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The Rescaled Bootstrap Applied to the Variance Estimation of Relative Poverty Lines: Results Derived from the EU-SILC

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Abstract: *The poverty line is a threshold that divides a given population into poor and nonpoor. It is unknown in practice, and for this reason it is estimated by using survey data. The variance estimation and the construction of confidence intervals are common problems when estimating a given parameter of interest. This paper discusses the variance estimation of the customary estimator of the relative poverty line. For this purpose, we propose to use the rescaled bootstrap. Assuming real data extracted from the European Union Survey on Income and Living Conditions (EU-SILC), Monte Carlo simulation studies have been carried out. Results indicate that this method has a good performance in terms of bias and coverage rates. Different scenarios and data collected from various countries have been used in this study.*

Keywords:*poverty line,rescaled bootstrap method, Monte Carlo simulation, Survey on Income and Living Conditions.*

JEL Classification: *A10, C00, I3*

1. INTRODUCTION

The problem of measuring the poverty is a topic which has received quite attention in the last years. For example, many researchers have contributed to the literature by defining new poverty indicators, comparing poverty among different regions or countries, etc. In addition, the study and the reduction of poverty is the main objective of development policy-making and a key commitment of many statistical agencies interested in poverty. For example, this interest can be seen by the MillenimumDevelopment Goal, set by the United Nation, or the Europe 2020 Strategy, set by the European Union. Relevant references related to poverty study are Aber *et al.* (2007), Addabbo, Di Tommaso and Maccagnan (2014), Atkinson (1987), Bastos *et al.* (2009), Bishop, Chow and Zheng (1995), Bishop, Fromby and Zheng (1997), Clark,

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Hemming and Ulph (1981), Foster, Greer and Thorbecke (1984), Haughton and Khandker (2009), Molina and Rao (2010), Muñoz *et al.* (2014), Rueda and Muñoz (2011) and Sen (1976).

A key issue in poverty measurement is to establish a threshold to separate poor from nonpoor. That is the poverty line or level of income to leave poverty (see for instance Ravallion, 1998). Poverty lines can be classified as absolute and relative.The former prevail fixed over time and they are only adjusted for inflation. The World Bank uses absolute poverty lines: $1.25 a day (in 2005 PPP$, Purchasing Power Parity) for the world's poorest countries and $2 a day poverty line (in 2005 PPP$) for developing countries. However, most countries and Government Agencies utilize relative poverty lines. For example Eurostat (Statistical Office of the European Communities) defines the poor as those whose income fall bellow 60% of the median. An exhaustive discussion between absolute and relative poverty measures can be seen by Foster (1998), Jolliffe (2001), Khandker (2005) and Madden (2000). This paper is based on relative poverty lines.

The calculation of a relative poverty line is not a simple issue. First, we have to decide the suitable quantitative variable used for the problem of calculating it. In general, the income of individuals or household is the considered variable of interest.Obviously, it is impossible to collect the information of this variable for the whole population of interest. For this reason, a random sample is selected from the population to collect the required information for the poverty study. Indeed, all poverty studies rely on household survey data. This fact leads us to be aware of several problems when interpreting poverty measurement from a survey such as sampling errors.In other words, the estimation of the poverty line as well as other poverty indicators are based on survey data, which implies that they can suffer from sampling errors. Though it is a common praxis to say that the percentage of individuals below thepoverty line is for example 20%, it would be more rigorous to say that we are 95% confident that the true percentage of individuals below the poverty line is between 18% and 22% and our best point estimation is 20%.

An additional aim in poverty studies is to compare the results to the conclusions derived from other regions and useful comparisons need considering the sampling errors. For these reasons, the variances of the estimators of the various poverty indicators are required in practice. The problem of obtaining a theoretical expression for the variance of a poverty indicator is a complexissue. This is due to the fact that the poverty indicators are not generally simple measures, and the definition of a theoretical expression for the variance is complex. In addition, the random samples are usually based on complex sampling designs, and expressions for the theoretical variances are not given in this situation, or it is quite difficult to obtain them. For example, Zheng (2001) proposed theoretical expressions for a class of poverty measures, but this study is limited to basic sampling designs such as simple random sampling, stratified sampling and cluster sampling. They also require the knowledge of the population density, which is unknown, so an additional estimation is necessary.

Many alternative variance estimation methods can be used when the study of interest is based on complex situation such as the presence of complex parameters or complex sampling designs. The most popular methods are based on the bootstrap technique (Biewen, 2002; Davidson and Flachaire, 2007; Kovacevic,Yung andPandher, 1995; Rueda and Muñoz, 2011; Stine, 1989; Thompson, 2013). We can observe that some of the previous references are also related to the estimation of variances in poverty studies.

