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EFFECT OF TEMPERATURE MODULATION ON THE ONSET OF RAYLEIGH-BÉNARD CONVECTION IN A COUPLE STRESS FLUID WITH MAXWELL-CATTANEO LAW

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Abstract: The linear stability of a horizontal layer of couple stress fluid heated from below is considered. A time-dependent periodic perturbation is applied to the wall temperatures in addition to a steady temperature difference between the walls of the fluid layer. The classical Fourier heat law is replaced by the Maxwell-Cattaneo law. The critical Rayleigh number, correction Rayleigh number and wave number for small amplitude of the modulation are calculated using the perturbation method. Only infinitesimal disturbances are considered. The effect of Cattaneo number, couple stress parameter and the Prandtl number is discussed.

Key words: Couple Stress Fluid; Maxwell-Cattaneo Law; Temperature Modulation.

INTRODUCTION

The classical Fourier's law governs the process of heat transfer by diffusion. Along with the First Law of Thermodynamics Fourier's law gives rise to a parabolic temperature field which implies an infinite speed of heat propagation. However, a disturbance wave in the temperature will travel at a finite speed since it is transferred by molecular interaction ([1]). Maxwell-Cattaneo (MC) law characterizes this behaviour. It has a transient term multiplied by the thermal relaxation time of the medium. This relaxation time is the required time for the heat flux to reach a steady state following a perturbation to the temperature gradient and thereby establishing a hyperbolic temperature field ([2]). The propagation of heat as a wave is not just a low-temperature phenomenon but is observed in ultra-fast heating [3], heat transfer in biological materials [4], convection in nanodevices and complex fluids [5] to name a few. Within the context of thermal convection Straughan and Franchi [6] , Straughan [7,8], Pranesh and Kiran [9], Papanicolaou *et al*. [10], Stranges *et al*. [2], Uribe *et al*. [11], Bissell [12], Shivakumara *et al*. [13] have studied various fluid systems employing the MC law.

The characteristics of the fluid flow encountered in many practical problems such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plates, exotic lubricants and colloidal fluids are different from that of a Newtonian fluid. These practical applications of non-Newtonian fluids in modern technology have motivated the study of convective instability of such fluids. The modelling of these problems is convoluted and at the same time mathematically and physically challenging. The constitutive equations for couple stress fluids were given by Stokes [14]. The theory proposed by Stokes is the simplest one for micro-fluids which allow polar effects such as the presence of couple stress, body couple, and non-symmetric tensors. Stokes deduced that couple stresses could be seen in noticeable magnitudes in liquids with large molecules. The couple stress fluid according to Eringen [15] is a particular case of micropolar fluid when micro-rotation balances with the natural vorticity of fluid. Couple stress fluid for Rayleigh-Bénard situation was studied by Siddheshwar and Pranesh [16] and for a porous medium in the presence of magnetic field by Sharma and Thakur [17]. Later many authors Pranesh and Sangeetha [18,19], Malashetty *et al.* [20], Shivakumara and Naveen [21], Jaimala and Kumar [22] , Kumar at al.[23] and Sameena and Pranesh [24] have investigated the Rayleigh-Bénard situation in couple stress fluid along with various components such as magnetic field, rotation, non-uniform temperature gradients, modulations, diffusing components, hall currents and in porous medium.

Externally modulated hydrodynamic systems have been receiving growing interest because it can alter the behaviour of the system and can induce novel dynamic states, particularly near a point of instability. Venezian [25] in his work considered the thermal analogue of Donnelly's experiments [26] by modulating the wall temperature with an additional perturbation in addition to the fixed temperature difference between the walls. A review of the relevant results of this system was given by Davis [27]. Gershuni and Zhukhovitskii [28], Rosenblat and Herbert [29], Rosenblat and Tanaka [30] have studied temperature modulation on various systems. Pranesh and Sangeetha [18,19] analyzed the effect of temperature modulation on the onset of magneto-convection in electrically conducting Boussinesq-Stokes suspensions and electroconvection under AC electric field in dielectric couple stress liquids. Bhadauria and Kiran [31] and Vasudha *et al.* [32] studied the combined effect of internal heating and temperature modulation in a Newtonian fluid and micropolar fluid respectively. Suthar *et al.* [33] and Umavathi at al. [34] have studied Rayleigh-Bénard convection in a densely packed porous layer and Maxwell fluid-nanofluid subjected to time-periodic temperature modulations respectively.

We study the effect of thermal modulation on the onset of Rayleigh-Bénard convection employing the non-classical Maxwell-Cattaneo law in this paper. The eigenvalue problem obtained is solved by perturbation technique with the amplitude of the temperature modulation as a perturbation parameter.

MATHEMATICAL FORMULATION

We consider a layer of couple stress fluid confined between two horizontal walls as shown in Figure 1. A Cartesian coordinate system (x, y, z) is chosen such that the origin is at the lower wall and z-axis is vertically upward. A vertical gravitational force acts on the fluid. The lower wall at $z = 0$ and the upper wall at $z = d$ are subjected to the temperatures

$$T(t) = T_0 + \frac{1}{2}\Delta T[1 - \varepsilon \cos\omega t] \tag{1}$$

and

$$T(t) = T_0 - \frac{1}{2}\Delta T[1 - \varepsilon \cos(\omega t + \phi)] \tag{2}$$

respectively, where T_0 is the reference temperature, ΔT is the temperature difference between the walls, ε is the amplitude of the modulation, ϕ is the phase angle and ω is the frequency of the modulation.

