International Review of Pure and Applied Mathematics
Volume 14 • Number 2 • July-December 2018

ON A TYPE OF SEMI-SYMMETRIC METRIC CONNECTION IN A LORENTZIAN BETA-KENMOTSU MANIFOLD

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Abstract: In this paper, we consider a semi-symmetric connection in a Lorentzian beta-Kenmotsu manifold. We investigate the curvature tensor and the Ricci tensor of a Lorentzian beta-Kenmotsu manifold with a semi-symmetric connection. Also we investigate the condition for which Lorentzian beta-Kenmotsu manifolds with a semi-symmetric connection are eta-Einstein manifold. In last we discuss î-projectively flat Lorentzian beta-Kenmotsu manifolds.

Keywords: Kenmotsu manifold, semi-symmetric metric connection, η-Einstein manifold, ξ-projectively flat.

2010 Mathematics Subject Classification: Primary 53C05, 53D15, 53C25.

I. INTRODUCTION

In, 1969 S.Tanno [23] classified connected almost contact metric manifolds whose automorphism groups possess the maximum dimension. For this type of manifold, the sectional curvature of the plane sections containing ξ is a constant, say c. He showed that the sectional curvature of plain sections can be divided three classes:-

1. Homogeneous normal contact Riemannian manifold with $c > 0$.
2. Global Riemannian products of a line or a circle with a Kähler manifold of constant holomorphic sectional curvature if $c = 0$ and last
3. A warped product space RX_fC if $c < 0$.

The manifold of class (1) are characterized by admitting a Sasakian structure. Kenmotsu [19] characterized the differential geometric properties of the manifold of the class (3), the structure so obtained is now known as Kenmotsu structure. In general these structures are not Sasakian [19]. In [16], Gray and Hervella, classification of almost Hermitian manifolds there appears a class W_4 of Hermitian manifolds which are closely related to locally conformal Kählerian manifold [14]. An almost contact metric structure on the manifold M is called a Trans-Sasakian structure [22] if the product manifold $M \times R$ belongs to the class W_4. The class $C_6 \oplus C_5$ ([20], [21]) coincides with the class of trans-Sasakian structure of type (α, β). We note that trans-Sasakian structure of type (0,0), $(0,\beta)$ and $(\alpha,0)$ are cosymplectic [7], β-Kenmotsu [19] and α-Sasakian [19], respectively.

In 2010, Bagewadi, Venkatesha and Sreenivasa [2], studies Lorentzian β-Kenmotsu manifolds. In 1932, Hayden [18] introduced semi-symmetric linear connection on a Riemannian manifold.

Let M be a n-dimensional Riemannian manifold of class C^{-} endowed with the Riemannian metric g and ∇ be the Levi-Civita connection on (M^n, g). In 1924, Friedmann and Schouten [15] introduced a linear connection $\overline{\nabla}$ defined on (M^n, g) is said to be semi-symmetric if its torsion tensor T is the form

$$T(X, Y) = \eta(Y)X - \eta(X)Y, \tag{1.1}$$

where η is 1- form and ξ is a vector field given by

$$\eta(X) = g(X, \xi), \tag{1.2}$$

For all vector fields $X \in \chi(M^n)$, $\chi(M^n)$ is the set of all differentiable vector field on M^n.

A semi-symmetric connection $\overline{\nabla}$ is called a semi-symmetric metric connection [18] if it further satisfies [11].

$$\overline{\nabla} g = 0. \tag{1.3}$$

A relation between the semi-symmetric connection $\overline{\nabla}$ and the Levi-Civita connection ∇ on (M^n, g) has been obtained by Yano [25] which is given by

$$\overline{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X, Y)\xi. \tag{1.4}$$

We also have

$$(\overline{\nabla}_X \eta)(Y) = (\nabla_X \eta)Y - \eta(X)\,\eta(Y) + \eta(\xi)\, g\,(X, Y). \tag{1.5}$$

