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CASP FOR TRUNCATED TRANSMUTED WEIBULL DISTRIBUTION BASED ON CUSUM SCHEMES AND ITS OPTIMIZATION

Abstract: *Acceptance Sampling is a form of inspection used to decide either to accept or reject the lots of finished goods. It helps in the improvement of the quality of the product but not to control. In this direction, many optimization techniques are developed under the assumption that variable with regard to the quality characteristic is distributed according to certain probability law. In the present paper, we assume that variable under study follows the Truncated Transmuted Weibull Distribution and its Optimization of CASP-CUSUM Schemes and critical comparison made based on the obtained numerical results.*

Keywords: *CASP - CUSUM Schemes, O C Curve, Optimal Truncated Point, Truncated Transmuted Weibull Distribution*

1. INTRODUCTION

It is not easy to define Quality. Even today there is some ambiguity in describing Quality. It refers to the grade of service, product, reliability, safety, consistency and consumer's perception. Broadly it includes fitness for use. A Quality of a product is also defined as it is being manufactured as intended purpose to satisfy consumer expectations. Ultimately the customer satisfaction has a lot of significance as it is known that customer is King and some say Customer is God.

However, Quality products help to maintain Customer satisfaction, loyalty and reduce the risk of replacing the faulty goods. Companies can build a reputation for Quality by gaining accreditation with the recognized quality standard. Hence the manufacturer shows a keen interest in maintaining the Quality of the product.

Of all the features of the product, the most important one is Durability. In other words, Durability is the lifetime of a product. Quality depends on the performance of the product. Consistency in the performance of product overtime is Reliability and the lifespan of products is Durability. Testing the Lifetime of a product is a complex process as some products are explosive i.e., products can be used only once. Here 100% inspection is not possible. Even in case of other products like Bulbs, Batteries etc., the manufacturer has to wait till the product becomes functional.

In such cases, the Acceptance sampling plans are one of the statistical tools in testing the quality of a product. It is the middle path between 100% inspection and no inspection. They used to decide whether to accept or reject a lot of finished goods. These techniques may not have a direct impact in the controlling the quality but it involves indirect effect in improving the quality of the products. Continuous Accepting Sampling Plans - Cumulative sum schemes (CASP-CUSUM Schemes) are widely used in the Industries in life testing of the Products. It is a powerful, versatile and diagnostic tool in Statistical Quality Control as it reduces time, cost, and machinery.

Hawkins, D. M. [4] proposed a fast accurate approximation for ARL's of a CUSUM Control Charts. This approximation can be used to evaluate the ARL's for Specific parameter values and the out of control ARL's location and scale CUSUM Charts.

Kakoty. S., Chakravarthy A.B. [6] determined CASP-CUSUM charts under the assumption that the variable under study follows a Truncated Normal Distribution. Generally, truncated distributions are employed in many practical phenomena where there is a constraint on the lower and upper limits of the variable under study. For example, in the production of engineering items, the sorting procedure eliminates items above or bellows designated tolerance limits. It is worthwhile to note that any continuous variable is first approximated as an exponential variable.

Vardeman S., Di-ou Ray [10] introduced CUSUM control charts under the restriction that the values are regard to quality is exponentially distributed. Further, the phenomena under study are the occurrence of the rate of rare events and the inter-arrival times for a homogenous poison process are identically independently distributed exponential random variables.

Lonnie. C. Vance [7], considered Average Run Length of Cumulative Sum Control Charts for controlling normal means and to determine the parameters of a

CUSUM Chart. To determine the parameters of CUSUM Chart the acceptable and rejectable quality levels along with the desired respective ARL's are considered.

Muhammed Riaz, Nasir Abbas[8] and Ronald J.M.M Does propose two Runs rule schemes for the CUSUM Charts. The performance of the CUSUM and EWMA Charts are compared with the usual CUSUM and weighted CUSUM, the first initial response CUSUM compared with usual EWMA Schemes. This comparison stated that the proposed schemes perform better for small and moderate shifts.

Mohammed Akhtar. P and Sarma K.L.A.P [1] analyzed and Optimization of CASP-CUSUM Schemes based on truncated two parametric Gamma distributions and evaluates L (0), L? (O) and a probability of Acceptance and also Optimized CASP-CUSUM Schemes based numerical results.

Narayana Murthy, B.R. and Mohammed Akhtar.P[11] proposed an Optimization of CASP CUSUMSchemes based on Truncated Log-logistic distribution and evaluate the probability of acceptance for different Parameter values.