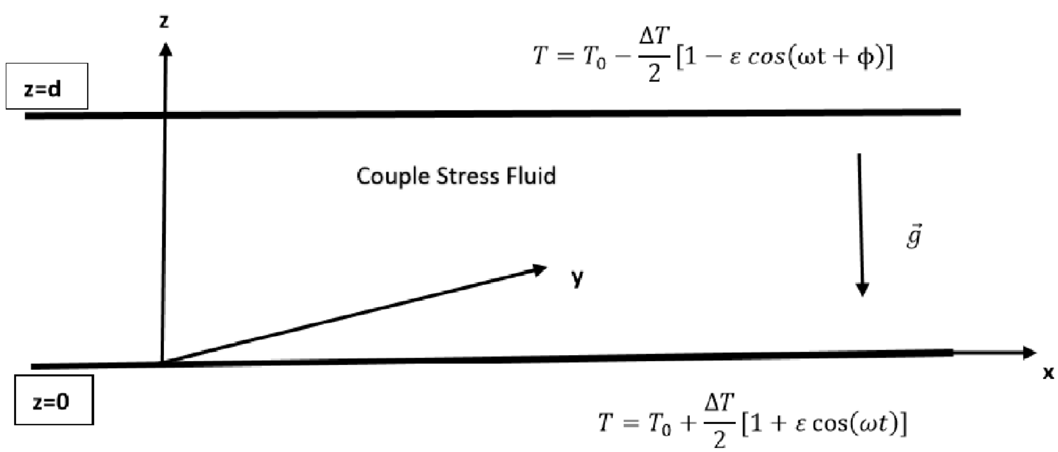

Figure 1: Physical configuration of the problem

We assume that the amplitude is very small and can be used to control convection. We use the amplitude as a parameter to find the solution of the basic equation. The following three types of temperature modulations are considered:

- when oscillating field is symmetric so that wall temperatures are modulated in-phase with $\phi = 0$,
- when oscillating field is asymmetric so that wall temperatures are modulated out-of-phase with $\phi = \pi$,

- when only the temperature of the bottom wall is modulated, the upper wall being held at constant temperature with $\phi = -i\infty$.

Under the Boussinesq approximation, the governing equations are given by [16,35]

$$\nabla.\vec{q} = 0, \tag{3}$$

$$\rho_0\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right) = -\nabla p + \rho\vec{g} + \mu\nabla^2\vec{q} - \mu'\nabla^4\vec{q}, \tag{4}$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = -\nabla.\vec{Q}, \tag{5}$$

$$\tau\left[\frac{\partial \vec{Q}}{\partial t} + \overrightarrow{\omega_1} \times \vec{Q}\right] = -\vec{Q} - \kappa\nabla T, \tag{6}$$

$$\rho = \rho_0\,[1 - \alpha(T - T_0)], \tag{7}$$

where $\vec{q}$ is the velocity, T is the temperature, p is the pressure, ρ is the density, ρ_0 is the density at $T = T_0$, $\vec{g}$ is the acceleration due to gravity, $\vec{Q}$ is the heat flux vector, μ is the dynamic viscosity, μ' is the couple stress viscosity, τ is the relaxation time, κ is the thermal conductivity, α is the coefficient of thermal expansion and $\overrightarrow{\omega_1} = \frac{1}{2}\nabla \times \vec{q}.$ The basic state of the fluid is quiescent and is given by

$$\overrightarrow{q_b}(z) = \vec{0},\ \rho = \rho_b(z,t), p = p_b(z,t),\ T = T_b(z,t),\ \vec{Q} = \vec{Q}_b(z). \tag{8}$$

Substituting (8) in (3)-(7), the pressure p_b, heat flux $\vec{Q}_b$, temperature T_b, and density ρ_b satisfy the following equations:

$$\frac{\partial p_b}{\partial z} = -\rho_b g \tag{9}$$

$$\overrightarrow{Q_b} = -\kappa\frac{\partial T_b}{\partial z} \tag{10}$$

$$\frac{\partial T_b}{\partial t} = \kappa\frac{\partial^2 T_b}{\partial z^2} \tag{11}$$

$$\rho_b = \rho_0 \, [1 - \alpha(T_b - T_0)] \tag{12}$$

The solution of (11) that satisfies the boundary conditions (1) and (2) is

$$T_b = T_s\,(z) + \varepsilon T_1\,(z, t) \tag{13}$$

where

$$T_s(z) = T_0 + \left(\frac{\Delta T}{2d}\right)(d - 2z),$$

$$T_1(z,t) = \mathrm{Re}\{[a(\lambda)e^{\lambda z/d} + a(-\lambda)e^{-\lambda z/d}]e^{-i\omega t}\},$$

$$\lambda = (1-i)\left(\frac{\omega d^2}{2\kappa}\right)^{\frac{1}{2}},$$

$$a(\lambda) = \left(\frac{\Delta T}{2}\right)\left[\frac{e^{-i\phi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right]$$

and Re stands for the real part.

Let the basic state be disturbed by an infinitesimal thermal perturbation. We assume a solution for $\vec{q}$, T, p, ρ and $\vec{Q}$ in the form

$$\vec{q} = \overrightarrow{q_b} + \vec{q}', \rho = \rho_b + \rho', p = p_b + p', T = T_b + T', \vec{Q} = \overrightarrow{Q_b} + \vec{Q}' \tag{14}$$

where $\vec{q}'$, ρ', p', T', $\vec{Q}'$ represents the perturbed quantities which are assumed to be small. Substituting (14) in (3)-(7) and using the basic state equations we obtain the following linearized equations for the infinitesimal perturbations:

$$\nabla.\vec{q}' = 0, \tag{15}$$

$$\rho_0 \frac{\partial \overrightarrow{q'}}{\partial t} = -\nabla p' + \mu \nabla^2 \overrightarrow{q'} - \mu' \nabla^4 \overrightarrow{q'} - \rho' g \hat{k}, \tag{16}$$

