 2 to 5.

Case (i): If $u_{10} < u_{40} < u_{20} < u_{30}$ and $a_2 < a_1 < a_4 < a_3$

In this case the Host (S_4) of S_2 has the least natural birth rate. Initially it is dominated over by the Prey (S_1) till the time instant t^*_{14} and there after the dominance is reversed. Also the Predator (S_2) dominates over the Prey (S_1) till the time instant t^*_{12} and the dominance gets reversed thereafter.

Case (ii): If $u_{20} < u_{30} < u_{40} < u_{10}$ and $a_2 < a_3 < a_4 < a_1$

In this case the Host (S_4) of S_1 has the least natural birth rate. Initially it is dominated over by the Host (S_3) of S_1, Predator (S_2) till the time instant t^*_{34}, t^*_{24} respectively and there after the dominance is reversed.

Case (iii): If $u_{30} < u_{10} < u_{40} < u_{20}$ and $a_4 < a_2 < a_1 < a_3$

In this case the Host (S_4) of S_2 has the least natural birth rate. Initially it is dominated over by the Prey (S_1), Host (S_3) of S_1 till the time instant t^*_{14}, t^*_{34} respectively and thereafter the dominance is reversed. Also the Predator (S_2) dominates over the Prey (S_1), Host (S_3) of S_1 till the time instant t^*_{12}, t^*_{32} respectively and thereafter the dominance is reversed. Similarly the Prey (S_1) dominates its Host (S_3) till the time instant t^*_{31} and the dominance gets reversed thereafter.

Case (iv): If $u_{40} < u_{30} < u_{20} < u_{10}$ and $a_3 < a_1 < a_4 < a_2$

In this case the Host (S_4) of S_2 has the least natural birth rate. Initially the Prey (S_1) dominates over the Predator (S_3) till the time instant t^*_{21} and thereafter the dominance is reversed.

Case (B): When the roots λ_1 and λ_2 have same signs.

The solutions in this case are same as in Case (A) and the solution curves are illustrated in the Figs. 6 to 9.

Case (i): If $u_{10} < u_{20} < u_{30} < u_{40}$ and $a_1 < a_2 < a_3 < a_4$

In this case the Host (S_4) of S_2 has the least natural birth rate. Initially it is dominated over by the Predator (S_2), Host (S_3) of S_1, Prey (S_1) till the time instant t^*_{24} t^*_{14}, t^*_{34} respectively and thereafter the dominance is reversed. Also the Predator (S_2) dominates over the Host (S_3) of S_1 till the time instant t^*_{32} and the dominance gets reversed thereafter.

Case (ii): If $u_{20} < u_{40} < u_{30} < u_{10}$ and $a_3 < a_4 < a_1 < a_2$

In this case the Host (S_4) of S_2 has the least natural birth rate. Initially it is dominated over by the Predator (S_2) till the time instant t^*_{24} and thereafter the dominance is reversed. Also the Prey (S_1) and its Host (S_3) dominates over the Predator (S_2) till the time instant t^*_{21}, t^*_{23} respectively and the dominance gets reversed thereafter.

Case (iii): If $u_{30} < u_{10} < u_{40} < u_{20}$ and $a_2 < a_3 < a_4 < a_1$

In this case the Host (S_4) of S_2 has the least natural birth rate. Initially it is dominated over by the Prey (S_1), Host (S_3) of S_1 till the time instant t^*_{14}, t^*_{34} respectively and thereafter the dominance is reversed. Also the Predator (S_2) dominates over the Prey (S_1) and its Host (S_3) till the time instant t^*_{12}, t^*_{32} respectively and the dominance gets reversed thereafter.

Case (iv): If $u_{40} < u_{20} < u_{10} < u_{30}$ and $a_2 < a_1 < a_3 < a_4$

In this case the Host (S_4) of S_2 has the least natural birth rate and the Host of S_1 dominates the Prey (S_1), Predator (S_2), Host (S_4) of S_2 in natural growth rate as well as in its population strength.