Sainath.B and Mohammed Akhtar .P [18] studied an Optimization of CASP-CUSUM Schemes based on truncated Burr distribution and the results were analyzed at different values of the parameters.

Venkatesulu.G and Mohammed Akhtar.P[16] determined Truncated Gompertz Distribution and its Optimization of CASP-CUSUM Schemes by changing the values of the parameters and finally, critical comparisons are drawn based on the obtained numerical results.

Venkatesulu.G and Mohammed Akhtar.P[17] determined Truncated Lomax Distribution and its Optimization of CASP-CUSUM Schemes by changing the values of the parameters and finally, critical comparisons are drawn based on the obtained numerical results.

In the present paper, we assume that variable under study follows the Truncated Transmuted Weibull Distribution and its Optimization of CASP-CUSUM Schemes and critical comparison made based on the obtained numerical results. We determine the Type-C OC Curves of CASP-CUSUM Schemes. Thus it is more worthwhile to study some interesting characteristics of Type-C OC Curves based on this distribution.

2. TRANSMUTED WEIBULL DISTRIBUTION

It is well known that Weibull distribution is widely used for analyzing lifetime data in reliability engineering. It has wide applications in the field of Automotives, aerospace, Electronics etc. There are several generalization forms of Weibull distribution. Transmuted Weibull distribution is a one of such generalization of Weibull distribution has some similar properties of another Weibull family of distributions.

A continuous random variable X is said to follow Transmuted Weibull distribution if it assumes non-negative values with parameters η, σ and λ and its probability density function is given by

$$f(x) = \frac{\eta}{\sigma}\left(\frac{x}{\sigma}\right)^{\eta-1} e^{-\left(\frac{x}{\sigma}\right)^{\eta}} \left(1-\lambda+2\lambda e^{\left(1-\left(\frac{x}{\sigma}\right)\right)^{\eta}}\right) \tag{2.1}$$

Truncated Transmuted Weibull distribution

It is defined as the ratio of the probability density function of Transmuted Weibull distribution to its probability cumulative distribution function at the point B.

A random variable X is said to follow Truncated Transmuted Weibull distribution if its probability density function is given by

$$f_B(X) = \frac{\frac{\eta}{\sigma}\left(\frac{x}{\sigma}\right)^{\eta-1} e^{-\left(\frac{x}{\sigma}\right)^{\eta}} \left(1-\lambda+2\lambda e^{\left(1-\left(\frac{x}{\sigma}\right)\right)^{\eta}}\right)}{\left[1-e^{-\left(\frac{B}{\sigma}\right)^{\eta}}\right]\left[1+\lambda e^{-\left(\frac{B}{\sigma}\right)^{\eta}}\right]} \tag{2.2}$$

Where 'B' is the upper truncated point of Truncated Transmuted Weibull distribution

3. DESCRIPTION OF THE TYPE-C OC CURVE

Beattie [3] has suggested the method for constructing the continuous acceptance sampling plans. The procedure, suggested by him consists of a chosen decision interval namely, "Return interval" with the length h“, above the decision line is taken. We plot on the chart the sum $S_m = \Sigma(X_i - k_1)$ $X_i's$ (i = 1, 2, 3) are distributed independently and k1 is the reference value. If the sum lies in the area of the normal chart, the product is accepted and if it lies on the return chart, then the product is rejected, subject to the following assumptions.

1. When the recently plotted point on the chart touches the decision line, then the next point to be plotted at the maximum. i.e., $h = h'$.
2. When the decision line is reached or crossed from above, the next point on the chart is to plot at the baseline.

When the CUSUM falls in the return chart, network or a change of specification may be employed rather than outright rejection.

The procedure, in brief, is given below

1. Start plotting the CUSUM at 0.
2. The product is accepted when $S_m = \Sigma(X_i - k) < h$; when $S_m < 0$, return cumulative to 0
3. When $h < S_m < h = h'$ the product is rejected; when Sm crossed h, i.e., when $S_m > h + h'$ and continue rejecting Product until $S_m > h + h'$ return cumulative to h+h'

The type-C, OC function, which is defined as the probability of acceptance of an item as the function of incoming Quality, when the sampling rate is same in acceptance and rejection regions. Then the probability of acceptance

$P(A)$ is given by

$$P(A) = \frac{L(0)}{L(0) + L'(0)} \tag{3.1}$$

Where $L(0)$ = Average Run Length in acceptance zone and

$L'(0)$ = Average Run Length in rejection zone.

