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A Five-Term 3-D Novel Conservative Chaotic System and its Generalized Projective Synchronization via Adaptive Control Method

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Abstract: First, this paper announces a five-term novel 3-D conservative chaotic system with a quadratic nonlinearity and a sextic nonlinearity. The conservative chaotic systems are characterized by the important property that they are volume conserving. The phase portraits of the novel conservative chaotic system are displayed and the mathematical properties are discussed. The Lyapunov exponents of the novel conservative chaotic system are obtained as $L_1 = 0.0846$, $L_2 = 0$ and $L_3 = -0.0846$. Since the sum of the Lyapunov exponents is zero, the 3-D novel chaotic system is conservative. The maximal Lyapunov exponent (MLE) for the novel chaotic system is obtained as $L_1 = 0.0846$ and Lyapunov dimension as $D_L = 3$. Next, an adaptive controller is designed to achieve generalized projective synchronization (GPS) of two identical novel conservative chaotic systems with unknown system parameters. MATLAB simulations have been shown to demonstrate the phase portraits of the conservative system and the GPS results derived via the adaptive control method.

Keywords: Chaos, chaotic systems, conservative systems, generalized projective synchronization, adaptive control.

1. INTRODUCTION

There is significant interest in the chaos literature in the finding of chaotic behaviour in nature and physical systems. A *chaotic system* is commonly defined as a nonlinear dissipative dynamical system that is highly sensitive to even small perturbations in its initial conditions. The sensitivity of a nonlinear chaotic system in response to small changes in the initial conditions is commonly called as *butterfly effect* [1] and this is one of the characterizing features of a chaotic system.

A significant development in chaos theory occurred when Lorenz discovered a 3-D chaotic system of a weather model in 1963 [2]. The Lyapunov exponent of a dynamical system is a quantitative measure that characterizes the rate of separation of infinitesimally close trajectories of the system. Thus, a chaotic system is also defined mathematically as a dynamical system having at least one positive Lyapunov exponent.

In the last four decades, many chaotic systems have been found in the literature using modelling and other techniques. Some paradigms of chaotic systems can be listed as Rössler system [3], Shimizu-Morioka system [4], Shaw system [5], Chen system [6], Lü system [7], Chen-Lee system [8], Cai system [9], Tigan system [10], Li system [11], etc. Many new 3-D chaotic systems have been discovered in the recent years such as Sundarapandian systems [12-13], Vaidyanathan systems [14-20], Vaidyanathan-Madhavan system [21], Vaidyanathan-Azar system [22], Vaidyanathan-Volos system [23-24], Pehlivan-Moroz system [25], Pham system [26], etc.

Hyperchaotic systems are the chaotic systems with more than one positive Lyapunov exponent. They have important applications in control and communication engineering. Some recently discovered 4-D

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hyperchaotic systems are hyperchaotic Vaidyanathan systems [27-28], hyperchaotic Vaidyanathan-Azar system [29], etc. A 5-D hyperchaotic system with three positive Lyapunov exponents was also recently found [30].

Chaos control and chaos synchronization are important research problems in the chaos literature, which have been studied extensively in the last four decades. There are several applications of chaos theory in a variety of fields such as lasers [31], oscillators [32-33], chemical reactors [34-35], biology [36-38], ecology [39-40], neural networks [41-43], robotics [44-45], memristors [46-48], fuzzy systems [49-50], etc.

The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [51-52]. Some popular methods for chaos control are active control [53-57], adaptive control [58-59], sliding mode control [60-62], etc.

Chaos synchronization problem can be stated as follows. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Because of the butterfly effect which causes exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, the synchronization of chaotic systems is a very challenging research problem in the chaos literature.

The synchronization of chaotic systems has applications in secure communications [63-65], cryptosystems [66-67], encryption [68-70], etc. The chaos synchronization problem has been paid great attention in the literature and a variety of impressive approaches have been proposed.

Since the pioneering work by Pecora and Carroll [71-72] for the chaos synchronization problem, many different methods have been proposed in the control literature such as active control method [73-80], adaptive control method [81-107], sampled-data feedback control method [108-109], time-delay feedback approach [110], backstepping method [111-122], sliding mode control method [123-131], etc.

In the chaos literature, there is also an active interest in the discovery of conservative chaotic systems [132], which have the special property that the volume of the flow is conserved. A practical way of checking whether a chaotic system is conservative or dissipative is via adding all the Lyapunov exponents of the system. If the sum of the Lyapunov exponents is zero, then the chaotic system is conservative and if the sum is negative, then the chaotic system is dissipative.

A classical example of a conservative chaotic system is the Nosé-Hoover system [132-133], which is modelled by the system of differential equations

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + x_2 x_3 \\ \dot{x}_3 &= 1 - x_2^2 \end{aligned} \tag{1}$$

The Nosé-Hoover system (1) has the Lyapunov exponents $L_1 = 0.014$, $L_2 = 0$, and $L_3 = -0.014$. The system (1) is chaotic as it has a positive Lyapunov exponent and is conservative as the sum of the Lyapunov exponents is zero. Thus, the system (1) is volume-conserving.

In this research work, we modify the dynamics of Nosé-Hoover system (1) and obtain a 5-term 3-D novel conservative system with Lyapunov exponents $L_1 = 0.0846$, $L_2 = 0$ and $L_3 = -0.0846$. Thus, it is clear that the maximal Lyapunov exponent (MLE) of the novel conservative chaotic system is $L_1 = 0.0846$, which is greater than the maximal Lyapunov exponent (MLE) of the Nosé-Hoover chaotic system (1). The Lyapunov dimension of all 3-D conservative chaotic systems is equal to three.