Further, a relation between the curvature tensor $\overline{R}$ of the semi-symmetric metric connection $\overline{\nabla}$ and the curvature tensor R of the Levi-Civita connection ∇ is given by

$$\begin{aligned}\overline{R}(X, Y)Z = R(X, Y)Z &+ \gamma(X, Z)Y - \gamma(Y, Z)X \\ &+ g(X, Z)QY - g(Y, Z)QX,\end{aligned} \tag{1.6}$$

where γ is a tensor field of type (0,2) and a tensor field Q of type (1,1) is given by

$$\begin{aligned}\gamma(Y, Z) = g(QY, Z) = (\nabla_X \eta)Z &- \eta(Y)\,\eta(Z) \\ &+ (1/2)\,\eta\,(\xi)g(Y, Z)\end{aligned} \tag{1.7}$$

The study of semi-symmetric connection was further developed by Ajit Barman [5], A. Haseeb and R. Prasad [17], Amur and Pujar [1], Binh [6], Chaki and Konar [8], De ([9], [10]), De and Biswas [11], De and De [12], De and De [13] and many others.

The Projective curvature tensor is an important tensor from the differential geometric point of view. Let M be a (2n+1)-dimensional Riemannian manifold. If there exists a one-to-one correspondence between each coordinate neighborhood of M and a domain in Euclidian space such that any geodesic of the Riemannian manifold corresponds to a straight line in the Euclidean space, then M is said to be locally projectively flat. For $n \geq 1$, M is locally projectively flat if and only if the projective curvature tensor vanishes. Here the projectively curvature tensor $\bar{P}$ with respect to the semi-symmetric metric connection is defined by

$$\bar{P}(X,Y)Z = \bar{R}(X,Y)Z - (1/(2n))[\bar{S}(Y,Z)X - \bar{S}(X,Z)Y] \tag{1.8}$$

for $X, Y, Z \in \chi(M)$, where $\bar{S}$ is the Ricci tensor with respect to the semi-symmetric connection.

The present paper is organized as follow: The section 2 is equipped with some preliminaries about Lorentzian β-Kenmotsu manifold. In section 3, we establish the relation of the curvature tensor between the Levi-Civita connection and the semi-symmetric metric connection of a Lorentzian β-Kenmotsu manifold. In section 4, we consider ξ-projectively flat Lorentzian β-Kenmotsu manifolds with respect to the semi-symmetric metric connection.

II. PRELIMINARIES

A differentiable manifold M of dimension $(2n + 1)$ is called Lorentzian β-Kenmotsu manifold if it admits a (1,1)-tensor field φ, a contravariant vector field ξ, a covariant vector field η and Lorentzian metric g which satisfy ([11], [12]):

$$\varphi(\xi) = 0,\ \eta(\varphi X) = 0,\ \eta(\xi) = -1,\ g(X, \xi) = \eta(X), \tag{2.1}$$

$$\varphi^2(X) = X + \eta(X) \tag{2.2}$$

$$g(\varphi X, \varphi Y) = g(X, Y) + \eta(X)\,\eta(Y), \tag{2.3}$$

for any vector field X, Y on M.

Also, Lorentzian β-Kenmotsu manifold M is satisfying:

$$\nabla_X \xi = \beta[X - \eta(X)\xi], \tag{2.4}$$

$$(\nabla_X \eta)(Y) = \beta[g(X,Y) - \eta(X)\,\eta(Y)], \tag{2.5}$$

where denotes the operator of covariant differentiation with respect to the Lorentzian metric g.

Further, On Lorentzian β-Kenmotsu manifold M the following relation holds:

$$\eta(R(X, Y)Z) = \beta^2[g(X, Z)\eta(Y) - g(Y, Z)\eta(X)], \tag{2.6}$$

$$R(\xi, X)Y = \beta^2[\eta(Y)X - g(X, Y)\xi], \tag{2.7}$$

$$R(X, Y)\xi = \beta^2[\eta(X)Y - \eta(Y)X], \tag{2.8}$$

$$S(X, \xi) = -2n\hat{a}^2\varsigma(X), \tag{2.9}$$

$$Q\xi = -2n\beta^2\xi, \tag{2.10}$$

$$S(\xi, \xi) = 2n\beta^2, \tag{2.11}$$

where S is the Ricci tensor of the Levi-Civita connection.

