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A bulk queue with unreliable server, with compulsory server vacation and with two stages of service

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Abstract : The congestion situatins encountered in computer, communication, manufacturing, production system, etc, can be modelled as queueing system with vacation and with unreliabe server. A bulk arrival Poisson queue with two stages of batch service has been considered. In addition the server take compulsory vacation, also the server may breaks down. The objective is to analyze the model in time independent domine. The supplementary variable technique has been used to find the probability generating function of number of customers in the queue at various server states. Using the properties of probability generating function some operating characteristics have been derived and numerical examples are given to realize the model.

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1. INTRODUCTION

The congestion situatins encountered in computer, communication, manufacturing, production system, etc, can be modelled as queueing system with vacation. Several researchers have contributed significantly on vacation models [Takagi[19], Lee et al[16], Bacot et al[1] and Choudhury [2], Ke [13]]. Doshi [6] and Takagi [19] are the two excellent survey works on vacation queues. In many real life situations, the server may break downs, so that a more realistic queueing model is that which incorporates the assumption of unreliable server. Many researchers have contributed on queue with unreliable servers [Li et al[17], Wang[21], Wang[22]]. Some notable works on queuing with break down are Wang [21,22], Wang et al[23] and Ke [12]. Many researchers have studied the queueing model with unreliable server in different frameworks and suggested ways and means to tackles related situations. Grey et al [8] incorporated the server breakdown on vacation queueing model. Haridass and Arumuganathan [9] studied $M^{[X]}/G/1$ queuing system with an unreliable server and with single vacation. Choudhury and Deka [4] investigated an M/G/1 unreliable server Bernoulli vacation queue with two phases of service . In 2013, the same authors[5] studied a batch arrival unreliable server Bernoulli vacation queue with two phases of service and delayed repair. Ke et al [14], analyzed an $M^{[X]}/G/1$ queuing system with an unreliable server and repair, in which the server operates with a randomized vacation policy. Kalyanaraman and Nagarajan [11] have analyzed a single unreliable server bulk arrival, fixed batch service queue with Bernoulli vacation. The motivation of queueing model with two phases of service mainly comes from communication networks, in which messages are processed in two stages by a single server. This types of model has been first studed by Krishna and Lee [15]. Some notable works are Doshi [7], Selvam and Sivasankaran [18], Kalyanaraman and Ayappan [10], Choudhury et al [3], Thangaraj and Vanitha [20].

In this article we consider an $M^{[X]}/G^{K}/1$ queue with unreliable server and with compulsory server vacation. In addition the server provides two stages of service. This type of queuing system exists in manufacturing industries, Transportation system etc. In manufacturing industries, after products are approved for transportation to customer

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shops, they are transported to the shops in bulks by truck. After transporting the products, if no batch is available for transportation, the truck will be used for other work or the truck is sent for maintenance (vacation period). During the service period (transportation period), the trucks may break down. During the production stage, the product undergo serveral stages of service like compiling, quality testing ect., The above situation can be modeled as an $M^{[X]}/G^K/1$ queue with unreliable server and compulsory vacation and with two stages of service.

The remainder of this article is organized as follows: Section 2 provides the model description and mathematical analysis. In section 3, we obtain some queuing characteristics of the model discussed in this paper. In section 4, we present some particular models. In section 5, we illustrate the model by some numerical examples. Finally, In section 6 we present a conclusion.

2. THE MODEL AND ANALYSIS

We condisder and $M^{[X]}/G^K/1$ queueing system, where the number batches of customers arrives to the system follow a compound Poisson process with arrival rate λ. The size of the successive arriving batches is a random vabirable with probability $P\{X = j\} = C_j$, whose probability generating function is defined by $C(z) = \sum_{j=1}^{\infty} C_j z^j$.

The services are given in batchs of fixed size 'K'. Each batch undergoes two stages of heterogenous service provided by a single server on a first come first served basis. The two stages of service time are random periods follow different gernerally distributed random variables with distribution function $G_i(x)$ and density function $g_i(x)$ for $i = 0,1$.

After completion of second stage of service, the server takes a compulsory vacation of random duration. The vacation period is also generally distributed with distribution function $B(x)$.

In addition, the server may breakdown during a service and the number of breakdowns are assumed to occur according to a Poisson process with rate 'α'. Once the server breakdown, the customer whose service is interrupted goes to the head of the queue and the repair to server starts immediately. The duration of the repair period is generally distributed with distribution function $H(x)$. Immediatly after the broken server is repaired, the server is ready to start its service. Further, we assume that the input process, server life time, server repair time, sevice time and vacation times are independent of each other.

The analysis of this model is based on supplementary variable technique and the supplementary variable is elapsed service time / elapsed vacation time / elapsed repair time.

We define the following probabilities and conditional probabilities:

$\mu_i(x) = \dfrac{g_i(x)}{1 - G_i(x)}$ for $i = 1, 2$. is the conditional probability that the completion of i^{th} phase service during the interval $(x, x+dx)$, given that the elapsed service time is 'x'.

