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IRROTATIONAL τ-CURVATURE TENSOR IN *S*-MANIFOLDS

K.R. Vidyavathi* and C.S. Bagewadi**

Abstract

In the present paper, we study the irrotational τ-curvature tensor in s-manifolds, where τ-curvature tensor is a generalization of quasi-conformal, conformal, conharmonic, concircular, pseudo-projective, projective, M-projective, W_0, W_0^*, W_1, W_1^*, W_2, W_3, W_4, W_5, W_6, W_7, W_8, W_9 curvature tensors.

Keywords: *S-manifold, η-Einstein manifold, Einstein manifold, Ricci soliton.*

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1. INTRODUCTION

The notion of f-structure on a $(2n+s)$-dimensional manifold M, i.e., a tensor field of type $(1,1)$ on M of rank $2n$ satisfying $f^3 + f = 0$, was firstly introduced in 1963 by K. Yano [13] as a generalization of both (almost) contact (for s = 1) and (almost) complex structures (for s = 0). During the subsequent years, this notion has been furtherly developed by several authors [4], [5], [7], [8], [9], [10], [11]. Among them, H. Nakagawa in [10] and [11] introduced the notion of framed f-manifold, later developed and studied by S.I. Goldberg and K. Yano ([7], [8]) and others with the denomination of globally framed f-manifolds.

The authors C.S.Bagewadi and N.B.Gatti [1], [6], C.S.Bagewadi, E.Gireesh Kumar and Venkatesha [2] have studied irrotational projective curvature, quasi-conformal curvature tensor and D-conformal curvature tensor in K-Contact, Kenmotsu and Trans-Sasakian manifolds and they have shown that these manifolds are Einstein and also studied some properties like flatness and space of

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* Department of Mathematics, Kuvempu University, Shankaraghatta - 577 451, Shimoga, Karnataka, India. *Email: vidyarsajjan@gmail.com*

** Department of Mathematics, Kuvempu University, Shankaraghatta - 577 451, Shimoga, Karnataka, India. *Email: prof_bagewadi@yahoo.co.in*

constant curvature. Further the authors C.S.Bagewadi, Gurupadavva Ingalahalli and K.T.Pradeepkumar [3] extended the notion to C-Bochner curvature tensor for K-Contact and Kenmotsu manifolds and they have proved that these manifolds are η-Einstein.

Motivated by the above work, we study the irrotational τ-curvature tensor in s-manifolds and we derive the result for the particular cases of τ. Further we discuss about Ricci soliton.

2. PRELIMINARIES

Let M be a $(2n+s)$-dimensional manifold with an f-structure of rank $2n$. If there exists global vector fields $\xi_\alpha, \alpha = (1,2,3,......,s)$ on M such that;

$$f^2 = -I + \sum \xi_\alpha \otimes \eta_\alpha, \ \ \eta_\alpha(\xi_\beta) = \delta^\alpha_\beta, \tag{2.1}$$

$$f\xi_\alpha = 0, \ \ \eta_\alpha \circ f = 0, \tag{2.2}$$

$$g(X, \xi_\alpha) = \eta_\alpha(X), \ \ g(X, fY) = -g(fX, Y), \tag{2.3}$$

where, η_α are the dual 1-forms of ξ_α, we say that the f-structure has complemented frames. For such a manifold there exists a Riemannian metric g such that

$$g(X,Y) = g(fX, fY) + \sum \eta_\alpha(X)\eta_\alpha(Y) \tag{2.4}$$

for any vector fields X and Y on M.

An f-structure f is normal, if it has complemented frames and

$$[f, f] + 2\sum \xi_\alpha \otimes d\eta_\alpha = 0,$$

where, $[f, f]$ is Nijenhuis torsion of f.

Let F be the fundamental 2-form defined by $F(X,Y) = g(X, fY)$, $X, Y \in T(M)$. A normal f-structure for which the fundamental form F is closed, $\eta_1 \wedge, ..., \eta_s \wedge (d\eta_\alpha)^n \neq 0$ for any α, and $d\eta_1 = ... = d\eta_s = F$ is called to be an s-structure. A smooth manifold endowed with an s-stucture will be called an s-manifold. These manifolds introduced by Blair [4].

We have to remark that if we take $s=1$, s-manifolds are natural generalizations of Sasakian manifolds. In the case $s\geq 2$ some interesting examples are given.