III. CURVATURE TENSOR OF A LORENTZIAN BETA- KENMOTSU MANIFOLD WITH RESPECT TO THE SEMI-SYMMETRIC CONNECTION

Apply (2.1) and (2.5) in (1.7), we get

$$\begin{aligned}\gamma(X, Y) = g(QX, Y)\\ &= (\nabla_X\eta)(Y) - \eta(X)\eta(Y) + (1/2)\eta(\xi)g(X, Y)\\ &= \beta[g(X, Y) - \eta(X)\eta(Y)] - \eta(X)\eta(Y) - (1/2)g(X, Y)\\ &= (\beta-(1/2))g(X,Y) - (\beta+1)\eta(X)\eta(Y).\end{aligned} \tag{3.1}$$

From (3.1), it follows that,

$$QX = (\beta - (1/2))X - (\beta + 1)\eta(X)\xi. \tag{3.2}$$

Again using (3.1) and (3.2) in (1.6), we have

$$\begin{aligned}\bar{R}(X, Y)Z &= R(X, Y)Z + \gamma(X, Z)Y - \gamma(Y, Z)X\\ &\quad + g(X, Z)QY - g(Y, Z)QX\\ &= R(X, Y)Z + (\beta-(1/2))g(X, Z)Y\\ &\quad - (\beta + 1)\eta(X)\eta(Z)Y\\ &\quad - (\beta-(1/2))g(Y, Z)X + (\beta + 1)\eta(Y)\eta(Z)X\\ &\quad + (\beta-(1/2))g(X, Z)Y - (\beta + 1)g(X, Z)\eta(Y)\xi\\ &\quad -(\beta-(1/2))g(Y, Z)X + (\beta + 1)g(Y, Z)\eta(X)\xi,\end{aligned}$$

or,

$$\begin{aligned}\bar{R}(X, Y)Z &= R(X, Y)Z + (2\beta-1)[g(X, Z)Y\\ &\quad - g(Y, Z)X] + (\beta - 1)[\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y\\ &\quad + g(Y, Z)\eta(X)\xi - g(X, Z)\eta(Y)\xi].\end{aligned} \tag{3.3}$$

Taking the inner product of (3.3) with W, we get

$$\bar{R}(X, Y, Z, W) = R(X, Y, Z, W)$$

$$+ (2\beta - 1)[g(X, Z)g(Y, W) - g(Y, Z)g(X, W)]$$
$$+ (\beta - 1)[\eta(Y)\eta(Z)g(X, W) - \eta(X)\eta(Z)g(Y,W)$$
$$+ g(Y, Z)\eta(X)\eta(W) - g(X, Z)\eta(Y)\eta(W)], \tag{3.4}$$

Taking a frame field from (3.4), we obtain

$$\bar{S}(Y, Z) = S(Y, Z) + [(\beta-1)-2n(2\beta-1)]g(Y, Z)$$
$$+ (\beta - 1)(2n + 3)\eta(Y)\eta(Z). \tag{3.5}$$

Putting $Z = \xi$, in (3.5) and using (2.1) and (2.9), we get

$$\bar{S}(Y, \xi) = S(Y, \xi) + [(\beta - 1) - 2n(2\beta - 1)]g(Y, \xi)$$
$$+ (\beta - 1)(2n + 3)\eta(Y)\eta(\xi)$$
$$= -2n\beta^2\eta(Y) + [(\beta-1)-2n(2\beta-1)]\eta(Y)$$
$$- (\beta-1)(2n + 3)\eta(Y).$$
$$\bar{S}(Y, \xi) = [-2n\beta^2 - 6n\beta - 2\beta - 4]\eta(Y). \tag{3.6}$$

Now, contracting the equation (3.5), we get

$$= \bar{r} + (2n + 1) [(\beta - 1) - 2n(2\beta-1)] + (\beta-1)(2n + 3), \tag{3.7}$$

where and $\bar{r}$ are the scalar curvatures of the connections and $\bar{\nabla}$ and ∇ respectively.

