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TEST FOR STRUCTURAL CHANGE UNDER DEPENDENT ERRORS: THE CASE OF SUCCESSIVE REGRESSIONS

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Abstract: *The Chow test is not robust under dependency of errors. The presence of dependency of errors will affect level of significance as well as power of the test, especially when the sizes of the samples are small. The present paper not only resolves the problem of dependency in the error terms, but also extends the existing method of comparing two regression equations to many equations in order to make comparisons of the successive coefficients to be possible, thus enabling one to detect structural changes, if any. The procedure is then illustrated through detection of structural change, by comparing the successive decadal growth rates of population, using State level data of India.*

JEL Classification: *C18, C51.*

Key words: *Chow test, Dependency of errors, Comparisons of successive coefficients.*

1. INTRODUCTION

It is a common practice to test the equality between sets of coefficients in two linear regressions by Chow Test (Chow 1960) [1]. In the Chow Test, if the null hypothesis of equality between the sets of coefficients is not rejected then there is no problem (as in the examples in his paper). But if rejected, then, naturally, one is probed to the questions: a). at which component/s the sets differ, and b). for each of those components, between the two coefficients of the two regressions concerned, which one is larger/smaller. Chow test does not provide answer to any of these questions. This problem can be resolved with some modifications of the model (Saha and Pal 2014) [2]. Saha and Pal introduced the concept of "component wise complete comparison" (CCC)[1] in order to overcome this problem. The test procedure for CCC between every two successive regressions out of any number of given successive regressions was developed. Also, however, Chow assumed independence of the regression errors. If the regression errors are dependent then the estimates may not be efficient. The presence of dependent errors will affect level of significance as well as power of the test of the regression coefficients and the test may result into wrong conclusion especially when the sizes of the samples

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are small. If errors are dependent then the problem of CCC aggravates and needs further modifications. The present paper extends the earlier paper by incorporating dependency of errors in the model and developing test procedure for CCC between every two successive regressions out of any number of given successive regressions and thus enabling one to detect structural changes, if any. This is the purpose of the paper.

Successive regressions arise when the regression equations are ordered. E.g., in time series data we may be interested to know whether the regression coefficient changes over time. In such a case each time point will have a regression equation and one may test the changes in the coefficients of the regressions between successive points of time.

Quandt (1972) [3] considered the problem of discontinuous shifts in regression regimes. But he assumed that nature chooses between regimes with probabilities p and 1 – p, $0 \le p \le 1$. Thus it is not known apriori whether an observation falls in regime 1 or regime 2. In other words, it is some sort of a classification problem. Though he chose only two regimes, it is possible it to extend it to more than two regimes. Our formulation here is completely different. We assume that the change is fixed from one regime to the next regime, but it may be different from one regime to the other and the observations fall in one of the two regimes known apriori.

Successive regressions arise not only in time series data, but also in spatially distributed data. Spatially dependent error models are introduced in spatial regression analysis in which the errors are assumed to be spatially correlated. Many specification tests for spatially dependent error models are available in the literature (Cliff and Ord 1981 [4], Burridge 1980 [5], Anselin 1988 [6] and Anselin et al. 1996 [7]). But in most of the cases the test statistics had degrees of freedom only 1.

We may now straight go to the problem and discuss how we can arrive at a solution.

2. The Model: We consider the problem of finding test procedure for CCC between every two successive regressions out of m given successive regressions:

$$y^{(1)} = a_1^{(1)} + a_2^{(1)} x_2^{(1)} + a_3^{(1)} x_3^{(1)} + \ldots.. + a_k^{(1)} x_k^{(1)} + u^{(1)},$$

$$y^{(2)} = a_1^{(2)} + a_2^{(2)} x_2^{(2)} + a_3^{(2)} x_3^{(2)} + \ldots.. + a_k^{(2)} x_k^{(2)} + u^{(2)},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$y^{(m)} = a_1^{(m)} + a_2^{(m)} x_2^{(m)} + a_3^{(m)} x_3^{(m)} + \ldots.. + a_k^{(m)} x_k^{(m)} + u^{(m)}, 1 \qquad (1)$$

the superscripts denoting the individual regressions and $n_1, n_2, \ldots, n_m$ being the nos. of observations for these regressions, when the errors are dependent. In order to introduce dependency of the errors we consider the Model consisting of the following assumptions:

i) $E(u^{(i)}) = 0_{nx1}$, $\forall$ i = 1, 2, ………, m,

ii) $E((u^{(i)})(u^{(i)})') = \sigma^2 I_{nxn}$, $\forall$ i = 1, 2, ………, m,

iii) $E((u^{(i)})(u^{(j)})') = \sigma_{ij} I_{nxn}$, $\forall$ i ≠ j = 1, 2, ………, m,

iv) $\Sigma_{mxm} = (\sigma_{ij})_{mxm}$, say, is a positive definite matrix, (2)

where, $\sigma_{ii} = \sigma^2$, i = 1, ……, m, and, $n_i = n$, $\forall$i = 1, 2, ………, m,

(i.e., the sample sizes for the different regressions are the same, say, n).