$$\frac{\partial T'}{\partial t} + W' \frac{\partial T_b}{\partial z} = -\nabla.\overrightarrow{Q'}, \tag{17}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\overrightarrow{Q'} = \frac{1}{2}\kappa\tau\left(\frac{\partial T_b}{\partial z}\right)\left[\frac{\partial \overrightarrow{q'}}{\partial z} - \nabla W'\right] - \kappa\nabla T', \tag{18}$$

$$\rho' = -\rho_0\,\alpha T' \tag{19}$$

We eliminate p' from (16) by operating curl twice and eliminate $\overrightarrow{Q'}$ between (17) and (18) by operating divergence on (18). The perturbation equations are then non-dimensionalized using:

$$(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right);\ \overrightarrow{q^*} = \frac{\vec{q}'}{\left(\frac{\kappa}{d}\right)};\ t^* = \frac{t}{\left(\frac{d^2}{\kappa}\right)};T^* = \frac{T'}{\Delta T};\ \omega^* = \frac{\omega}{\left(\frac{\kappa}{d^2}\right)}; \tag{20}$$

to obtain, ignoring the asterisks:

$$\frac{1}{\Pr}\frac{\partial}{\partial t}(\nabla^2 W) = \nabla^4 W - C\nabla^6 W + R\nabla_1^2 T, \tag{21}$$

$$\left(1+2M\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} = M\left(\frac{\partial T_0}{\partial z}\right)\nabla^2 W + \nabla^2 T - \left(\frac{\partial T_0}{\partial z}\right)\left(1+2M\frac{\partial}{\partial t}\right)W. \tag{22}$$

where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\nabla^2 = \nabla_1^2 + \frac{\partial^2}{\partial z^2}$, $\Pr = \frac{\mu}{\rho_0 \kappa}$ (Prandtl number),

$$R = \frac{\rho_0\,\alpha\, g\, d^3\,\Delta T}{\mu\kappa} \text{ (Rayleigh number)},\quad M = \frac{\tau\,\kappa}{2\,d^2} \text{ (Cattaneo number)},$$

$$C = \frac{\mu'}{\mu\, d^2} \text{ (Couple stress parameter)}.$$

$\frac{\partial T_0}{\partial z}$ in (22) is the non-dimensional form of $\frac{\partial T_b}{\partial z}$ where

$$\frac{\partial T_0}{\partial z} = -1 + \varepsilon\, f \tag{23}$$

where f is the modulation temperature gradient given by

$$f = \mathrm{Re}\{[A(\lambda)e^{\lambda z} + A(-\lambda)e^{-\lambda z}]e^{-i\omega t}\},$$

with

$$A(\lambda) = \left(\frac{\lambda}{2}\right)\left[\frac{e^{-i\phi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right].$$

(21) and (22) are solved for stress-free, isothermal, vanishing couple-stress boundary conditions

$$W = \frac{\partial^2 W}{\partial z^2} = T = 0 \; at \; z = 0 \; and \; z = 1.$$

Eliminating T from (21) and (22), we obtain

$$\left\{X_1 X_2 + \left(\frac{\partial T_0}{\partial z}\right) X_3 R \; \nabla_1^2\right\} W = 0 \tag{24}$$

with the boundary conditions

$$W = \frac{\partial^2 W}{\partial z^2} = \frac{\partial^4 W}{\partial z^4} = \frac{\partial^6 W}{\partial z^6} = 0 \; at \; z = 0,1. \tag{25}$$

where

$$X_1 = \left[\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}\nabla^2 - \nabla^4 + C\;\nabla^6\right];\; X_2 = \left[\nabla^2 - \left(1 + 2M\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\right];$$
$$X_3 = \left[M\nabla^2 - \left(1 + 2M\frac{\partial}{\partial t}\right)\right]. \tag{26}$$

Solution

We seek eigenfunctions W and eigenvalues R of (24) for the basic temperature profile (23) that departs from the linear profile $\frac{\partial T_o}{\partial z} = -1$ by quantities of order ε. Thus the eigenvalues of the present problem differ from the classical Rayleigh-Bénard problem by quantities of order ε.

We assume the solution of (24) in the form

$$(W, R) = (W_0, R_0) + \varepsilon(W_1, R_1) + \varepsilon^2 (W_2, R_2) + \dots \tag{27}$$

where W_0 and R_0 are the eigenfunction and eigenvalue respectively for the unmodulated system and W_i and R_i, for $i \geq 1$, are the correction to W_0 and R_0 in the presence of modulation.

Substituting (27) in (24) and equating the corresponding terms, we obtain the following system of equations:

$$LW_0 = 0, \tag{28}$$

$$LW_1 = (R_1 - R_0 f)\, X_3\, \nabla_1^2\, W_0, \tag{29}$$

$$LW_2 = (R_1 - R_0 f) X_3\, \nabla_1^2 W_1 + (R_2 - R_1 f) X_3\, \nabla_1^2 W_0. \tag{30}$$

where

$$L = X_1 X_2 - R_0 X_3\, \nabla_1^2 \tag{31}$$

Each of W_0, W_1, W_2 is required to satisfy the boundary conditions in (25). The stability of the system in the absence of thermal modulation is investigated by introducing vertical velocity perturbation W_0, lowest mode of convection, as

$$W_0 = \sin(\pi z) \exp\,[i(lx + my)] \tag{32}$$

where l and m are the wave numbers in the x and y directions respectively such that $l^2 + m^2 = a^2$.