If M is an S-manifold, then the following relations holds true;

$$\nabla_X \xi_\alpha = -fX, \quad X\in T(M), \alpha=1,2,....,s \tag{2.5}$$

$$(\nabla_X \eta)(Y) = -g(fX,Y), \tag{2.6}$$

$$(\nabla_X f)Y = \sum\{g(fX,fY)\xi_\alpha + \eta_\alpha(Y) f^2 X\}, \; X,Y\in T(M), \tag{2.7}$$

where, ∇ is the Riemannian connection of g. Let Ω be the distribution determined by the projection tensor- f^2 and let N be the complementry distribution which is determined by f^2+I and spanned by $\xi_1,.....\xi_s$. It is clear that if $X\in\Omega$ then $\eta_\alpha(X)=0$ for any α, and if $X\in N$, then fX =0. A plane section π on M is called an invariant f-section if it is determined by a vector $X\in\Omega(x), x\in M$, such that $\{X, fX\}$ is an orthonormal pair spanning the section. The sectional curvature of π is called the f-sectional curvature. If M is an s-manifold of constant f-sectional curvature k, then its curvature tensor has the form

$$\begin{aligned} R(X,Y,Z,W) = \sum_{\alpha,\beta}\{ & g(fX,fW)\eta_\alpha(Y)\eta_\beta(Z) - g(fX,fZ)\eta_\alpha(Y)\eta_\beta(W) \\ & + g(fY,fZ)\eta_\alpha(X)\eta_\beta(W) - g(fY,fW)\eta_\alpha(X)\eta_\beta(Z)\} \\ & + \frac{1}{4}(k+3s)\{g(fX,fW)g(fY,fZ) - g(fX,fZ)g(fY,fW)\} \\ & + \frac{1}{4}(k-s)\{F(X,W)F(Y,Z) - F(X,Z)F(Y,W) \\ & - 2F(X,Y)F(Z,W)\}, \end{aligned} \tag{2.8}$$

where, $X,Y,Z,W\in T(M)$. Such a manifold $N(K)$ will be called an *s*-space form. The Euclidean space E^{2n+s} and the hiperbolic space H^{2n+s} are examples of *s*-space forms.

Definition 1: *s-manifold $(M, f, \eta_\alpha, g, \xi_\alpha)$ is said to be η-Einstein if the Ricci tensor S of M is of the form*

$$S = ag + b\sum_{\alpha=1}^{s} \eta_\alpha \otimes \eta_\alpha,$$

where, a, b are constants on M.

Now contracting equation (2.8) we get

$$S(Y,Z) = b_1 g(Y,Z) + b_2 \eta_\alpha(Y)\eta_\alpha(Z), \tag{2.9}$$

$$S(Y,\xi_\alpha) = b_3 \eta_\alpha(Y). \tag{2.10}$$

where,

$$b_1 = \left[\frac{4s + (k+3s)(2n-1) + 3(k-s)}{4}\right],$$

$$b_2 = \left[\frac{(2n+s-2)(4-k-3s) - 3(k-s)}{4}\right],$$

$$b_3 = \left[\frac{s^2(13-6n-k-3s) + 2s(7n-5) + k(2-s) + 2nk(1-s)}{4}\right].$$

From (2.8) we have

$$R(X,Y)\xi_\alpha = s\sum_\alpha \{\eta_\alpha(Y)X - \eta_\alpha(X)Y\}, \tag{2.11}$$

$$R(\xi_\alpha, Y)Z = s\sum_\alpha \{g(Y,Z)\xi_\alpha - \eta_\alpha(Z)Y\}, \tag{2.12}$$

$$\eta_\alpha(R(X,Y)Z) = s\sum_\alpha \{g(Y,Z)\eta_\alpha(X) - g(X,Z)\eta_\alpha(Y)\}. \tag{2.13}$$

In a $(2n+1)$-dimensional Riemannian manifold M, the τ-curvature tensor [12] is given by

$$\begin{aligned}\tau(X,Y)Z = {} & a_0 R(X,Y)Z + a_1 S(Y,Z)X + a_2 S(X,Z)Y + a_3 S(X,Y)Z \\ & + a_4 g(Y,Z)QX + a_5 g(X,Z)QY + a_6 g(X,Y)QZ \\ & + a_7 r[g(Y,Z)X - g(X,Z)Y]. \end{aligned} \tag{2.14}$$

where, R, S, Q and r are the curvature tensor, the Ricci tensor, the Ricci operator and the scalar curvature, respectively.