The main objective of this paper is to estimate the variance of the customary estimator of the relative poverty line by using the rescaled bootstrap estimator proposed by Rao, Wu and Yue (1992). We propose this method because it is simple. The methodology of this approach consists in substituting the survey weights by a set of new weights obtained from the bootstrapping samples. This implies that the variance is obtained by using the customary estimator of the poverty line, but such estimator is weighted by using the new weights. This methodology is evaluated by using real data extracted from the 2011 European Union Survey on Income and Living Conditions (EU-SILC), i.e., we evaluated the rescaled bootstrap on the problem of estimating the relative poverty line and using real data sets.

This paper is organized as follows. In Section 2 we define the relative poverty line and present the customary estimator of this poverty indicator. The rescaled bootstrapmethod is also analyzed in this section. In Section 3 the confidence intervals for the relative poverty line are shown. Then, we present the results derived from the various simulation studies in Section 4, which are based onreal data extracted from the EU-SILC. Various possible scenarios for the relative poverty line are discussed, and they are evaluated by using the rescaled bootstrap estimator. Finally, the main conclusions are summarized in Section 5.

2. THE RELATIVE POVERTY LINE AND THE RESCALED BOOTSTRAP METHOD

Many poverty indicators such as the poverty line are unknown in practice. In order to solve this, many statistical agencies carry out surveys to collect information on income, expenditures, etc, of individuals and households. Assume that U, with N individuals, is the population of interest where we are interested in estimating the relative poverty line L. Note that the relative poverty line is based on a suitable variable of well-being, which we denote as y. For example, the variable of interest$y$ can be the income of individuals. The quantity y_i denotes the value of y for the ith individual in the population. The real relative poverty line $L_{\alpha,\beta}$ for the population U is defined as

$$L_{\alpha,\beta} = \alpha Y_{\beta}, \tag{1}$$

where

$$Y_{\beta} = \inf\,(t\colon F(t) \geq \beta) \tag{2}$$

is the β quantile of the variable of interest y, and

$$F(t) = \frac{1}{N}\sum_{i=1}^{N}\delta(y_i \leq t) \tag{3}$$

is the distribution function evaluated at the argument t. The function $\delta(\cdot)$ is the indicator variable, which takes the value 1 if its argument is true, and 0 otherwise. Moreover, α and β can take different values, according to the criteria established to define the poverty line $L_{\alpha,\beta}$. For example, Eurostat defines the relative poverty line as the 60% of the median of the equivalised net income, i.e., Eurostat uses the relative poverty line $L_{\alpha,\beta}$ with $\alpha = 0.6$ and $\beta = 0.5$.

In practice, the poverty line $L_{\alpha,\beta}$ is unknown, and it is estimated by using survey data. Let s be a random sample, with size n, selected from the population U. We assume that the sample s is selected by using a general sampling design with first inclusion probabilities given by π_i. The survey weights are defined as $d_i = \pi_i^{-1}$. The customary estimator of the poverty line $L_{\alpha,\beta}$ is defined as

$$\hat{L}_{\alpha,\beta} = \alpha\hat{Y}_{\beta}, \tag{4}$$

where

$$\hat{Y}_{\beta} = \inf(t : \hat{F}(t) \geq \beta) \tag{5}$$

and

$$\hat{F}(t) = \frac{1}{\hat{N}}\Sigma_{i=1}^{n} d_i\delta(y_i \leq t) \tag{6}$$

is the Hájek (1964) estimator of the population distribution function $F(t)$, and where

$$\hat{N} = \Sigma_{i=1}^{n} d_i \tag{7}$$

The aim of this paper is to obtain the variance of the estimator $\hat{L}_{\alpha,\beta}$ that is derived by using the rescaled bootstrap variance method proposed by Rao *et al.* (1992). We use this bootstrap method because it is simpler than alternative resampling methods, and this is due to the fact that the bootstrapping estimators are similarly defined to the estimator $\hat{L}_{\alpha,\beta}$, but using the new bootstrap weights. We now describe the rescaled bootstrap when it is applied to the estimation of the variance of $\hat{L}_{\alpha,\beta}$. This paper also contributes to the literature by evaluating numerically the suggested variance estimator. For this purpose, we carry out a Monte Carlo simulation study based on real data set extracted from the EU-SILC. Results derived from this study can be seen in Section 4.