$\beta(x) = \dfrac{b(x)}{1 - B(x)}$ is the conditional probability that the completion of vacation during the interval $(x, x+dx)$, given that the elapsed vacation time is 'x'.

$\gamma(x) = \dfrac{h(x)}{1 - H(x)}$ is the conditional probability that the completion of repair during the interval $(x, x+dx)$, given that the elapsed repair time is 'x'.

The Markov process related to this model is $\{(N(t), S(t)) : t \geq 0\}$ where $N(t)$ be the number of customer in the queue and $S(t)$ be the supplementary variable at time t. and

$$
\begin{aligned}
S(t) &= S_1(t), \text{ the elapsed } 1^{st} \text{ stage service time} \\
&= S_2(t), \text{ the elapsed } 2^{nd} \text{ stage service time} \\
&= S_3(t), \text{ the elapsed vacation time} \\
&= S_4(t), \text{ the elapsed repair time}
\end{aligned}
$$

$P_n^{(i)}(t,x)$ = Probability that, at time 't', there are 'n' customers in the queue, the server provides the 'i' stage of service (excluding the customer in service) and the elapsed service time is 'x'. where $i = 1, 2$.

$V_n(t,x)$ = Probability that, at time 't', there are 'n' customers in the queue and the elapsed vacation time is 'x'

$R_n(t,x)$ = Probability that, at time 't', there are 'n' customers in the queue and the elapsed repair time is 'x'

$Q_n(t)$ = Probability that, at time 't', there are n customers in the queue and the server is idle.

The differential-difference equations for this model are

$$\frac{dP_0^{(1)}(x)}{dx} = -(\lambda + \mu_1(x) + \alpha)P_0^{(1)}(x) \tag{1}$$

$$\frac{dP_n^{(1)}(x)}{dx} = -(\lambda + \mu_1(x) + \alpha)P_n^{(1)}(x) + \lambda\sum_{j=1}^{n} C_j P_{n-j}^{(1)}(x), \text{ for } n \ge 1 \tag{2}$$

$$\frac{dP_0^{(2)}(x)}{dx} = -(\lambda + \mu_2(x) + \alpha)P_0^{(2)}(x) \tag{3}$$

$$\frac{dP_n^{(2)}(x)}{dx} = -(\lambda + \mu_2(x) + \alpha)P_n^{(2)}(x) + \lambda\sum_{j=1}^{n} C_j P_{n-j}^{(2)}(x), \text{ for } n \ge 1 \tag{4}$$

$$\frac{dV_0(x)}{dx} = -(\lambda + \beta(x))V_0(x) \tag{5}$$

$$\frac{dV_n(x)}{dx} = -(\lambda + \beta(x))V_n(x) + \lambda\sum_{j=1}^{n} C_j V_{n-j}(x), \text{ for } n \ge 1 \tag{6}$$

$$\frac{dR_0(x)}{dx} = -(\lambda + \gamma(x))R_0(x) \tag{7}$$

$$\frac{dR_n(x)}{dx} = -(\lambda + \gamma(x))R_n(x) + \lambda\sum_{j=1}^{n} C_j R_{n-j}(x), \text{ for } n \ge 1 \tag{8}$$

$$0 = -\lambda Q_n + \lambda(1-\delta_{n,K})\sum_{j=1}^{n} c_j Q_{n-j} + \int_0^\infty R_n(x)\gamma(x)dx + \int_0^\infty V_n(x)\beta(x)dx \tag{9}$$

The boundary conditions are

$$P_n^{(1)}(0) = \int_0^\infty V_{n+K}(x)\beta(x)dx + \int_0^\infty R_{n+K}(x)\gamma(x)dx + \lambda\sum_{j=0}^{K-1} C_{n+K-j}Q_j,$$

$$\text{for } n = 0, 1. \tag{10}$$

$$P_n^{(2)}(0) = \int_0^\infty P_n^{(1)}(x)\mu_1(x)dx, \text{ for } n = 0,1, \tag{11}$$

$$V_n(0) = \int_0^\infty P_n^{(2)}(x)\mu_2(x)dx, \text{ for } n = 0,1, \tag{12}$$

$$R_n(0) = \alpha\int_0^\infty (P_{n-K}^{(1)}(x) + P_{n-K}^{(2)}(x))dx, \text{ for } n = K, K+1, \ldots \tag{13}$$

$$R_n(0) = 0, \text{ for } n < K \tag{14}$$

and the normalization condition is

$$\sum_{n=0}^{K-1} Q_n + \int_0^\infty \sum_{n=0}^{\infty}[P_n^{(1)}(x) + P_n^{(2)}(x) + V_n(x) + R_n(x)]dx = 1 \tag{15}$$

For the analysis, we define the following probability generating functions.