It is admitted that a particular type of dependency has been considered. Observe that the model is similar to that adopted in the Zellner's (1962) [8] SURE Estimation Procedure (ZSEP), and the solution here is, also, similar to that of Zellner's.

In order to utilize the Model on dependency of the error terms just introduced, i.e., Model (2), we combine the above m regressions into a single regression equation model as follows:

$$\begin{pmatrix} y_1^{(1)} \\ \vdots \\ y_n^{(1)} \\ y_1^{(2)} \\ \vdots \\ y_n^{(2)} \\ \vdots \\ y_1^{(m)} \\ \vdots \\ y_n^{(m)} \end{pmatrix} = \begin{pmatrix} 1 & x_{21}^{(1)} & \cdots & x_{k1}^{(1)} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{2,n}^{(1)} & \cdots & x_{k,n}^{(1)} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & x_{21}^{(2)} & \cdots & x_{k1}^{(2)} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & x_{2,n}^{(2)} & \cdots & x_{k,n}^{(2)} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 1 & x_{21}^{(m)} & \cdots & x_{k1}^{(m)} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 1 & x_{2,n}^{(m)} & \cdots & x_{k,n}^{(m)} \end{pmatrix} \begin{pmatrix} a_1^{(1)} \\ \vdots \\ a_k^{(1)} \\ a_1^{(2)} \\ \vdots \\ a_k^{(2)} \\ \vdots \\ a_k^{(m)} \\ \vdots \\ a_k^{(m)} \end{pmatrix} + \begin{pmatrix} u_1^{(1)} \\ \vdots \\ u_n^{(1)} \\ u_1^{(2)} \\ \vdots \\ u_n^{(2)} \\ \vdots \\ u_1^{(m)} \\ \vdots \\ u_n^{(m)} \end{pmatrix} \quad (3)$$

The solution for the single equation model is same as that of finding solution separately for each equation for the m equations model. The benefit of writing a single equation model is that we can now utilize the Model on the dependency of the error terms easily. In addition to introducing dependencies of the error terms we want to compare $a_j^{(i)}$ with $a_j^{(i+1)}$, for all j = 1, 2, ..., k and i = 1, 2, ..., m-1. That is also possible if we slightly change the model further.

Notice that the above model does not have an intercept term. We may now introduce the intercept term in (3) and rewrite (3) as follows:

$$\begin{pmatrix} y_1^{(1)} \\ \vdots \\ y_n^{(1)} \\ y_1^{(2)} \\ \vdots \\ y_n^{(2)} \\ \vdots \\ y_1^{(m)} \\ \vdots \\ y_n^{(m)} \end{pmatrix} = \begin{pmatrix} 1 & x_{21}^{(1)} & \cdots & x_{k1}^{(1)} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{2,n}^{(1)} & \cdots & x_{k,n}^{(1)} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 1 & x_{21}^{(2)} & \cdots & x_{k1}^{(2)} & 1 & x_{21}^{(2)} & \cdots & x_{k1}^{(2)} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{2,n}^{(2)} & \cdots & x_{k,n}^{(2)} & 1 & x_{2,n}^{(2)} & \cdots & x_{k,n}^{(2)} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{21}^{(m)} & \cdots & x_{k1}^{(m)} & 0 & 0 & \cdots & 0 & \cdots & 1 & x_{21}^{(m)} & \cdots & x_{k1}^{(m)} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & x_{2,n}^{(m)} & \cdots & x_{k,n}^{(m)} & 0 & 0 & \cdots & 0 & \cdots & 1 & x_{2,n}^{(m)} & \cdots & x_{k,n}^{(m)} \end{pmatrix} \begin{pmatrix} c_{11} \\ \vdots \\ c_{1k} \\ c_{21} \\ \vdots \\ c_{2k} \\ \vdots \\ c_{m1} \\ \vdots \\ c_{mk} \end{pmatrix} + \begin{pmatrix} u_1^{(1)} \\ \vdots \\ u_n^{(1)} \\ u_1^{(2)} \\ \vdots \\ u_n^{(2)} \\ \vdots \\ u_1^{(m)} \\ \vdots \\ u_n^{(m)} \end{pmatrix} \quad (4)$$

