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ILLUSTRATIONS TO COMPUTE HCF AND LCM USING A NEW METHOD

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Abstract

Many a ways are there in literature to find HCF and LCM for given numbers. In this paper, new methods and number of illustrations are given to compute these differently. These methods to compute HCF and LCM seems to be simple and more quick than our usual ones.

Keywords: *HCF, LCM, Factors, Prime Factorization.*

1. HISTORY

The HCF of two or more numbers is the largest common factor among all their common factors. The HCF of two or more given numbers can be obtained by any one of the following methods:

1. By finding first all the factors, then common factors and finally the largest/highest common factor.
2. Prime factorization method.
3. Division Method (based on Euclidean algorithm) [1, 2].

The LCM of two or more numbers is the smallest common multiple among all their common multiples. The LCM of two or more given numbers can be obtained by any one of the following methods:

1. By finding first multiples, then common multiples and finally the smallest/least common multiple.
2. Prime Factorization Method.
3. Division Method[1,2].

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2. ILLUSTRATIONS TO COMPUTE HCF USING A DIFFERENT METHOD

Based on the following step by step procedure, we have a different method to compute HCF.

Method to Compute HCF

Step 1. Take any number, preferably the smallest or prime number, among the given numbers.

Step 2. List all the factors of this number and arrange these in descending order.

Step 3. Pick up the largest/ highest factor (the number itself).

Step 4. Check whether this factor is a factor of all the other numbers (i.e. whether it divides all the other numbers or not?). If yes, then this factor is the HCF of the given numbers otherwise go to Step 5.

Step 5. Move to the next factor in the list of Step 2 and go to Step 4.

Here are some illustrations to compute HCF using the above method.

Illustration 1:

Let us compute the HCF of 135, 75, 25 and 100.

Smallest number = 25.

Factors of 25: 1, 5 & 25.

Factors in descending order: 25, 5 & 1.

We start with 25.

Now 25 doesn't divide 135.

Discard it and move to next factor 5.

Now 5 divides the rest of the numbers viz. 135, 75 & 100.

Hence , HCF = 5.

Illustration 2:

Let us compute the HCF of 47, 17, 87 and 27.

Prime number = 17 (That too the smallest between 47 & 17).

Factors of 17: 1 & 17.

Factors in descending order: 17 & 1.

We start with 17.

Now 17 doesn't divide 47.

Discard it and move to next factor 1.

Now 1 divides all other numbers viz. 47, 87 & 27.

Hence, HCF = 1.

(It's obvious also that HCF = 1, moment we reached to the factor 1).

Illustration 3:

Let us compute the HCF of 48, 14, 24, 32 and 66.

Smallest number = 14.

Factors of 14: 1, 2, 7, 14.

Factors in descending order: 14, 7, 2 & 1.

We start with 14.

Now 14 doesn't divide 48.

Discard it and move to next factor 7.

Now again 7 doesn't divide 48.

Discard it and move to next factor 2.

Now 2 divides the rest of the numbers viz. 48, 24, 32 & 66.

Hence , HCF = 2.

Illustration 4:

Let us compute the HCF of 96, 144, 24, 72 and 168.

Smallest number = 24.

Factors of 24: 1, 2, 3, 4, 6, 8, 12 & 24.

Factors in descending order: 24, 12, 8, 6, 4, 3, 2 & 1.

We start with 24.

Now 24 divides the rest of the numbers viz. 96, 144, 72 & 168.

Hence , HCF = 24.

Illustration 5:

Let us compute the HCF of 250, 370, 100, 170 and 150.

Smallest number = 100.

Factors of 100: 1, 2, 4, 5, 10, 20, 25, 50 & 100.

Factors in descending order: 100, 50, 25, 20, 10, 5, 4, 2 & 1.

We start with 100.

Now 100 doesn't divide 250.

Discard it and move to next factor 50.

Now 50 doesn't divide 370.

Discard it and move to next factor 25.

Now 25 doesn't divide 370.

Discard it and move to next factor 20.

Now 20 doesn't divide 370.

Discard it and move to next factor 10.

Now 10 divides the rest of the numbers viz. 250, 370, 170, & 150.

Hence , HCF = 10.

3. ILLUSTRATIONS TO COMPUTE LCM DIFFERENTLY

Likewise we have a different method to compute LCM.