$$P_1(x,z) = \sum_{n=0}^{\infty} P_n^{(1)}(x)z^n, P_2(x,z) = \sum_{n=0}^{\infty} P_n^{(2)}(x)z^n, R(x,z) = \sum_{n=0}^{\infty} R_n(x)z^n, C(z) = \sum_{j=0}^{\infty} C_j z^j$$

$$Q(z) = \sum_{n=0}^{K-1} Q_n z^n, V(x,z) = \sum_{n=0}^{\infty} V_n(x)z^n,$$

Multiplying equation (2) by z^n and applying $\sum_{n=1}^{\infty}$, we have

$$\frac{\partial \sum_{n=1}^{\infty} P_n^{(1)}(x)z^n}{\partial x} = -(\lambda + \mu_1(x) + \alpha)\sum_{n=1}^{\infty} P_n^{(1)}(x)z^n + \lambda \sum_{n=1}^{\infty}\sum_{j=1}^{n} C_j P_{n-j}^{(1)}(x)z^n$$

Adding the above equation with equation (1), we have

$$\frac{\partial P_1(x,z)}{\partial x} + (\lambda - \lambda C(z) + \mu_1(x) + \alpha)P_1(x,z) = 0 \tag{16}$$

Multiplying equation (4) by $z^n$and applying $\sum_{n=1}^{\infty}$, we have

$$\frac{\partial \sum_{n=1}^{\infty} P_n^{(2)}(x)z^n}{\partial x} = -(\lambda + \mu_2(x) + \alpha)\sum_{n=1}^{\infty} P_n^{(2)}(x)z^n + \lambda \sum_{n=1}^{\infty}\sum_{j=1}^{n} C_j P_{n-j}^{(2)}(x)z^n$$

Adding the above equation with equation (3), we have

$$\frac{\partial P_2(x,z)}{\partial x} + (\lambda - \lambda C(z) + \mu_2(x) + \alpha)P_2(x,z) = 0 \tag{17}$$

Multiplying equation (6) by z^n and applying $\sum_{n=1}^{\infty}$, we have

$$\frac{\partial \sum_{n=1}^{\infty} V_n(x)z^n}{\partial x} = -(\lambda + \beta(x))\sum_{n=1}^{\infty} V_n(x)z^n + \lambda \sum_{n=1}^{\infty}\sum_{j=1}^{n} C_j V_{n-j}(x)z^n$$

Adding the above equation with equation (5), we have

$$\frac{\partial V(x,z)}{\partial x} + (\lambda - \lambda C(z) + \beta(x))V(x,z) = 0 \tag{18}$$

Multiplying equation (8) by $z^n$and applying $\sum_{n=1}^{\infty}$, we have

$$\frac{\partial \sum_{n=1}^{\infty} R_n(x)z^n}{\partial x} = -(\lambda + \gamma(x))\sum_{n=1}^{\infty} R_n(x)z^n + \lambda \sum_{n=1}^{\infty}\sum_{j=1}^{n} C_j R_{n-j}(x)z^n$$

Adding the above equation with equation(7) , we have

$$\frac{\partial R(x,z)}{\partial x} + (\lambda - \lambda C(z) + \gamma(x))R(x,z) = 0 \tag{19}$$

Multiplying equation (10) by z^{n+k} and applying $\sum_{n=0}^{\infty}$, we have

$$\sum_{n=0}^{\infty} P_n^{(1)}(0)z^{n+K} = \lambda \sum_{n=0}^{\infty}\sum_{j=0}^{K-1} Q_j C_{n+K-j} z^{n+K} + \int_0^{\infty} \gamma(x) \sum_{n=0}^{\infty} R_{n+K}(x)z^{n+K}dx + \int_0^{\infty} \beta(x) \sum_{n=0}^{\infty} V_{n+K}(x)z^{n+K}dx$$

$$z^K P_1(0,z) = \int_0^{\infty} \gamma(x) \sum_{n=K}^{\infty} R_n(x)z^n dx + K(z) + \int_0^{\infty} \beta(x) \sum_{n=K}^{\infty} V_n(x)z^n dx \tag{20}$$

where $$K(z) = \lambda\sum_{n=0}^{\infty}\sum_{j=0}^{K-1} Q_j C_{n+K-j} z^{n+K}$$

We multiplying equation (9) by z^n and applying $\sum_{n=0}^{K-1}$, we get

$$0 = -\lambda\sum_{n=0}^{K-1} Q_n z^n + \int_0^{\infty}\gamma(x)\sum_{n=0}^{K-1} R_n(x) z^n dx + \int_0^{\infty}\beta(x)\sum_{n=0}^{K-1} V_n(x) z^n dx + \lambda\sum_{n=0}^{K-1}(1-\delta_{n,K})\sum_{j=1}^{n} C_j Q_{n-j} z^n$$

$$0 = -\lambda Q(z) + \int_0^{\infty}\gamma(x)\sum_{n=0}^{K-1} R_n(x) z^n dx + \int_0^{\infty}\beta(x)\sum_{n=0}^{K-1} V_n(x) z^n dx + \lambda L(z) \tag{21}$$

where $$L(z) = \sum_{n=0}^{K-1}(1-\delta_{n,K})\sum_{j=1}^{n} C_j Q_{n-j} z^n$$

We add the equations (20) and (21), we get

$$z^K P_1(0,z) = \int_0^{\infty}\beta(x)V(x,z)dx + \int_0^{\infty}\gamma(x)R(x,z)dx$$

where $$K(z) = \lambda[C(z)Q(z) - L(z)]$$

$$z^K P_1(0,z) = \int_0^{\infty}\beta(x)V(x,z)dx + \int_0^{\infty}\gamma(x)R(x,z)dx - \lambda Q(z) + [\lambda C(z)Q(z) - \lambda L(z)] + \lambda L(z)$$

$$z^K P_1(0,z) = \int_0^{\infty}\beta(x)V(x,z)dx + \int_0^{\infty}\gamma(x)R(x,z)dx + \lambda[C(z)-1]Q(z) \tag{22}$$