Furthermore, we derive an adaptive control law that achieves generalized projective synchronization (GPS) of the identical 3-D novel conservative chaotic systems when the system parameters are unknown. Generalized projective synchronization [134] is a general type of synchronization which generalizes complete synchronization, anti-synchronization, hybrid synchronization, and projective synchronization of chaotic systems. The main synchronization result is proved using adaptive control theory and Lyapunov stability theory. MATLAB simulations are shown in detail to validate and demonstrate the generalized projective synchronization of the identical 3-D novel conservative chaotic systems.

2. A FIVE-TERM 3-D NOVEL CONSERVATIVE CHAOTIC SYSTEM

In this section, we describe the equations and properties of a 3-D novel conservative chaotic system with a quadratic nonlinearity and a sextic nonlinearity.

The proposed novel conservative chaotic system is modelled by the 3-D dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + a x_2 x_3 \\ \dot{x}_3 &= b - x_2^6 \end{aligned} \tag{2}$$

where x_1, x_2, x_3 are the states and a, b are constant, positive parameters of the system.

The system (2) exhibits a *conservative chaotic attractor* for the values

$$a = 1,\ b = 1 \tag{3}$$

For numerical simulations, we take the initial state as $x_1(0) = 0.4,\ x_2(0) = 0,$ and $x_3(0) = 0.4.$ Figure 1 shows the conservative chaotic attractor of the system (2). Figures 2-4 show the 2-D view of the chaotic attractor of the system (2) in (x_1, x_2), (x_2, x_3) and (x_1, x_3) planes respectively.