4.2 Trajectories

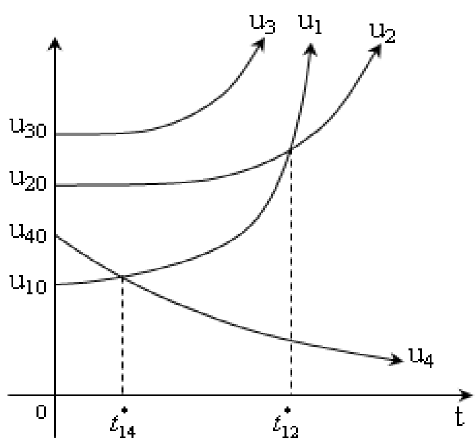

Figure 2

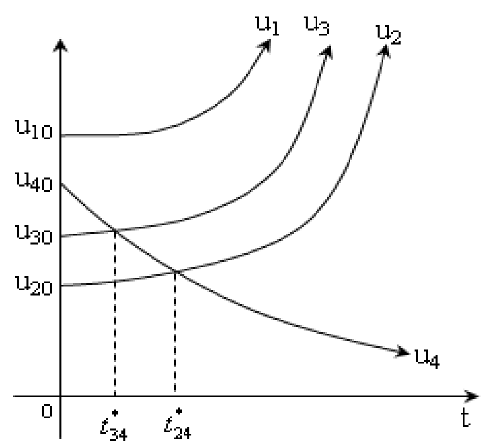

Figure 3

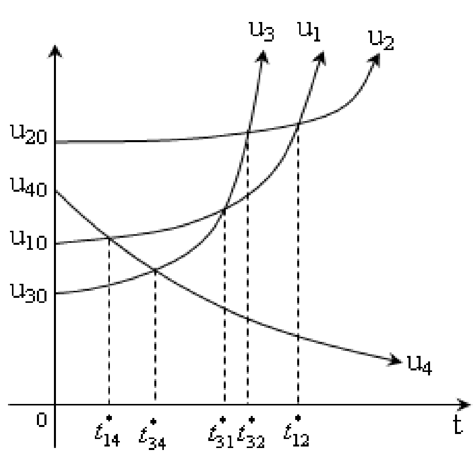

Figure 4

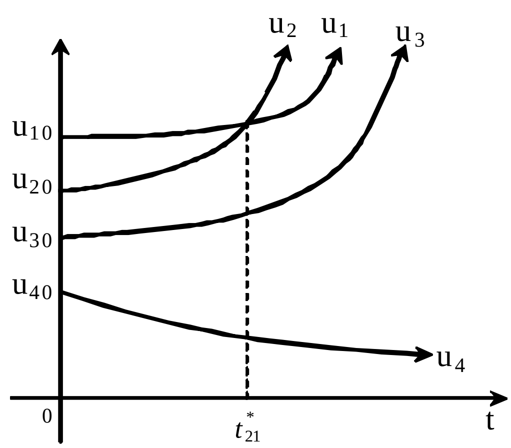

Figure 5

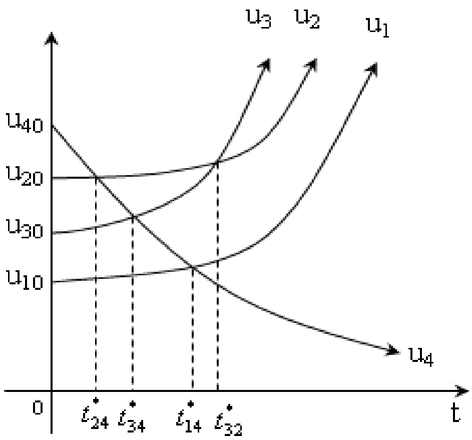

Figure 6

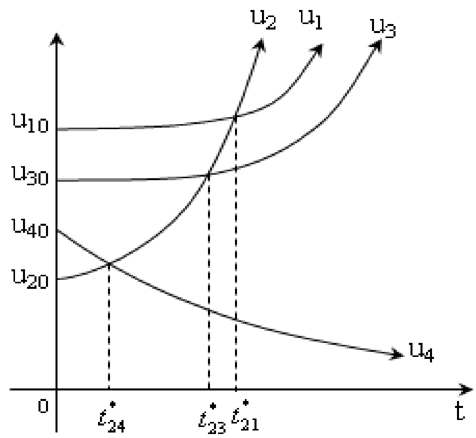

Figure 7

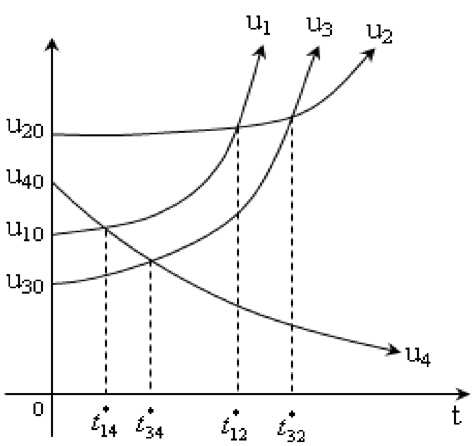

Figure 8

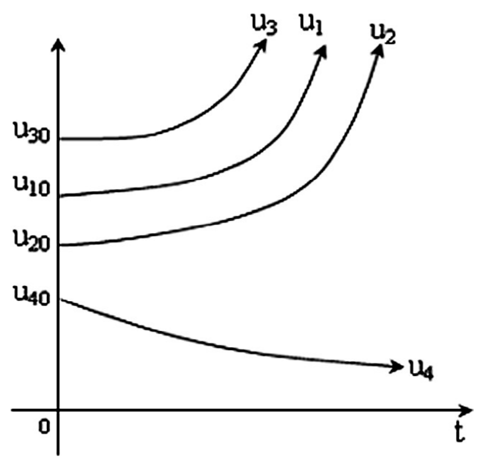

Figure 9

REFERENCES

[1] Archana Reddy R., "On the Stability of Some Mathematical Models in BioSciences – Interacting Species", *Ph.D.Thesis*, J.N.T.U., (2010).

[2] Bhaskara Rama Sharma B., "Some Mathematical Models in Competitive Eco-Systems", *Ph.D. Thesis*, Dravidian University, (2010).

[3] Hari Prasad B., and Pattabhi Ramacharyulu N. Ch., "On the Stability of a Four Species: A Prey-Predator-Host-Commensal-Syn Eco-System-I" (Fully Washed out State), *International eJournal of Mathematics and Engineering*, **11**, (2010), 122-132.

[4] Hari Prasad B., and Pattabhi Ramacharyulu N. Ch., "On the Stability of a Four Species: A Prey-Predator-Host-Commensal-Syn Eco-System-II" (Prey and Predator Washed out States), *International eJournal of Mathematics and Engineering*, **5**, (2010), 60-74.

[5] Hari Prasad B, and Pattabhi Ramacharyulu N. Ch., "On the Stability of a Four Species: A Prey-Predator-Host-Commensal-Syn Eco-System-III" (Co-Existent State), *International eJournal of Mathematics and Engineering*, **16**, (2010), 163-173.