Page E.S. [9] has introduced the formulae for $L(0)$ and $L'(0)$ as

$$L(0) = \frac{N(0)}{1 - P(0)} \tag{3.2}$$

$$L'(0) = \frac{N'(0)}{1 - P'(0)} \tag{3.3}$$

Where P (0) = Probability for the test starting from zero on the normal chart,

N (0) = ASN for the test starting from zero on the normal chart,

P' (0) = Probability for the test on the return chart and

N' (0) = ASN for the test on the return chart

He further obtained integral equations for the quantities P (0), N (0), P' (0), N' (0) as follows:

$$P(Z) = F(k_1 - z) + \int_0^h P(y) f(y + k_1 - z) dy \tag{3.4}$$

$$N(Z) = 1 + \int_0^h N(y) f(y + k_1 - z) dy \tag{3.5}$$

$$P'(Z) = \int_{k_1+z}^h f(Y) dy + \int_0^h P'^{(y)} f(-y + k_1 + z) dy \tag{3.6}$$

$$N'(Z) = 1 + \int_0^h N'(y) f(-y + k_1 - z) dy \tag{3.7}$$

$$F(x) = 1 + \int_1^h f(x) dx$$

$$F(k_1 - z) = 1 + \int_A^{k_{1^-}} of\ (Y) dy$$

And z is a distance of the starting of the test in the normal chart from zero.

4. METHOD OF SOLUTION

We first express the integral equation (3.4) in the form

$$F(X) = Q(X) + \int_c^d R(x,t) F(t) dt \tag{4.1}$$

Where $F(X) = P\ (z)$, $Q(x) = F\ (k\text{-}z)$ and $R(X, t) = f(y + k\text{-}z)$

Let the integral $I = \int_c^d f(x)$ dx be transformed to

$$I = \frac{d-c}{2} \int_c^d f(y) dy = \frac{d-c}{2} \Sigma\, a_i f(ti) \tag{4.2}$$

Where $y = \dfrac{2x-(c-d)}{d-c}$, $a_i's$ and $t_i's$ respectively the weight factor and for the Gauss-Chebyshev polynomial, given in Jain M.K. and *et al* [4] using (3.1) and (3.2), (2.4) can be written as

$$F(X) = Q(X)\frac{d-c}{2}\Sigma a_i R(x,t_i)F(t_i)dt \tag{4.3}$$

Since equation (4.3) should be valid for all values of x in the (c, d), it must be true for $x = t_i$, $i = 0$ to n then obtain

$$F(t_i) = Q(Xt_i) + \frac{d-c}{2}\Sigma\, a_i R(t_j,t_i)F(t_i)dt,\ j = 0(1)n \tag{4.4}$$

Substituting $F\,(t_i) = F_i$, $Q\,(t_i) = Q_i$, $i = 0(1)$n in (4.4), we get

$$F_0 = Q_0 + \frac{d-c}{c}[a_0R(t_0,t_0)F_0 + a_1R(t_0,t_1)F_1 + \ldots\ldots\ldots\ldots + a_nR(t_0,t_n)F_n]$$

$$F_1 = Q_1 + \frac{d-c}{c}[a_0R(t_1,t_0)F_0 + a_1R(t_1,t_1)F_1 + \ldots\ldots\ldots\ldots + a_nR(t_1,t_n)F_n]$$

....

....

$$F_n = Q_n + \frac{d-c}{c}[a_0R(t_n,t_0)F_0 + a_1R(t_0,t_1)F_1 + \ldots\ \ldots\ \ldots\ \ldots + a_nR(t_n,t_n)F_n] \tag{4.5}$$

In this system of equations expect F_i, $i = 0,1,2$.........., n are known and hence can be solved for Fi, we Solved the system of equations by the method of Iteration. For this, we write the system (3.5) as

$$[1 - Ta_0R(t_0,t_0)F_0 = Q_0 + T[a_0R(t_0,t_0)F_0 + a_1R(t_0,t_1)F_1\ \ldots\ \ldots\ \ldots\ \ldots + a_nR(t_0,t_n)F_n]$$

$$[1 - Ta_0R(t_0,t_0)F_0 = Q_0 + T[a_0R(t_0,t_0)F_0 + a_1R(t_0,t_1)F_1\ \ldots\ \ldots\ \ldots\ \ldots + a_nR(t_0,t_n)F_n]$$

....