The rescaled bootstrap method consists in calculating the bootstrap weights $d^*_{i(b)}$, with $i = \{1,..., N\}$ and b = $\{1,..., B\}$, by using the scale adjustment on the original survey weights d_i. The number of bootstrap samples is given by B. For the bth bootstrap sample, the bootstrap estimator of the poverty line $L_{\alpha,\beta}$ is defined as

$$\hat{L}_{\alpha,\beta(b)} = \alpha\hat{Y}_{\beta(b)} \tag{8}$$

where

$$\hat{Y}_{\beta(b)} = \inf\left(t : \hat{F}_{(b)}(t) \geq \beta\right), \tag{9}$$

$$\hat{F}_{(b)}(t) = \frac{1}{\hat{N}_{(b)}}\Sigma_{i=1}^{n} d^*_{i(b)}\delta(y_i \leq t) \tag{10}$$

and

$$\hat{N}_{(b)} = \Sigma_{i=1}^{n} d^*_{i(b)} \tag{11}$$

The previous values of $\hat{L}_{\alpha,\beta(b)}$, with $b = \{1, ..., B\}$, are used to obtain the rescaled bootstrap variance estimator of the customary estimator $\hat{L}_{\alpha,\beta}$, which is defined as

$$\hat{V}_{boot}(\hat{L}_{\alpha,\beta}) = \frac{1}{B}\Sigma_{b=1}^{B}\left(\hat{L}_{\alpha,\beta(b)} - \hat{L}_{\alpha,\beta}\right)^2 \tag{12}$$

3. CONFIDENCE INTERVALS FOR THE RELATIVE POVERTY LINE

The $(1-\gamma) \times 100\%$ confidence interval can easily be derived by using the previous rescaled bootstrap variance estimator. In particular, the confidence interval based on the Normal approximation is given by $CI_N\left(\hat{L}_{\alpha,\beta}\right) = [L_N, U_N]$, where the lower ($L_N$) and the upper ($U_N$) limits are defined, respectively, by

$$L_N = \hat{L}_{\alpha,\beta} - z_{1-\gamma/2}\sqrt{\hat{V}_{boot}\left(\hat{L}_{\alpha,\beta}\right)} \tag{13}$$

and

$$U_N = \hat{L}_{\alpha,\beta} + z_{1-\gamma/2}\sqrt{\hat{V}_{boot}\left(\hat{L}_{\alpha,\beta}\right)} \tag{14}$$

Alternative procedures can be used for the purpose of constructing confidence intervals. For instance, the $(1-\gamma) \times 100\%$ confidence interval based on the percentile bootstrap method is defined as $CI_P\left(\hat{L}_{\alpha,\beta}\right)=[L_p,U_P]$, where the lower ($L_p$) and the upper ($U_p$) limits are given, respectively, by

$$L_P=\hat{L}_{\alpha,\beta}[\gamma/2] \tag{15}$$

and

$$U_P=\hat{L}_{\alpha,\beta}\left[1-\frac{\gamma}{2}\right], \tag{16}$$

where $\hat{L}_{\alpha,\beta}[\alpha]$ denotes the ath quantile of the bootstrapped values $\hat{L}_{\alpha,\beta(b)}$. Finally, we can also use the studentized bootstrap approximation, which yields to the confidence interval $CI_t\left(\hat{L}_{\alpha,\beta}\right)=[L_t,U_t]$, where the lower ($L_t$) and the upper ($U_t$) limits are defined, respectively, by

$$L_t=\hat{L}_{\alpha,\beta}+\hat{T}_{\alpha,\beta}[\gamma/2]\sqrt{\hat{V}_{boot}(\hat{L}_{\alpha,\beta})} \tag{17}$$

and

$$U_t=\hat{L}_{\alpha,\beta}+\hat{T}_{\alpha,\beta}[1-\gamma/2]\sqrt{\hat{V}_{boot}\left(\hat{L}_{\alpha,\beta}\right)}, \tag{18}$$

where

$$\hat{T}_{\alpha,\beta}=\frac{\hat{L}_{\alpha,\beta(b)}-\hat{L}_{\alpha,\beta}}{\sqrt{\hat{V}_{boot}(\hat{L}_{\alpha,\beta})}}. \tag{19}$$

4. MONTE CARLO SIMULATION STUDIES

In this section, we evaluate numerically the performance of the rescaled bootstrap variance estimator $\hat{V}_{boot}\left(\hat{L}_{\alpha,\beta}\right)$ suggested in Section 2. In addition, we also analyze the performance of the various confidence intervals proposed in Section 3. We can observe that both estimators $\hat{L}_{\alpha,\beta}$ and $\hat{V}_{boot}(\hat{L}_{\alpha,\beta})$ are required for the construction of confidence intervals, hence the performance of the confidence intervals can be affected by the performance of the estimator $\hat{L}_{\alpha,\beta}$. For this reason, we also evaluate numerically the performance of the customary estimator $\hat{L}_{\alpha,\beta}$ of the population relative poverty line $L_{\alpha,\beta}$.