Substituting (32) in (28), we obtain

$$R_0 = \frac{k_1^6 + C\, k_1^8}{a^2 (1 + M\, k_1^2)} \tag{33}$$

where

$$k_1^2 = \pi^2 + a^2.$$

Then (29) for W_1 becomes

$$LW_1 = a^2\, (R_1 - R_0 f)\, (1 + M k_1^2) W_0 \tag{34}$$

The solubility condition requires that the time independent part of the right-hand side of (34) should be orthogonal to W_0. Since f varies sinusoidally with time, the only time independent term in the right-hand side of (34) is $a^2 R_1 (1 + M k_1^2) W_0$. Hence $R_1 = 0$ and it follows that all the odd coefficients R_3, R_5 and so on in (27) are zero. Expanding the right-hand side of (29) using Fourier series and by inverting the operator L term by term we obtain W_1 as

$$W_1 = -a^2 R_0 (1 + M k_1^2) \, \mathrm{Re}\left\{ \sum_{n=1}^{\infty} \frac{B_n(\lambda)}{L_1(\omega, n)} e^{-i\omega t} \sin(n\pi z) \right\}, \tag{35}$$

where

$$B_n(\lambda) = A(\lambda)\, g_{n1}(\lambda) + A(-\lambda) g_{n1}(-\lambda)$$

$$= \frac{-2n\pi^2 \lambda^2 [e^{\lambda} - e^{-\lambda} + (-1)^n (e^{-\lambda - i\phi} - e^{\lambda - i\phi})]}{(e^{\lambda} - e^{-\lambda})(\lambda^2 + (n+1)^2 \pi^2)(\lambda^2 + (n-1)^2 \pi^2)}$$

$$k_n^2 = n^2 \pi^2 + a^2$$

$$\begin{aligned} L_1(\omega, n) = {} & [(-k_n^2 + 2\omega^2 M)(-k_n^4 - C\, k_n^6) - \frac{\omega^2 k_n^2}{\mathrm{Pr}} - R_0 a^2 (M k_n^2 + 1)] \\ & i\omega[-k_n^4 - C\, k_n^6 + \frac{k_n^2}{P_r}(-k_n^2 + 2\omega^2 M) + 2 M R_0 a^2] \end{aligned} \tag{36}$$

(30) for W_2 becomes

$$L W_2 = R_0\, a^2 f (1 + M k_1^2) W_1 + a^2 R_2 X_4 W_0, \tag{37}$$

Where

$$X_4 = -1 - M k_n^2 + i 2 M \omega. \tag{38}$$

We use (37) to determine R_2, the first non-zero correction to R_0. The solubility condition requires that the steady part of the right-hand side of (37) should be orthogonal to W_0. This gives

$$R_2 = \frac{a^2\ R_0^2}{4} \sum_{n=1}^{\infty} \left\{ \frac{|B_n(\lambda)|^2}{|L_1(\omega, n)|^2} [N(\omega, n) + N^*(\omega, n)] \right\} \tag{39}$$

where * denotes a complex conjugate and

$$N(\omega, n) = L_1^*(\omega, n) X_4$$

Results

The value of Rayleigh number R obtained by this procedure is the eigenvalue corresponding to the eigenfunction W which although oscillating remains bounded in time. Since R is a function of the horizontal wave number a and the amplitude of perturbation ε, we have

$$R(a, \varepsilon) = R_0(a) + \varepsilon^2 R_2(a) + \dots \qquad (40)$$

The critical value is computed up to $O(\varepsilon^2)$ by evaluating R_0 and R_2 at $a = a_0$. It is only when we need to evaluate R_4 that a_2 should be taken into account where $a = a_2$ minimizes R_2.

We now evaluate R_{2c} for the three cases:

- In-phase modulation with $\phi = 0$. Then

$$B_n(\lambda) = \begin{cases} b_n, n\, even \\ 0, \ n\, odd \end{cases}$$

- Out-of-phase modulation with $\phi = \pi$. Then

$$B_n(\lambda) = \begin{cases} 0, \ n\, even \\ b_b, \ n\, odd \end{cases}$$

- When only the temperature of the bottom wall is modulated with $\phi = -i\infty$. Then

$B_n(\lambda) = \dfrac{b_n}{2}$, for all integer values of n

where

$$b_n = \frac{4n\pi^2\lambda^2}{[\lambda^2 + (n+1)^2\pi^2][\lambda^2 + (n-1)^2\pi^2]} \qquad (41)$$

and

$$\lambda = (1-i)\left(\frac{\omega}{2}\right)^{1/2}.$$

Thus

$$|b_n|^2 = \frac{16n^2\pi^4\omega^2}{[\omega^2 + (n+1)^4\pi^4][\omega^2 + (n-1)^4\pi^4]} \qquad (42)$$

From (40) the critical Rayleigh number is in the form

$$R_c = R_{0c} + \varepsilon^2 R_{2c} \tag{43}$$

where R_{0c} and R_{2c} can be obtained from (33) and (39) respectively. If R_{2c} is positive, supercritical instabilities exists and R_c has a minimum at $\varepsilon = 0$. If R_{2c} is negative, subcritical instabilities are possible. In such a case we have from (43)

$$\varepsilon^2 < \frac{R_{0c}}{R_{2c}} \tag{44}$$

From (44) the range of the amplitude of modulation ε which causes subcritical instabilities can be examined and can be explained by assigning values to the physical parameters involved in the problem.

DISCUSSION

This paper presents an analytical study of the effect of time-periodic temperature modulation on the onset of convection in couple stress fluid using Maxwell-Cattaneo law. An approximate linear stability analysis proposed by Venezian [25] is used to find the critical Rayleigh number as a function of frequency of the modulation, Cattaneo number, couple stress parameter and Prandtl number.

The solution obtained is based on the assumption that the amplitude of the modulation is small. The validity of the results depends on the value of the modulating frequency ω. When $\omega << 1$, the period of modulation is large and it affects the entire volume of the fluid. For large frequencies the effect of modulation disappears because the buoyancy force takes a mean value leading to the equilibrium state of the unmodulated case. Thus we have chosen only moderate values of ω in our study. A positive R_{2c} means the modulation effect is stabilizing while a negative R_{2c} means the modulation effect is destabilizing compared to the unmodulated system.