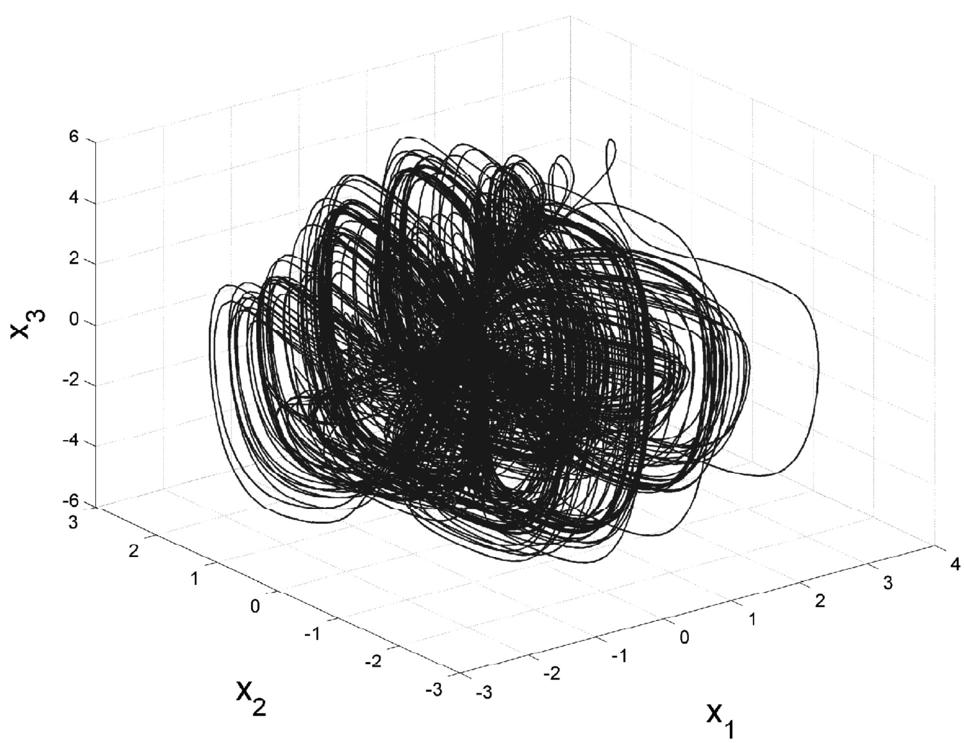

Figure 1: Strange attractor of the novel conservative chaotic system

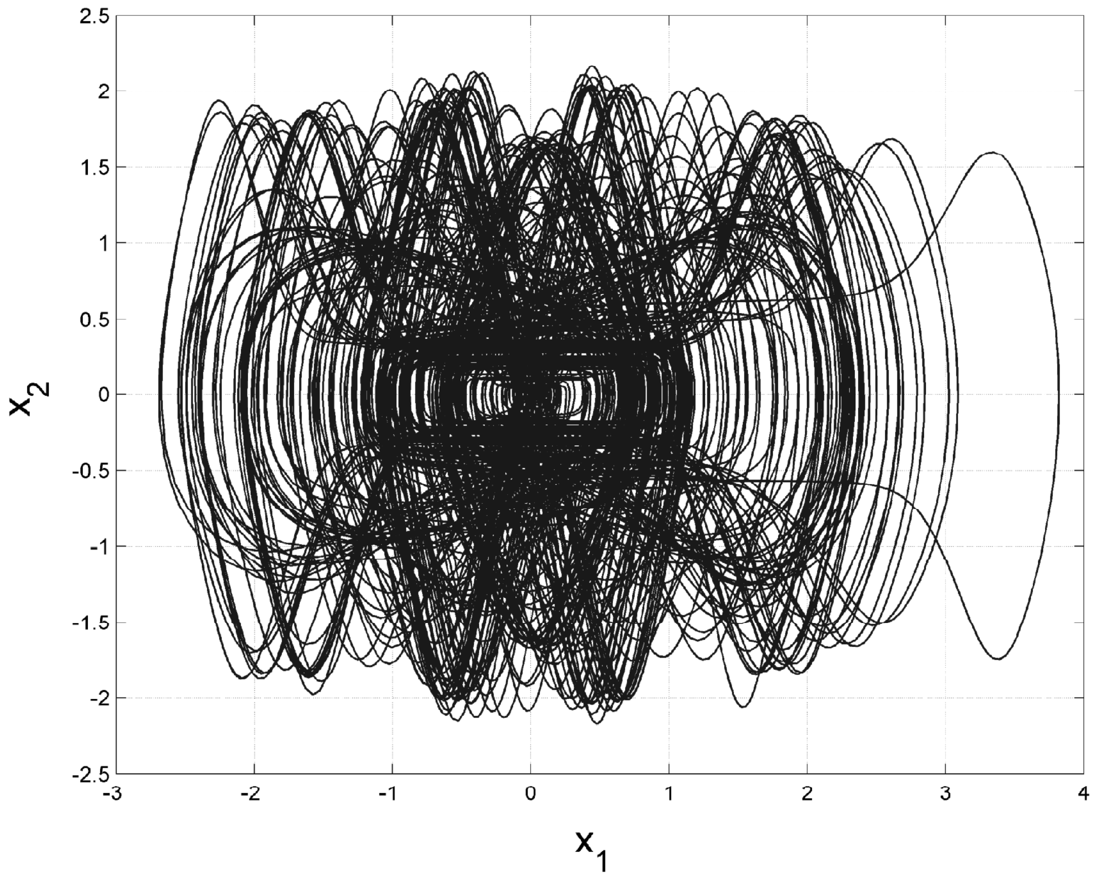

Figure 2: 2-D view of the novel conservative chaotic system in (x_1, x_2) plane

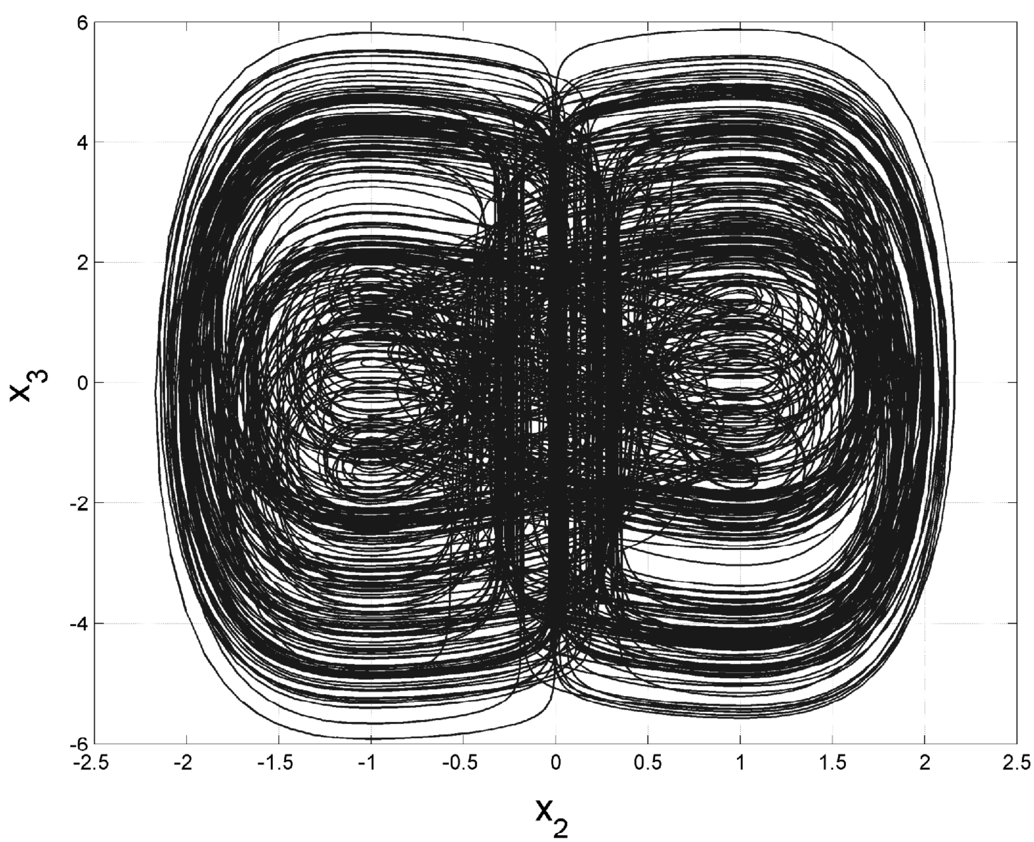

Figure 3: 2-D view of the novel conservative chaotic system in (x_2, x_3) plane

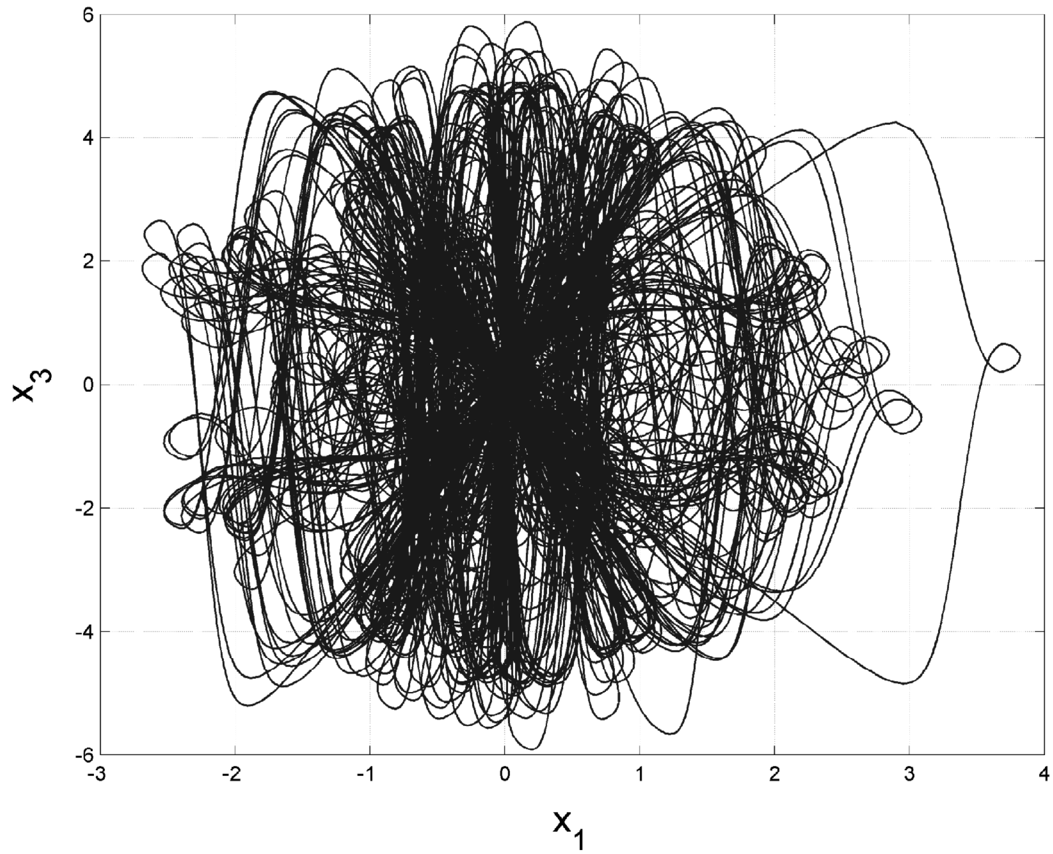

Figure 4: 2-D view of the novel conservative chaotic system in (x_1, x_3) plane

3. ANALYSIS OF THE NOVEL CONSERVATIVE CHAOTIC SYSTEM

A. Volume Conservation of the Flow

In vector notation, we may express the system (2) as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix} \tag{4}$$

where

$$\begin{aligned} f_1(x_1, x_2, x_3) &= x_2 \\ f_2(x_1, x_2, x_3) &= -x_1 + a x_2 x_3 \\ f_3(x_1, x_2, x_3) &= b - x_2^6 \end{aligned} \tag{5}$$

We take the parameter values as in the chaotic case, viz. $a = 1$ and $b = 1$.

Let Ω be any region in R^3 with a smooth boundary and also $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f. Furthermore, let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, we have

$$\frac{dV}{dt} = \int_{\Omega(t)} (\nabla \cdot f)\, dx_1 dx_2 dx_3 \tag{6}$$

The divergence of the novel chaotic system (2) is easily calculated as

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = 0 + 0 + 0 = 0 \tag{7}$$

Substituting (7) into (6), we get

$$\frac{dV}{dt} = 0 \tag{8}$$

which has the unique solution

$$V(t) \equiv V(0) \text{ for all } t \in R \tag{9}$$

This shows that the 3-D novel chaotic system (2) is volume-conserving.