[6] Kapur J. N., Mathematical Modelling in Biology and Medicine, *Affiliated East West*, (1985).

[7] Kapur J. N., *Mathematical Modelling*, Wiley Easter, (1985).

[8] Kushing J.M., Integro-Differential Equations and Delay Models in Population Dynamics, *Lecture Notes in Bio-Mathematics*, Springer Verlag, **20**, (1977).

[9] Lakshmi Narayan K., A. Mathematical Study of a Prey-Predator Ecological Model with a Partial Cover for the Prey and Alternate Food for the Predator, *Ph.D. Thesis*, J.N.T.U., (2005).

[10] Lakshmi Narayan K., and Pattabhiramacharyulu N. Ch., "A Prey-Predator Model with Cover for Prey and Alternate Food for the Predator and Time Delay," *International Journal of Scientific Computing*, **1**, (2007), 7-14.

[11] Lotka A. J., *Elements of Physical Biology*, Williams & Wilking, Baltimore, (1925).

[12] Meyer W. J., *Concepts of Mathematical Modeling*, Mc. Grawhill, (1985).

[13] Phani Kumar, Seshagiri Rao, and Pattabhi Ramacharyulu N. Ch., "On the Stability of a Host – A Flourishing Commensal Species Pair with Limited Resources", *International Journal of Logic Based Intelligent Systems*, **3**(1), January-June 2009.

[14] Srinivas N. C., "Some Mathematical Aspects of Modeling in Bio-Medical Sciences" *Ph.D Thesis*, Kakatiya University, (1991).

[15] Volterra V., Leconssen La Theorie Mathematique De La Leitte Pou Lavie, Gauthier-Villars, Paris, (1931).

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