....

$$[1 - Ta_nR(t_n,t_n)F_n = Q_n + T[a_0R(t_n,t_0)F_0 + a_1R(t_n,t_1)F_1\ \ldots\ \ldots\ \ldots\ \ldots + a_nR(t_n,t_n)F_n] \tag{4.6}$$

Where $T = \dfrac{d-c}{c}$

To start the Iteration process, let us put $F_1 = F_2 = ... = F_n = 0$ in the first equation of (3.6), we obtain a rough value of F_0. Putting this value of F_0 and $F_1 = F_2 = ... = F_n = 0$ on the second equation, we get the rough value F_1 and so on. This gives the first set of values F_i $i = 0, 1, 2...$ n which are just the refined values of F_i $i = 0, 1, 2 ...n$. The process is continued until two consecutive sets of values are obtained up to a certain degree of accuracy. In the similar way solutions P' (0), N (0), N' (0) can be obtained.

5. COMPUTATION OF ARL'S AND P (A)

We developed computer programs to solve these equations (3.4), (3.5), (3.6) and (3.7) and we get the following results given in the tables (5.1) to (5.24).

Table 5.1
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 2, \lambda = 1, k = 1, h = 0.01, h'=0.01$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	2.65937	1.0126318	0.724228859
1.4	2.96936	1.0134054	0.745552599
1.3	3.51053	1.0144242	0.775815785
1.2	4.62703	1.0157652	0.819988966
1.1	8.03966	1.0175406	0.887654006
1.0	7787.7012	1.0199178	0.999869049

Table 5.2
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 2, \lambda = 1, k = 1, h = 0.02, h'=0.02$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	2.62489	1.0250356	0.719162405
1.4	2.92828	1.0265466	0.740431786
1.3	3.45776	1.0285306	0.770739019
1.2	4.54948	1.031132	0.815229654
1.1	7.88084	1.0345564	0.883958459
1.0	1948.3662	1.0391033	0.999466956

Table 5.3
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 2, \lambda = 1, k = 1, h = 0.03, h'=0.03$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	2.59028	1.0371979	0.714072168
1.4	2.88694	1.0394069	0.735273659
1.3	3.4044	1.0422984	0.765601873
1.2	4.47029	1.0460727	0.810369194
1.1	7.71374	1.0510088	0.880086899
1.0	850.26758	1.0574976	0.998757839

Table 5.4
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 2, \lambda = 1, k = 1, h = 0.04, h'= 0.04$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	2.55554	1.0491058	0.708957672
1.4	2.84533	1.051971	0.730077267
1.3	3.35047	1.0557082	0.760402501
1.2	4.38948	1.0605618	0.805403173
1.1	7.53883	1.066862	0.876028299
1.0	467.90268	1.0750487	0.997707665

Table 5.5
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 3, \lambda = 1, k = 1, h = 0.01, h'= 0.01$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	2.06764	1.0147216	0.670797229
1.4	2.33567	1.0162519	0.69681555
1.3	2.78995	1.0181582	0.732634068
1.2	3.71047	1.0205636	0.784283102
1.1	6.49686	1.0236446	0.863886118
1.0	7306.0732	1.02766	0.999859333

Table 5.6
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 3, \lambda = 1, k = 1, h = 0.02, h' = 0.02$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	2.03688	1.0290917	0.664350748
1.4	2.29718	1.0320594	0.690001011
1.3	2.73824	1.0357364	0.725558221
1.2	3.63158	1.040341	0.777320266
1.1	6.33192	1.0461757	0.858205199
1.0	1819.3737	1.0536575	0.999421179

Table 5.7
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 3, \lambda = 1, k = 1, h = 0.03, h' = 0.03$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	2.00643	1.0430933	0.657948613
1.4	2.25904	1.0473995	0.68322438
1.3	2.68694	1.0527018	0.718501687
1.2	3.55292	1.0592839	0.770330369
1.1	6.16448	1.0675179	0.852389693
1.0	787.47577	1.0778676	0.998633087

Table 5.8
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 3, \lambda = 1, k = 1, h = 0.04, h'=0.04$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	1.97626	1.0567106	0.651592016
1.4	2.22124	1.0622507	0.676487267
1.3	2.636	1.0690242	0.711466074
1.2	3.47446	1.0773484	0.763313949
1.1	5.99465	1.0876045	0.846432507
1.0	429.45294	1.1001863	0.997444689