Figs. 2, 3 and 4 show the variation of R_{2c} with respect to ω for various governing parameters for the case of in-phase modulation. Figures show that the critical Rayleigh number R_{2c} is negative for small frequencies indicating a destabilized flow. For moderate frequencies R_{2c} is positive indicating that the effect of in-phase modulation is to stabilize the system. It can also be seen that as the frequency increases from zero to infinity R_{2c} decreases to its minimum value producing maximum destabilization and then increases to its maximum stabilizing value and thereafter decreases to zero. Hence in the presence of in-phase modulation convection occurs at lower values of the Rayleigh number compared to the unmodulated system.

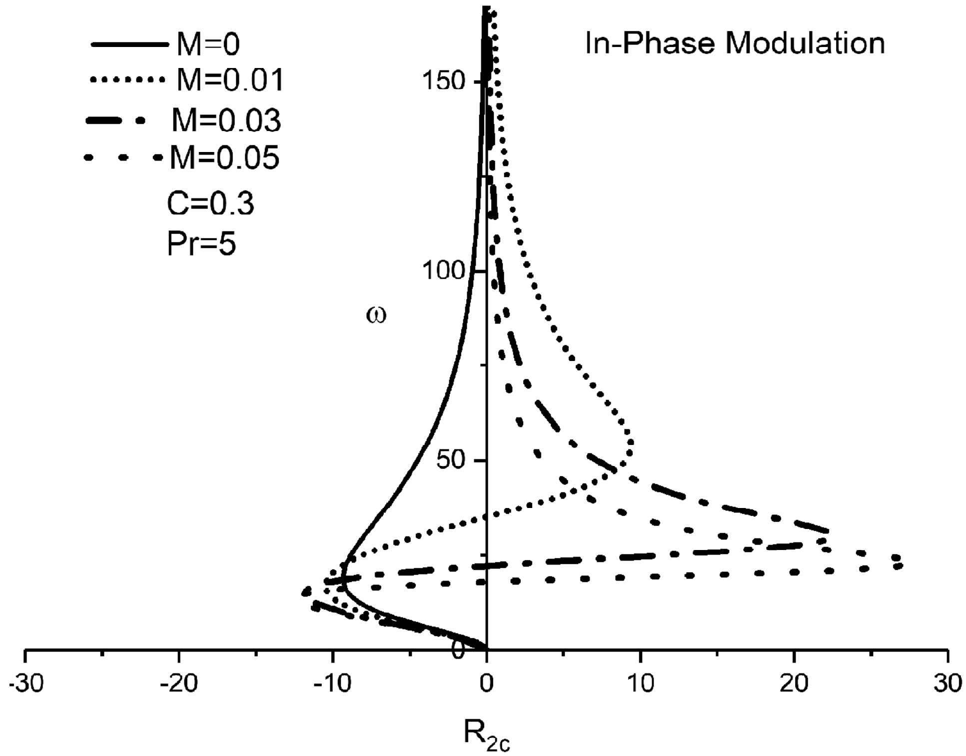

Figure 2: Variation of R_{2c} with ω for various values of Cattaneo number for in-phase modulation

Figure 2 shows the variation of R_{2c} with the modulating frequency ω for different values of Cattaneo number, M. We see that as M increases, R_{2c} is becoming more negative for small values of ω and increasing positively for moderate values of ω. Hence the effect of Cattaneo number is to destabilize the system for small frequencies while it stabilizes for moderate frequencies. The figure also shows that the influence of Cattaneo number is dominant for small values of M. This is because the convection cells have fixed aspect ratio. Increase in Cattaneo number leads to narrowing of the convection cells and thus advancing the convection. In the absence of the Cattaneo number, the results obtained agree with the one by Pranesh and Sangeetha [19] and Malashetty and Basavaraja [20].

According to Einstein [36] the presence of suspended particles is to increase the viscosity of the fluid by a factor of 2.5ϕ where ϕ is the volume fraction of the suspended particles. Therefore we assume that the viscosity of a suspension is higher than that of the carrier fluid([16]).