Multiplying equation (11) by z^n and applying $\sum_{n=0}^{\infty}$, we have

$$\sum_{n=0}^{\infty} P_n^{(2)}(0) z^n = \int_0^{\infty}\mu_1(x)\sum_{n=0}^{\infty} P_n^{(1)}(x) z^n dx$$

$$P_2(0,z) = \int_0^{\infty}\mu_1(x)P_1(x,z)dx \tag{23}$$

Multiplying equation (12) by z^n and applying $\sum_{n=0}^{\infty}$, we have

$$V(0,z) = \int_0^{\infty}\mu_2(x)P_2(x,z)dx \tag{24}$$

Multiplying equation (13) by z^n and applying $\sum_{n=K}^{\infty}$, we have

$$\sum_{n=K}^{\infty} R_n(0) z^n = \alpha\int_0^{\infty}\left(\sum_{n=K}^{\infty} P_{n-K}^{(1)}(x) z^n + \sum_{n=K}^{\infty} P_{n-K}^{(2)}(x) z^n\right)dx$$

Adding the above equation with equation (14), we have

$$R(0,z) = \alpha z^K\int_0^{\infty}(P_1(x,z) + P_2(x,z))dx = \alpha z^K(P_1(z) + P_2(z)) \tag{25}$$

Integrating the equation (16) partially with respect to 'x' with the limits from '0' to 'x', we have

$$P_1(x,z) = P_1(0,z)e^{-ax-\int_0^x \mu_1(x)dx} dx\text{, where } a = \lambda - \lambda C(z) + \alpha \tag{26}$$

Integrating equation (26) partially with respect 'x' with the limits from '0' to '∞', we have

$$P_1(z) = \frac{P_1(0,z)[1 - G^*_1(a)]}{a} \tag{27}$$

Multiplying equation (26) by $\mu(x)$ and integrating partially with respect to 'x', with the limits from '0' to '∞', we have

$$\int_0^\infty \mu_1(x)P_1(x,z)\partial x = P_1(0,z)G^*{}_1(a) \quad (28)$$

Integrating equation (17) partially with respect to 'x' with the limits from '0' to 'x', we have

$$P_2(x,z) = P_2(0,z)e^{-ax-\int_0^x \mu_2(x)dx} dx \quad (29)$$

Integrating equation (29) partially with respect to 'x' with the limits from '0' to '∞', we have

$$P_2(z) = \frac{P_2(0,z)[1-G^*{}_2(a)]}{a} \quad (30)$$

Multiplying equation (29) by $\mu(x)$ and integrating partially with respect to 'x', with the limits from '0' to '∞', we have

$$\int_0^\infty \mu_2(x)P_2(x,z)\partial x = P_2(0,z)G^*{}_2(a) \quad (31)$$

Substituting equation (31) in (24), we have

$$V(0,z) = P_2(0,z)G^*{}_2(a) \quad (32)$$

Integrating equation (18) partially with respect to 'x', with the limits from '0' to 'x', we have

$$V(x,z) = V(0,z)e^{-mx-\int_0^x \beta(x)dx} \quad (33)$$

Substituting equation (32) in equation (33), we have

$$V(x,z) = P_2(0,z)G^*{}_2(a)e^{-mx-\int_0^x \beta(x)dx} \quad (34)$$

Integrating equation (34) partially with respect to 'x', with the limits form '0' to '∞', we have

$$V(z) = \frac{P_2(0,z)G^*{}_2(a)[1-B^*(m)]}{m} \quad (35)$$

Multiplying equation (34) by $\beta(x)$ and integrating partially with respect to x, with limits from 0 to ∞.

$$\int_0^\infty V(x,z)\beta(x)dx = P_2(0,z)G^*{}_2(a)B^*(m) \quad (36)$$

Integrating equation (19) partially with respect to 'x', with the limits from '0' to 'x', we have

$$R(x,z) = R(0,z)e^{-mx-\int_0^x \gamma(x)dx} \quad (37)$$

Substituting equation (25),(27) and (30) in (29), we have

$$R(x,z) = \frac{\alpha z^K e^{-mx-\int_0^x \gamma(x)dx}}{a}\left[P_1(0,z)[1-G^*{}_1(a)] + P_2(0,z)[1-G^*{}_2(a)]\right] \quad (38)$$