From the above discussions, we can state the following theorem:

Theorem 1. For a Lorentzian β-Kenmotsu manifold M with respect to the semi-symmetric metric connection $\bar{\nabla}$

a) The Curvature Tensor $\bar{R}$ is given by

$$\bar{R}(X, Y)Z = R(X, Y)Z + (2\beta-1)[g(X, Z)Y$$
$$- g(Y, Z)X] + (\beta-1)[\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y$$
$$+ g(Y, Z)\eta(X)\xi - g(X, Z)\eta(Y)\xi].$$

b) The Ricci tensor S is given by

$$\bar{S}(Y, Z)= S(Y, Z) + [(\beta-1) - 2n(2\beta - 1)]g(Y, Z)$$
$$+ (\beta-1)(2n + 3)\eta(Y)\eta(Z).$$

c) The Ricci tensor $\bar{S}$ is symmetric.

d) $\bar{S}(Y, \xi) = [-2n\beta^2-6n\beta-2\beta-4]\eta(Y)$.

e) $\bar{r} = r + (2n + 1) [(\beta-1) - 2n(2\beta-1)] + (\beta-1)(2n + 3)$, Where $\bar{r}$ and r are the scalar curvatures of the connections $\bar{\nabla}$ and ∇ respectively.

Writing two more equation by the cyclic permutation of X, Y and Z in equation (3.3), we get

$$\bar{R}(Y, Z)X = R(Y, Z)X + (2\beta-1)[g(Y, X)Z - g(Z, X)Y] + (\beta-1)[\eta(Z)\eta(X)Y - \eta(Y)\eta(X)Z + g(Z, X)\eta(Y)\xi - g(Y, X)\eta(Z)\xi], \tag{3.8}$$

and

$$\bar{R}(Z, X)Y = R(Z, X)Y + (2\beta-1)[g(Z,Y)X - g(X,Y)Z] + (\beta-1)[\eta(X)\eta(Y)Z - \eta(Z)\eta(Y)X + g(X, Y)\eta(Z)\xi - g(Z, Y)\eta(X)\xi]. \tag{3.9}$$

Adding equations (3.3), (3.8) and (3.9) and using Bianchi's first identity. We get,

$$\bar{R}(X,Y)Z + \bar{R}(Y,Z)X + \bar{R}(Z,X)Y = 0 \tag{3.10}$$

Thus, we can state as follows:

Theorem 2. An β-Kenmotsu manifold M^{2n+1} with semi-symmetric metric connection satisfies the equation (3.10).

Now, from equation (3.4), we have

$$\bar{R}(X,Y,Z,W) = R(X,Y,Z,W) + (2\beta-1)[g(X,Z)g(Y,W) - g(Y, Z)g(X, W)] + (\beta-1)[g(X,W)\eta(Y) - g(Y, W)\eta(X)]\eta(Z) + (\beta-1)[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)]\eta(W). \tag{3.11}$$

Now interchanging X and Y in the above equation, we get

$$\bar{R}(X, Y, Z, W) = R(Y, X, Z, W) + (2\beta-1)[g(Y, Z)g(X,W) - g(X, Z)g(Y, W)] + (\beta-1)[g(Y, W)\eta(X) - g(X, W)\eta(Y)]\eta(Z) + (\beta-1)[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)]\eta(W). \tag{3.12}$$

Adding equation (3.11) and (3.12) with the fact that $R(X, Y, Z, W) + R(Y, X, Z, W) = 0$, we get

$$\bar{R}(X,Y,Z,W) + \bar{R}(Y,X,Z,W) = 0. \tag{3.13}$$

Again interchanging Z and W in the equation (3.11), we get

$$\bar{R}(X, Y, W, Z) = \bar{R}(X, Y, W, Z) + (2\beta-1)[g(X,W)g(Y,Z) - g(Y, W)g(X, Z)] + (\beta-1)[g(X, Z)\eta(Y) - g(Y, Z)\eta(X)]\eta(W) + (\beta-1)[g(Y, W)\eta(X) - g(X, W)\eta(Y)]\eta(Z). \tag{3.14}$$

