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Financial Instability and Effects of Monetary Policy

TOSHIO WATANABE

Abstract: Keynes and Minsky emphasize the effects of instability in the financial markets. We represent bank behavior and household portfolio preferences explicitly and investigate monetary policy effects on economic stabilization. Our model comprises dynamic equations for both the debt-capital ratio and the interest rate monetary policy. We show that the economy becomes unstable when the equity demand from households is sensitive to the debt-capital ratio. Further, we indicate that it is hard to change an unstable state into a stable state by changing monetary policy alone. We point out the need for financial regulations to make central bank policy effective.

Keywords: Bank behavior, Debt-capital ratio, Minsky's financial instability hypothesis, Monetary policy

JEL classification: E12, E44, E52

INTRODUCTION

Due to several global financial crises, there is discussion on the interactions between the financial markets and real economy. Keynes (1936) rejected accounts from classical economics and emphasized the effects of instability in the financial markets on the real economy. Minsky (1975) developed his own ideas on financial crises based on his interpretations of Keynes' theory. He developed a new theory, known as the financial instability hypothesis, which states that the financially dominated capitalist economy is inherently unstable. His theory covers both microeconomic and institutional aspects.

In earlier contributions, Taylor and O'Connell (1985) incorporate Minsky's ideas into a mathematical model, which takes into account long-run expectations and household portfolios. More recent studies include Hein (2007) and Charles (2008, 2016), who set the Kaleckian investment function

* Fukui Prefectural University, 4-1-1 Matsuoka-Kenjojima, Eiheiji-Town, Fukui 910-1195, Japan. Email: toshio@fpu.ac.jp

with financial factors to show the endogenous instability of the economy. In this study, we focus on the following three points based on the ideas of Keynes and Minsky.

First, Minsky stressed the role of financial asset structures in a capitalist economy, saying, "In a capitalist economy, one way every economic unit can be characterized is by its portfolio: the set of tangible and financial assets it owns, and the financial liabilities on which it owes" (Minsky 1975, p. 70). Moreover, "[t]he higher leverage ratio of banks was part of the process that moved the economy towards financial fragility because it facilitated an increase in short-term borrowing (and leverage) by bank customers" (Minsky 1986, p. 238). In light of these considerations, we integrate a stock-flow monetary accounting framework and present a consistent set of sectoral and national balance sheets where every financial asset is exposed to counterpart liability. [1] This formulation is consistent with the stock-flow consistent (SFC) approach.

Second, Keynes and Minsky emphasized the change in risk estimation and the influence of the interest rate on the real economy. They insisted that booms induce investors and banks to adopt more speculative financial arrangements. According to Keynes, "During a boom, the popular estimation of both borrower's risk and lender's risk is apt to become unusually and imprudently low" (Keynes 1936, p. 145). Minsky also makes the following remark:

> The instability of a financial regime heavily weighted by speculative and Ponzi finance is due to the impact of changing interest rate[s] that develop as an investment boom matures. … Rising interest rates diminish or eliminate the margins of safety that make the financing of investment possible. This tends to force units to decrease investments or sell out positions. (Minsky, 1986, p. 214)

Furthermore, Minsky focused on the role of the bank as a lender and emphasized that economic fluctuations are magnified through the expansion or contraction of bank credit. Thus, this study formulates the credit creation of the bank.

Finally, we analyze the effects of the monetary policy. We use the discount rate of the central bank as a policy variable.[2] This model sets up a dynamic equation of the discount rate and investigates the effects on economic stability. The discount rate is determined according to the countercyclical policy of the central bank.

We first construct a short-term static model and then extend it to a dynamic model incorporating equations for both the debt-capital ratio and the monetary policy rule. Using the static model, we demonstrate that when the debt-capital ratio rises, the rate of profit decreases. This result fits "the paradox of debt" theory and is similar to Lavoie (1995) but different from Ryoo (2013a,b). However, the effect of the debt-capital ratio on the bank lending rate is undetermined in general and depends on the bank's behavior. From the dynamic model, we show that the economy becomes unstable when equity demand from households is sensitive to the debt-capital ratio. To make the economy stable, both the government and central bank must implement regulations to promote a moderate reaction by the households to shocks. Moreover, even if the reaction of the households is normal, the economy remains unstable if the central bank does not choose an appropriate adjustment speed of the discount rate. The adjustment speed depends on several economic factors, including the sensitivity of the investment and the bank lending with respect to the profit rate. The central banker should recognize that the economy becomes stable when both a moderate action by economic agents and an appropriate monetary policy are simultaneously performed.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature and presents an overview of our model. Section 3 discusses the behavior of firms, banks, and households and explicitly derives the borrowing demand function and the bank lending function. Section 4 examines equilibrium in the commodity market, bank lending market, and equity market, and analyzes the short-run equilibrium of the economy. Section 5 investigates economic instability and constructs a dynamic system considering the effects of monetary policy. Section 6 summarizes the results.

LITERATURE REVIEW AND MODEL FRAMEWORK

Literature Review and Features of Our Model

Minsky's ideas led to the development of various mathematical models. Some early contributions are Taylor and O'Connell (1985), Dutt (1995), and Lavoie (1995). Taylor and O'Connell (1985) show that an economy will fall into a financial crisis if the decline in the expected profit rate deteriorates the financial condition of firms and increases household preferences for liquidity. More recent studies include Hein (2007), Lima and Meirelles (2007), Isaac and Kim (2013), and Charles (2008, 2016). They set the Kaleckian investment function with financial factors, such as the interest rate and debt-capital ratio, to show the inherent instability of the economy. Ryoo (2013a) contributes to the post-Keynesian literature by incorporating an active role of profit-seeking banks.[3] Ryoo builds the model which includes dynamic equations for both the net worth ratio of the bank and the debt ratio of firms to explain the emergence of long waves through the financial interaction between firms and banks. However, Kindleberger (1978) and Lavoie (1997) characterize Minsky's financial instability hypothesis as inherited from classical economics.[4] Regarding the interest rate, their ideas are different from the post-Keynesian economists, which generally belong to the "horizontalist" tradition and involve models where the interest rate is exogenous.

We formalize the risk estimation, bank behavior, and financing of the firm for a better understanding of the relationship between the real economy and the financial economy. We use the Keynesian investment function with long-run expectations and financial factors and examine the importance of financial intermediaries in accelerating booms and crises. We follow the insights from classical economics and endogenize the bank lending rate.

Finally, we investigate the effects of monetary policy. Hein (2007) examines the influence of interest policy on several variables in the steady state, showing that the effect of interest rate variations on the steady state depends on not only the parameter values in the saving and investment functions but also the interest elasticity of the distribution and the initial conditions regarding the interest rate and the debt-capital ratio. Watanabe (2016) investigates the effects on economic stability and shows that the adjustment of the discount rate can shift the saddle path such that it is consistent with the initial condition of the economy. However, the supposed monetary policy is temporary. Furthermore, it is still fragile, as the economy essentially remains a knife-edge property. Hence, we consider a more convincing monetary policy to explicitly specify the adjustment of the discount rate over time, depending on the state of the economy, and transform the economy from instability to stability.

Model framework

Our approach is similar to Lavoie (1995) and Ryoo (2013a), with some distinctive features. We model the bank lending and equity market explicitly and assume that the market determines the interest rate. The economic system in this study comprises four sectors (firms, banks, households, and the central bank) and six markets (commodities, bank lending, equity, deposits, cash currency, and central bank advances).

A firm first decides on a capital accumulation rate. It then raises equity and normally pays a dividend. The investment is financed through retained earnings, issuance of new equity shares, or bank borrowing. We suppose that the firm raises the equity to maintain a constant equity-capital ratio level for the medium-and long-term. Regarding borrowing, we assume that there is a time lag between borrowing and interest payments, and the firm pays interest on the previous borrowing in the current period. The bank makes lending decisions to maximize profits. We assume that the deposit and discount rates are exogenous. The bank accepts all deposits from households. The discount rate is exogenously determined as a tool of monetary policy, and the central bank supplies fund that bank's request. We assume that the profit of the bank is distributed to the households via the bank's labor costs and other factors. Moreover, the revenue of the central bank and the transaction costs of the banks also belong to the households.[5] The entire national income, except retained earnings, belongs to the households. The household saves money via deposits, equity, or cash currency.