Integrating equation (38) partially with respect to 'x', with the limits from 0 to ∞, we have

$$R(z) = \frac{\alpha z^K[1-H^*(m)]}{am}\left[P_1(0,z)(1-G^*{}_1(a)) + P_2(0,z)(1-G^*{}_2(a))\right] \quad (39)$$

Multiplying equation (38) by $\gamma(x)$ and integrating partially with respect to 'x', with the limits from 0 to ∞, we have

$$\int_0^\infty R(x,z)\gamma(x)dx = \frac{\alpha z^K H^*(m)}{a}[P_1(0,z)(1-G^*{}_1(a)) + P_2(0,z)(1-G^*{}_2(a))] \quad (40)$$

Substitute equation (28) in (23), we have

$$P_2(0,z) = P_1(0,z)G_1^*(a) \tag{41}$$

Substitute equation (41) in (40), we have

$$\int_0^\infty R(x,z)\gamma(x)dx = \frac{\alpha z^K H^*(m)}{a}[P_1(0,z)(1-G^*{}_1(a)) + P_1(0,z)G_1^*(a)(1-G^*{}_2(a))] \tag{42}$$

Now using equation (40), (41), (36) and (31) in equation (22), we have

$$P_1(0,z) = \frac{aQ(z)m}{D} \tag{43}$$

where $$D = \alpha z^K[1-G^*{}_1(a)G^*{}_2(a)]H^*(m) - a[z^K - B^*(m)G^*{}_1(a)G^*{}_2(a)]$$

Substituting $P_1(0,z)$ in the equation (41), we have

$$P_2(0,z) = \frac{aQ(z)mG_1^*(a)}{D} \tag{44}$$

Substituting $P_1(0,z)$ in the equation (27), we have

$$P_1(z) = \frac{mQ(z)[1-G^*{}_1(a)]}{D} \tag{45}$$

Substituting $P_2(0,z)$ in the equation (30), we have

$$P_2(z) = \frac{mQ(z)[1-G^*{}_2(a)]G_1^*(a)}{D} \tag{46}$$

Substituting $P_2(0,z)$ in the equation (35), we have

$$V(z) = \frac{aG^*{}_1(a)G_2^*(a)[1-B^*(m)]Q(z)}{D} \tag{47}$$

Substituting $P_1(0,z)$ & $P_2(0,z)$ in the equation (39), we have

$$R(z) = \frac{\alpha z^K Q(z)[1-G^*{}_1(a)G_2^*(a)][1-H^*(m)]}{D} \tag{48}$$

Now adding (44) ,(45), (46) and (47), we have

$$S(z) = P_1(z) + P_2(z) + V(z) + R(z)$$

Here S(z) represent the probability generating function of number of customer in the queue, independent of server state

$$S(z) = \frac{Q(z)N}{D} \tag{49}$$

where $$N = aG^*{}_1(a)G_2^*(a)[1-B^*(m)] + [m + \alpha z^K(1-H^*(m))][1-G_1{}^*(a)G_2^*(a)]$$

and $$m = \lambda - \lambda\, C(z),\ a = \lambda - \lambda - C(z) + \alpha$$

We know that S(z) is probability generating function, it has the property that it must converge inside the unit circle $|z|=1$. Here it can be seen that the expression in the denominator of S(z) has 'K' zeros. By Rouches theorem, we notice that K–1 zeros of this expression lies inside the unit circle $|z|=1$, and must coinside with K–1 zero's of numerator of S(z), and one zero lies out side the unit circle $|z|=1$. Let z_0 be the zero which lies outside the circle $|z|=1$. As S(z) converges, K–1 zero's of numerator and denominator of S(z) will be cancelled. Therefore we have

$$S(z) = \frac{A}{z - z_0} \tag{50}$$

By substituting $z=1$,we have

$$A = (1-z_0)S(1) = (1-z_0)\frac{QN'_1}{D'_1} \quad (51)$$

$$N'_1 = \lambda E(X)\{[1-G^*_1(\alpha)G^*_2(\alpha)][1+\alpha E(R)] + \alpha G^*_1(\alpha)G^*_2(\alpha)E(V)\}$$

$$D'_1 = \alpha G^*_1(\alpha)G^*_2(\alpha)[K-\lambda E(X)E(V)] - \lambda E(X)[1-G^*_1(\alpha)G^*_2(\alpha)][1+\alpha E(R)]$$

Substituting the value of (51) in the equation (50), we have

$$S(z) = \frac{(z_0-1)QN'_1}{z_0 D'_1}\sum_{n=0}^{\infty}\left(\frac{z}{z_0}\right)^n \quad (52)$$

Which is probability generating function of number of customer in the queue.

3. SYSTEM PERFORMANCE MEASURES

In this section, the system performance measures, the mean number of customers in the queue and idle probability have been calculated.

1. The mean number of customers in the queue

$$E(N) = S'(1) = \frac{QN'_1}{(z_0-1)D'_1} \quad (53)$$

2. The idle probability

Since $Q + S(1) = 1$,

where $Q = \sum_{n=0}^{K-1} Q_n$,

which leads to

$$Q = 1-\lambda E(X)\{\frac{1}{\alpha K G^*_1(\alpha)G^*_2(\alpha)} - \frac{1}{\alpha K} + \frac{E(R)}{KG^*_1(\alpha)G^*_2(\alpha)} - \frac{E(R)}{K} + \frac{E(V)}{K}\} \quad (54)$$

4. SOMR PARTICULAR MODELS

In this section, three particular models have been derived by assigning particular forms to the parameters and to the distribution function.