In particular, the τ -curvature tensor is reduced to be

1. the quasi-conformal curvature tensor C_* if

$$a_1 = -a_2 = a_4 = -a_5; a_3 = a_6 = 0; a7 = -\frac{1}{2n+1}\left(\frac{a_0}{2n} + 2a_1\right);$$

2. the conformal curvature tensor C if

$$a_0 = 1; a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{2n-1}; a_3 = a_6 = 0; a_7 = \frac{1}{2n(2n-1)};$$

3. the conharmonic curvature tensor L if

$$a_0 = 1; a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{2n-1}; a_3 = a_6 = 0; a_7 = 0;$$

4. the concircular curvature tensor V if

$$a_0 = 1; a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = 0; a_7 = \frac{-1}{2n(2n+1)};$$

5. the pseudo-projective curvature tensor P_* if

$$a_1 = -a_2; a_3 = a_4 = a_5 = a_6 = 0; a_7 = -\frac{1}{2n+1}\left(\frac{a_0}{2n} + a_1\right);$$

6. the projective curvature tensor P if

$$a_0 = 1; a_1 = -a_2 = -\frac{1}{2n}; a_3 = a_4 = a_5 = a_6 = 0 = a_7 = 0;$$

7. the M -projective curvature tensor if

$$a_0 = 1; a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{4n}; a_3 = a_6 = a_7 = 0;$$

8. the W_0 -curvature tensor if

$$a_0 = 1; a_1 = -a_5 = -\frac{1}{2n}; a_2 = a_3 = a_4 = a_6 = a_7 = 0;$$

9. the W_0^*-curvature tensor if

$$a_0 = 1; a_1 = -a_5 = \frac{1}{2n}; a_2 = a_3 = a_4 = a_6 = a_7 = 0;$$

10. the W_1-curvature tensor if

$$a_0 = 1; a_1 = -a_2 = \frac{1}{2n}; a_3 = a_4 = a_5 = a_6 = a_7 = 0;$$

11. the W_1^*-curvature tensor if

$$a_0 = 1; a_1 = -a_2 = -\frac{1}{2n}; a_3 = a_4 = a_5 = a_6 = a_7 = 0;$$

12. the W_2-curvature tensor if

$$a_0 = 1; a_4 = -a_5 = -\frac{1}{2n}; a_1 = a_2 = a_3 = a_6 = a_7 = 0;$$

13. the W_3-curvature tensor if

$$a_0 = 1; a_2 = -a_4 = -\frac{1}{2n}; a_1 = a_3 = a_5 = a_6 = a_7 = 0;$$

14. the W_4-curvature tensor if

$$a_0 = 1; a_5 = -a_6 = \frac{1}{2n}; a_1 = a_2 = a_3 = a_4 = a_7 = 0;$$

15. the W_5-curvature tensor if

$$a_0 = 1; a_2 = -a_5 = -\frac{1}{2n}; a_1 = a_3 = a_4 = a_6 = a_7 = 0;$$

16. the W_6-curvature tensor if

$$a_0 = 1; a_1 = -a_6 = -\frac{1}{2n}; a_2 = a_3 = a_4 = a_5 = a_7 = 0;$$

17. the W_7-curvature tensor if

$$a_0 = 1; a_1 = -a_4 = -\frac{1}{2n}; a_2 = a_3 = a_5 = a_6 = a_7 = 0;$$

18. the W_8 -curvature tensor if

$$a_0 = 1; a_1 = -a_3 = -\frac{1}{2n}; a_2 = a_4 = a_5 = a_6 = a_7 = 0;$$

19. the W_9 -curvature tensor if

$$a_0 = 1; a_3 = -a_4 = \frac{1}{2n}; a_1 = a_2 = a_5 = a_6 = a_7 = 0;$$

Definition 2: *The rotation (curl) of τ -curvature tensor on a Riemannian manifold is given by*

$$Rot\tau = (\nabla_U \tau)(X, Y, Z) + (\nabla_X \tau)(U, Y, Z) + (\nabla_Y \tau)(U, X, Z)$$
$$-(\nabla_Z \tau)(X, Y, U). \tag{2.15}$$

By virtue of second Bianchi identity

$$(\nabla_U \tau)(X, Y, Z) + (\nabla_X \tau)(U, Y, Z) + (\nabla_Y \tau)(U, X, Z) = 0. \tag{2.16}$$