Table 5.9
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 4, \lambda = 1, k = 1, h = 0.01, h' = 0.01$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	1.92293	1.015772	0.654346824
1.4	2.17515	1.017681	0.681260645
1.3	2.60115	1.02004	0.718313158
1.2	3.46243	1.022996	0.771929204
1.1	6.06627	1.026756	0.855244339
1.0	7159.84	1.031622	0.999855936

Table 5.10
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 4, \lambda = 1, k = 1, h = 0.02, h' = 0.02$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	1.89241	1.031117	0.647303104
1.4	2.13632	1.034804	0.673679292
1.3	2.54823	1.039328	0.710296869
1.2	3.38073	1.04494	0.763891041
1.1	5.89429	1.051979	0.848554909
1.0	1776.576	1.060894	0.999403179

Table 5.11
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 4, \lambda = 1, k = 1, h = 0.03, h' = 0.03$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	1.86231	1.046016	0.640337646
1.4	2.09806	1.051341	0.66617775
1.3	2.49608	1.057821	0.702349484
1.2	3.29996	1.065767	0.755878866
1.1	5.72192	1.075565	0.841770053
1.0	765.5163	1.087638	0.998581231

Table 5.12
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 4, \lambda = 1, k = 1, h = 0.04, h' = 0.04$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	1.83262	1.060449	0.633452535
1.4	2.06033	1.067264	0.658758938
1.3	2.44463	1.075481	0.694475055
1.2	3.22005	1.085422	0.747897089
1.1	5.54911	1.097426	0.834887505
1.0	415.4289	1.111717	0.997331083

Table 5.13
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 5, \lambda = 1, k = 1, h = 0.01, h'=0.01$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	1.86365	1.016326	0.647106051
1.4	2.1086	1.018438	0.674312234
1.3	2.52196	1.02104	0.71181494
1.2	3.35725	1.024293	0.766225636
1.1	5.88171	1.02842	0.85117203
1.0	7093.48047	1.033745	0.999854267

Table 5.14
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 5, \lambda = 1, k = 1, h = 0.04, h'=0.04$

B	*L(0)*	*L'(0)*	*P(A)*
1.5	1.77338	1.062401	0.625358641
1.4	1.99304	1.06988	0.650699139
1.3	2.3635	1.078844	0.686596334
1.2	3.11076	1.089606	0.740592301
1.1	5.35515	1.102462	0.829277277
1.0	408.6619	1.117492	0.997272968

Table 5.15
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 3, \lambda = 1, k = 2, h = 0.01, h' = 0.01$

B	*L(0)*	*L'(0)*	*P(A)*
2.5	5.96899	1.008649	0.855445564
2.4	6.86566	1.008872	0.871881604
2.3	8.41155	1.009146	0.892879903
2.2	11.57283	1.009477	0.919770122
2.1	21.18182	1.009876	0.954493046
2.0	126741.836	1.010356	0.999992013

Table 5.16
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 3, \lambda = 1, k = 2, h = 0.04, h' = 0.04$

B	*L(0)*	*L'(0)*	*P(A)*
2.5	5.81349	1.033956	0.84900111
2.4	6.68172	1.034813	0.865896642
2.3	8.17824	1.035859	0.887579024
2.2	11.23721	1.037123	0.915504754
2.1	20.52385	1.038643	0.951831043
2.0	9318.058	1.040464	0.999888361

Table 5.17
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 4, \lambda = 1, k = 2, h = 0.01, h' = 0.01$

B	*L(0)*	*L'(0)*	*P(A)*
2.5	3.98472	1.007634	0.798164487
2.4	4.66522	1.008002	0.822323024
2.3	5.81035	1.008425	0.852110445
2.2	8.11664	1.008915	0.889440715
2.1	15.06681	1.009483	0.937206745
2.0	164423.6	1.010144	0.999993861

Table 5.18
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 4, \lambda = 1, k = 2, h = 0.04, h' = 0.04$

B	*L(0)*	*L'(0)*	*P(A)*
2.5	3.89392	1.030024	0.790813386
2.4	4.55371	1.031439	0.815324724
2.3	5.66386	1.033066	0.845740378
2.2	7.89928	1.034939	0.884160101
2.1	14.63222	1.037105	0.933813095
2.0	14643.12	1.039615	0.999929011

Table 5.19
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 5, \lambda = 1, k = 2, h = 0.01, h' = 0.01$

