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ON Q-FINITISTIC SPACES

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There are many types of open covers in fuzzy topology (One can see page 187-188 of Liu-Luo[20]).One among these open covers is called Q-open cover. In this paper, we have introduced the concept of Q-finitistic space by using Q-open cover and studied its various properties.

Key Words and Phrases: *Finitistic Space, Q-open cover, Good extension property, Continuity.*

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INTRODUCTION AND PRELIMINARIES

The concept of finitistic space in general topology was introduced in 1960 by R.G. Swan [10]. The purpose of considering these spaces was to extend Borel Proofs[1] of P.A. Smith fixed point theorems for spheres (Smith [6], [7], [8], [9]). Of course Swan did not name these spaces as finitistic. The term "Finitistic" was used by Bredon in his book [5] and since than it has become a firmly established term. Let (X, δ) be a L-fuzzy topological space and A be an L-fuzzy subset of X. A subfamily μ of δ is said to be an Q-open cover of A in (X, δ) if $\forall$ $x \in Supp(A)$, there exists some $U \in \mu$ such that $x_{A(x)}$ is not less than or equal to U'.μ is said to be Q-open cover of (X, δ) if μ is Q-open cover of $\underline{1}$ in (X, δ). Clearly the definition of Q-open covers in fuzzy topology i.e. when $I = [0,1]$ can be written as: Let (X, δ) be a fuzzy topological space and A be a fuzzy subset of X. A subfamily μ of δ is said to be an Q-open cover of A in (X, δ) if $\forall$ $x \in Supp(A)$,there exists some $U \in \mu$ such that $A'(x) < U(x)$.

The order[2] of a family $\{U_\lambda : \lambda \in \Delta\}$ of subsets, not all empty, of some set X is the largest integer n for which there exists a subset M of Δ with n +1 elements such that $\cap_{\lambda \in M} U_\lambda$ is nonempty, or is ∞ if there is no such largest integer. In general topology, a topological space X is said to be Finitistic ([5], p.111) if each open cover of X has a finite order open refinement.

Let $\Delta \neq \varnothing$ and $A = \{A_\lambda : \lambda \in \Delta\}$ be a family of fuzzy subsets of a nonempty set X. Then order ([3], [4]) of A is defined as under:

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Case-I. When $A_\lambda \neq \underline{0}$ for atleast one value of λ in Δ. Then the order of A is the largest nonnegative integer n for which there exists a subset M of Δ having n+1 elements such that $\wedge_{\lambda\in M} A_\lambda \neq \underline{0}$ or is ∞ if there is no such largest integer n.

Case-II. When $A_\lambda = \underline{0}$ for all $\lambda\in\Delta$. Then the order of A is –1. An L-fuzzy topological space (or simply L-topological space) (X, δ) is said to be finitistic [4] if each open cover of (X, δ) has a finite order open refinement. All the other undefined terms on fuzzy topology which have used in this paper can easily be seen in [19] or in [20].

MAIN DEFINITIONS AND RESULTS

Definition 2.1. Let (X, δ) be a fuzzy topological space and A be a fuzzy subset of X. A is said to be Q-finitistic in (X, δ) if every Q-Open Cover of A in (X, δ) has a finite order Q-open refinement. A fuzzy topological space (X, δ) is said to be Q-finitistic if $\underline{1}$ is Q-finitistic in (X, δ).

Theorem 2.2. Let (X, T) be a general topological space and $A\subset X$. Then A is finitistic in (X, T) if and only if χ_A is Q-finitistic in (X, χ(T)) where χ is characteristic Functor from **Top** to F-**Top.**

Proof. Here (X, T) be a general topological space and $A\subset X$ and (X, χ(T)) is fuzzy topological space where χ_A is fuzzy subset of X and $\chi(T) = \{\chi_{U:} U\in T\}$.Let $\mu = \{\chi_{U\lambda:}\ \lambda\in\Lambda\}$ be any Q-open cover of χ_A in (X, χ(T)). We show that $\nu = \{ U_\lambda: \chi_{U\lambda}\in\mu\}$ is an open cover of A in (X, T). Let $x\in A$. Then $\chi_A(x) = 1 > 0$. But $\chi_A(x) > 0 \Rightarrow x\in Supp(\chi_A)$ and μ is Q –open cover of χ_A in (X, χ(T)) $\Rightarrow$there exists some $\chi_{U\lambda x}\in\mu$ such that $\chi'_A(x) < \chi_{U\lambda x}(x)$. But $\chi'_A(x) < \chi_{U\lambda x}(x) \Rightarrow \chi_{U\lambda x}(x) = 1 \Rightarrow x\in U_{\lambda x} \Rightarrow \nu$ is an open cover of A in (X, T). Since A is finitistic in (X, T), therefore ν has a finite order open refinement say $\nu_1 = \{V_\alpha: \alpha\in\Delta\}$. We can easily show that $\mu_1 = \{\chi_{V\alpha}: V_\alpha\in\nu_1\}$ is finite order Q-open refinement of μ. Hence χ_A is Q-finitistic in (X, χ(T)). Similarly it can be easily shown that if χ_A is Q-finitistic in (X, χ(T)), then A is finitistic in (X, T).