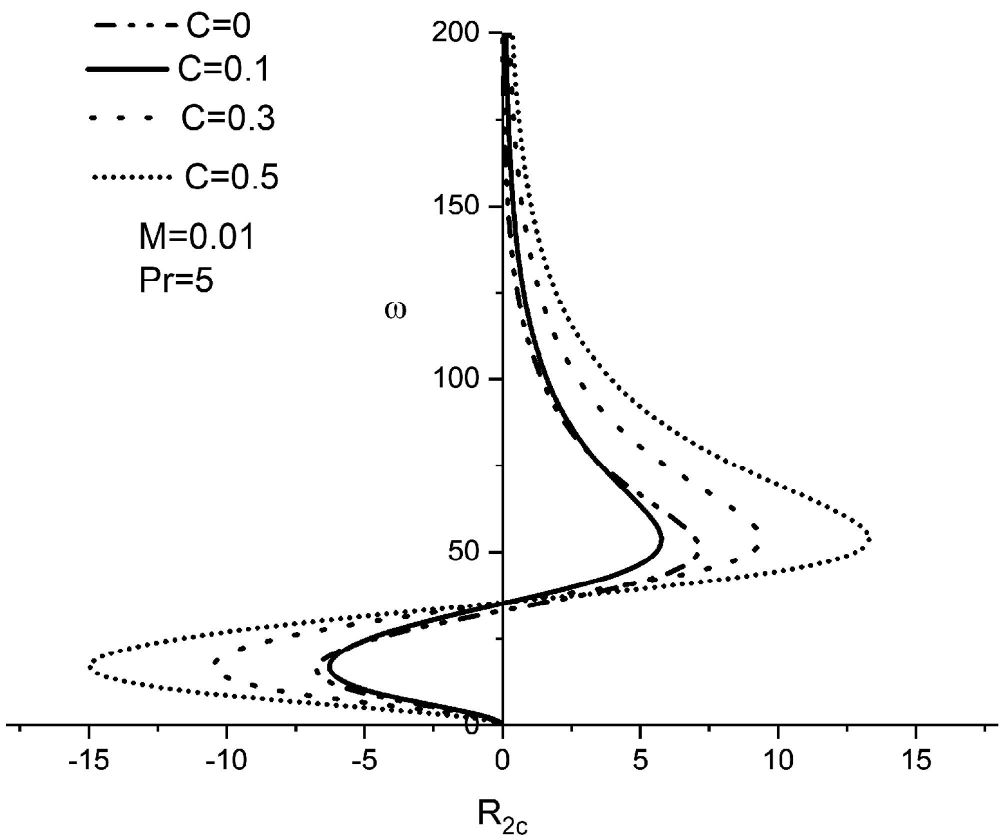

Figure 3: Variation of R_{2c} with ω for various values of couple stress parameter for in-phase modulation

Figure 3 is the plot of R_{2c} with the modulating frequency ω for different values of the couple stress parameter, *C*. We see that R_{2c} is negative for small values of ω, but is positive for moderate values of ω. This indicates that the couple stress parameter has a destabilizing effect on the system for small values of ω and stabilizing effect for moderate values of ω.

Figure 4 is the plot of R_{2c} with the modulating frequency ω for different values of Prandtl number, *Pr*. R_{2c} is negative for small values of the frequency but is positive for moderate and large values of the frequency. Hence Prandtl number has a destabilizing effect on the system for small values of ω and stabilizing effect for moderate values of ω. It is also found that the effect of Prandtl number is insignificant in case of in-phase modulation of the wall temperature in systems with couple stress fluid.

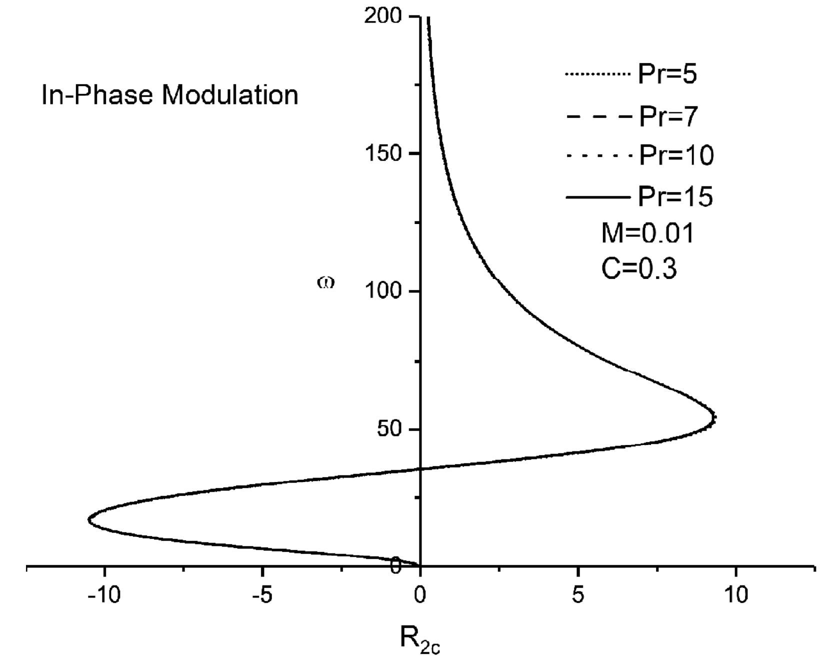

Figure 4: Variation of R_{2c} with ω for various values of Prandtl number for in-phase modulation

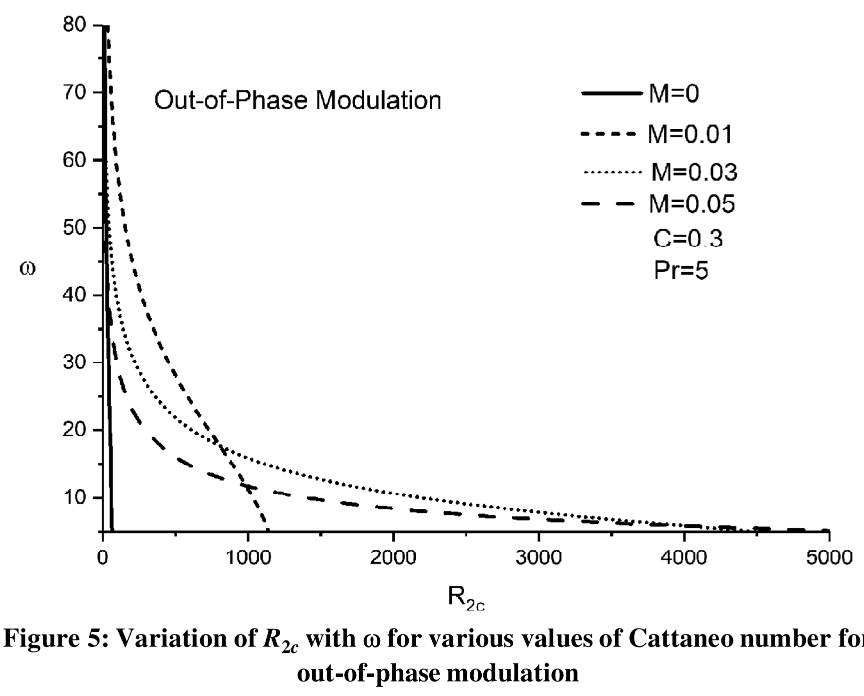

Figure 5: Variation of R_{2c} with ω for various values of Cattaneo number for out-of-phase modulation

The results obtained for the case of out-of-phase modulation are presented in Figures 5, 6 and 7. It can be seen from Figure 5 that Cattaneo number has stabilizing effects for small values of the frequency and has destabilizing effect for moderate values. For the values of M we have considered in this problem it has a destabilising effect for $\omega \geq 18$. The value of ω which leads to the destabilising effect is dependent on the values of the other governing parameters.