The budget constraints for each sector describe how the balance between expenditure flows, factor income, and transfers generates counterpart changes in the stocks of assets and liabilities. Table 1 presents the balance sheet matrix of this economy. Table 2 provides the flow matrix that describes the transactions between the four sectors of the economy.[6]

We build portfolios for each sector and investigate their effects on the economy. Our dynamic model explicitly accounts for time flows and the relationship between stock and flow variables. We then construct a discrete-time model that incorporates microeconomic foundations. [7]

Table 1. Balance Sheets

Central Bank		*Firm*	
Central Bank Loans A	*Bank Reserves* R	*Capital* pK	*Debt* L
	Cash Currency J		*Equity* qE
			Net Worth Z
Private Bank		***Household***	
Loans L	*Deposits* D	*Cash Currency* J	*Wealth* W
Bank Reserves R	*Borrowings from Central Bank* A	*Deposits* D	
		Equity qE	

Table 2. Transaction Matrix

		Firms				
	Households	*Current*	*Capital*	*Banks*	*Central Bank*	Σ
Consumption	$-pC$	$+pC$				*0*
Investment		$+pI$	$-pI$			*0*
Wage	$+wN+G$	$-wN$		$-G$		*0*
Net Profit		$-F$	$+F$			*0*
Interest on Loans		$-iL$		$+iL$		*0*
Interest on Deposits	$+i^dD$			$-i^dD$		*0*
Interest on Borrowings from Central Bank				$-i^aA$	$+i^aA$	*0*
Dividends	$+Div$	$-Div$				*0*
Transfers	Π^b+i^aA			$-\Pi^b$	$-i^aA$	*0*
Δ in Loans			$+\Delta L$	$-\Delta L$		*0*
Δ in Deposits	$-\Delta D$			$+\Delta D$		*0*
Δ in Cash Currency	$-\Delta J$				$+\Delta J$	*0*
Issue of Equities	$-q\Delta E$		$+q\Delta E$			*0*
Δ in Borrowings from Central Bank				$+\Delta A$	$-\Delta A$	*0*
Δ in Bank Reserves				$-\Delta R$	$+\Delta R$	*0*
Σ	*0*	*0*	*0*	*0*	*0*	*0*

INVESTMENT STRUCTURE AND BEHAVIOR

Investment Decisions by Firms[8]

At the beginning of each period, a firm inherits a real capital level K_{t-1}, a real equity level E_{t-1}, a nominal debt level L_{t-1}, and a nominal net worth level Z_{t-1}. We assume an imperfectly competitive firm with markup pricing over labor costs at a constant rate τ. The nominal wage is ω, labor is N_t, output is Y_t, and the labor-output ratio is n. The price level p is given by

$$p = (1+\tau)\omega n, \quad n = N_t/Y_t.^9 \tag{1}$$

The profit rate r_t on capital evaluated at the current price level is defined as

$$r_t = (pY_t - \omega n Y_t)/pK_{t-1} = \{\tau/(1+\tau)\} \cdot (Y_t/K_{t-1}). \qquad (2)$$

For simplicity, we assume that τ, ω, and n are constant, and the relative price of capital goods to consumer goods remains constant throughout this study.

The firm makes an investment, given its existing capital stock. Since the firm cannot accurately know the value of returns over the periods during which the newly installed equipment will be used in the future, the firm's investment decisions are made based on the expected returns. Besides, the firm cannot realize returns when it makes bad investment decisions and goes bankrupt.

Let us denote the expected returns from investment I_t by

$$\{Q^e_{t+1}, Q^e_{t+2}, \ldots, Q^e_{t+n}, \ldots\}$$

and the bank lending rate during the current period by i_t. The bank lending rate discounts the expected returns. The lifetime of capital goods is assumed to be infinite, and the capitalized value of expected earnings for investment, PV_t, is defined as

$$PV_t = \sum_{j=1}^{\infty} Q^e_{t+j}/(1+i_t)^j. \qquad (3a)$$

We may assume, for simplicity, that the sequence of anticipated returns from investment $\{Q^e_{t+j}\}$ is represented by a constant series $\{Q^e\}$ satisfying

$$\sum_{j=1}^{\infty} Q^e_{t+j}/(1+i_t)^j = \sum_{j=1}^{\infty} Q^e/(1+i_t)^j. \qquad (3b)$$

Let Q^e denote the average expected returns. Hence, the present value of expected returns from investments is written as

$$PV_t = Q^e/i_t. \qquad (3c)$$

As for the factors that determine Q^e, we make the following assumptions in the spirit of Keynes's theory of investment. First, we assume that the ratio of prospective yields to investment Q^e/pI_t is a function of the capital accumulation rate k_t and the state of expectations of the firm ε^f.

$$Q^e/pI_t = \phi(k_t, \varepsilon^f),\ \phi_{k_t} < 0,\ \phi_{\varepsilon^f} > 0,\ \eta = -k\phi_k/\phi < 1,\ k_t = I_t/K_{t-1}. \qquad (4a)$$

Note that ϕ corresponds to what we call the marginal efficiency of investment.[10] We, therefore, assume that ϕ decreases as k_t increases. We express the elasticity of ϕ with respect to k_t as η and assume it is constant.[11]

Furthermore, Keynes emphasized the effects of the state of expectations ε^f on investments. Expectation improvements related to ε^f raises the expected return on investment ϕ. The firm formed its expectations based on the current economic variable.[12] When the current profit rate is higher, expectations will improve. Moreover, when the debt to capital ratio l_{t-1} is high, interest payments will increase, and net profit will decrease, thus worsening expectations. We assume that expectations are a function of the profit rate and the debt-capital ratio,

$$\varepsilon^f = \varepsilon^f(r_t, l_{t-1}),\ \varepsilon^f_{r_t} > 0,\ \varepsilon^f_{l_{t-1}} < 0,\ l_{t-1} = L_{t-1}/pK_{t-1}. \tag{4b}$$

In view of this last expression, equation (4a) may be written as

$$Q^e/pI_t = \phi(k_t, \varepsilon^f(r_t, l_{t-1})). \tag{4c}$$

Substituting equation (4c) into equation (3c), the expected net cash flows from the investment are given by

$$\Pi^f_t = (Q^e/i_t) - pI_t = \{[k_t\phi(k_t, \varepsilon^f(r_t, l_{t-1}))/i_t] - k_t\}pK_{t-1}\,. \tag{5a}$$

We assume that the firm determines its investment level to maximize the value of Π^f_t. Maximizing equation (5a) with respect to k_t yields

$$\phi(k_t, \varepsilon^f(r_t, l_{t-1}))(1-\eta) = i_t. \tag{5b}$$

The left-hand side of equation (5b) shows the marginal efficiency of investment, and the right-hand side indicates the marginal cost.[13] Further, the capital accumulation rate k_t may be expressed as

$$k_t = k(r_t, i_t, l_{t-1}),\ k_{r_t} > 0,\ k_{i_t} < 0,\ k_{l_{t-1}} < 0. \tag{6}$$

The capital accumulation rate is, therefore, a decreasing function of the bank lending rate and the debt-capital ratio and an increasing function of the profit rate.[14]

Financing Investment

Let us now consider the relationship between investment and financing. First, we set up the firm's budget constraints. We assume that the firm has three types of fund inflows: gross earnings, bank loans, and issuance of equity shares. These are distributed among investments, dividends, and interest payments to the bank. This relationship may be represented as

$$pI_t + Div_t + i_{t-1}L_{t-1} = p_tY_t - \omega N_t + \Delta L^d_t + q_t\Delta E^s_t, \tag{7a}$$

where ΔL^d_t and ΔE^s_t represent new borrowing from the bank and new equity issues in the current period, respectively, q_t is the equity price, and Div_t stands for dividends. The total borrowing L^d_t and equity E^s_t can be written as

$$L_t^d = L_{t-1} + \Delta L_t^d, \tag{7b}$$

$$E_t^s = E_{t-1} + \Delta E_t^s. \tag{7c}$$

Let us consider the relationship between investment and financing. The sources of financing for investment consist of retained earnings, bank loans, and equity shares. Their relationship may be represented as

$$pI_t = F_t + \Delta L_t^d + q_t \Delta E_t^s, \tag{7d}$$

where F_t stands for retained earnings.[15] We suppose that the firm holds a constant gross earnings ratio as retained earnings. Hence,

$$F_t = v(pY_t - \omega N_t), \tag{7e}$$

where v denotes the retained earnings rate. Substituting equation (7e) into equation (7d), we obtain

$$pI_t = v(pY_t - \omega N_t) + \Delta L_t^d + q_t \Delta E_t^s. \tag{7f}$$[16]