Equation (2.15) reduces to

$$curl\tau = -(\nabla_Z \tau)(X, Y, U). \tag{2.17}$$

If the τ -curvature tensor is irrotational then $curl\,\tau = 0$ and by (2.17) we have

$$(\nabla_Z \tau)(X, Y)U = 0. \tag{2.18}$$

Which implies,

$$\nabla_Z \{\tau(X, Y)U\} = \tau(\nabla_Z X, Y)U + \tau(X, \nabla_Z Y)U + \tau(X, Y)\nabla_Z U. \tag{2.19}$$

Put $U = \xi$ in the above equation, we have

$$\nabla_Z \{\tau(X, Y)\xi\} = \tau(\nabla_Z X, Y)\xi + \tau(X, \nabla_Z Y)\xi + \tau(X, Y)\nabla_Z \xi. \tag{2.20}$$

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Put $Z = \xi$ in (2.14) and using (2.9), (2.10) and (2.11) we get,

$$\tau(X, Y)\xi = k_1\eta(Y)X + k_2\eta(X)Y + k_3 g(X, Y)\xi + k_4\eta(X)\eta(Y)\xi, \tag{3.1}$$

where, $k_1 = a_0 s + a_1 b_3 + a_4 b_3 + a_7 r$, $k_2 = -a_0 s + a_2 b_3 + a_5 b_3 - a_7 r$,

$k_3 = a_3 b_1 + a_6 b_3$, $\qquad k_4 = a_3 b_2$

Theorem 1: *If the τ-curvature tensor in s-manifold is irrotational, then the manifold is η-Einstein.*

Proof: Using equation (3.1) in (2.20) we get

$$-\tau(X,Y)fZ = k_1(\nabla_Z\eta)(Y)X + k_2(\nabla_Z\eta)(X)Y + k_3 g(X,Y)(-fX)$$
$$+k_4\{(\nabla_Z\eta)(X)\eta(Y)\xi + (\nabla_Z\eta)(Y)\eta(X)\xi - \eta(X)\eta(Y)\xi\}. \quad (3.2)$$

By virtue of (2.5) in (3.2) we have

$$-\tau(X,Y)fZ = -k_1 g(fZ,Y)X - k_2 g(fZ,X)Y - k_3 g(X,Y)fZ$$
$$+k_4\{-g(fZ,X)\eta(Y)\xi - g(fZ,Y)\eta(X)\xi - \eta(X)\eta(Y)fZ\} \quad (3.3)$$

Replace Z by fZ in (3.3) and using (2.1) we have

$$\tau(X,Y)Z = k_1 g(Y,Z)X + k_2 g(X,Z)Y + k_3 g(X,Y)Z$$
$$+k_4\{g(X,Z)\eta(Y)\xi + g(Y,Z)\eta(X)\xi + \eta(X)\eta(Y)Z\} \quad (3.4)$$

Using (2.14) and (3.4) we can write

$$a_0 R(X,Y,Z,W) = k_1 g(Y,Z)g(X,W) + k_2 g(X,Z)g(Y,W)$$
$$+k_3 g(X,Y)g(Z,W) + k_4\{g(X,Z)\eta(Y)\eta(W) + g(Y,Z)\eta(X)\eta(W)$$
$$+\eta(X)\eta(Y)g(Z,W)\} - a_1 S(Y,Z)g(X,W) - a_2 S(X,Z)g(Y,W)$$
$$-a_3 S(X,Y)g(Z,W) - a_4 g(Y,Z)g(QX,W) - a_5 g(X,Z)g(QY,W)$$
$$-a_6 g(X,Y)g(QZ,W) + a_7 r[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)]. \quad (3.5)$$

Let $e_i, i = 1,2,........(2n+s)$ be an orthonormal basis of the tangent space. Then summing for $1 \le i \le (2n+s)$ of the relation (3.5) with $X = W = e_i$ yields the Ricci tensor S is given by

$$S(Y,Z) = Ag(X,Y) + B\eta_\alpha(X)\eta_\alpha(Y). \quad (3.6)$$

where, $A = \dfrac{(2n+s)k_1 + k_2 + k_3 + k_4 - (2n+s-1)ra_7 - ra_4}{a_0 + (2n+s)a_1 + a_2 + a_3 + a_5 + a_6}$,

$$B = \frac{2k_4}{a_0 + (2n+s)a_1 + a_2 + a_3 + a_5 + a_6}.$$

The above theorem 3.1 is shown in tabular form for different curvatures which can be obtained independently for s manifold.