Hence, the system (2) is a conservative chaotic system.

B. Symmetry and Invariance

It is easy to see that the system (2) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) \mapsto (-x_1, -x_2, x_3). \tag{10}$$

Thus, the system (2) has rotation symmetry about the x_3-axis and any non-trivial trajectory of the system (2) must have a twin trajectory.

The x_3- axis ($x_1 = 0,\ x_2 = 0$) is invariant for the system (2).

Hence, all orbits of the system (2) starting on the x_3-axis stay in the x_3-axis for all values of time. Also, this invariant motion is unstable.

C. Equilibrium Points

It is easy to verify that there is no equilibrium point for the system (2).

D. Lyapunov Exponents and Lyapunov Dimension

We take the parameter values of the system (2) as

$$a = 1, \quad b = 1 \tag{11}$$

The Lyapunov exponents of the system (2) are numerically obtained with MATLAB as

$$L_1 = 0.0846, \quad L_2 = 0, \quad L_3 = -0.0846 \tag{12}$$

Eq. (13) shows that the system (2) is chaotic, since it has a positive Lyapunov exponent.

Also, the maximal Lyapunov exponent (MLE) of the system (1) is obtained as $L_1 = 0.0846$.

Since $L_1 + L_2 + L_3 = 0,$ it is immediate that (2) is a conservative chaotic system.

The dynamics of the Lyapunov exponents is depicted in Figure 5. The Lyapunov dimension of the chaotic system (1) is determined as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 3 \tag{13}$$

which is high value. This shows that the chaotic behaviour of the system (2) is very complex.