Now, consider new equity issues. Equity is one of the financial assets that is based on the capital of the firm. It neither has a term of redemption nor includes requirements to pay interest. We suppose that the firm has medium- and long-term perspectives and decides to raise the equity to maintain a constant equity-capital ratio. This formulation is similar to Lavoie and Godley (2001), namely,

$$\frac{E_{t-1}}{pK_{t-1}} = \frac{E_t}{pK_t} = e, \tag{7g}$$

where e denotes the equity-capital ratio, which is assumed constant. From this equation, we obtain

$$\frac{\Delta E^s}{E_{t-1}} = \frac{pI_t}{pK_{t-1}} = k_t. \tag{7h}$$

Substituting equations (7f) and (7g) into (7d), and taking into account equations (6) and (7h), we obtain the borrowing demand function.

$$l_t^d = (1 - q_t e)k(r_t, i_t, l_{t-1}) - vr_t + l_{t-1}, \tag{8}$$

$$l_{r_t}^d \lesseqgtr 0,\ l_{i_t}^d < 0,\ l_{q_t}^d < 0,\ l_{l_{t-1}}^d \lesseqgtr 0,\ l_t^d = L_t^d / pK_{t-1}.$$

Additionally, from equation (7h), we obtain the equity supply function.

$$q_t e_t^s = q_t e\{1 + k(r_t, i_t, l_{t-1})\}, \tag{9}$$

$$e_{r_t}^s > 0,\ e_{i_t}^s < 0,\ e_{l_{t-1}}^s < 0,\ e_t^s = E_t^s / pK_{t-1}.$$

The above results can be summarized as follows. The borrowing from the bank is a decreasing function of the bank lending rate and the equity price. The effect of the rate of profit is undetermined since an increase in the rate of profit influences the increase in investment as well as retained earnings. Similarly, the effect of the debt-capital ratio is also undetermined. The equity supply depends on the capital accumulation rate.

Bank Behavior

Similar to firms, banks seek profits. Banks earn revenue from lending to firms and make interest payments to depositors and the central bank. We account for transaction costs G_t, which include losses in the event of corporate failures and operating costs. For simplicity, we assume that the deposit interest rate and the discount rate are exogenous.[17] Moreover, the discount rate has already been determined by the central bank. The bank accepts all deposits from households, while the central bank supplies the funds requested by the banks. Thus, the bank's profit per unit of capital π_t^b is given by

$$\pi_t^b = i_t l_t^s - i^d d_t^d - i_{t-1}^a a_t - g_t,$$

$$\pi_t^b = \Pi_t^b / pK_{t-1},\ l_t^s = L_t^s / pK_{t-1},\ d_t^d = D_t^d / pK_{t-1},\ a_t = A_t / pK_{t-1},\ g_t = G_t / pK_{t-1}, \quad (10a)$$

where Π_t^b, L_t^s, i^d, D_t^d, i_{t-1}^a, and A_t denote bank profit, bank lending, the deposit rate, deposits from the household, the discount rate, and borrowings from the central bank, respectively.[18]

Let us consider the transaction cost function g_t. This cost is more plausibly dependent on a subjective risk assessment by the lender and, hence, a subjective risk premium.[19] This premium becomes high when the bank estimates that the likelihood of firm bankruptcy is high. We assume that the risk premium depends on the bank's subjective evaluation of the firm's debt-capital ratio and the profit rate. When the debt-capital ratio is high, and the profit rate is low, the bank's subjective evaluation becomes worse, and transaction costs increase. Hence, we express the subjective evaluation ε^b as

$$\varepsilon^b = \varepsilon^b(r_t, l_{t-1}),\ \varepsilon_{r_t}^b > 0, \varepsilon_{l_{t-1}}^b < 0. \quad (10b)$$

In addition, the bank's operating costs increase as bank lending increases. We assume that the cost g_t is an increasing function of the ratio of bank lending to capital. The cost function g_t may then be written as

$$g_t = g(l_t^s, \varepsilon^b) = g(l_t^s, \varepsilon^b(r_t, l_{t-1})), \quad (10c)$$

$$g_{l_t^s} > 0,\ g_{r_t} < 0,\ g_{l_{t-1}} > 0,\ g_{l_t^s l_t^s} > 0,\ g_{l_t^s r_t} < 0\ g_{l_t^s l_{t-1}} > 0.$$

We assume that the marginal cost of bank lending increases more than proportionally, as l_t^s increases, and decreases, as l_{t-1} decreases, and the profit rate r_t increases. The profit of the bank per unit of capital π_t^b is given by

$$\pi_t^b = i_t l_t^s - i^d d_t^d - i_{t-1}^a a_t - g(l_t^s, \varepsilon^b(r_t, l_{t-1})). \tag{10d}$$

From the bank's balance sheet, we have

$$L_t^s + R_t = D_t^d + A_t, \tag{11a}$$

where R_t denotes bank reserves, which must satisfy reserve requirements. This is represented by

$$R_t = \theta D_t^d, \tag{11b}$$

where θ denotes the legal reserve rate, which is assumed to remain constant. Substituting equation (11b) into the bank's balance sheet equation (11a) and dividing by pK_{t-1} yields

$$l_t^s = (1-\theta) d_t^d + a_t. \tag{11c}$$

To summarize, the bank's problem is to maximize equation (10d), subject to the constraints in (11c). In this problem, bank lending l_t^s and the borrowing from the central bank a_t are controlled by the bank.[20] The first-order condition to maximize π_t^b is

$$i_t = g_{l_t^s}(l_t^s, \varepsilon^b(r_t, l_{t-1})) + i_{t-1}^a. \tag{11d}$$

The left-hand side of equation (11d) indicates the marginal revenue of lending, and the right-hand side indicates marginal cost. Bank lending can be expressed as

$$l_t^s = l^s(i_t, r_t, l_{t-1}, i_{t-1}^a),\ l_{i_t}^s > 0,\ l_{r_t}^s > 0,\ l_{l_{t-1}}^s < 0,\ l_{i_{t-1}^a}^s < 0. \tag{12}$$

Hence, bank lending is an increasing function of the bank lending rate and the rate of profit, and a decreasing function of the debt-capital ratio and the discount rate.[21]

Household Behavior

We then formalize the portfolio behavior of households. The household holds wealth in the form of equity $q_t E_t^d$, deposits D_t^d, and cash currency J_t^d. We denote household wealth that is invested in financial assets by W_t. We define money demand M_t^d as the sum of deposits and cash currency. We then assume that the household divides the wealth into equity and money and allocates money to deposits and cash currency at a constant rate, which may be expressed as

$$W_t = q_t E_t^d + M_t^d, \tag{13h}$$

$$M_t^d = D_t^d + J_t^d. \quad (13i)[22]$$

The household allocates its wealth W_t to financial assets to maximize expected utility. We shall not consider the derivation of the asset demand function in-depth but simply assume that the demand for assets is directly proportional to its rate and inversely proportional to other rates. The portfolio behavior functions then become the usual representative form of substitutability between assets.[23] We express the ratio of equity demand to household wealth as α. The demand function of equity is expressed as

$$q_t E_t^d = \alpha \cdot W_t. \quad (14a)[24]$$

We also assume that the return rate of equity is measured by the anticipated dividend, which depends on the expectations of prospective yields in the household ε^h. We suppose that yield expectation is an increasing function of the current profit rate and a decreasing function of the debt-capital ratio,

$$\varepsilon^h = \varepsilon^h(r_t, l_{t-1}),\ \varepsilon^h_{r_t} > 0,\ \varepsilon^h_{l_{t-1}} < 0. \tag{14b}$$

Therefore, we may express the ratio of equity demand to household wealth as

$$\alpha = \alpha(\varepsilon^h) = \alpha(r_t, l_{t-1}),\ \alpha_{r_t} > 0,\ \alpha_{l_{t-1}} < 0. \tag{14c}$$

However, the wealth of the household at period t (W_t) is the sum of the wealth at period $t-1$ (W_{t-1}) and the household saving in period t (pS_t^h), expressed as

$$W_t = pS_t^h + W_{t-1}. \tag{15a}$$

This term is a stock variable. Let us consider equation (15a) in-depth. First, let us focus on W_{t-1}, which is equal to the value of wealth at the beginning of the current period. Considering the whole economy, the assets one has are the liabilities of another, thus satisfying the following equation.