Curvature tensor	*Manifold*	*Ricci tensor S*
Quasi conformal curvature tensor	Einstein	$S=\left\{\frac{(2n+s-1)(a_0 s+2a_1 b_3)-a_1 r}{a_0+(2n+s-1)a_1}\right\}g$
Conformal curvature tensor	Einstein	$S=\left(\frac{2n-1}{1-s}\right)\left\{(2n+s-1)\left(s-\frac{2b_3}{2n-1}+\frac{r}{2n(2n-1)}\right)+\frac{r}{2n(2n-1)}\right\}g$
Conharmonic curvature tensor	Einstein	$S=\left(\frac{2n-1}{1-s}\right)\left\{(2n+s-1)\left(s-\frac{2b_3}{2n-1}\right)+\frac{r}{2n-1}\right\}g$
Concircular curvature tensor	Einstein	$S=s(2n+s-1)g$
Projective curvature tensor	Einstein	$S=\left\{\frac{2n(2n+s-1)}{(1-s)}\right\}\left(s-\frac{b_3}{2n}\right)g$
Pseudo projective curvature tensor	Einstein	$S=\left\{\frac{(2n+s-1)(a_0 s+a_1 b_3)}{a_0+(2n+s-1)a_1}\right\}g$
M-projective curvature tensor	Einstein	$S=\left(\frac{4n}{2n-s+2}\right)\left\{(2n+s-1)\left(s-\frac{b_3}{2n}\right)+\frac{r}{4n}\right\}g$
W_0 -curvature tensor	Einstein	$S=\left(\frac{2n(2n+s-1)}{(1-s)}\right)\left(s-\frac{b_3}{2n}\right)g$
W_0^* -curvature tensor	Einstein	$S=\left(\frac{2n(2n+s-1)}{(4n+s-1)}\right)\left(s+\frac{b_3}{2n}\right)g$
W_1 -curvature tensor	Einstein	$S=\left(\frac{2n(2n+s-1)}{(4n+s-1)}\right)\left(s+\frac{b_3}{2n}\right)g$
W_1^* -curvature tensor	Einstein	$S=\left(\frac{2n(2n+s-1)}{(1-s)}\right)\left(s-\frac{b_3}{2n}\right)g$
W_2 -curvature tensor	Einstein	$S=\left(\frac{2n(2n+s-1)}{(2n+1)}\right)\left(s-\frac{b_3}{2n}\right)g$
W_3 -curvature tensor	Einstein	$S=\left(\frac{2n}{2n-1}\right)\left\{(2n+s-1)\left(s+\frac{b_3}{2n}\right)-\frac{r}{2n}\right\}g$
W_4 -curvature tensor	Einstein	$S=s(2n+s-1)g$

Curvature tensor	*Manifold*	*Ricci tensor S*
W_5 -curvature tensor	Einstein	$S = s(2n+s-1)g$
W_6 -curvature tensor	Einstein	$S = \left\{\frac{2n(2n+s-1)}{1-s}\right\}\left(s-\frac{b_3}{2n}\right)g$
W_7 -curvature tensor	Einstein	$S = -\left(\frac{2n}{s}\right)\left\{(2n+s)\left(s-\frac{b_3}{2n}\right)-s-\frac{r}{2n}\right\}g$
W_8 -curvature tensor	η-Einstein	$S = \left(\frac{2n}{1-s}\right)\left\{(2n+s)\left(s-\frac{b_3}{2n}\right)-s+\frac{b_1}{2n}-\frac{b_2}{2n}\right\}g + \sum_\alpha\left(\frac{2b_2}{1-s}\right)\eta_\alpha\otimes\eta_\alpha$
W_9 -curvature tensor	η-Einstein	$S = \left(\frac{2n}{2n+1}\right)\left\{(2n+s)\left(s-\frac{b_3}{2n}\right)-s+\frac{b_1}{2n}+\frac{b_2}{2n}+\frac{r}{2n}\right\}g + \sum_\alpha\left(\frac{2b_2}{2n+1}\right)\eta_\alpha\otimes\eta_\alpha$

4. RICCI SOLITON IN IRROTATIONAL τ -CURVATURE TENSOR IN *s*-MANIFOLDS

Definition 3: *A Ricci soliton is a natural generalization of an Einstein metric and is defined on a Riemannian manifold* (M,g). *A Ricci soliton is a triple* (g,V,λ) *with g is a Riemannian metric, V is a vector field and* λ *is a real scalar such that*

$$(L_V g)(X,Y)+2S(X,Y)+2\lambda g(X,Y)=0. \tag{4.1}$$

where, S is a Ricci tensor of M and L_V denotes the Lie derivative operator along the vector field V. The Ricci soliton is said to be shrinking, steady and expanding according as λ is negative, zero and positive respectively.