c_{11} is the intercept term in (4). This is same as $a_1^{(1)}$, the intercept term in the first regression equation in (1). Similarly, c_{12}, c_{13}, ..., c_{1k}, are also same as $a_2^{(1)}$, $a_3^{(1)}$, ..., $a_k^{(1)}$. The regression coefficients c_{21}, c_{22}, …, c_{2k} are the changes in the intercept term and the coefficients of other variables in the second equation from the corresponding values of the first equation. c_{31}, c_{32}, …, c_{3k} are the changes in the intercept term and the coefficients of other variables in the third equation from the corresponding values of the second equation, and so on. We can thus write $c_{1j} = a_j^{(1)}$, for all j = 1, 2, 3, ….., k and $c_{ij} = a_j^{(i)} – a_j^{(i-1)}$, for all j = 1, 2, 3, ….., k and for all i = 2, 3, ….., m.

Let us, for convenience, rewrite (4) as:

$$Y = X\, c + U, \quad (5)$$

where, Y_{Nx1} = the Y-vector in (4), X_{NxK} = the X-matrix in (4), c_{Kx1} = the coefficient-vector in (4) and U_{Nx1} = the disturbance-vector in (4), N = nm and K = km.

We can now estimate c as well as perform test for H_0: $c_{ij} = 0$ vs. H_A: $c_{ij} \neq 0$ or H_A: $c_{ij} < 0$ or H_A: $c_{ij} > 0$, for all j = 1, 2, 3, ….., k and for all i = 2, 3, ….., m, i.e., perform CCC between every two successive regressions in m-regression equation model, since $c_{ij} = a_j^{(i)} – a_j^{(i-1)}$. In fact, any of the coefficients c_{21}, c_{22}, …, c_{2k}, c_{31}, c_{32}, …, c_{3k}, ….., c_{mk}, or any combination of these coefficients can be tested. It thus can be seen as a generalization of Chow test in two directions, because we assumed that the errors are dependent.

3. The Methodology: For Model (5), the variance covariance matrix of the regression error is given as:

$$(D(\mathrm{U}))_{N\times N} = V, say, = \begin{pmatrix} \sigma^2 I_{n\times m} & \sigma_{12} I_{n\times n} & \cdots & \cdots & \cdots & \sigma_{1m} I_{n\times n} \\ \sigma_{21} I_{n\times n} & \sigma^2 I_{n\times n} & \cdots & \cdots & \cdots & \sigma_{2m} I_{n\times n} \\ \vdots & & & & & \\ \vdots & & & & & \\ \vdots & & & & & \\ \sigma_{m1} I_{n\times n} & \sigma_{m2} I_{n\times n} & \cdots & \cdots & \cdots & \sigma^2 I_{n\times n} \end{pmatrix}, \quad (6)$$

or, $V = \Sigma_{m\times m} \otimes I_{n\times n}$,

where, Σ_{mxm} is:

$$\Sigma_{m\times m} = \begin{pmatrix} \sigma^2 & \sigma_{12} & \cdots & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma^2 & \cdots & \cdots & \sigma_{2m} \\ \cdots & & & & \\ \cdots & & & & \\ \sigma_{m1} & \sigma_{m2} & & & \sigma^2 \end{pmatrix}$$

So the model (5) is a Generalised Least Squares Model (GLSM) [9] with the variance covariance matrix of the regression error as given by (6). The GLS estimator of c based on (5) is given as:

$$c^* = (X' V^{-1}X)^{-1}X' V^{-1}Y, \quad (7)$$

and its dispersion matrix is:

$$D(c^*) = (X' V^{-1}X)^{-1}. \quad (8)$$

But due to (6), V is unknown as Σ_{mxm} is so. So it is not possible to use (7) and (8) in practice, particularly for the testing purposes we are aimed at. Henceforth let us proceed following Zellner [8].

Firstly, we need to estimate V. For that we need to estimate Σ_{mxm} and that is done as follows. The steps are:

i) Apply OLS separately to each of the regressions in (1); let the residual vector for the i-th regression be denoted as e^i, for all i = 1, 2, …, m,

ii) Estimate σ^2 as: $s^2 = [\,(e^{1\prime}e^1) + \ldots + (e^{m\prime}e^m)\,]/(m(n-k))$.

iii) Estimate σ_{ij} as: $s_{ij} = (e^{i\prime}e^j)/(n-k), \forall\, i \neq j = 1, 2, \ldots, m$.