Adding equation (3.11) and (3.14) with the fact that $R(X, Y, Z, W) + R(X, Y, W, Z) = 0$, we get

$$\bar{R}(X,Y,Z,W) + \bar{R}(X,Y,W,Z) = 0. \tag{3.15}$$

Again interchanging the pair of slots in the equation (3.11), we get

$$\bar{R}(Z, W, X, Y) = R(Z, W, X, Y) + (2\beta-1)[g(Z, X)g(W, Y) - g(W, X)g(Z,Y)] + (\beta-1)[g(Z,Y)\eta(W) - g(W,Y)\eta(Z)]\eta(X) + (\beta-1)[g(W, X)\eta(Z) - g(Z, X)\eta(W)]\eta(Y). \tag{3.16}$$

Now subtracting the equation (3.16) from the equation (3.11) with the fact that $R(X, Y, Z, W) - R(Z, W, X, Y) = 0$, we get

$$\bar{R}(X,Y,Z,W) - \bar{R}(Z,W,X,Y) = 0. \tag{3.17}$$

Thus, in the view of equation (3.13), (3.15) and (3.17), we can state as follows:

Theorem 3. The curvature tensor of type (0,4) of semi-symmetric connection in Lorentzian β-Kenmotsu manifold is

1. $\bar{R}(X, Y, Z, W) + \bar{R}(Y, X, Z, W) = 0.$
2. $\bar{R}(X, Y, Z, W) + \bar{R}(X, Y, W, Z) = 0.$
3. $\bar{R}(X, Y, Z, W) - \bar{R}(Z, W, X, Y) = 0.$

Now, Let $\bar{R}(X, Y)Z = 0$, which by virtue of the equation (3.3) yields

$$R(X,Y)Z = (2\beta-1)[g(X,Z)Y - g(Y,Z)X + (\beta-1)[\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y + g(Y, Z)\eta(X)\xi - g(X, Z)\eta(Y)\xi].$$

Taking the inner product of the above equation with ξ, we get

$$\eta(R(X,Y)Z) = (2\beta-1)[g(X,Z)\eta(Y) - g(Y,Z)\eta(X) + (\beta-1)[\eta(Y)\eta(Z)\eta(X) - \eta(X)\eta(Z)\eta(Y) - g(Y, Z)\eta(X) + g(X, Z)\eta(Y)],$$

or,

$$\eta(R(X, Y)Z) = (3\beta-2)[g(X, Z)\eta(Y) - g(Y, Z)\eta(X)]. \tag{3.18}$$

By virtue of equation (2.1) yields

$$R(X, Y)Z = (3\beta - 2)[g(X, Z)Y - g(Y, Z)X].$$

The above equation can be written as:

$$R(X, Y, Z, W) = (3\beta - 2)[g(X, Z)g(Y, W)$$

$$- g(Y, Z)g(X, W)].$$

Thus, we can state as follows:

Theorem 4. An β-Kenmotsu manifold M^{2n+1} with semi-symmetric connection satisfy the above result.

Now, Suppose Ricci tensor of a semi-symmetric metric connection in M vanishes, i.e. $\bar{S}(Y, Z) = 0$, then from the equation (3.5), we have

$$S(Y, Z) = [2n(2\beta-1) - (\beta-1)]g(Y, Z)$$
$$-[(\beta-1)(2n + 3)]\eta(Y)\eta(Z),$$

which is of the form

$$S(Y, Z) = ag(Y, Z) + b\eta(Y)\eta(Z),$$

where $a = 2n(2\beta-1) - (\beta-1)$ and $b = -(\beta-1)(2n+3)$

Thus, from the above discussion, we can state the following:

Theorem 5. If the Ricci tensor of a semi-symmetric metric connection $\bar{\nabla}$ in a Lorentzian β-Kenmotsu manifold vanishes, then the manifold M is η-Einstein manifold.