Theorem 2.3. Let (X, T) be a general topological space. Then (X, T) is finitistic if and only if (X, χ(T)) is Q-finitistic where χ is characteristic Functor from **Top** to F-**Top.**

Proof. Since $\underline{1} = \chi_X$ and $\underline{1}_{(0)} = X$. Hence proof follows by Theorem 2.2.

Theorem 2.4. Let (X, δ) be a weakly induced fuzzy topological space and A be a fuzzy subset of X. Then A is Q-finitistic in (X, δ) if and only if $A_{(0)}$ is finitistic in (X, [δ]).

Proof. Let A be Q-finitistic in (X, δ). We have to show that $A_{(0)}$ is finitistic in (X, [δ]). Let $\mu = \{U_\lambda: \lambda\in\Lambda\}$ be any open cover of $A_{(0)}$ in (X, [δ]). By definition of [δ] each $\chi_{U\lambda}$ is a fuzzy open subset of X in (X, δ). We show that $\nu = \{\chi_{U\lambda}: U_\lambda\in\mu\}$is an Q-open cover of A in (X, δ). Let $x \in Supp(A)$. Then $A(x) > 0$. But $A(x) > 0 \Rightarrow x \in A_{(0)}$. Since $\mu = \{U_\lambda : \lambda \in \Lambda\}$ be any open cover of $A_{(0)}$ in $(X, [\delta])$, there exists some $U_{\lambda x} \in \mu$ such that $x \in U_{\lambda x}$. Now $x \in U_{\lambda x} \Rightarrow \chi_{U\lambda x}(x) = 1 \Rightarrow A'(x) < 1 = \chi_{U\lambda x}(x) \Rightarrow A'(x) < \chi_{U\lambda x}(x)$. Clearly $\chi_{U\lambda x} \in \nu$. Hence ν is Q-open cover of A in (X, δ). Since A is Q-finitistic in (X, δ), therefore ν has a finite order open refinement say $\nu_1 = \{W_\alpha : \alpha \in \Delta\}$. We show that $\mu_1 = \{(W_\alpha)_{(0)} : W_\alpha \in \nu_1\}$ is finite order open refinement of μ. Since (X, δ) is weakly induced, therefore each $(W_\alpha)_{(0)}$ is an open subset of X in $(X, [\delta])$. Let $x \in A_{(0)}$. Then $A(x) > 0$. But $A(x) > 0 \Rightarrow x \in Supp(A)$. Since ν_1 is Q- open cover of A in (X, δ), therefore there exists some $W_\alpha \in \nu_1$ such that $A'(x) < W_\alpha(x) \Rightarrow W_\alpha(x) > A'(x) \geq 0 \Rightarrow W_\alpha(x) > 0 \Rightarrow x \in (W_\alpha)_{(0)} \Rightarrow \mu_1$ is an open of $A_{(0)}$ in $(X, [\delta])$. Now we show that order of μ_1 is finite. Here order of ν_1 is finite. Let order of $\nu_1 = m$. Let $\{(W_1)_{(0)}, (W_2)_{(0)}, (W_3)_{(0)}, \ldots, (W_{m+2})_{(0)}\}$ be any subfamily of μ_1 having m+2 elements. We have to show that $\cap^{m+2}_{i=1}(W_i)_{(0)} = \emptyset$. Let $\cap^{m+2}_{i=1}(W_i)_{(0)} \neq \emptyset$. Then there exists some $x \in \cap^{m+2}_{i=1}(W_i)_{(0)}$. But $x \in \cap^{m+2}_{i=1}(W_i)_{(0)} \Rightarrow x \in (W_i)_{(0)}$ for all $i = 1, 2, 3, \ldots \ldots, m+2 \Rightarrow W_i(x) > 0$ for all $i = 1,2,3,\ldots\ldots,m+2 \Rightarrow \wedge^{m+2}_{i=1} W_i(x) > 0 \Rightarrow \wedge^{m+2}_{i=1} W_i(x) \neq 0 \Rightarrow$ order of ν_1 is exceeding m. This is a contradiction. Hence order of μ_1 is not exceeding m. It means $A_{(0)}$ is finitistic in $(X, [\delta])$.