Then, estimated Σ_{mxm}, say, S_{mxm}, is: $S_{mxm} = (s_{ij})_{mxm}$, where $s_{ii} = s^2$, for all i = 1, 2, ..., m. Then, V is estimated as:

$$\hat{V} = S_{mxm} \otimes I_{nxn} \tag{9}$$

Now we replace V in (7) by $\hat{V}$ as given by (9) and form the estimator:

$$c^{**} = (X' (\hat{V})^{-1}X)^{-1}X' (\hat{V})^{-1}Y. \tag{10}$$

Then it follows that $(n^{1/2})$ $(c^{**} - c)$ has asymptotic normal distribution and the dispersion matrix of c^{**} is:

$$D(c^{**}) = (X' (\hat{V})^{-1}X)^{-1} + {}_0(n^{-1}),$$

where ${}_0(n^{-1})$ denotes terms of high order of smallness than n^{-1}.

So, for large value of n, c^{**} is normally distributed. Also, evidently, for large n, ${}_0(n^{-1})$ is negligible and then,

$$D(c^{**}) \simeq (X' (\hat{V})^{-1}X)^{-1}.$$

So, for large n, we have:

$$c^{**}\ N_K(c_{Kx1}, (X' (\hat{V})^{-1}X)^{-1}). \tag{11}$$

Now, the tests that we require are obvious, provided that n is sufficiently large which we assume for rest of the paper. Representing,

$$c^{**} = (c^{**}_1\ c^{**}_2\ c^{**}_3 \ldots\ldots c^{**}_K)',$$

$$c = (c_1\ c_2\ c_3 \ldots\ldots c_K)', \text{ and}$$

$$(X' (\hat{V})^{-1}X)^{-1} = (a_{ij})_{KxK,}$$

we have: $(c^{**}_i - c_i) / (a_{ii})^{1/2} \sim N(0, 1)$, for all i = 1, 2, 3, …… , K.

Hence for the null hypothesis: $H_0 : c_i = 0$,

the test statistic is: $T = c^{**}_i / (a_{ii})^{1/2}$, and (12)

$T \sim N(0, 1)$, under H_0, for all i = 1, 2, 3, … , K.

Needless to say that each of the tests here is a normal test.

4. Illustration: In the context of rate of growth of population in India, we consider three regression equations as follows (m=3). With State level population of India, we first define the following four variables:

X_1 = size of the population in a State of India in 1981,

X_2 = size of the population in a State of India in 1991,

X_3 = size of the population in a State of India in 2001,

X_4 = size of the population in a State of India in 2011, the sources of the data being Census of India: 1981, 1991, 2001, 2011 [10].

Let us now define variables $Y_{1,}$ $Y_{2,}$ Y_3 as follows:

$Y_1 = X_2 - X_1$ (i.e., growth/increase of population during: 1981 to 1991),

$Y_2 = X_3 - X_2$ (i.e., growth/increase of population during: 1991 to 2001),

$Y_3 = X_4 - X_3$ (i.e., growth/increase of population during: 2001 to 2011).

Then, the regressions considered are as follows:

$$\left.\begin{array}{l} Y_1 = \beta_1 X_1 + U_1 \\ Y_2 = \beta_2 X_2 + U_2 \\ Y_3 = \beta_3 X_3 + U_3 \end{array}\right] \tag{13}$$

β_1, β_2 and β_3 are nothing but the rates of growth of population over the decades: 1981 to 1991, 1991 to 2001 and 2001 to 2011 respectively (to be referred afterwards as, respectively, first decade, second decade and so on). Our task is to perform CCC between every two successive regressions out these three regressions with a view to detect structural changes, if any.

Now, following the Methodology described above, we apply OLS separately to each of the above three regressions in (13). (It may be noted that each of these regressions is a regression without an intercept term.)

(The no. of observations for each regression is n = 32 (no. of States in India). So, we have: n = 32, k = 1, m = 3, and hence: N = nm = 96, K = km = 3.)

The Residual vectors of these three regressions are first obtained. It is now a routine calculation to get the sum of squares and the sum of products of the residual vectors and hence the estimates of σ^2 and σ_{ij} (s^2 and s_{ij}) using the formula as given already. We then use the following steps to get the estimate c** as given by (10), the variance covariance matrix of c**, i.e., $(X'(\hat{V})^{-1}X)^{-1}$), as given by (11), and thence value of the test statistic T as given by (12). It should be noted that the first component of c** gives the estimate of the growth rate in the first decade (β_1), and the second and the third components of c** give respectively the estimates of changes in the growth rates over first decade to second one ($\beta_2 - \beta_1$) and over second decade to third one ($\beta_3 - \beta_2$).