Method to Compute LCM

Step 1. Take any number, preferably the largest number, among the given numbers.

Step 2. List multiples of this number and arrange these multiples in ascending order.

Step 3. Pick up the lowest multiple (the number itself).

Step 4. Check whether this multiple is a multiple of other numbers (i.e. whether this multiple is divisible by all the other numbers?). If yes, then this number is the LCM otherwise go to Step 5.

Step 5. Move to the next multiple in the list of Step 2 and go to Step 4.

Now we give illustrations to compute LCM

Illustration 1:

Let us find the LCM of 24, 36 and 16.

Largest number = 36.

Multiples of 36: 36, 72, 108, 144, 180, 216 etc…

We start with 36.

Now 36 is not a multiple of 24 (in other words 24 doesn't divide 36).

Discard 36 and move to next multiple 72.

Again 72 is a multiple of 24 but not of 16.

Discard 72 and move to next multiple 108.

Now 108 is not a multiple of 24 (in other words 24 doesn't divide 108).

Discard 108 and move to next multiple 144.

Now 144 is a multiple of 24 and as well as a multiple of 16.

Hence, LCM = 144.

Illustration 2:

Let us find the LCM of 9, 27, 12 and 24.

Largest number = 27.

Multiples of 27 in ascending order: 27, 54, 81, 108, 135, 162, …

We start with 27.

Now 27 is a multiple of 9 but not of 12.

Discard 27 and move to next multiple 54.

Now 54 is a multiple of 9 but not of 12.

Discard 54 and move to next multiple 81.

Now 81 is a multiple of 9 but not of 12.

Discard 81 and move to next multiple 108.

Now 108 is a multiple of 9 & 12 but not of 24.

Discard 108 and move to next multiple 135.

Now 135 is a multiple of 9 but not of 12.

Discard 135 and move to next multiple 162.

Now 162 is a multiple of 9 but not of 12.

Discard 162 and move to next multiple 189.

Now 189 is a multiple of 9 but not of 12.

Discard 189 and move to next multiple 216.

Now 216 is a multiple of 9, 12 & 24 (all other numbers).

Hence, LCM = 216.

Illustration 3:

Let us find the LCM of 50, 250, 100 and 125.

Largest number = 250.

Multiples of 250 in ascending order: 250, 500, 750, 1000, 1250 ,…

We start with 250.

Now 250 is a multiple of 50 but not of 100.

Discard 250 and move to next multiple 500.

Now 500 is a multiple of 50, 100 & 125.

Hence, LCM = 500.

Illustration 4:

Let us find the LCM of 24, 18 and 30.

Largest number = 30.

Multiples of 30 in ascending order: 30, 60, 90, 120, 150, 180, 210, 240, 270, 300 . . ..

We start with 30.

Now 30 is not a multiple of 24.

Discard 30 and move to next multiple 60.

Now 60 is not a multiple of 24.

Discard 60 and move to next multiple 90.

Now 90 is not a multiple of 24.

Discard 90 and move to next multiple 120.

Now 120 is a multiple of 24 but not of 18.

Discard 120 and move to next multiple 150.

Now 150 is not a multiple of 24.

Discard 150 and move to next multiple 180.

Now 180 is not a multiple of 24.

Discard 180 and move to next multiple 210.

Now 210 is not a multiple of 24.

Discard 210 and move to next multiple 240.

Now 240 is a multiple of 24 but not of 18.

Discard 240 and move to next multiple 270.

Now 270 is not a multiple of 24.

Discard 270 and move to next multiple 300.

Now 300 is not a multiple of 24.

Discard 300 and move to next multiple 330.

Now 330 is not a multiple of 24.

Discard 330 and move to next multiple 360.

Here 24 and 18 (all the other numbers) divide 360.

Hence, LCM = 360.

Illustration 5:

Let us find the LCM of 18, 24 and 12.

Largest number = 24.

Multiples of 24 in ascending order: 24, 48, 72, 96, 120, …

We start with 24.

Now 18 doesn't divide 24.

Discard 24 and move to next multiple 48.

Now 18 doesn't divide 48.

Discard 48 and move to next multiple 72.

Here 18 and 12 (the other numbers) both divide 72.

Hence, LCM = 72.

REMARK

These new methods to compute HCF and LCM are different from the known methods which we have been using for so many decades. Also these methods seems to be less time consuming and simpler to the existing methods.

REFERENCES

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