Converse. Suppose $A_{(0)}$ is finitistic in $(X, [\delta])$. We have to show that A is Q-finitistic in (X, δ). Let $\mu = \{U_\lambda : \lambda \in \Lambda\}$ be any Q-open cover of A in (X, δ). Let $x \in A_{(0)}$. Then $A(x) > 0$. But $A(x) > 0 \Rightarrow x \in Supp(A)$. Since μ is Q-open cover of A, there exists some $U_{\lambda x} \in \mu$ such that $A'(x) < U_{\lambda x}(x)$. Let $\mu_1 = \{U_{\lambda x} : x \in Supp(A)\}$. Then clearly $\mu_1 \subset \mu$ and μ_1 is also a Q-open cover of A in (X, δ). Let $\nu = \{(U_{\lambda x})_{(A'(x))} : U_{\lambda x} \in \mu_1\}$. We show that ν is an open cover of $A_{(0)}$ in $(X, [\delta])$. Clearly each $(U_{\lambda x})_{A'(x)} \in [\delta]$ because (X, δ) is weakly induced. Let $x \in A_{(0)}$. Then $A(x) > 0$. But $A(x) > 0 \Rightarrow x \in Supp(A) \Rightarrow A'(x) < U_{\lambda x}(x) \Rightarrow U_{\lambda x}(x) > A'(x) \Rightarrow x \in (U_{\lambda x})_{(A'(x))}$. It means ν is an open cover of $A_{(0)}$ in $(X, [\delta])$. Since $A_{(0)}$ is finitistic in $(X, [\delta])$, therefore ν has a finite order open refinement say $\nu_1 = \{W_\alpha : \alpha \in D\}$. Let $x \in Supp(A)$. Then $A(x) > 0 \Rightarrow x \in A_{(0)}$. Since ν_1 is an open cover of $A_{(0)}$, there exists some $W_{\alpha x} \in \nu_1$ such that $x \in W_{\alpha x}$. Let $\mu_2 = \{\chi_{W\alpha x} \wedge U_{\lambda x} : x \in Supp(A)\}$. Without loss of generality we can assume that $\chi_{W\alpha x}$ is not repeated in μ_2. Then it can be easily checked that μ_2 is a finite order Q-open refinement of μ in (X, δ). Hence A is Q-finitistic in (X, δ).

Theorem 2.5. Let (X,T) be a general topological space. Then (X,T) is finitistic if and only if $(X, \chi(T))$ is Q-finitistic where χ is characteristic Functor from **Top** to F-**Top.**

Proof. We know that $\chi_X = \underline{1}$. By Theorem 2.2, X is finitistic in (X, T) if and only if χ_X is Q-finitistic in $(X, \chi(T))$. It means (X,T) is finitistic if and only if $(X, \chi(T))$ is Q-finitistic.

Theorem 2.6. Let (X, T) be a general topological space and A be a fuzzy subset of X in $(X,\omega(T))$. A is Q-finitistic in $(X, \omega(T))$ if and only if $A_{(0)}$ is finitistic in (X, T) where ω is the Lowen Functor from **Top** to F-**Top.**

Proof. Since (X, ω(T)) is induced fuzzy topological space and we know every induced fuzzy topological space is weakly induced, therefore (X, ω(T)) is weakly induced. Here [ω(T)] = T. Hence by Theorem 2.4, $A_{(0)}$ is finitistic in (X, T) if and only if A is Q-finitistic in (X, ω(T)).

Theorem 2.7. Q-finitisticness is good extension property of finitisticness in general topology.

Proof. We know that $1_{(0)}$ = X. By Theorem 2.6, $\underline{1}$ is Q-finitistic in (X, ω(T)) if and only if $\underline{1}_{(0)}$ is finitistic in (X,T). It means (X,T) is finitistic if and only if (X, ω(T)) is Q-finitistic.

Theorem 2.8. Every closed fuzzy subset of a Q-finitistic space is Q-finitistic.