IV. XI-PROJECTIVELY FLAT LORENTZIAN BETA-KENMOTSU MANIFOLDS WITH RESPECT TO THE SEMI-SYMMETRIC METRIC CONNECTION

A Lorentzian β-Kenmotsu manifolds M with respect to the semi-symmetric metric connection is said to be ξ-projectively flat if

$$\bar{P}(X, Y)\xi = 0, \tag{4.1}$$

for all vector fields X, Y on M. This notion was first defined by Tripathi and Dwivedi [24]. If equation (4.1) just holds for X, Y orthogonal to ξ, we called such a manifold a horizontal ξ-projectively flat manifold.

Using (3.3) in (1.8), we get

$$\begin{aligned}\bar{P}(X, Y)Z &= R(X, Y)Z + (2\beta-1)[g(X, Z)Y - g(Y, Z)X] \\ &+ (\beta-1)[\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y + g(Y, Z)\eta(X)\xi \\ &- g(X, Z)\eta(Y)\xi] - (1/(2n))\,[\bar{S}(Y, Z)X - \bar{S}(X,Z)Y].\end{aligned} \tag{4.2}$$

Putting $Z = \xi$, using (2.1), (2.8) and (3.6) in equation (4.2), we get

$$\begin{aligned}\bar{P}(X, Y)\xi &= R(X, Y)\xi + (2\beta - 1)[g(X, \xi)Y - g(Y, \xi)X] \\ &+ (\beta-1)[\eta(Y)\eta(\xi)X - \eta(X)\eta(\xi)Y + g(Y, \xi)\eta(X)\xi - g(X, \xi)\eta(Y)\xi] \\ &- (1/(2n))[\bar{S}(Y, \xi)X - \bar{S}(X, \xi)Y],\end{aligned}$$

this implies

$$\bar{P}(X, Y)\xi = [\beta^2 + 3\beta - 2 - (1/(2n))(2n\beta^2 + 6n\beta + 2\beta + 4)]\,(\eta(X)Y - \eta(Y)X). \tag{4.3}$$

From (4.3), implies that

$$\bar{P}(X, Y)\xi = 0,\ \forall\, X, Y \text{ orthogonal to } \xi, \tag{4.4}$$

we called such a manifold a horizontal ξ-projectively flat manifold.

Hence, we state the following theorem:

Theorem 6. A (2n+1)-dimensional Lorentzian β-Kenmotsu manifold is horizontal ξ-projectively flat with respect to the semi-symmetric metric connection.

Again using (3.5) in (4.2), we have

$$\begin{aligned}\bar{P}(X,Y)Z = {} & P(X, Y)Z + (2\beta-1)[g(X, Z)Y - g(Y, Z)X] + (\beta-1)\\ & [\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y + g(Y, Z)\eta(X)\xi - g(X, Z)\eta(Y)\xi]\\ & -(1/(2n))[\{(\beta-1) - 2n(2\beta-1)\}\{g(Y, Z)X - g(X, Z)Y\} +\\ & \{(\beta-1)(2n + 3)\}\{\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y\}],\end{aligned} \tag{4.5}$$

where P be the projective curvature tensor with respect to the Levi-Civita connection.

Putting $Z = \xi$ in (4.5) and using (2.1), it follows that

$$\begin{aligned}\bar{P}(X, Y)\xi = {} & P(X, Y)\xi + (2\beta-1)[\eta(X)Y - \eta(Y)X] + (\beta-1)[\eta(X)Y-\eta(Y)X]\\ & - (1/(2n))[\{2n(3\beta-2) + 2\beta-2\}\{\eta(X)Y - \eta(Y)X\}].\end{aligned} \tag{4.6}$$

From (4.6), implies that

$$\bar{P}(X, Y)\xi = P(X, Y)\xi,\ \forall\, X, Y \text{ orthogonal to } \xi. \tag{4.7}$$

In view of above, we state the following theorem:

Theorem 7. A $(2n+1)$-dimensional Lorentzian β-Kenmotsu manifold is horizontal ξ-projectively flat with respect to the semi-symmetric metric connection if and only if the manifold is ξ-projectively flat with respect to the Levi-Civita connection.

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