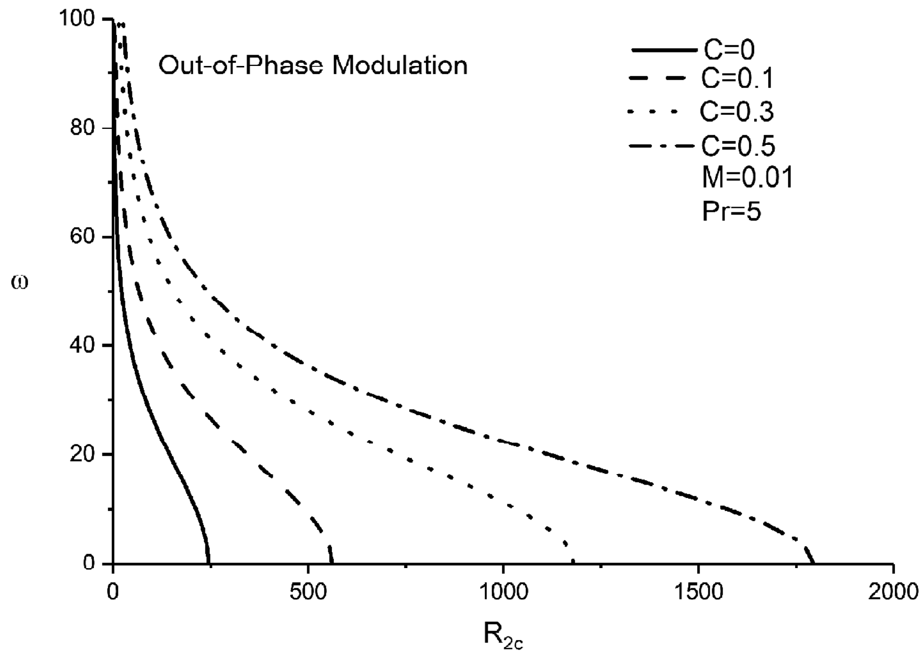

Figure 6: Variation of R_{2c} with ω for various values of couple stress parameter for out-of-phase modulation

The effect of couple stress parameter on the onset of convection is shown in Figure 6. R_{2c} is positive in the case of out-of-phase modulation. It is found that the effect of increasing the couple stress parameter is to make the system more stable. This is in contrast to its effect in the case of in-phase modulation. This can be anticipated because R_c increases with increase in C as C is indicative of the concentration of the suspended particles.

Figure 7 depicts the variation of R_{2c} with ω for different values of Prandtl number. We notice from the figure that the effect of Prandtl number is insignificant. However Prandtl number tends to stabilize the system for moderate values of ω. Variation of all the governing parameters for the case of only lower wall temperature modulation produce similar effects as for out-of-phase modulation. The results are illustrated in Figures 8, 9 and 10.

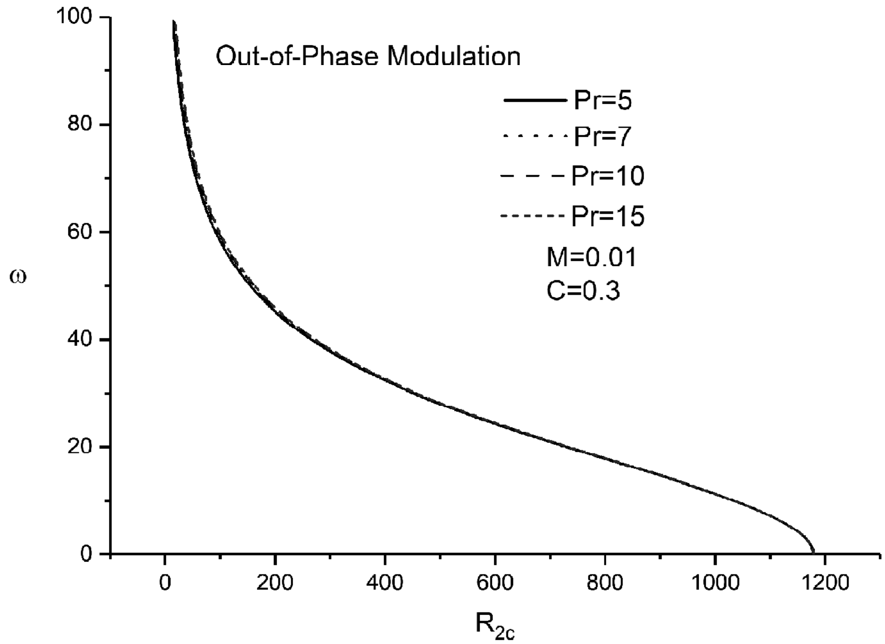

Figure 7: Variation of R_{2c} with ω for various values of Prandtl number for out-of-phase modulation