1. Construct the matrix S_{3x3} and compute $(S_{3x3})^{-1}$,
2. Compute $(\hat{V})^{-1}_{96x96} = S_{3x3}^{-1} \otimes I_{32x32}$,
3. Compute the matrix: $(a_{ij})_{3x3} = (X'(\hat{V})^{-1} X)^{-1}$,
4. Compute c** as: $c^{**}_{3x1} = (X'(\hat{V})^{-1} X)^{-1} X'(\hat{V})^{-1}Y$,

5. Compute the test statistic $T = c^{**}_i / (a_{ii})^{1/2}$.

Now, it comes out that: $c^{**}_{3x1} = (0.231\ -0.107\ 0.057)'$, and

$T = -3.857$, for $H_0 : \beta_2 - \beta_1 = 0$, and

$T = 2.412$, for $H_0 : \beta_3 - \beta_2 = 0$.

Hence, the estimate of the growth rate in the first decade is 0.231 and the estimates of the changes concerned are respectively -0.107 and 0.057. Now, comparing the test statistic T-values with table values, we get that $\beta_2 < \beta_1$ and $\beta_2 < \beta_3$[2]. These evidently indicate that there are structural changes twice , the first one being negative and the second one being positive signifying that there is a decline in growth rate as one moves from the first decade to the second decade while there is a rise when one moves from the second decade to the third one.

Observe that we treated all the states equally. But we should have given weights proportional to the population of the states respectively.

5. CONCLUSIONS

Our procedure extends the existing method from comparing two regression equations to many equations making comparisons of the successive coefficients to be possible, enabling one to detect structural changes, if any, and from the assumption of the independency of errors to the dependency of errors. This obviously can be seen as a generalization of Chow test in two directions.

Firstly, we can compare whether any two coefficients are equal against the alternative hypotheses of inequality of any direction i.e., ‘<’ or ‘>’, instead of only ‘≠’. This can further be extended to vector of regression coefficients with similar alternative hypotheses for each component of vector.

Secondly, our procedure enables one to perform component wise complete comparison between the vectors of coefficients of every two successive regressions out of several given successive regressions. Now, one of the important implications of this is as follows. Suppose each one of the given regressions pertains to a time period/point and the regressions are arranged in increasing order of time and the investigator is in search of a) existence of structural breakthrough and b) detection of the point/s (here, by a point we mean a time period or a time point) where it occurs, if there is any such at all. Not only the point/s of structural breakthrough, if there is any at all, through our procedure we get something more. For every such point we get component wise complete comparison, or, in other words, component wise complete picture, so to say, of the vectors of coefficients of the two regressions associated with that point. Actually, it is not necessary that the regressions need to be ordered in increasing/decreasing order of time; it is sufficient for the regressions to be ordered in a well defined sense, e.g., (i) in order of space, i.e., regressions pertain to some states of India arranged from North to South, (ii) in increasing order of income, i.e., regressions pertain to some groups of peoples arranged in increasing order of income, etc. It seems that the concept of "Structural Change" can be extended, not pertaining to only "order of time" but pertaining to any well defined order in which the regressions can be meaningfully arranged.

Thirdly, consider the test provided by Gujarati (Gujarati 1970) [11], called Generalised Dummy Variable Approach, in order to find out whether a given set of regressions differ from one another. A moment's reflection shows that the purpose of this test is also served by our test simply because if we arrange these regressions successively (with or without any definite meaning) then we can say that these regressions do not differ from one another iff the two vectors of coefficients of every two successive regressions coincide with each other and this is easily verifiable by our procedure. But, needless to say, the objective of this paper, i.e., developing test procedure for CCC between every two successive regressions out of any number of given successive regressions, is not served by the test due to Gujarati.

Notes

1. By complete comparison between any two parameters a and b we mean to decide whether $a < b$ or $a = b$ or $a > b$. By component wise complete comparison (CCC) between two vectors of parameters of the same size ($a_1\ a_2 \dots a_m$) and ($b_1\ b_2 \dots b_m$) we mean complete comparison between (a_1 and b_1), (a_2 and b_2), ... and (a_m and b_m). By CCC between/of/for two regressions with same no. of parameters we mean CCC between the two vectors of parameters of these regressions. In the paper by Saha and Pal, CCC is done between every two successive regressions out of any number of given successive regressions with same no. of parameters.
2. All tests are done at 5% level of significance.

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