PARTICULAR MODEL-01

In the above model, we assume that batch arrival size random variable X follows geometric distributin with probablilty $C_n = (1-s)^{n-1}s$ for $n \geq 1$ and $s = 1-t$, then $E(X) = \frac{1}{s}$. Also we assume that the service time random variables follows exponential distribution with $E(S) = \frac{1}{\mu_i}$ then $G^*(\alpha) = \frac{\mu_i}{\alpha+\mu_i}$, for $i = 1, 2$ and the repair time random variable R follows exponential distribution with $E(R) = \frac{1}{\gamma}$, In addition, we assume that vacation time random variable V follows exponential distribution with $E(V) = \frac{1}{\beta}$,

The probability generating function of number of customers in the queue

$$S(z) = \frac{(z_0-1)Q\lambda[\beta(\alpha+\gamma)(\alpha+\mu_1+\mu_2)+\mu_1\mu_2\gamma]}{z_0\{\mu_1\mu_2\gamma(Ks\beta-\lambda)-\lambda\beta(\alpha+\gamma)(\alpha+\mu_1+\mu_2)\}}\sum_{n=0}^{\infty}\left(\frac{z}{z_0}\right)^n$$

The idle probability is $$Q = \frac{\{\beta\mu_1\mu_2\gamma Ks - \lambda[\beta(\alpha+\gamma)(\alpha+\mu_1+\mu_2)+\mu_1\mu_2\gamma]}{\beta\mu_1\mu_2\gamma Ks}$$

The mean number of customers in the queue is

$$E(N) = \frac{Q\lambda[(\alpha+\gamma)\beta(\alpha+\mu_1+\mu_2)+\mu_1\mu_2\gamma]}{(z_0-1)\{\mu_1\mu_2\gamma[Ks\beta-\lambda]-\beta\lambda(\alpha+\gamma)(\alpha+\mu_1+\mu_2)\}}$$

PARTICULAR MODEL -02

If we put K = 1,we get a model with batch service of size one.

Thr probability generating function of number of customers in the queue is

$$S(z) = \frac{QJ_1}{J_2}$$

where $$J_1 = \{[m+\alpha z(1-H^*(m))][1-G^*_1(a)G^*_2(a)] + aG^*_1(a)G^*_2(a)[1-B^*(m)]\}$$

$$J_2 = \alpha z[1-G^*_1(a)G^*_2(a)]H^*(m) - a[z-B^*(m)G^*_1(a)G_2(a)]$$

and $$m = \lambda-\lambda C(z), a = \lambda - \lambda C(z) + \alpha$$

The Idle probability is

$$Q = 1-\lambda E(X)\{\frac{1}{\alpha G^*_1(\alpha)G^*_2(\alpha)} - \frac{1}{\alpha} - E(R) + \frac{E(R)}{G^*_1(\alpha)G^*_2(\alpha)} + E(V)\}$$

The mean number of customers in the queue is

$$E(N) = \frac{Q\lambda E(X)L_1}{(z_0-1)L_2}$$

$$L_1 = \{[1-G^*_1(\alpha)G^*_2(\alpha)][1+\alpha E(R)]+\alpha G^*_1(\alpha)G^*_2(\alpha)E(V)\}$$

$$L_2 = \{\alpha G^*_1(\alpha)G^*_2(\alpha)[1-\lambda E(X)E(V)]-\lambda E(X)[1-G^*_1(\alpha)G^*_2(\alpha)][1+\alpha E(R)]\}$$

PARTICULAR MODEL -03

If we put K = 1, and X = 1, we get a model with single arrival queue and the customers are served single.

The probability generating function of number of customers in the queue

$$S(z) = \frac{QJ_1}{J_2}$$

where $$J_1 = \{[m+\alpha z(1-H^*(m))][1-G^*_1(a)G^*_2(a)] + aG^*_1(a)G^*_2(a)[1-B^*(m)]\}$$

$$J_2 = \alpha z[1-G^*_1(a)G^*_2(a)]H^*(m) - a[z-B^*(m)G^*_1(a)G^*_2(a)]$$

and $$m = \lambda - \lambda z, a = \lambda - \lambda z + \alpha$$

The idle probability

$$Q = 1-\lambda\{\frac{1}{\alpha G_1^*(\alpha)G^*_2(\alpha)} - \frac{1}{\alpha} + \frac{E(R)}{G_1^*(\alpha)G^*_2(\alpha)} - E(R) + E(V)\}$$