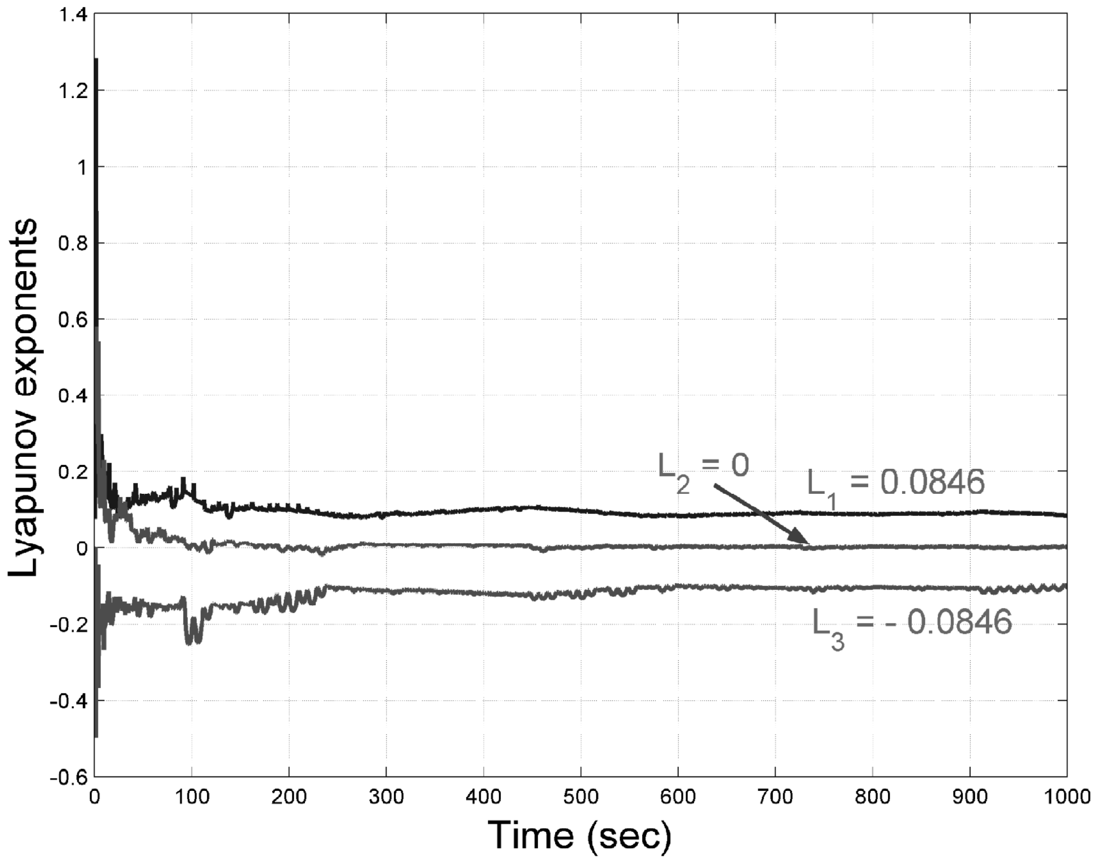

Figure 5: Dynamics of the Lyapunov exponents of the novel chaotic system

4. ADAPTIVE CONTROLLER DESIGN FOR THE GENERALIZED PROJECTIVE SYNCHRONIZATION OF THE IDENTICAL NOVEL CONSERVATIVE CHAOTIC SYSTEMS

In this section, we derive new results for an adaptive controller to achieve generalized projective synchronization (GPS) of the identical 3-D novel conservative chaotic systems.

As the master system, we take the novel 3-D conservative chaotic system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + a x_2 x_3 \\ \dot{x}_3 &= b - x_2^6 \end{aligned} \tag{14}$$

where x_1, x_2, x_3 are state variables and a, b are constant, unknown, parameters of the system.

As the slave system, we take the controlled novel 3-D conservative chaotic system

$$\begin{aligned} \dot{y}_1 &= y_2 + u_1 \\ \dot{y}_2 &= -y_1 + a y_2 y_3 + u_2 \\ \dot{y}_3 &= b - y_2^6 + u_3 \end{aligned} \tag{15}$$

where y_1, y_2, y_3 are state variables and u_1, u_2, u_3 are adaptive controllers to be designed.

The GPS error is defined by

$$\begin{aligned} e_1 &= y_1 - \mu_1 x_1 \\ e_2 &= y_2 - \mu_2 x_2 \\ e_3 &= y_3 - \mu_3 x_3 \end{aligned} \tag{16}$$

In (16), μ_1, μ_2, μ_3 are real, scaling, constants, which can take both positive and negative values. When $\mu_i = 1$, the GPS error (16) represents the complete synchronization error. When $\mu_i = -1$, the GPS error (16) represents the anti-synchronization error. When $\mu_1 = 1$, $\mu_2 = -1$, and $\mu_3 = 1$, the GPS error (16) represents the hybrid synchronization error.

The GPS error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= y_2 - \mu_1 x_2 + u_1 \\ \dot{e}_2 &= -y_1 + \mu_2 x_1 + a(y_2 y_3 - \mu_2 x_2 x_3) + u_2 \\ \dot{e}_3 &= b(1 - \mu_3) - y_2^6 + \mu_3 x_2^6 + u_3 \end{aligned} \tag{17}$$

We consider the adaptive control law defined by

$$\begin{aligned} u_1 &= -y_2 + \mu_1 x_2 - k_1 e_1 \\ u_2 &= y_1 - \mu_2 x_1 - \hat{a}(t)(y_2 y_3 - \mu_2 x_2 x_3) - k_2 e_2 \\ u_3 &= -\hat{b}(t)(1 - \mu_3) + y_2^6 - \mu_3 x_2^6 - k_3 e_3 \end{aligned} \tag{18}$$

where k_1, k_2, k_3 are positive gain constants, and $\hat{a}(t), \hat{b}(t)$ are estimates of the unknown parameters $a, b,$ respectively.