The various estimators and confidence intervals are evaluated by using Monte Carlo simulation studies based on real data sets extracted from the EU-SILC. We considered the survey data obtained for the following countries: Belgium, Bulgaria, Italy, Lithuania, Poland, Slovenia, Spain and United Kingdom (UK). In this study, the survey data of a given country are considered as population from which R samples are selected, and where R denotes the number of simulation runs in the Monte Carlo simulation study. It is quite common to consider the values $\alpha = \{0.3, 0.5, 0.6, 0.8\}$ and $\beta = 0.5$, and for this reason they are considered in this paper. Note that the various samples are selected by using simple random sampling without replacement.

The empirical performance of the estimator $\hat{L}_{\alpha,\beta}$ is evaluated by using the empirical relative bias (RB) and the empirical relative root mean square error (RRMSE), which are given by

$$RB\left[\hat{L}_{\alpha,\beta}\right] = \frac{E[\hat{L}_{\alpha,\beta}] - L_{\alpha,\beta}}{L_{\alpha,\beta}} \tag{20}$$

and

$$RRMSE\left[\hat{L}_{\alpha,\beta}\right] = \frac{\sqrt{MSE\left[\hat{L}_{\alpha,\beta}\right]}}{L_{\alpha,\beta}}, \tag{21}$$

where the empirical expectation $E\left[\hat{L}_{\alpha,\beta}\right]$ and the empirical mean square error $MSE\left[\hat{L}_{\alpha,\beta}\right]$ based on the $R = 1000$ simulation runs are defined by

$$E\left[\hat{L}_{\alpha,\beta}\right] = \frac{1}{R}\Sigma_{i=1}^{R}\hat{L}_{\alpha,\beta}(i) \tag{22}$$

and

$$MSE\left[\hat{L}_{\alpha,\beta}\right] = \frac{1}{R}\Sigma_{i=1}^{R}\left(\hat{L}_{\alpha,\beta}(i) - L_{\alpha,\beta}\right)^2 \tag{23}$$

The value $\hat{L}_{\alpha,\beta}(i)$ denotes the estimator $\hat{L}_{\alpha,\beta}$ obtained at the ith simulation run. Note that the empirical measures RB and RRMSE are very popular when analyzing the precision of estimators. Relevant references are Deville and Sarndal (1992), Rao, Kovar and Mantel (1990), Silva and Skinner (1995), etc.

The relative bias is also used for the problem of evaluating the performance of the rescaled bootstrap variance estimator $\hat{V}_{boot}(\hat{L}_{\alpha,\beta})$. In this situation, the relative bias is called RB_V.

Finally, the various confidence intervals are evaluated by using the empirical coverage rate (CR), which is given by

$$CR\left[\hat{L}_{\alpha,\beta}\right] = \frac{1}{R}\Sigma_{i=1}^{R}\delta\left(L(i) \leq L_{\alpha,\beta} \leq U(i)\right) \tag{24}$$

where $L(i)$ and $U(i)$ denote, respectively, the lower and the upper limit of the confidence interval $CI\left(\hat{L}_{\alpha,\beta}\right)$. The confidence intervals $CI_N\left(\hat{L}_{\alpha,\beta}\right)$, $CI_P\left(\hat{L}_{\alpha,\beta}\right)$, and $CI_t\left(\hat{L}_{\alpha,\beta}\right)$ defined

Table 1

Empirical Measures to Evaluate the Performance of the Estimators $\hat{L}_{\alpha,\beta}$ and $\hat{V}_{boot}\left(\hat{L}_{\alpha,\beta}\right)$, and Coverage Rates Related to the Various Confidence Intervals (β = 0.5)