The mean number of customers in the queue

$$E(N) = \frac{Q\lambda L_1}{(z_0-1)L_2}$$

$$L_1 = \{[1-G_1^*(\alpha)G^*_2(\alpha)][1+\alpha E(R)]+\alpha G_1^*(\alpha)G^*_2 E(V)\}$$

$$L_2 = \{\alpha G_1^*(\alpha)G^*_2(\alpha)[1-\lambda E(V)]-\lambda[1-G^*_1(\alpha)G^*_2(\alpha)][1+\alpha E(R)]\}$$

5. NUMERICAL EXAMPLE

In this section, we present some numerical examples related to the models in section 4. We fix the values of β, α, γ, μ_1, μ_2, K, *s*, *p* and we vary the values of the arrival rate λ. For various values of z_0, we find the values of E(N). Also we find the vlaues of Q. The results are presented in tables 1 to 4. From the values, it is clear that, as the arrival rate increases, the idle probability decreases. Which is very much coincide with our expectations. Aslo the mean number of customers in the queue increases, for increasing values of arrival rate. Again, which is very much coincide with our expectation. Surprisingly in all the models, If the zero z_0 increases from 1.00001 to 15, the mean number of customers in the queue considerably decreases. In addition, if K value increases E(N) value decreases, which is again vermuch coincide with our expectations.

Table 1. The Mean arrival rate versus Q and E(N)

$(\beta = 20, \alpha = 10\ \gamma = 10, \mu_1 = 60, \mu_2 = 60, \mathrm{K} = 10, s = 0.7)$

		E(N)					
		Z_0 values					
λ	Q	1.0001	1.00001	1.5	5	10	15
1	0.9825	174	1746	0.0349	0.0044	0.0019	0.0012
2	0.9651	349	3498	0.0698	0.0087	0.0039	0.0025
3	0.9476	523	5238	0.1048	0.0131	0.0058	0.0037
4	0.9302	698	6984	0.1397	0.0175	0.0078	0.0050
5	0.9127	873	8730	0.1746	0.0218	0.0097	0.0062
6	0.8952	1047	10476	0.2095	0.0262	0.0116	0.0075
7	0.8778	1222	12222	0.2444	0.0306	0.0136	0.0087
8	0.8603	1396	13968	0.2794	0.0349	0.0155	0.0100
9	0.8429	1571	15714	0.3143	0.0393	0.0175	0.0122
10	0.8254	1746	17460	0.3492	0.0437	0.0194	0.0125

Table 2. The mean arrival rate versus Q and E(N)

$(\beta = 20, \alpha = 10\ \gamma = 10, \mu_1 = 60, \mu_2 = 60, \mathrm{K} = 5, s = 0.7)$

		E(N)					
		Z_0 values					
λ	Q	1.0001	1.00001	1.5	5	10	15
1	0.9651	349	3492	0.0698	0.0087	0.0039	0.0025
2	0.9302	698	6984	0.1397	0.0175	0.0078	0.0050
3	0.8952	1047	10476	0.2095	0.0262	0.0116	0.0075
4	0.8603	1397	13968	0.2794	0.0349	0.0155	0.0100
5	0.8254	1746	17640	0.3492	0.0437	0.0194	0.0125
6	0.7905	2095	20953	0.4190	0.0524	0.0233	0.0150
7	0.7556	2444	24445	0.4889	0.0611	0.0272	0.0175
8	0.7206	2793	27937	0.5587	0.0698	0.0310	0.0200
9	0.6857	3143	31429	0.6286	0.0786	0.0349	0.0224
10	0.6508	3492	34921	0.6984	0.0873	0.0388	0.0249

Table 3. The mean arrival rate versus Q and E(N)

$(\beta = 20, \alpha = 10\ \gamma = 10, \mu_1 = 60, \mu_2 = 60, K = 15, s = 0.7)$

		E(N)					
		Z_0 values					
λ	Q	1.0001	1.00001	1.5	5	10	15
1	0.9932	67	679	0.0136	0.0017	0.0007	0.0004
2	0.9864	135	1358	0.0272	0.0034	0.0015	0.0009
3	0.9796	203	2037	0.04407	0.0051	0.0023	0.0015
4	0.9728	271	2716	0.0543	0.0068	0.0030	0.0019
5	0.9660	339	3395	0.0679	0.0085	0.0038	0.0024
6	0.9593	407	4074	0.0815	0.0102	0.0045	0.0029
7	0.9525	475	4753	0.0951	0.0119	0.0053	0.0034
8	0.9457	543	5432	0.1086	0.0136	0.0060	0.0039
9	0.9389	611	6111	0.1222	0.0153	0.0068	0.0044
10	0.9321	679	6790	0.1358	0.0170	0.0075	0.0049

Table 4. The mean arrival rate versus Q and E(N)

$(\beta = 20, \alpha = 10\ \gamma = 10, \mu_1 = 60, \mu_2 = 60, K = 20, s = 0.7)$