Substituting (18) into (17), we get the closed-loop error dynamics as

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= (a - \hat{a}(t))(y_2 y_3 - \mu_2 x_2 x_3) - k_2 e_2 \\ \dot{e}_3 &= (b - \hat{b}(t))(1 - \mu_3) - k_3 e_3 \end{aligned} \tag{19}$$

To simplify the error dynamics (35), we define the parameter estimation error as

$$\begin{aligned} e_a(t) &= a - \hat{a}(t) \\ e_b(t) &= b - \hat{b}(t) \end{aligned} \tag{20}$$

Using (20), we can simplify the error dynamics (19) as

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= e_a(y_2 y_3 - \mu_2 x_2 x_3) - k_2 e_2 \\ \dot{e}_3 &= e_b(1 - \mu_3) - k_3 e_3 \end{aligned} \tag{21}$$

Differentiating the parameter estimation error (20) with respect to t, we get

$$\dot{e}_a(t) = -\dot{\hat{a}}(t)$$
$$\dot{e}_b(t) = -\dot{\hat{b}}(t) \tag{22}$$

Next, we find an update law for parameter estimates using Lyapunov stability theory.

Consider the quadratic Lyapunov function defined by

$$V(e_1, e_2, e_3, e_a, e_b) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2\right), \tag{23}$$

which is positive definite on R^5.

Differentiating V along the trajectories of (21) and (22), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[e_2(y_2 y_3 - \mu_2 x_2 x_3) - \dot{\hat{a}}\right] + e_b\left[e_3(1-\mu_3) - \dot{\hat{b}}\right] \tag{24}$$

In view of (24), we define an update law for the parameter estimates as

$$\dot{\hat{a}} = e_2(y_2 y_3 - \mu_2 x_2 x_3)$$
$$\dot{\hat{b}} = e_3(1-\mu_3) \tag{25}$$

Theorem 1. The adaptive control law (18) and the parameter update law (25) achieve generalized projective synchronization (GPS) between the identical novel conservative chaotic systems (14) and (15) with unknown system parameters for all initial conditions, where k_i, $(i = 1, 2, 3)$ are positive gain constants.

Proof. The result is proved using Lyapunov stability theory [135]. We consider the quadratic Lyapunov function V defined by (23), which is a positive definite function on R^5.

Substituting the parameter update law (25) into (24), we obtain $\dot{V}$ as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{26}$$

which is a negative semi-definite function on R^5.

Thus, it can be concluded that the synchronization vector $e(t)$ and the parameter estimation error are globally bounded, i.e.

$$\left[e_1(t) \quad e_2(t) \quad e_3(t) \quad e_a(t) \quad e_b(t)\right]^T \in L_\infty. \tag{27}$$

We define

$$k = \min\left\{k_1, k_2, k_3\right\}. \tag{28}$$

Then it follows from (26) that

$$\dot{V} \le -k\|e\|^2 \text{ or } k\|e\|^2 \le -\dot{V}. \tag{29}$$

Integrating the inequality (29) from 0 to t, we get

$$k\int_o^t \|e(\tau)\|^2 \, d\tau \;\le\; -\int_0^t \dot{V}(\tau)d\tau = V(0) - V(t) \tag{30}$$

Therefore, we can conclude that $e(t) \in L_2$.

Using (21), we can conclude that $\dot{e}(t) \in L_\infty$.

Hence, using Barbalat's lemma, we can conclude that $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in R^3$.

This completes the proof.

Numerical Results

For the novel chaotic systems, the parameter values are taken as in the chaotic case, *viz.* $a = 1$, and $b = 1$. We take the feedback gains as $k_i = 10$ for $i = 1, 2, 3$.

The GPS scale constants are taken as

$$\mu_1 = 1.2, \quad \mu_2 = -0.7, \quad \mu_3 = 1.8 \tag{31}$$

The initial values of the master system (30) are taken as

$$x_1(0) = 8.5, \; x_2(0) = 2.3, \; x_3(0) = -5.1 \tag{32}$$

The initial values of the slave system (31) are taken as

$$y_1(0) = 4.8, \; y_2(0) = -2.9, \; y_3(0) = 6.4 \tag{33}$$

The initial values of the parameter estimates are taken as

$$\hat{a}(0) = 4.1, \; \hat{b}(0) = 0.2 \tag{34}$$

Figures 6-8 depicts the generalized projective synchronization (GPS) of the identical novel conservative chaotic systems.

Figure 9 depicts the time-history of the GPS errors.