Proof. Let (X, δ) be a Q-finitistic space and A be a closed fuzzy subset of X in (X, δ). We have to show that A is Q-finitistic. Let $\mu = \{U_\lambda : \lambda \in \Lambda\}$ be any Q-open cover of A. We show that $\nu = \{U_\lambda : \lambda \in \Lambda\} \cup \{A'\}$ is a Q-open cover of (X, δ). Let $x \in X$. Then two cases arise:

Case-I. When $x \in$ Supp(A). Then A(x) > 0. But A(x) >0 $\Rightarrow$ A′(x) < 1. But $x \in$ Supp(x) $\Rightarrow$ there exists some $U_{\lambda x} \in \mu$ such that $A'(x) < U_{\lambda x}(x) \Rightarrow U_{\lambda x}(x) > A'(x) \geq 0 = \underline{1}'(x) \Rightarrow \underline{1}'(x) < U_{\lambda x}(x)$.

Case-II. When $x \notin$ Supp(A). Then A(x) = 0. But A(x) = 0$\Rightarrow$ A′(x) = 1>0 = $\underline{1}'(x) \Rightarrow \underline{1}'(x) < A'(x)$. From cases-I and II, we conclude that for all $x \in$ Supp($\underline{1}$) = X, there exists some $U \in \nu$ such that $\underline{1}'(x) < U(x)$. Hence ν is Q-open cover of (X,δ). Since (X, δ) is Q-finitistic, therefore ν has a finite order Q-open refinement say $\nu_1 = \{W_\alpha : \alpha \in \Delta\}$. Let $\mu_1 = \nu_1 - \{W_\alpha \in \nu_1 : W_\alpha \leq A'\}$. It can be easily shown that μ_1 is a finite order Q-open refinement of μ. Hence A is Q-finitistic in (X, d).

Theorem 2.9. Every closed subspace of a Q-finitistic space is Q-finitistic.

Proof. Let (X, δ) be a Q-finitistic space and (Y, $\delta|_Y$) be a closed subspace of (X, δ). We have to show that (Y, $\delta|_Y$) Q-finitistic.Let $\mu = \{U_\lambda : \lambda \in \Lambda\}$ be any Q-open cover of (Y, $\delta|_Y$). Then for all $U_\lambda \in \mu$, there exists V_λ such that $U_\lambda = V_\lambda|_Y$. We show that $\nu = \{V_\lambda : U_\lambda = V_\lambda|_Y$ where $U_\lambda \in \mu\} \cup \{\chi_{Y'}\}$ is Q-open cover of (X, δ). Clearly each member of ν is fuzzy open subsets of X in (X, δ). Let $x \in$ Supp($\underline{1}$)= X. Then two cases arise:

Case-I. When $x \in Y$. Then there exists some $U_{\lambda x} \in \mu$ such that $\underline{1}'(x) < U_{\lambda x}(x)$.

Case-II. When $x \notin Y$. Then $x \in Y'$. But $x \in Y' \Rightarrow \chi_{Y'}(x) = 1>0 = \underline{1}'(x) \Rightarrow \underline{1}'(x) < \chi_{Y'}(x)$. Thus ν is a Q-open cover of (X, δ). Since (X, δ) is Q-finitistic, therefore ν has a finite order Q-open refinement say $\nu_1 = \{W_\alpha : \alpha \in \Delta\}$. Then clearly $\mu_1 = \{W_\alpha|_Y : W_\alpha \in \nu 1\}$ is finite order Q-open refinement of μ. Hence (Y, $\delta|_Y$) is Q-finitistic.

Theorem 2.10. Every Q-compact fuzzy subset is Q-finitistic.

Proof. Let (X, δ) be a fuzzy topological space and A be a Q-compact fuzzy subset of X in (X, δ). Let $\mu = \{U_\lambda : \lambda \in \Lambda\}$ be any Q-open cover of A in (X, δ). Since A is Q-compact, therefore μ has a finite Q-open subcover say $\nu = \{U_1, U_2, U_3, \ldots, U_n\}$. Then clearly $\nu_1 = \{\underline{0}, U_1, U_2, U_3, \ldots, U_n\}$ is finite order Q-open refinement of μ. Hence A is Q-finitistic.

Remark 2.11. Converse of above theorem is not true. See following example:

Example 2.12. Let X be an infinite set. Let $\delta = \{\chi_U : U \subset X\}$. Then clearly (X, δ) is a fuzzy topological space and $\underline{1}$ is Q-finitistic in (X, δ) because the family $\{\chi_{\{x\}} : x \in X\}$ is zero order Q-open refinement of every Q-open cover of $\underline{1}$ in (X, δ). But $\underline{1}$ is not Q-compact in (X, δ) because the Q-open cover $\{\chi_{\{x\}} : x \in X\}$ of $\underline{1}$ has no finite Q-open subcover.