$$W_{t-1} = pK_{t-1} - Z_{t-1}, \tag{15b}$$

In addition, from the firm's balance sheet, its capital is expressed by

$$pK_{t-1} = L_{t-1} + q_t E_{t-1} + Z_{t-1}. \tag{15c}$$

Substituting equation (15c) into equation (15b), we obtain

$$W_{t-1} = q_t E_{t-1} + L_{t-1}. \tag{15d}$$

Next, let us consider pS_t^h. The household earns wages ωN and dividends from a firm. We assume that the costs of bank G and profit Π_t^b are distributed to the household via a bank's labor cost and other factors. Since a time lag exists between borrowing and interest payment, the household earns the transaction costs and the bank's profit for the previous period. In addition, the household can earn interest from deposits. Finally, we suppose that the revenue of the central bank also belongs to the household. Above all, we may represent household income pY_t^h in the current period as

$$pY_t^h = pY_t - v(pY_t - \omega N_t). \qquad (15e)[25]$$

We assume that all wages are spent on consumption, as expressed by

$$pC_t = \omega N_t. \qquad (15f)$$

From equations (15e) and (15f), we obtain the household saving function,

$$pS_t^h = (1 - v)(pY_t - \omega N_t) \qquad (15g)$$

Substituting equations (15a), (15d) and (15g) into equation (14a) and dividing by pK_{t-1}, we obtain the equity demand function,

$$q_t e_t^d = \alpha(r_t, l_{t-1})[(1 - v)r_t + q_t e_{t-1} + l_{t-1}], \qquad (16)[26]$$

$$w_t = (1 - v)r_t + q_t e_{t-1} + l_{t-1}$$

$$q_t e_{r_t}^d > 0,\ q_t e_{q_t}^d > 0,\ q_t e_{l_{t-1}}^d \gtreqless 0,\ q_t e_{e_{t-1}}^d > 0,\ e_t^d = E_t^d / pK_{t-1},\ w_t = W_t / pK_{t-1}.$$

A rise in the profit rate increases equity demand. The effect of the debt-capital ratio on equity demand in the current period is undetermined. Finally, since a rise in the equity price and equity-capital ratio increases household wealth, the demands for equity increases.[27]

MARKET EQUILIBRIUM

The economic system consists of six markets: commodities, bank lending, equity, deposits, cash currency, and central bank advances. We assume that the deposit rate is exogenous and that banks accept all the deposits that households wish to make. The supply of deposits is constrained by demand. Similarly, we assume that the discount rate is exogenously determined as a monetary policy tool, and the central bank supplies all funds requested by banks. We consider equilibrium in four markets: commodities, bank lending, equity, and cash currency. Applying Walras' law, we exclude the cash currency market.

Let us first consider the commodity market. The commodity market achieves equilibrium when investments and savings are equal. The savings of the economy pS_t are equal to the sum of household savings pS_t^h and the firm's retained earnings F_t as follows.

$$pS_t = pS_t^h + F_t = (1-v)(pY_t - \omega N_t) + v(pY_t - \omega N_t) = pY_t - \omega N_t. \quad (17a)$$

Dividing this equation by pK_{t-1} and taking into account equation (2), we obtain

$$s_t = \frac{pS_t}{pK_{t-1}} = r_t. \quad (17b)$$

From equations (6) and (17b), we may express the balance equation of the commodity market as follows.

$$k(r_t, i_t, l_{t-1}) = r_t. \quad (18a)$$

Furthermore, we view the bank lending and equity markets as the whole financial market. Using equations (8), (9), (12), and (16), we obtain the balance equations for the bank lending and equity markets as follows.

$$(1-q_t e)k(r_t, i_t, l_{t-1}) - vr_t + l_{t-1} = l^s(i_t, r_t, l_{t-1}, i_{t-1}^a), \quad (18b)$$

$$\alpha(r_t, l_{t-1})[(1-v)r_t + l_{t-1} + q_t e_{t-1}] = q_t e[1 + k(r_t, i_t, l_{t-1})]. \quad (18c)[28]$$

We now consider a short-run equilibrium. The system comprises equations (18a), (18b), and (18c), and determines the rate of profit r_t, bank lending rate i_t, and equity price q_t. We start with the coefficient matrix M_s given by

$$M_s = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$= \begin{pmatrix} k_{r_t} - 1 & k_{i_t} & 0 \\ (1-q_t e)k_{r_t} - v - l_{r_t}^s & (1-q_t e)k_{i_t} - l_{i_t}^s & -ek_t \\ \alpha_{r_t} w + (1-v)\alpha - qek_{r_t} & -qek_{i_t} & -e(1-\alpha+k_t) \end{pmatrix}. \quad (19a)[29]$$

We assume that short-run equilibrium is stable and the following conditions are satisfied,

$$a_{11} < 0 \quad (19b)$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0 \quad (19c)$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} < 0. \quad (19d)$$

In this case, the equilibrium may be represented as follows.

$$r_t = r(l_{t-1}, i_{t-1}^a), \quad (20a)$$

$$r_{l_{t-1}} < 0,\ r_{i_{t-1}^a} < 0,$$

$$i_t = i(l_{t-1}, i^a_{t-1}), \tag{20b}$$

$$i_{l_{t-1}} \gtreqless 0,\ i_{i^a_{t-1}} > 0,$$

$$q_t = q(l_{t-1}, i^a_{t-1}), \tag{20c}$$

$$q_{l_{t-1}} \gtreqless 0,\ \ q_{i^a_{t-1}} \gtreqless 0.$$

Let us consider the effect of the debt-capital ratio l_{t-1} on the short-run equilibrium. A rise in l_{t-1} will lead to a decrease in the profit rate. This result contrasts with that of Ryoo (2013b). Ryoo assumes that the accumulation rate is independent of the debt-capital ratio. In his model, a rise in the debt-capital ratio l_{t-1} leads to an increase in the profit rate and, therefore, does not produce what we call "the paradox of debt." In addition, in our model, the effects on the bank lending rate and equity price are undetermined. However, when the absolute values of $l^s_{r_t}$, $l^s_{l_{t-1}}$, α_{r_t} and $\alpha_{l_{t-1}}$ are large, a rise in the debt-capital ratio increases the bank lending rate and decreases the equity price. Thus, the effects on the bank lending rate and the equity price depend on bank behavior and the degree of substitutability between the equity and money at the household level.[30]

We further consider the effects on bank lending rates. The change in the debt-capital ratio affects both supply and demand in the bank lending market. On the supply side, the rise in the debt-capital ratio decreases bank lending. On the demand side, although the rise in the debt-capital ratio decreases investment, total borrowing demand, including the stock of debt, is nevertheless undetermined. When the decrease in supply is larger than that for demand, the bank lending rate will increase. In addition, when bank lending significantly decreases in response to the profit rate, the rise in the banking rate is larger.

By comparing our model with the standard investment-savings and liquidity preference-money supply model (IS-LM), equation (18b) is analogous to the LM equation, which is the key equation to determine interest rates in the financial market. The classical LM equation shows a positive relationship between the profit rate and the interest rate. In contrast, this relationship is undetermined in our model, as it depends on bank behavior. If the debt-capital ratio increases, when bank lending is sensitive to the profit rate, the bank lending rate will rise largely and exacerbate the situation.

Finally, we consider the effect of the discount rate i^a_{t-1}. On one hand, when the discount rate rises, the profit rate decreases, and the bank lending rate increases. On the other hand, the effect on the equity price is undetermined. However, when the absolute values of α_{r_t} and α are large, the equity price decreases. The effect of the discount rate is similar to the effect of the debt-capital ratio on equity prices; that is, it depends on its influence on the profit rate. The decrease in the profit rate causes a decrease in the supply and demand for equity. When the decrease in the profit rate causes financial assets to significantly shift to money due to a strong substitution effect, the demand for equity will decrease. When the decrease in demand surpasses the decrease in supply, the equity price will also decrease.

The above results show that bank behavior, household portfolio selection, and the balance sheet elements affect the real economy in agreement with the insights by Minsky and Keynes. In the next section, we investigate the unstable economy by using a dynamic model. The following propositions summarize the findings of this section.