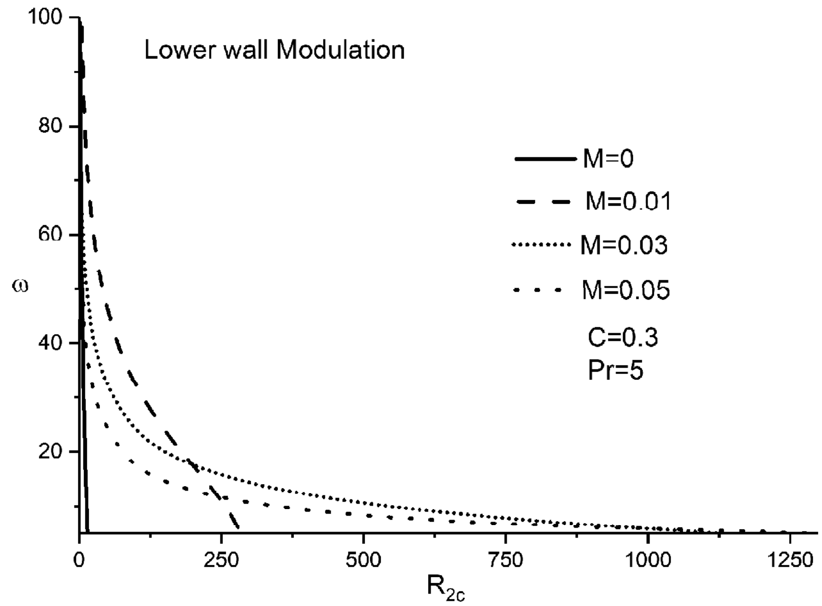

Figure 8: Variation of R_{2c} with ω for various values of Cattaneo number for only lower wall modulation

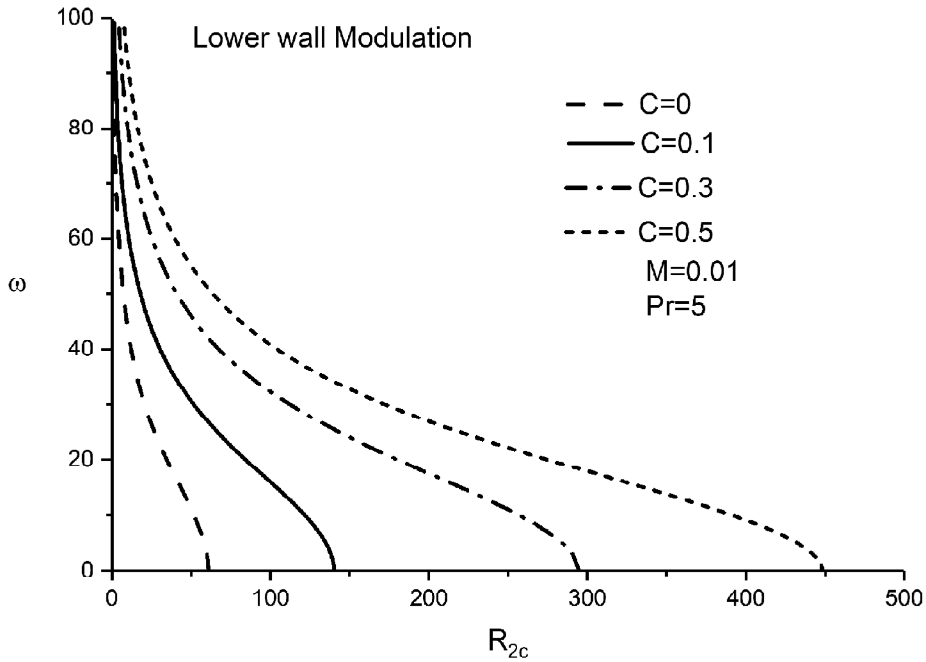

Figure 9:Variation of R_{2c} with ω for various values of couple stress parameter for only lower wall modulation

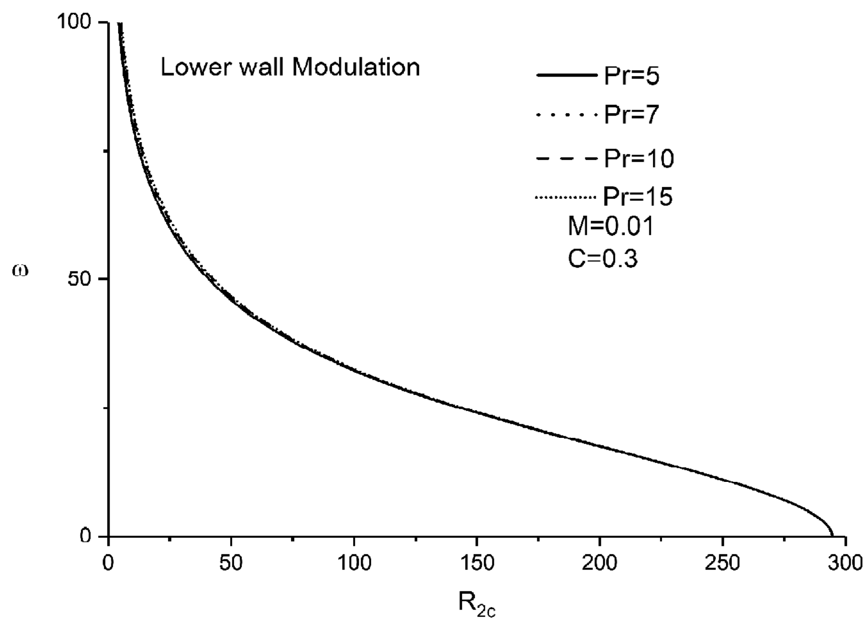

Figure 10: Variation of R_{2c} with ω for various values of Prandtl number for only lower wall modulation

CONCLUSION

The effect of thermal modulation on the onset of convection in a couple stress fluid with Maxwell-Cattaneo law is studied using a linear stability analysis. It is found that the system is more stable when boundary temperature is modulated out-of-phase. In-phase temperature modulation leads to sub-critical motions. Cattaneo number is stabilizing for moderate frequencies and destabilizing for small frequencies in the case of in-phase modulation. It has a destabilizing effect for moderate frequencies and stabilizing effect for a range of small frequencies in the cases of out-of-phase and lower wall modulation. Couple stress parameter stabilizes the system in the case of out-of-phase and lower wall modulations, and for moderate frequencies in the case of in-phase modulation. The results show the effect of temperature modulation on smaller time scales. It is shown that non-Fourier effects in fluids varies significantly from that of Fourier case. The problem gives insight into external means of controlling convection. The study is particularly relevant to low-temperature fluids.

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