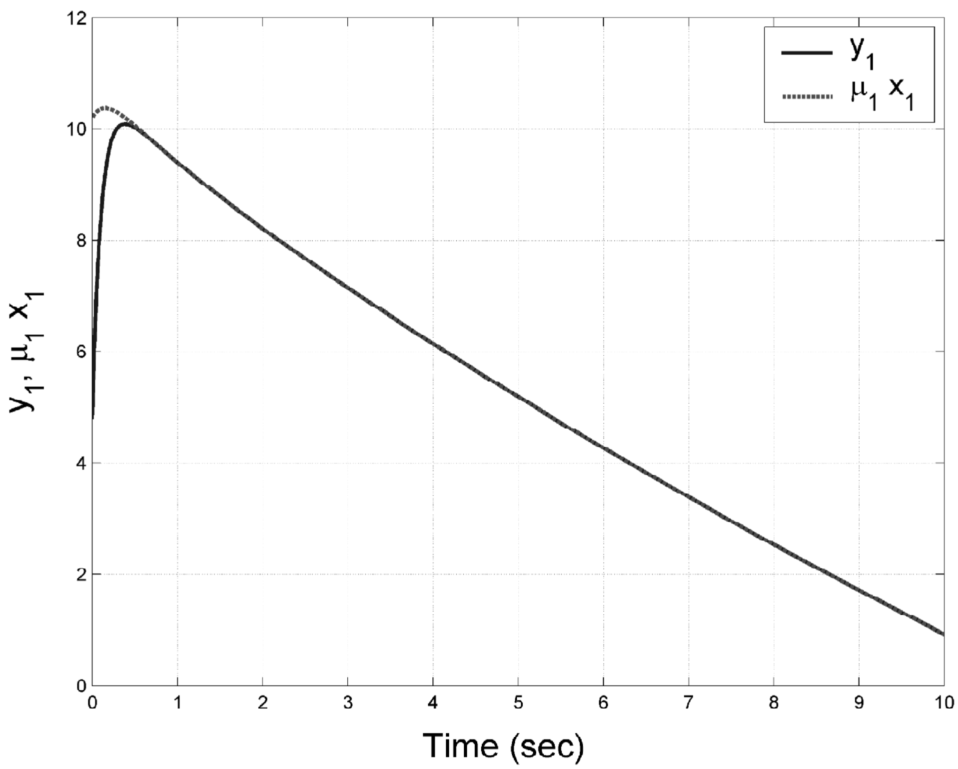

Figure 6: GPS of the states x_1 and y_1 of the novel chaotic system

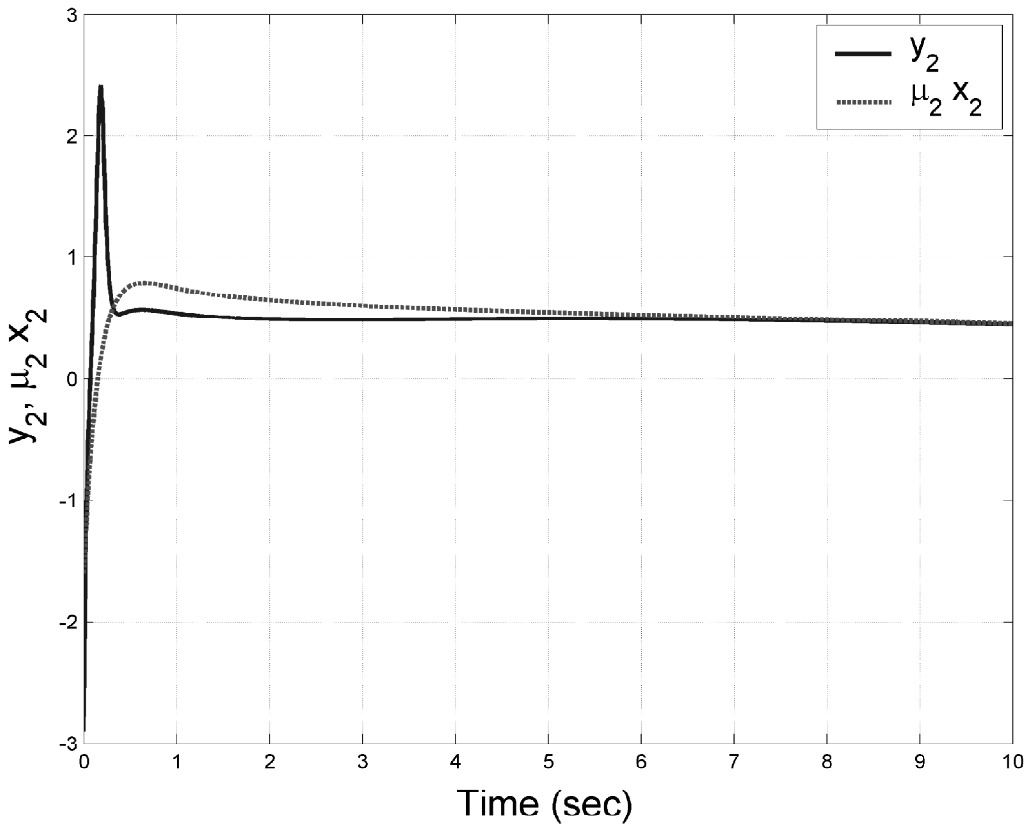

Figure 7: GPS of the states x_2 and y_2 of the novel chaotic system

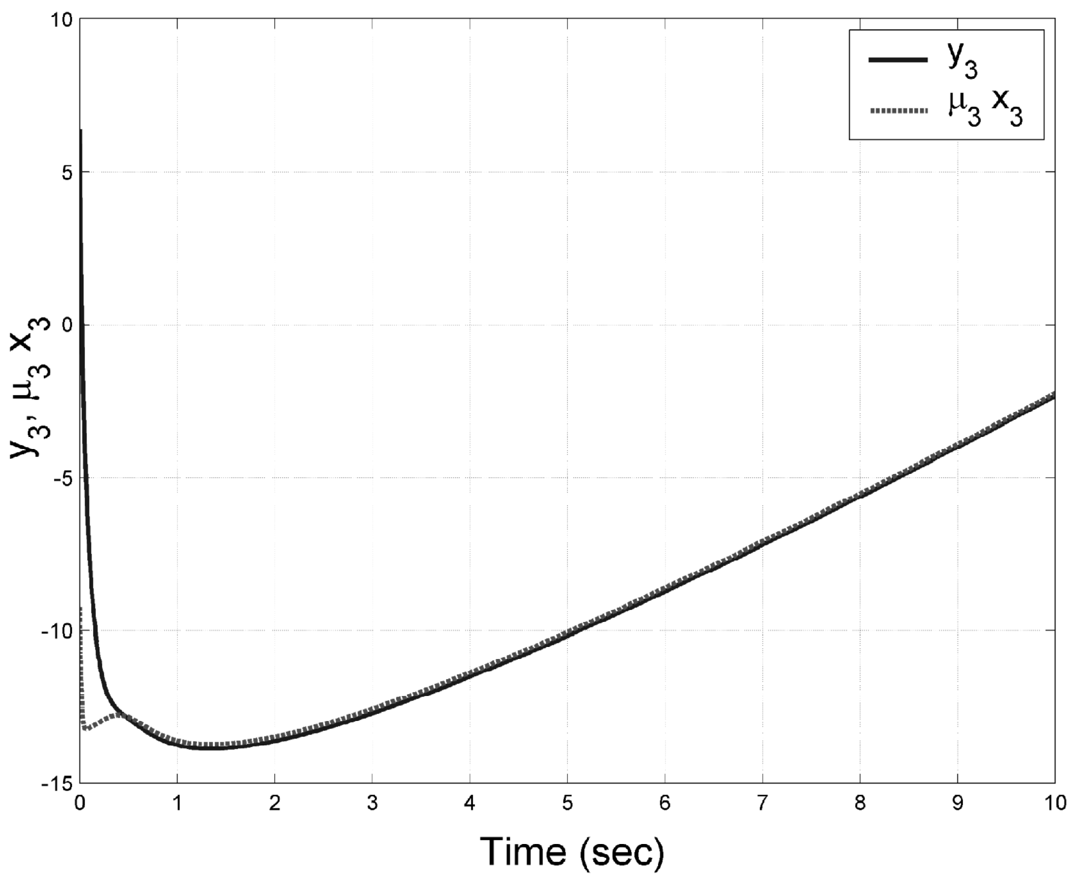

Figure 8: GPS of the states x_3 and y_3 of the novel chaotic system

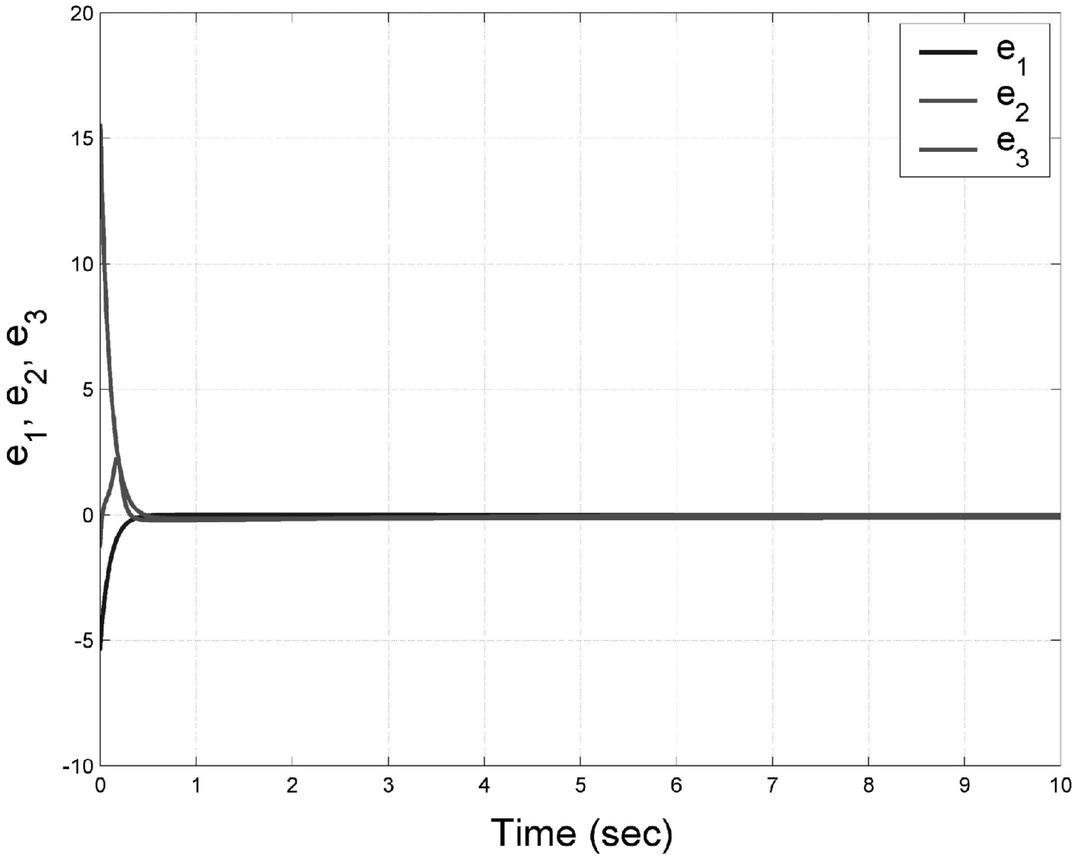

Figure 9: Time history of the GPS errors e_1, e_2, e_3

6. CONCLUSIONS

In this paper, we have proposed a five-term 3-D novel conservative chaotic system with a quadratic nonlinearity and a sextic nonlinearity. The conservative chaotic systems have the important property that they are volume conserving. Also, the Lyapunov dimension of any 3-D conservative chaotic system is equal to 3. The Lyapunov exponents of the 3-D novel chaotic system have been obtained as $L_1 = 0.0846$, $L_2 = 0$ and $L_3 = -0.0846$. The phase portraits of the novel chaotic system were simulated using MATLAB. We also showed that the 3-D novel conservative chaotic system has no equilibrium points and discussed its symmetry and invariance properties. Next, an adaptive controller was designed to achieve generalized projective synchronization (GPS) of two identical novel chaotic systems with unknown system parameters. The adaptive GPS synchronization result was established using Lyapunov stability theory. Finally, MATLAB simulations were shown to validate and demonstrate the GPS result derived in this work for identical 3-D novel conservative chaotic systems.

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