SILC	α	*RB*	*RRMSE*	RB_V	CR_N	CR_P	CR_t
Belgium	0.3	0.3	6.9	8.9	93.6	95.6	95.6
	0.5	0.1	7.0	8.6	94.3	95.2	95.2
	0.6	-0.1	6.8	11.5	93.6	95.5	95.5
	0.8	-0.2	7.1	2.4	93.4	94.0	94.0
Bulgaria	0.3	0.0	6.9	14.1	96.3	95.7	95.7
	0.5	-0.2	6.8	15.7	94.2	94.7	94.7
	0.6	0.0	6.7	21.4	95.4	94.8	94.8
	0.8	-0.3	6.8	15.4	95.6	95.1	95.1
Italy	0.3	-0.3	4.1	-1.5	91.8	92.4	92.4
	0.5	0.0	4.1	-4.7	92.7	94.1	94.1
	0.6	-0.1	4.0	4.2	94.0	94.3	94.3
	0.8	-0.3	4.0	2.5	93.9	94.9	94.9
Lithuania	0.3	-0.7	6.4	10.4	93.8	94.0	94.0
	0.5	-0.8	6.4	11.4	94.1	93.5	93.5
	0.6	-0.6	6.6	6.0	94.2	92.8	92.8
	0.8	-0.7	6.7	5.1	93.4	93.3	93.3
Poland	0.3	-0.1	4.5	11.9	95.2	95.0	95.0
	0.5	0.0	4.4	13.9	95.1	94.7	94.7
	0.6	0.2	4.7	1.8	95.0	94.4	94.4
	0.8	-0.1	4.7	-0.2	93.9	93.8	93.8
Slovenia	0.3	-0.2	3.7	-5.0	92.1	93.2	93.3
	0.5	-0.3	3.8	-6.6	92.0	94.2	94.2
	0.6	-0.3	3.7	-3.8	91.5	92.9	93.0
	0.8	-0.3	3.7	-2.9	92.7	94.0	94.0
Spain	0.3	-0.3	4.3	6.7	93.6	95.0	95.0
	0.5	-0.2	4.6	-9.8	93.1	93.4	93.4
	0.6	-0.1	4.3	6.0	94.3	94.7	94.7
	0.8	-0.3	4.5	-2.5	93.5	93.2	93.2
UK	0.3	-0.3	6.8	-3.3	92.1	93.4	93.4
	0.5	-0.4	6.8	-2.7	92.8	94.1	94.1
	0.6	-0.2	6.6	1.5	93.4	94.2	94.2
	0.8	-0.5	6.4	10.8	93.4	94.3	94.3

in Section 3 are considered in this study. The corresponding coverage rates are denoted as CR_N, CR_P and CR_t, respectively. We considered a confidence level of 95%.

The empirical results of this simulation study can be seen in Table 1. First, we use the measures RB and RRMSE to analyze the performance of the customary estimator $\hat{L}_{\alpha,\beta}$. We observe that this estimator has reasonable biases, since all the values of RB are less than 1%. The value of does not have a clear impact on the efficiency of the estimator $\hat{L}_{\alpha,\beta}$, since the values of RRMSE are very similar for a given country. The efficiency of $\hat{L}_{\alpha,\beta}$ is different for the various countries. We observe that the most efficient results are obtained by the data derived from Slovenia, and the worst results can be seen in Belgium, Bulgaria and UK.

The relative biases of the rescaled bootstrap estimator $\hat{V}_{boot}\left(\hat{L}_{\alpha,\beta}\right)$ are generally less than 10%. However, we observe large biases with the data extracted from Bulgaria. The values $\alpha = \{0.3, 0.5\}$ also give large biases in Lithuania and Poland.

In general, the various confidence intervals have a good performance, since the corresponding coverage rates are close to the nominal level of 95%. The largest value of the various coverage rates is 96.3%, which can be observed in Bulgaria when $\alpha = 0.3$. On the other hand, the smallest value of the various coverage rates is 91.8%, which can be observed in Italy when $\alpha = 0.3$. The results derived from the percentile bootstrap (CI_p) and the studentized bootstrap (CI_t) methods are very similar. In general, the values of CR_N (based on the normal approximation) are slightly smaller than the values of CR_p and CR_t.

5. CONCLUSION

This paper discusses the variance estimation of relative poverty lines. We describe the customary estimator of this poverty indicator and suggest the rescaled bootstrap method (Rao *et al.*, 1992) for the problem of estimating the variance of the estimator of the relative poverty line. The corresponding confidence intervals for the relative poverty line are alsoanalyzed. We propose to use confidence intervals based on the normal approximation, the percentile bootstrap method, and the studentized bootstrap method.

Monte Carlo simulation studies have been carried out in order to evaluate the various estimation methods discussed in this paper. Real data sets obtained from the 2011 EU-SILC have been used in these simulation studies. In general, we have observed that the biases of the variance estimator are smaller than 10%. The various confidence intervals give coverage rate close to the nominal 95%. It seems that the best results are obtained when we consider the percentile bootstrap method and the studentized bootstrap method, which give similar results.

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