Proposition 1. *In the short run, the effect of the debt-capital ratio on the bank lending rate and the equity price will largely depend on the behaviors of banks and households. When bank lending is sensitive to the profit rate, and the degree of substitutability between the financial assets in the household is high, the bank lending rate will increase, and the equity price decrease.*

Proposition 2. *In the short run, the effect of the discount rate on the equity price will largely depend on its indirect influence via the profit rate. When the degree of substitutability between equity and the other financial assets in the household is high, the rise in the discount rate will decrease the equity price.*

DYNAMIC MODEL

Stability of the Steady State

In this section, we construct a dynamic, discrete model to investigate the mechanism of financial instability and then analyze the characteristics of the steady state. We have so far treated the debt-capital ratio l_{t-1} and the discount rate i_{t-1}^a as exogenous variables. Changes in these variables generate shifts in the short-run equilibrium. First, we focus only on the dynamics of the debt-capital ratio and analyze the characteristics of the steady state, specifically, its instability. We then incorporate the dynamic equation for the discount rate into the original model to obtain a dynamic model comprising two equations. We subsequently examine the influence of monetary policy on the stability of the economy.

Let us first derive the dynamic equation for the debt-capital ratio l_{t-1}. From the definition of the debt-capital ratio, we have

$$l_t = \frac{L_t}{pK_t} = \frac{L_t^d}{pK_{t-1}} \cdot \frac{pK_{t-1}}{pK_t}. \qquad (21a)[31]$$

We assume that the discount rate is constant in this section. Substituting equations (20a), (20b), and (20c) into equation (21a), we obtain

$$l_t = f(l_{t-1}) = \frac{1}{1+k_t(r(l_{t-1}), i(l_{t-1}), l_{t-1})} \cdot [(1 - q_t(l_{t-1})e)k(r(l_{t-1}), i(l_{t-1}), l_{t-1}) - vr(l_{t-1}) + l_{t-1}] \qquad (21b)$$

Since the commodity market is balanced at the steady state, we can express the steady state as

$$(1 - q(l^*)e - l^* - v)k(r(l^*), i(l^*), l^*) = 0, \qquad (21c)$$

where l^* denotes the value of debt-capital ratio at the steady state. We suppose that the capital accumulation rate k^* is positive at the steady state. Therefore, the following equation is satisfied at the steady state,

$$(1 - q(l^*)e - l^* - v) = 0. \qquad (21d)$$

At the steady state (l^*), investment is positive, and the debt-capital ratio remains constant.

We now consider the movement of the debt-capital ratio around the steady state. From equation (21b), we have

$$\frac{dl_t}{dl_{t-1}} = \frac{df(l_{t-1})}{dl_{t-1}} = \frac{1}{1+k^*}[-q_{l_{t-1}}ek^* + 1]. \qquad (22a)[32]$$

We focus on the case where the steady state becomes unstable, and the debt-capital ratio cumulatively increases when it is larger than the steady state level, as shown in Figure 1. This occurs when the following condition is satisfied.

$$\frac{1}{1+k^*}[-q_{l_{t-1}}ek^* + 1] > 1. \qquad (22b)$$

In other words, we obtain

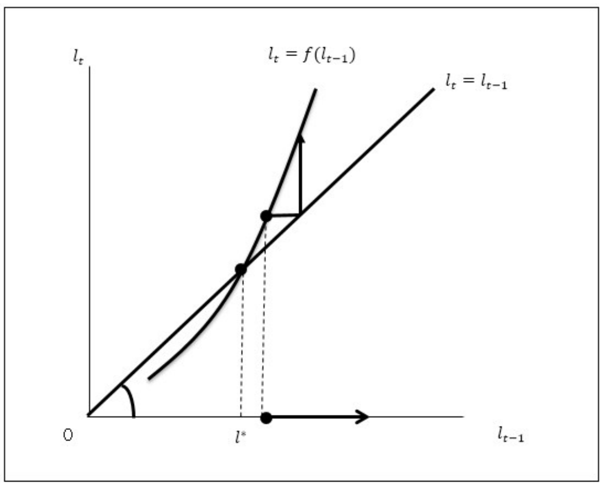
$$-(q_l e + 1)k^* > 0. \qquad (22c)$$

We may point out that when q_l is negative, and the absolute value is large, the economy becomes unstable. Thus, considering the results of the static model, this condition means that the degree of substitutability between equity and money in the household is high, and bank lending is sensitive to the profit rate and the debt-capital ratio.

The mechanism that generates instability may be described as follows.

In an unstable economy, a rise in the debt-capital ratio causes a fall in profit rates and equity prices and an increase in the interest rate. Since the fall in the profit rate leads to a decrease in retained earnings, the effect on investment financing is severe. The decline in the equity price makes it difficult for the firm to raise enough funds. Consequently, the firm increases bank loans, and the debt-capital ratio will be higher. When the debt-capital ratio is larger than in the steady state, it will continue to increase, and the economy will decline. The proposition below summarizes this finding.

Proposition 3. *When the degree of substitutability between equity and money in the household is high, and bank lending is sensitive to the profit rate and the debt-capital ratio, the system may be unstable, and the debt-capital ratio will cumulatively increase when it is larger than the level of the steady state.*



Monetary Policy and Stability of the Steady State

When the economy is unstable with respect to the debt-capital ratio, how much can the central bank contribute to the stability of the economy? Can the central bank completely remove economic instability? We next analyze the effects of monetary policy on the stability of the economy.

First, we formalize an interest rate monetary policy and construct a dynamic model that incorporates the debt-capital ratio and the discount rate. We examine the effectiveness of the introduced monetary policy on the economy. The discount rate is the policy variable that the central bank can control. Since the consumer goods price remains unchanged in our model, the policy primarily aims to accommodate business cycles. When the economic situation deteriorates, the central bank will reduce the interest rate. We assume that the central bank is able to recognize the current economic situation from observing the profit rate. When the actual profit rate is higher than the normal long-run level of the profit rate r^n, the central bank has some concerns about the economy overheating and will increase the discount rate. Then, the behavior of the discount rate i_t^a through time is given by

$$i_t^a = \beta(r - r^n) + i_{t-1}^a, \ \beta > 0, \tag{23}$$

where β is the adjustment speed of the discount rate.[33] The normal long-run level of the profit rate r^n is expected by the central bank, which is given exogenously.

Substituting equation (20a) into equations (21a) and (23), we obtain a dynamic model comprising two difference equations as follows.

$$l_t = T(l_{t-1}, i_{t-1}^a) = \frac{[(1-q_t(l_{t-1}, i_{t-1}^a)e)k(r(l_{t-1}, i_{t-1}^a), i(l_{t-1}, i_{t-1}^a), l_{t-1}) - vr(l_{t-1}, i_{t-1}^a) + l_{t-1}]}{1+k_t(r(l_{t-1}, i_{t-1}^a), i(l_{t-1}, i_{t-1}^a), l_{t-1})}, \tag{24a}$$

$$i_t^a = R(l_{t-1}, i_{t-1}^a) = \beta[r(l_{t-1}, i_{t-1}^a) - r^n] + i_{t-1}^a. \tag{24b}$$

The steady state of this dynamic model is given by

$$(1 - q(l^{**}, i^{a**})e - l^{**} - v) = 0, \tag{25a}$$

$$r(l^{**}, i^{a**}) = r^n. \tag{25b}$$

At the steady state (l^{**}, i^{a**}), the debt-capital ratio and the discount rate remain constant, and the profit rate is equal to the normal long-run level of the profit rate r^n.

Regarding the coefficient matrix M_D, the values of each element at the steady state are

$$M_D = \begin{pmatrix} T_1 & T_2 \\ R_1 & R_2 \end{pmatrix}, \tag{26a}$$

$$T_1 = \frac{1}{1+k^{**}}[-q_{l_{t-1}}ek^{**} + 1], \tag{26b}$$

$$T_2 = \frac{1}{1+k^{**}}[-q_{i_{t-1}^a}ek^{**}], \tag{26c}$$

$$R_1 = \beta r_{l_{t-1}}, \tag{26d}$$

$$R_2 = \beta r_{i_{t-1}^a} + 1. \tag{26e}$$

From Schur's condition, the system is stable if and only if the following conditions are satisfied,

$$T_1 \cdot R_2 - T_2 \cdot R_1 < 1, \tag{27a}$$

$$(T_1 - 1)(R_2 - 1) - R_1 \cdot T_2 > 0, \qquad \text{(27b)}$$

$$(T_1 + 1)(R_2 + 1) - R_1 \cdot T_2 > 0. \qquad \text{(27c)}$$[34]

Substituting the equations (26b)-(26e) into equation (27b) and taking into account the results of the comparative statics, the following condition must be fulfilled to satisfy equation (27b).

$$\alpha_{l_{t-1}} w^{**} + 1 + k^{**} > 0. \qquad \text{(28a)}$$[35]

This equation is satisfied when the absolute value of $\alpha_{l_{t-1}}$ is small. Besides, this condition is independent of the monetary policy. Hence, monetary policy is not directly effective to stabilize the economy, but effective if the substitution effect of the household portfolio selection in response to the debt-capital ratio is weak. In other words, the economy always has the potential to become unstable, endogenously, if portfolio behavior is left to the discretion of the household.