		E(N)					
		Z_0 values					
λ	Q	1.0001	1.00001	1.5	5	10	15
1	0.9913	87	873	0.0175	0.0022	0.0009	0.0006
2	0.9825	174	1746	0.0349	0.0044	0.0019	0.0012
3	0.9738	261	2619	0.0524	0.0065	0.0029	0.0019
4	0.9651	349	3492	0.0698	0.0087	0.0039	0.0025
5	0.9563	436	4365	0.0873	0.0109	0.0049	0.0031
6	0.9476	523	5238	0.1048	0.0131	0.0058	0.0034
7	0.9389	611	6111	0.1222	0.0153	0.0068	0.0044
8	0.9302	698	6984	0.1397	0.0175	0.0078	0.0050
9	0.9214	785	7857	0.1571	0.0196	0.0087	0.0056
10	0.9127	873	8730	0.1746	0.0218	0.0097	0.0062

6. CONCLUSTION

In this article, a single server bulk arrival, heterogenous two stage batch service(fixed) queue with compulsory vacation and with unreliable server has been completely analysed. To illustrate the analytical compatability of the model we present some numerical examples by taking particular values to the parameters and particular form to the probability distributions. The model can be extended by taking the break down period as generally distributed.

7. REFERENCES

1. Bacot J.B. and Dshalalow J.H, "A bulk input queueing system with batch gated service and multiple vacation policy", Mathematical and computer modelling, Vol 34, pp 873-886, 2001.
2. Choudhary G, "A batch arrival queue with a vacation time under single vacation policy", Computers and operation research, Vol 29, pp 1941-1955, 2002.
3. Choudhury G. Tads L and Paul M, "The steady state analysis of $M^X/G/1$ queue with two phase of service and Bernoulli vacation, under multiple vacation policy", Applied mathematical modelling, Vol 31(6), pp 1079-1091, 2007.
4. Choudhury G and Deka M, "A single server queueing system with two phases of service subject to server break down and Bernoulli vacation", Applied mathematical modelliing, Vol 36, pp 6050-6060, 2012.
5. Choudhury G and Deka M, "A batch arrival unreliable server Bernoulli vacation queue with two phases of servie delayed repair", International journal of operations research, Vol 10(3), pp 134-152, 2013.
6. Doshi B.T, "Single server queue with vacation; a survey", Queuing systems, Vol I, pp 29-66, 1986.
7. Doshi B.T, "Analysis of a two phase queueing system with general service times", Operation Research letter, Vol 10(5), pp 265-272, 1991.
8. Grey W.L, Wang P.P and Scatt M.K, "A vacation queueing model with service break down", Applied mathematical modelling , Vol 24, pp 391-400, 2000.
9. Haridass M and Arrumuganathan R, "Analysis of a bulk queue with unreliable server and single vacation", Internation journal of open problems compt.math. vol 1(2), pp 130-148, 2008.
10. Kalyanarman R and Ayyappan G, "A single server vacation queue with balking and with single and batch service ", Bulletin of pure and applied science, Vol 19 (2), pp 521-528, 2000.
11. Kalyanaraman R and Nagarajan P, "Bulk arrival, fixed batch service queue with unreliable server and with Bernoulli vacation", International journal of applied engineering research, Vol 11(1), pp 421-429, 2016.
12. Ke J.C, "Modified T Vacation policy for an M/G/1 queuing system with an unreliable server and start up", Mathematical and computer modelling, Vol 41, pp 1267-1277, 2005.
13. Ke J.C, "Operation characteristics analysis of $M^X/G/1$ system with varient vacation policy and balking", Applied mathematical modelling, Vol 31, pp 1321-1337, 2007.
14. Ke J.C, Huang K.B and Peam W.L., "A batch arrival queue under randomized multple vacation policy with unreliable server and repair". International Journal of system science,Vol 13(2), pp 552-565, 2012.
15. Krishna C.M and Lee Y.H, "A study of a two phase service, Operation Research Letter", Vol 9, pp 91-97, 1990.
16. Lee S.S, Lew H.W, Yoon S.H and Chae K.C, "Batch arrival queue with N Policy and single vacation", Computer and operations research, Vol 22, pp 173-189, 1995.
17. Li W, Shi D and Chao X, "Reliablity analysis of M/G/1 queueing system with server breakdown and vacations", Journal of applied probability, Vol 34, pp 546-555, 1997.
18. Selvam D.D and Sivasankaran V, "A two phase queueing system with server vacation", Operation Research letter, Vol 15(3), pp 163-168, 1994.
19. Takagi H, "Queueing Analysis vacation and priority system", North Holland, Amsterdam, Vol 1, 1991.
20. Thangaraj V. and Vanitha S, "M/G/1 queue with two stages of heterogenous service compulsory server vacation and random breakdowns", Internation journal of contemporary mathematical science, Vol 5(7), pp 307-322, 2010.
21. Wang K.H, "Optimal operation of a Markovian queuing system with a removable and non-reliable server", Micro electron Reliab., Vol 35, pp 1131-1136, 1995.
22. Wang K.H, "Optimal control of an $M/E_k/1$ queuing system with removable service station subject to breakdown", Operation research society, Vol 48, pp 936-942, 1997.
23. Wang K.H, Chang K.W and Sivazlian B.D, "Optimal control of removable and non-reliable server in an infinite and finite $M/H_2/1$ queueing system", Applied mathematical modelling, Vol 23, pp 651-666, 1999.