B	*L(0)*	*L'(0)*	*P(A)*
2.5	3.43756	1.007163	0.773402691
2.4	4.03931	1.007595	0.800353885
2.3	5.04809	1.008087	0.833543837
2.2	7.0746	1.00865	0.875217319
2.1	13.17236	1.009298	0.928830743
2.0	180526.3	1.010047	0.999994397

Table 5.20
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 5, \lambda = 1, k = 2, h = 0.04, h' = 0.04$

B	*L(0)*	*L'(0)*	*P(A)*
2.5	3.36421	1.028196	0.765914917
2.4	3.94788	1.029861	0.793106973
2.3	4.92631	1.031753	0.826830924
2.2	6.89162	1.033911	0.869546771
2.1	12.80299	1.036383	0.92511344
2.0	18167.15	1.039229	0.99994278

Table 5.21
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 5, \lambda = 1, k = 3, h = 0.01, h' = 0.01$

B	*L(0)*	*L'(0)*	*P(A)*
3.5	6.55346	1.004793	0.867060304
3.4	7.78403	1.004928	0.885660112
3.3	9.85068	1.005079	0.907415092
3.2	14.00657	1.005248	0.933036268
3.1	26.51746	1.005436	0.963469088
3.0	758292.6	1.005647	0.999998689

Table 5.22
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 5, \lambda = 1, k = 3, h = 0.04, h' = 0.04$

B	*L(0)*	*L'(0)*	*P(A)*
3.5	6.45966	1.018976	0.863748372
3.4	7.66941	1.019506	0.882666051
3.3	9.70107	1.020096	0.904852152
3.2	13.78638	1.020755	0.931063294
3.1	26.08313	1.021491	0.962313056
3.0	101247.1	1.022313	0.999989927

Table 5.23
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 5, \lambda = 1, k = 4, h = 0.01, h' = 0.01$

B	*L(0)*	*L'(0)*	*P(A)*
4.5	8.39329	1.002008	0.893350065
4.4	9.79226	1.002047	0.907168984
4.3	12.1918	1.002093	0.924048722
4.2	17.08144	1.002147	0.944582522
4.1	31.91208	1.002209	0.969550967
4.0	3364224	1.002281	0.999999702

Table 5.24
Values of ARL's AND TYPE-C OC CURVES when
$\eta = 2, \sigma = 5, \lambda = 1, k = 4, h = 0.04, h' = 0.04$

B	*L(0)*	*L'(0)*	*P(A)*
4.5	8.45164	1.008081	0.893434465
4.4	9.86167	1.008238	0.90724498
4.3	12.28017	1.008425	0.924113512
4.2	17.20846	1.008643	0.944632113
4.1	32.15635	1.008895	0.969579756
4.0	4238515	1.009187	0.999999762

6. NUMERICAL RESULTS AND CONCLUSIONS

At the hypothetical values of the parameters η, σ, λ, k, h and h' are given at the top of each table, we determine optimum truncated point B at which P (A) the probability of accepting an item is maximum and also obtained ARL's values which represents the acceptance zone L(0) and rejection zone L'(0) values. The values of truncated point B of random variable X, L(0), L'(0) and the values for Type-C Curve, i.e. P (A) are given in columns I, II, III, and IV respectively.

From the above tables 5.1 to 5.24 we made the following conclusions:

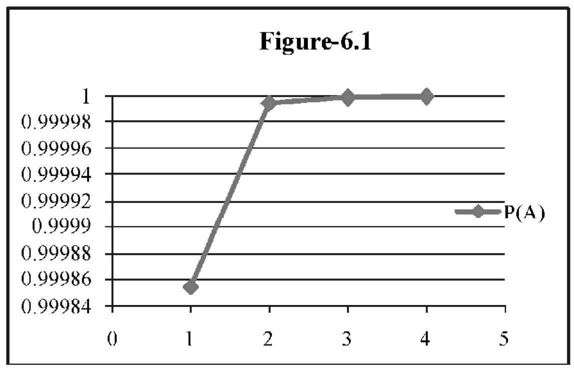
1. From the tables 5.1 to 5.24 it is observed that the values of P (A) are increased as the value of truncated point decreases. Thus, the truncated point of the random variable and the various parameters for CASP-CUSUM are related.
2. And also we observe that it can be minimized the truncated point B by increasing value of k.
3. From table 5.1 to 5.24 it is observed that at the maximum level of probability of acceptance P (A) the truncated point B from 4.5 to 1.0 as the value of h changes from 0.01 to 0.04.
4. From the table 5.1 to 5.24 it was observed that the value of L (0) and P (A) is increased as the value of truncated point decreases thus the truncated point of the random variable and the various parameters for CASP-CUSUM are related.
5. From the table 5.1 to 5.24 it was observed that the truncated point B changes from 4.5 to 1.0 and P (A) is as h?0.04 maximum i.e. 0.999999762. Thus

truncated point B and h are inversely related and hand P (A) are positively related.