If V is co-linear with ξ, then Ricci soliton along ξ is given by

$$(L_\xi g)(X,Y)+2S(X,Y)+2\lambda g(X,Y)=0$$

Definition 4: *Let* $(f,\xi_1,\xi_2,.......,\xi_s,\eta_1,\eta_2,.......,\eta_s,g)$ *is the contact* s*-frame manifold, if* V *is in the linear span (combination) of* $\xi_1,\xi_2,......\xi_s$ *then* $V=c_1\xi_1+c_2\xi_2+..........+c_s\xi_s$ *and the Ricci soliton is a triple* (g,ξ_α,λ) *with*

g is a Riemannian metric, $\xi_\alpha,(\alpha=1,2,...s)$ is a vector field and λ is a real scalar such that

$$(\sum_{i=1}^{s} c_i L_{\xi_i} g)(X,Y)+2S(X,Y)+2\lambda g(X,Y)=0.. \tag{4.2}$$

From (4.2) we have

$$c_i g(\nabla_X \xi_\alpha, Y)+c_i g(\nabla_Y \xi_\alpha, X)+2S(X,Y)+2\lambda g(X,Y)=0. \tag{4.3}$$

Using (2.5) in (4.3) we get

$$c_i g(-fX,Y)+c_i g(-fY,X)+2S(X,Y)+2\lambda g(X,Y)=0. \tag{4.4}$$

From (3.6) and (4.4) we have

$$(A+\lambda)g(X,Y)+B\eta_\alpha(X)\eta_\alpha(Y)=0. \tag{4.5}$$

Taking $X=Y=e_i$ in (4.5) and summing over $i=1,2,.......2n+s$, we get the value of λ

$$\lambda=-\left(A+\frac{B}{2n+s}\right) \tag{4.6}$$

thus we state the following theorem

Theorem 2: *The Ricci soliton in irrotational τ -curvature tensor in s manifolds is*

1. shrinking if $A,B>0$
2. steady if if $A,B=0$
3. expanding if if $A,B<0$.

References

[1] C.S. Bagewadi and N.B. Gatti, *On Einstein manifolds-II,* Bull. Cal. Math. Soc., 97(3), pp. 245-252, 2005.

[2] C.S. Bagewadi, E. Gireesh Kumar and Venkatesha, *On irrotational D-Conformal curvature tensor,* Novi Sad J. Math., 35(2), pp. 85-92, 2005.

[3] C.S. Bagewadi, Gurupadavva Ingalahalli and K.T. Pradeep Kumar, *On irrotatinal C-Bochner curvature tensor in K-Contact and Kenmotsu manifolds*, Acta Universitatis Apulensis, v0l. 32, pp. 221-232, 2012.

[4] D.E. Blair, *Geometry of manifolds with structural group* $U(n)\times O(s)$, J. Differential Geom. 4 (1970), 155–167

[5] D.E. Blair, G. Ludden and K. Yano, *Differential geometric structures on principal toroidal bundles,* Trans. Amer. Math. Soc. 181 (1973), 175–184.

[6] N.B. Gatti and C.S. Bagewadi, *On irrotational Quasi-conformal curvature tensor,* Tensor N.S., 64(3), pp. 248-258, 2003.

[7] S.I. Goldberg and K. Yano, *On normal globally framed* f *-manifolds,* Tohoku Math. J. 22 (1970), 362–370.

[8] S.I. Goldberg and K. Yano, *Globally framed* f *-manifolds,* Illinois J. Math. 15 (1971), 456–474.

[9] S. Ishihara, *Normal structure* f *satisfying* $f_3 + f = 0$, Kodai Math. Sem. Rep. 18 (1966), 36–47.

[10] H. Nakagawa, f *-structures induced on submanifolds in spaces, almost Hermitian or Kaehlerian,* Kodai Math. Sem. Rep. 18 (1966), 161–183.

[11] H. Nakagawa, *On framed* f *-manifolds,* Kodai Math. Sem. Rep. 18 (1966), 293–306.

[12] M. M. Tripati and P. Gupta, *On* τ *-curvature tensor in* K *-contact and Sasakian manifolds,* International Electronic Journal of Geometry Volume 4 No. 1 pp. 32-47 (2011).

[13] K. Yano, *On a structure defined by a tensor field* f *of type* $(1,1)$ *satisfying* $f_3 + f = 0$, Tensor N.S. 14 (1963), 99–109.