Theorem 2.13. If supp(A) is finite, then A is Q-finitistic.

Proof. Let (X, δ) be a fuzzy topological space and A be a fuzzy subset of X in (X, δ) such that supp(A) is finite. We have to show that A is Q-finitistic. Let $\mu = \{U_\lambda : \lambda \in \Lambda\}$ be any Q-open cover of A in (X, δ). Since supp(A) is finite, we can write supp (A) = $\{x_1, x_2, x_3, \ldots, x_n\}$. Since μ is a Q-open cover of A in (X, δ), for all $x_i \in$ supp(A), there exists some $U_{\lambda i} \in \mu$ such that $A'(x) < U_{\lambda i}(x)$. Then clearly $\nu = \{\underline{0}, U_{\lambda 1}, U_{\lambda 2}, U_{\lambda 3}, \ldots, U_{\lambda n}\}$ is finite order Q-open refinement of μ. Hence A is Q-finitistic.

Remark 2.14. Converse of above theorem is not true. See following example:

Example 2.15. Let X be an infinite set. Let $\delta = \{\chi_U : U \subset X\}$. Then clearly (X,δ) is a fuzzy topological space and $\underline{1}$ is Q-finitistic in (X, δ) because the family $\{\chi_{\{x\}} : x \in X\}$ is zero order Q-open refinement of every Q-open cover of $\underline{1}$ in (X, δ). But supp($\underline{1}$) = X which is not finite.

Theorem 2.16. Let X be a fuzzy topological space such that order of each element of supp(A) is finite in (X, δ). Then A is Q-finitistic in (X, δ).

Theorem 2.17. Join of two Q-finitistic fuzzy subsets of a set is again a Q-finitistic.

Proof. Let (X, δ) be a fuzzy topological space and A, B be two Q-finitistic fuzzy subsets of X in (X, δ). We have to show that A∨B is again a Q-finitistic subset of X in (X, δ). Let $\mu = \{U_\lambda : \lambda \in \Lambda\}$ be any Q-open cover of A∨B in (X,δ). It can be easily checked that μ is Q-open cover of A as well as Q-open cover of B. Since both A and B are Q-finitistic, therefore μ has two finite order Q-open refinements say μ_A and μ_B. Let $\nu = \mu_A \vee \mu_B$. Then clearly ν is finite order Q-open refinement of μ. Hence A∨B is Q-finitistic fuzzy subset of X in (X, δ).

Remark 2.18. Join of arbitrary family of Q-finitistic fuzzy subsets need not be Q-finitistic.

Example 2.19. Let X be an infinite set. Let T be a general topology on X such that (X, T) is not finitistic. Then $(X,\chi(T))$ is not Q-finitistic by Theorem 2.3. Let $x\in X$. Then $\chi_{\{x\}}$ is Q-finitistic fuzzy subset of X in $(X,\chi(T))$ but $\vee_{x\in X}\chi_{\{x\}} = \chi_X = \underline{1}$ is not Q-finitistic in $(X,\chi(T))$ because $(X,\chi(T))$ is not Q-finitistic.

Remark 2.20. Continuous image of Q-finitistic space need not be Q-finitistic.

Example 2.21. Let (X, D) be a discrete general topological space where X is an infinite set. Let T be a topology on X such that (X, T) is not finitistic. Then by Theorem 2.3, $(X, \chi(D))$ is Q-finitistic where $(X, \chi(T))$ is not Q-finitistic. Let I: $X\to X$ defined as I(x) = x. Then clearly I: $(X, \chi(D)) \to (X, \chi(T))$ is continuous and onto. It means $(X, \chi(T))$ is continuous image of $(X, \chi(D))$. Here $(X, \chi(D))$ is Q-finitistic but $(X, \chi(T))$ is not Q-finitistic.

Theorem 2.22. Homeomorphic image of Q-finitistic space is Q-finitistic.