6. From Table 5.1 to 5.24 it can be observed that the values of P (A) increased as the increases values of 'k'.
7. From the Table 5.1 to 5.24 it is observed that the values of Maximum Probabilities increased as the values of h and h' are increased.
8. It is observed that the Table -6.1 values of Maximum Probabilities increased as the increased values of 'k' as shown below the Figure 6.1.

Table 6.1
$\lambda = 1, \eta = 2, \sigma = 5, B = 1.0, h = 0.01, h'=0.01$

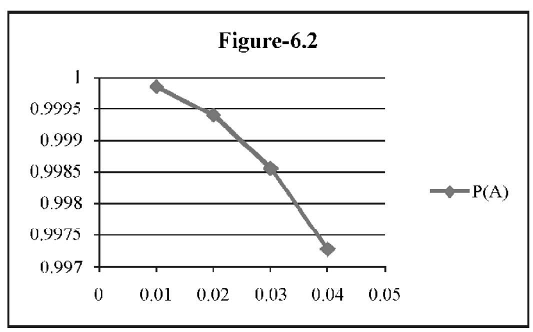
k	*P(A)*
1	0.999854267
2	0.999994397
3	0.999998689
4	0.999999702



9. It is observed that the Table 6.2 values of Maximum Probabilities increased as the decreased values of h and h' as shown below the Figure 6.2.

Table 6.2
$B = 1, \eta = 2, \sigma = 5, \lambda = 1, k = 1$

h and h'	*P(A)*
0.01	0.999854267
0.02	0.999394357
0.03	0.998555064
0.04	0.997272968



10. The various relations exhibited among the ARL's and Type-C OC curves with the parameters of the CASP-CUSUM based on the above tables 5.1 to 5.24 are observed from the following Table.

Table 6.3
Consolidated Table

B	η	σ	λ	h	h'	k	$P(A)$
1.0	2	2	1	0.01	0.01	1	0.999869049
1.0	2	2	1	0.02	0.02	1	0.999466956
1.0	2	2	1	0.03	0.03	1	0.998757839
1.0	2	2	1	0.04	0.04	1	0.997707665
1.0	2	3	1	0.01	0.01	1	0.999859333
1.0	2	3	1	0.02	0.02	1	0.999421179
1.0	2	3	1	0.03	0.03	1	0.998633087
1.0	2	3	1	0.04	0.04	1	0.997444689
1.0	2	4	1	0.01	0.01	1	0.999855936
1.0	2	4	1	0.02	0.02	1	0.999403179
1.0	2	4	1	0.03	0.03	1	0.998581231
1.0	2	4	1	0.04	0.04	1	0.997331083
1.0	2	5	1	0.01	0.01	1	0.999854267
1.0	2	5	1	0.04	0.04	1	0.997272968
2.0	2	3	1	0.01	0.01	2	0.999992013
2.0	2	3	1	0.04	0.04	2	0.999888361
2.0	2	4	1	0.01	0.01	2	0.999993861
2.0	2	4	1	0.04	0.04	2	0.999929011
2.0	2	5	1	0.01	0.01	2	0.999994397
2.0	2	5	1	0.04	0.04	2	0.99994278
3.0	2	5	1	0.01	0.01	3	0.999998689
3.0	2	5	1	0.04	0.04	3	0.999989927
4.0	3	5	1	0.04	0.04	4	0.999999762
4.0	3	5	1	0.01	0.01	4	0.999999702

From the above Table 6.3, we conclude that the optimum CASP-CUSUM schemes which have the values of ARL and P (A) reach their maximum i.e., 4238515, 0.999999762 respectively, is

$$\begin{bmatrix} B = 4 \\ \eta = 3 \\ \sigma = 5 \\ \lambda = 1 \\ k = 4 \\ h = 0.04 \\ h' = 0.04 \end{bmatrix}$$

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