Keynes writes (1936),

> When the capital development of a country becomes a by-product of the activities of a casino, the job is likely to be ill-done. The measure of success attained by Wall Street, regarded as an institution of which the proper social purpose is to direct new investment into the most profitable channels in terms of future yield, cannot be claimed as one of the outstanding triumphs of *laissez-faire* capitalism—which is not surprising, if I am right in thinking that the best brains of Wall Street have been in fact directed towards a different object. These tendencies are a scarcely avoidable outcome of our having successfully organized "liquid" investment markets (p.159).

Therefore, the government and central bank must implement regulations and rules to keep the financial markets transparent and avoid excessive portfolio reactions by households.

Furthermore, to make the economy stable, the central bank has to adjust the speed of the discount rate β to satisfy conditions (27a) and (27c). We may derive the following equation,

$$0 < \frac{(q_{l_{t-1}} e + 1) k^{**}}{\left(-q_{l_{t-1}} r_{i^a_{t-1}} + q_{i^a_{t-1}} r_{l_{t-1}}\right) e k^{**} + r_{i^a_{t-1}}} < \beta^T < \frac{2(q_{l_{t-1}} e k^{**} - 2 - k^{**})}{\left(-q_{l_{t-1}} r_{i^a_{t-1}} + q_{i^a_{t-1}} r_{l_{t-1}}\right) e k^{**} + r_{i^a_{t-1}} (2 + k^{**})}, \qquad \text{(28b)}$$

where β^T denotes the appropriate value to satisfy conditions (27a) and (27c). This shows that β^T depends on several economic factors, which include the sensitivity of the investment and the bank lending with respect to the profit rate. Above all, to stabilize the economy, the central bank must not only implement some regulations and rules but also simultaneously conduct an appropriate monetary policy. This means that monetary policy may have a stabilizing effect in theory, though, in practice, there would be considerable difficulties in achieving stability.

Minsky (1986) also alleges, “the financial system evolves from an initial robustness toward fragility, and continuous control and periodic reform of the banking system are needed to prevent the development of a financially unstable economy that cannot readily be contained” (p.319).

Mechanism of Business Cycle and Financial Instability

Finally, we consider the dynamics of the economy. The mechanism by which the economy converges to the steady state may be roughly described as follows. We suppose that the economy is in a boom. In this situation, the central bank raises the discount rate to cool down the economy. As the lending rate begins to rise, the investment decreases, which leads to a decrease in the profit rate and retained earnings of the firm. The equity price then declines. The firm raises funds for investment from bank borrowing. The discount rate and the bank lending rate continues to rise, and the debt-capital ratio increases, thus leading the economy into a recession. At this point, the central bank reduces the discount rate to help economic recovery. In a stable economy, although the debt-capital ratio increases during the recession, the extent of this increase is lower than that in an unstable economy. The range of change in the equity demand and the equity price is, therefore, small. If the central bank changes the discount rate appropriately, the debt-capital ratio will decrease by the end of the recession. Hence, the economy will reverse and converge to the steady state cyclically.

However, in an unstable economy, the rise in the debt-capital ratio leads households to decrease equity demand according to their sensitivity to financial asset substitutability. Equity prices will decrease dramatically, and the economy will shrink sharply and go into recession. To remedy this situation, the central bank will decrease the discount rate. Even if the economy recovers, the volatility is so large that the economy cannot return to a stable path. These findings are summarized by Propositions 4 and 5 below.

Proposition 4. *The economy always has the potential to become unstable endogenously if it leaves portfolio behavior to the discretion of households.*

Proposition 5. *To avoid financial instability, the central bank and the government must implement regulations to promote a moderate reaction of economic units to economic shocks. At the same time, the central bank should adopt an appropriate monetary policy.*

CONCLUSIONS

We construct a dynamic macroeconomic model by endogenizing the interest rate. We formalize the profit-bank behavior in a way similar to Ryoo (2013a) and demonstrate "the paradox of debt." While Ryoo assumes that the accumulation rate is independent of the debt-capital ratio, in our model, a rise in the debt-capital ratio l_{t-1} decreases investment. From a dynamic analysis, we find that the economy becomes unstable if the equity demand from households is sensitive to the debt-capital ratio. Financial instability may be inevitable unless the central bank designs a regulatory framework to make appropriate monetary policies effective.

Minsky (1975) and Keynes (1936) emphasize that the markets do not necessarily have self-correcting mechanisms, and financial instability may occur since economic agents operating under uncertainty and the real and financial factors are interdependent. Our results support these ideas. Relevant regulations for financial asset holders play a crucial role in stabilizing the economy.

There are some possible extensions to our model. The model of equity issues is perhaps somewhat stylized as we assume that the equity-capital ratio is constant. However, evidence from the 1990s economic boom in the US shows that this growth period was characterized by falling investment shares financed by equity issues and increasing indebtedness. Furthermore, asset prices and credit flows generally go hand in hand in economic booms. Therefore, future studies should aim to modify our financing model and make it closer to reality while including a strong micro-foundation. Second, additional adjustments are needed for monetary policy to develop a more comprehensive set of macroeconomic policies that may contribute to the stability of the steady state. Recently, in Japan and the US, the zero-interest policy has been retained, and quantitative easing is implemented. The conventional policy alone seems to be insufficient to make the economy stable. In addition, financial stability is often pursued through micro- and macro-prudential policies, instead of changes in the interest rate. We should also examine the effects of prudential policies on the economy. Third, Minsky (1986) argued that a big government would make the economy more stable.[36] However, this study does not deal with the role of government. Finally, we need to extend the model to an open economy model and investigate stability in the context of the global economy.

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NOTES

1. From the perspective of the stock-flow consistent (SFC) approach, it has been pointed out that traditional models, such as those suggested by Taylor and O'Connell (1985), often assume oversimplified hypotheses that do not reflect Minsky's analysis. See, for example, Dos Santos (2005).
2. The choice of interest rate rather than money supply as a monetary policy instrument is common in the post-Keynesian literature. See, for example, Asada (2014), Isaac (2009), and Lavoie (2006).
3. Bernanke and Blinder (1988) present a simple macro model in which credit supply by the bank plays a key role in determining the output.
4. Kindlberger (1978) describes that "[t]his model is in the tradition of the classical economists, including John Stuart Mill, Alfred Marshall, Knut Wicksell, and Irving Fisher, who also focused on the instability in the supply of credit" (p. 15). Lavoie (1997) points out that the financial instability hypothesis came from a Wicksellian model developed in the 1950s. Minsky (1957) describes that "the interest rate is determined by the demand for investment, the supply of saving, and the terms upon which individuals are willing to hold cash as an asset." This is a depiction of the theory of loanable funds.
5. For example, we suppose the central bank profits are provided to the government, which, in turn, transfers them to households in the form of social spending.
6. The flow matrix distinguishes between current and capital transactions by firms. The plus sign before a variable denotes a receipt (or source of funds), while minus sign denotes a payment (or use of funds).
7. Most dynamic models which reflect Minsky's ideas are continuous. See, for example, Taylor and 'Connell (1985), Lavoie (1995), and Ryoo (2013a,b). In contrast, from the perspective of the SFC approach, Dos Santos (2005) and Lavoie and Godley (2001) use the discrete model.
8. This formulation follows Adachi and Miyake (2015). However, there are differences in the definition of the marginal efficiency of investment.
9. The markup rate τ is taken to be constant. Therefore, the price is constant in our model.
10. Since we focus on the accumulation rate, the definition of the marginal efficiency is different from what Keynes (1936) actually meant. There are

many discussions about the marginal efficiency of capital or investment. Chick (1984) elaborates further on these issues.