Proof. Let (X, δ_1) be a Q-finitistic space and f : $(X, \delta_1) \to (Y, \delta_2)$ be a homoeomorphism. We have to show that (Y, δ_2) is Q-finitistic. Let $\mu = \{U_\lambda : \lambda \in \Delta\}$ be any Q-open cover of (X, δ_2). We show that $\nu = \{ U_\lambda f \,;\, U_\lambda \in \mu \}$ is a Q-open cover of (X, δ_1). Since f is continuous, therefore each $U_\lambda f$ is an L-fuzzy open set in (X, δ_1). Also let $x\in supp(\underline{1}) =X$. Then $f(x) \in Y = supp(\underline{1})$. Since $\mu = \{U_\lambda : \lambda \in\Delta\}$ is a Q-open cover of (Y, δ_2),there exists some $U_\lambda \in\mu$ such that $\underline{1}'(f(x))< U_\lambda(f(x))$. But $\underline{1}'(f(x))< U_\lambda(f(x)) \Rightarrow \underline{1}< U_\lambda f(x) \Rightarrow \underline{1}(x) < U_\lambda f(x) \Rightarrow \nu = \{U_\lambda f \,;\, U_\lambda \in \mu\}$ is an Q-open cover of (X, δ_1). Since (X, δ_1) is Q-finitistic, therefore $\nu = \{U_\lambda f : U_\lambda \in \mu \}$ has a finite order Q-open refinement say $\nu_1 = \{V_\beta : \beta \in \Delta_1\}$. Again since $f^{-1}: (Y, \delta_2) \to (X, \delta_1)$ is continuous, it can be easily checked that $\mu_1 = \{V_\beta f^{-1} : V_\beta \in V_1\}$ is a finite order Q-open refinement of μ. Hence (Y, δ_2) is Q-finitistic in (Y, δ_2).

Theorem 2.23. Let $\{(X_t , \delta_t), t \in T\}$ be a family of fuzzy topological spaces such that $(X, \oplus_{t\in T}\delta_t)$ is Q-finitistic. Then (X_t, δ_t) is Q-finitistic $\forall\, t\in T$.

Proof. Here $X= \cup_{t\in T} X_t$ where X_t's are disjoint. Suppose $(X,\oplus_{t\in T}\delta_t)$ is Q-finitistic. Let $\mu_t = \{U_\lambda : \lambda \in\Delta\}$ be any Q-open cover of (X_t, δ_t). $\forall\, U_\lambda \in\mu_t$, define $R_\lambda = U_\lambda$ on X_t and $R_\lambda = \underline{1}$ on $X\text{-}X_t$ where $X = \cup_{t\in T} X_t$. The clearly μ, the family of all R_λ's is an Q-open cover of $(X, \oplus_{t\in T}\delta_t))$. Since $(X, \oplus_{t\in T}\delta_t)$ is Q-finitistic, therefore μ has a finite order Q-open refinement say $V = \{V_\beta : \beta \in \Delta_1\}$. Then clearly $V_t = \{V_\beta|_{X_t} : V_\beta \in V\}$ is finite order Q-open refinement of μ_t. Hence (X_t, δ_t) is Q-finitistic $\forall\, t \in T$.

Theorem 2.24. Let (X, δ_1) and (Y, δ_2) be two Q-finitistic spaces. Then $(X\cup Y, \delta_1 \oplus \delta_2)$ is Q-finitistic.

Proof. Let $\mu =\{U_\lambda : \lambda\in\Delta\}$ be any Q-open cover of $(X\cup Y, \delta_1 \oplus\delta_2)$. Then clearly $\mu_x = \{U_\lambda|_X : U_\lambda \in\mu\}$ and $\mu_Y = \{U_\lambda|_Y : U_\lambda \in\mu\}$ are Q-open covers of (X, δ_1) and (Y, δ_2) respectively. Since both (X, δ_1) and (Y, δ_2) are Q-finitistic, therefore μ_X and μ_Y have finite order Q-open refinements say V_X and V_Y. Define $R_\alpha = V_\alpha$ on X and $R_\alpha = \underline{0}$ on Y and $S_\beta = W_\beta$ on Y and $S_\beta = \underline{0}$ on X where $V_\alpha \in V_X$ and $W_\beta \in V_Y$. Then clearly the family V of all $R_{\alpha's}$ and $S_{\beta's}$ defined above is a finite order Q-open refinement of μ. Hence $(X \cup Y, \delta_1 \oplus \delta_2)$ is Q-finitistic.

Theorem 2.25. The sum space $(X \cup Y, \delta_1 \oplus \delta_2)$ is Q-finitistic if and only if (X, δ_1) and (Y, δ_2) are Q-finitistic. Proof follows by Theorem 2.23 and Theorem 2.24.

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