11. This assumption $\eta < 1$ ensures that the maximization problem has a meaningful solution.
12. Keynes (1936) writes, "the considerations upon which expectations of prospective yields are based are partly existing facts which we can assume to be known more or less for certain, and partly future events which can only be forecasted with more or less confidence." For simplicity, we focus on "existing facts" and represent them by current profit and the debt-capital ratio.
13. Keynes (1936) writes, "the rate of investment will be pushed to the point on the investment demand schedule where the marginal efficiency of capital in general is equal to the market rate of interest" (p.136–137). However, in practice, it is hard for the firm to maximize profits under fundamental uncertainty. In this sense, we investigate the case in which the firm maximizes profits even though condition (5b) could only be achieved by mere luck, given the fundamental uncertainty.
14. We formalize the investment model in the spirit of Keynes's theory. In our model, we express the state of expectations explicitly. This factor plays an important role in Keynes' theory and causes the economy's instability. We formulate it as a function of the profit rate and debt-capital ratio. Therefore, the investment function depends on them. Thus, equation (6) is similar to the investment function based on the profit principle proposed by Kaldor and Kalecki.
15. We assume that investment exceeds retained earnings.
16. Substituting equations (7d) and (7e) into equation (7a), we obtain $Div_t = v(pY_t - \omega N_t) - i_{t-1}L_{t-1}$. The dividend is the surplus after deducting the interest payment from the constant gross profit rate v.
17. The deposit rate is closely tied to the discount rate controlled by the central bank. We assume that the deposit rate is represented by $i_t^d = \gamma i_{t-1}^a$, where γ is constant. This assumption is similar to Ryoo (2013a).
18. We assume that the bank pays interest to the central bank at the previous period's discount rate.
19. Keynes (1936) stressed the importance of the lender's risk. We note the following remarks made by Keynes: "But where a system of borrowing and lending exists, by which I mean the granting of loans with a margin of real or personal security, a second type of risk is relevant which we may call the lender's risk. This may be due either to moral hazard, i.e. voluntary default or other means of escape, possibly lawful, from the fulfilment of the obligation, or to the possible insufficiency of the margin of security, i.e. involuntary default due to the disappointment of expectation" (p144). In our model, the lender's risk has an indirect influence through the bank's cost function.
20. The bank does not hold excess reserves. When the bank wants to lend more, it borrows more from the central bank.

21. From equation (11c), we obtain $a_t = l^s(i_t, r_t, l_{t-1}, i_{t-1}^a) - (1-\theta)d_t^d$.The central bank supplies all funds a_t requested by banks. This formulation fits into the theory of endogenous money supply.
22. We can express the demand for deposits as $D_t^d = \lambda M_t^d$, where λ is constant.
23. The portfolio behavior functions follow Tobin (1969).
24. The households choose how many equities to hold and let money demand be determined by the rest of the transaction. Therefore, the money demand is represented by $M_t^d = W_t - q_t E_t^d = (1-\alpha)W_t$.
25. Appendix 1 outlines the derivation of equation (15e).
26. We assume that the capital gains or losses do not affect the household's income and consumption behavior. A rise in equity price increases wealth in the current period. The rise in equity price affects asset demands through the increase in wealth.
27. Once the household chooses how many equities hold, money demand is also determined from equation (13h). The money demand function is represented by

    $$m_t^d = (1-\alpha(r_t, l_{t-1})) \cdot w_t,\ \ m_{r_t}^d \gtreqless 0,\ m_{q_t}^d > 0,\ m_{l_{t-1}}^d > 0,\ m_{e_{t-1}}^d > 0,\ m_t^d = M_t^d / pK_{t-1}.$$

28. We omit the deposit rate because it is constant in this study.
29. The values of each element are evaluated in the short-run equilibrium.
30. Appendix 2 shows the mathematical results of the comparative statics.
31. Ryoo (2013a) expresses the dynamics of the debt-capital ratio of the firm as the supply function of bank loans.
32. Appendix 3 outlines the derivation of equation (22a).
33. Taylor (1993) proposed that the nominal interest rate should respond to the divergence of actual inflation rates from target inflation rates and of actual GDP from potential GDP. Since the consumer goods price remains unchanged in our model, equation (23) expresses a type of Taylor rule without the element of inflation.
34. For more information, see Gandolfo (1997).
35. Appendix 4 outlines the derivation of equations (28a) and (28b)
36. Minsky (1986) writes, "big government, with its potential for automatic massive deficits, puts a high floor under an economy's potential downward spiral. Although this high floor is important in itself, it is particularly important in a world with business and household debt because corporate gross profits and household savings are essential to validate such debt."

REFERENCES

Adachi, H., and Miyake, A. (2015) "A Macrodynamic Analysis of Financial Instability', in Adachi H., Nakamura T., and Osumi, Y. (eds.) *Studies in Medium-Run Macroeconomics*, Singapore: World Scientific Publishing.

Asada, T. (2014) "Mathematical Modeling of Financial Instability and Macroeconomic Stabilization Policies", in Dieci, R., He, X. Z., and Hommes, C. (eds.), *Nonlinear Economic Dynamics and Financial Modelling: Essays in Honour of Carl Chiarella* (pp. 41-63), Switzerland: Springer.

Bernanke, B.S., and Blinder, A.S. (1988) "Credit, Money, and Aggregate Demand", *American Economic Review*, 78(2), pp. 435-439.

Charles, S. (2008) "A Post-Keynesian Model of Accumulation with a Minskyan Financial Structure', *Review of Political Economy*, 20(3), pp. 319-331.

Charles, S. (2016) "Is Minsky's Financial Instability Hypothesis Valid?", *Cambridge Journal Economics*, 40(2), pp. 427-436.

Chick,V. (1983) *Macroeconomics after Keynes-A Reconsideration of the General Theory*, Oxford: Philip Allan

Dos Santos, C. (2005) "A Stock-Flow Consistent General Framework for Formal Minskian Analyzes of Closed Economies", *Journal of Post Keynesian Economics*, 27(4), pp. 711-735.

Dutt, A.K. (1995) "Internal Finance and Monopoly Power in Capitalist Economies: A Reformation of Steindl's Growth Model", *Metroeconomica*, 46(1), pp. 16-34.

Hein, E. (2007) "Interest Rate, Debt, Distribution and Capital Accumulation in a Post-Kaleckian Model", *Metroeconomica*, 58(2), pp. 310-339.

Isaac, A.G. (2009) "Monetary and Fiscal Interactions: Short-Run and Long-Run Implications", *Metroeconomica*, 60(1), pp. 197-223.

Isaac, A.G., and Kim, Y.K. (2013) "Consumer and Corporate Debt: A Neo-Kaleckian Synthesis", *Metroeconomica*, 64(2), pp. 244-271.

Keynes, J.M. (1936) *The General Theory of Employment, Interest, and Money*, London: Macmillan.

Kindleberger, C. (1978) *Manias, Panics, and Crashes: A History of Financial Crises*, New York: Macmillan.

Lavoie, M. (1995) "Interest Rate in Post-Keynesian Models of Growth and Distribution", *Metroeconomica*, 46 (2), pp. 146-177.

Lavoie, M. (1997) "Loanable Funds, Endogenous Money, and Minsky's Financial Fragility Hypothesis", in Cohen, A., Hagemann, H., and Smithin, J. (eds.), *Money, Financial Institutions and Macroeconomics* (pp 67-82), Boston: Kluwer Academic.

Lavoie, M., and Godley, W. (2001) "Kaleckian Models of Growth in a Coherent Stock-Flow Monetary Framework: A Kaldorian View", *Journal Post Keynesian Economics*, 24(2), pp. 277-311.

Lavoie, M. (2006) "A Post-Keynesian Amendment to the New Consensus on Monetary Policy", *Metroeconomica*, 57(2), pp. 165-192.

Lima G.T., and Meirelles, A.J. (2007) "Macrodynamics of Debt Regimes, Financial Instability and Growth", *Cambridge Journal Economics*, 31(4), pp. 563-580.

Minsky, H.P. (1957) "Monetary Systems and Accelerator Models", *American Economic Review*, 47(6), pp. 860-883. Reprinted in Minsky, H.P. (1982), pp. 231-257.

Minsky, H.P. (1975) *John Maynard Keynes*, New York: Columbia University Press.

Minsky, H.P. (1986) *Stabilizing an Unstable Economy*, New Haven: Yale University Press.

Ryoo, S. (2013a) "Bank Profitability, Leverage and Financial Instability: Minsky-Harrod Model", *Cambridge Journal of Economics*, 37(5), pp. 1127-1160.

Ryoo, S. (2013b) "The Paradox of Debt and Minsky's Financial Instability Hypothesis", *Metroeconomica*, 64(1), pp. 1-24.

Taylor, J. B. (1993) *Discretion versus Policy Rules in Practice*, Carnegie-Rochester Conference Series on Public Policy, 39, pp. 195–214.

Taylor, L., and O'Connell, S. A. (1985) "A Minsky Crisis", *Quarterly Journal of Economics*, 100(5), pp. 871-885.

Tobin, J. (1969) "A General Equilibrium Approach to Monetary Theory", *Journal of Money, Credit and Banking*, 1(1), pp. 15-29.

Watanabe T. (2016) "Net Worth Ratio, Bank Lending and Financial Instability", *Evolutionary and Institutional Economics Review*, 13(1), pp. 37-56.

APPENDIX 1. DERIVATION OF EQUATION (15E)

A household earns wages ωN_t and dividends Div_t from a firm. We assume that the costs of bank G and profit Π_{t-1}^{b} are distributed to the household via the bank's labor cost and other factors. Since there is a time lag between borrowing and interest payments, the household earns the transaction costs and the bank's profit for the previous period. In addition, the household can earn interest from deposits. Finally, we suppose that the revenue of the central bank also belongs to the household.

From equations (7a), (7d), and (7e), we derive the dividend equation,

$$Div_t = (1-v)(pY_t - \omega N_t) - i_{t-1}L_{t-1}. \tag{a1-1}$$

Considering that dividend equation, interest from deposits and transfers, the total income of household pY_t^h can be expressed as

$$pY_t^h = \omega N_t + [(1-v)(pY_t - \omega N_t) - i_{t-1}L_{t-1}] + i^d D_{t-1} + G_{t-1} + i_{t-2}^a A_{t-1} + \Pi_{t-1}^b. \tag{a1-2}$$

$$\Pi_{t-1}^b = iL_{t-1} - i^d D_{t-1}^d - i_{t-2}^a A_{t-1} - G_{t-1}. \tag{a1-3}$$

From equations (a1-2) and (a1-3), household income pY_t^h in the current period is expressed as

$$pY_t^h = pY_t - v(pY_t - \omega N_t). \tag{a1-4}$$

APPENDIX 2. THE MATHEMATICAL RESULTS OF COMPARATIVE STATICS

The mathematical results of comparative statics in the short-run model is expressed as

$$\frac{di_t}{dl_{t-1}} = \frac{-k_{l_{t-1}}(a_{21}a_{33}-a_{23}a_{31})+b_{2l}a_{11}a_{33}-b_{3l}a_{11}a_{23}}{\Delta}, \tag{a2-1}$$

$$\frac{dq_t}{dl_{t-1}} = \frac{-k_{l_{t-1}}(a_{21}a_{32}-a_{22}a_{31})+b_{2l}(a_{11}a_{32}-a_{12}a_{31})-b_{3l}(a_{11}a_{22}-a_{12}a_{21})}{\Delta}. \tag{a2-2}$$

When the following conditions are satisfied, a rise in the debt-capital ratio increases the bank lending rate and decreases the equity price.

$$a_{21} = (1-q_t e)k_{r_t} - v - l^s_{r_t} < 0, \tag{a2-3}$$

$$a_{31} = \alpha_{r_t} w_t + (1-v)\alpha - q_t e k_{r_t} > 0, \tag{a2-4}$$

$$b_{2l} = -(1-q_t e)k_{l_{t-1}} - 1 + l^s_{l_{t-1}} < 0, \tag{a2-5}$$

$$b_{3l=} - \alpha_{l_{t-1}} w_t - \alpha + q_t e k_{l_{t-1}} > 0. \tag{a2-6}$$

These conditions are satisfied when the absolute values of $l^s_{r_t}$, $l^s_{l_{t-1}}$, α_{r_t}, and $\alpha_{l_{t-1}}$ are large.

However, regarding the effect of the discount rate on the equity price, we have

$$\frac{dq_t}{di^a_{t-1}} = \frac{l^s_{i^a_{t-1}} k_{i_t}[\alpha_{r_t} w_t + (1-v)\alpha - q_t e]}{\Delta}. \tag{a2-7}$$

The equity price decreases if

$$\alpha_{r_t} w_t + (1-v)\alpha - q_t e > 0 \tag{a2-8}$$

is satisfied. This condition is satisfied when the absolute values of α_{r_t} and α are large.

APPENDIX 3. DERIVATION OF EQUATION (22A)

We start with

$$\frac{dl_t}{dl_{t-1}} = \frac{df(l_{t-1})}{dl_{t-1}} = \frac{[-q_{l_{t-1}} e k^* + (1-q(l^*)e)(k_{r_t} r_{l_{t-1}} + k_{i_t} i_{l_{t-1}} + k_{l_{t-1}}) - v r_{l_{t-1}} + 1](1+k^*)}{(1+k^*)^2} - \frac{(k_{r_t} r_{l_{t-1}} + k_{i_t} i_{l_{t-1}} + k_{l_{t-1}})[(1-q(l^*)e)k^* - vr^* + l^*]}{(1+k^*)^2}. \tag{a3-1}$$

At the steady state (l^*), the following condition is satisfied

$$(1-q(l^*)e)k^* - vr^* + l^* = l^*(1+k^*). \tag{a3-2}$$

In addition, from the balance equation of the commodity market, we obtain

$$k_{r_t} r_{l_{t-1}} + k_{i_t} i_{l_{t-1}} + k_{l_{t-1}} = r_{l_{t-1}}. \qquad \text{(a3-3)}$$

Substituting equations (a3-2) and (a3-3) into (a3-1), we derive equation (22a).

APPENDIX 4. DERIVATION OF EQUATIONS (28A) AND (28B)

Further, considering the comparative statics, the stability conditions from equation (27a) to (27c) are, respectively, rewritten as

$$\beta\left[\left(-q_{l_{t-1}} r_{i_{t-1}^a} + q_{i_{t-1}^a} r_{l_{t-1}}\right) e k^{**} + r_{i_{t-1}^a}\right] - (q_{l_{t-1}} e + 1) k^{**} < 0, \qquad \text{(a4-1)}$$

$$\frac{\beta k^{**}}{1+k^{**}}\left[\left(-q_{l_{t-1}} r_{i_{t-1}^a} + q_{i_{t-1}^a} r_{l_{t-1}}\right) e - r_{i_{t-1}^a}\right] > 0, \qquad \text{(a4-2)}$$

$$\beta\left[\left(-q_{l_{t-1}} r_{i_{t-1}^a} + q_{i_{t-1}^a} r_{l_{t-1}}\right) e k^{**} + r_{i_{t-1}^a}(2 + k^{**})\right] + 2\left(-q_{l_{t-1}} e k^{**} + 1\right) + 2(1 + k^{**}) > 0. \qquad \text{(a4-3)}$$

In addition, from comparative statics we obtain

$$-q_{l_{t-1}} r_{i_{t-1}^a} + q_{i_{t-1}^a} r_{l_{t-1}} = -\frac{l^s_{i_{t-1}^a}}{\Delta}\left(\alpha_{l_{t-1}} w^{**} + \alpha\right) k_{i_t} \qquad \text{(a4-4)}$$

$$r_{i^a} = \frac{l^s_{i_{t-1}^a}}{\Delta} e k_{i_t} (1 - \alpha + k^{**}). \qquad \text{(a4-5)}$$

At first, substituting the equations (a4-4) and (a4-5) into the equation (a4-2), we arrive at

$$\left(-q_{l_{t-1}} r_{i_{t-1}^a} + q_{i_{t-1}^a} r_{l_{t-1}}\right) e - r_{i_{t-1}^a} = -\frac{l^s_{i_{t-1}^a}}{\Delta} e k_{i_t}\left(\alpha_{l_{t-1}} w^{**} + 1 + k^{**}\right) > 0. \qquad \text{(a4-6)}$$

Hence, to satisfy equation (a4-6), the following condition should hold,

$$\alpha_{l_{t-1}} w^{**} + 1 + k^{**} > 0. \qquad \text{(a4-7)}$$

Furthermore, from equation (a4-1) and (a4-3), the value of β must satisfy the following equation,

$$0 < \frac{(q_{l_{t-1}} e + 1) k^{**}}{\left(-q_{l_{t-1}} r_{i_{t-1}^a} + q_{i_{t-1}^a} r_{l_{t-1}}\right) e k^{**} + r_{i_{t-1}^a}} < \beta^T < \frac{2(q_{l_{t-1}} e k^{**} - 2 - k^{**})}{\left(-q_{l_{t-1}} r_{i_{t-1}^a} + q_{i_{t-1}^a} r_{l_{t-1}}\right) e k^{**} + r_{i_{t-1}^a}(2 + k^{**})} \qquad \text{(a4-8)}$$

where β^T denotes the appropriate value to satisfy conditions (27a) and (27c).